Series Solution to the Radiation from Slotted Antenna on Elliptic Cylinder Coated by Biaxial Anisotropic Material

A-K. Hamid

Dept. of Electrical and Computer Engineering, College of Engineering
University of Sharjah, UAE
akhamid@sharrjah.ac.ae

Abstract ─ Radiation characteristics of an axially slotted circular or elliptical antenna coated by biaxial anisotropic material are investigated using series solution. The fields inside and outside the coating regions are expressed in terms of appropriate Mathieu functions with unknown field coefficients. The boundary conditions at the conducting and dielectric coating surfaces are invoked to obtain the unknown field expansion coefficients. Numerical results are presented graphically for the radiation pattern with various geometries and electrical parameters.

Index Terms ─ Axially slotted elliptic antenna, biaxial anisotropic material, Mathieu functions, series solution.

I. INTRODUCTION

Numerous scholars in the literature have studied the characteristic radiation by dielectric coated slotted circular and elliptical cylinders [1-6]. Investigation on the radiation by axially N slotted elliptical antenna coated with lossy dielectric material was conducted by Hussein and Hamid [7]. Recently, materials possessing both lossy and lossless metamaterials as well as chiral material have gained considerable attention by several researchers [8-10], while possessing anisotropic materials have been studied by [11-13]. The anisotropic material coating is important to investigate since the electrical characteristics of coated antenna are not uniform in all directions [14-15]. Antennas with anisotropic dielectric coating may have application in satellite communication, space aircraft and airplanes industries.

To the best of our knowledge, the radiation produced by an axially slotted circular or elliptical cylinder coated with biaxial anisotropic material has not been investigated using series solution.

This paper presents series solution to the radiation by an axially slotted antenna on a conducting elliptic cylinder coated with biaxial anisotropic material based on the boundary value method. The presented numerical results will show the effect of biaxial anisotropic material coating on the radiation pattern of slotted antenna.

II. FORMULATION

The geometry of conducting elliptic cylinder with axially slotted antenna covered by biaxial anisotropic material is shown in Fig. 1. The antenna is assumed to be infinite in the z-axis. The antenna parameters a_c and b_c represent the conducting core semi-major and semi-minor axes, respectively, while a and b are the semi-major and semi-minor axes of the dielectric coating material. The axial antenna slot coordinates are denoted by ν_1 and ν_2. The relation between the elliptical (u, v, z) and Cartesian coordinate systems (x, y, z) is:

$$x = F\cosh(u)\cos(\nu), \tag{1}$$

$$y = F\sinh(u)\sin(\nu), \tag{2}$$

where F is the semifocal length of the elliptical antenna. The radiated electric field exterior to the biaxial anisotropic dielectric coating (region I and $\xi > \xi_1$) can be expressed in terms of Mathieu functions as follows [16]:

$$E_z^I = \sum_{q,m}^{\infty} C_{qm} R_{qm}^{(4)}(c\ ,\xi) S_{qm}(c_0,\eta), \tag{3}$$

where $q = e, o$, C_{qm} are the unknown radiated field expansion coefficients, S_{qm} are the angular Mathieu functions of order m, while $R_{qm}^{(4)}$ are the radial Mathieu functions of the fourth kind. It should be noted that $\xi = \cosh u$, $\eta = \cos\nu$, $c = kF$, and $k = \omega\sqrt{\mu\varepsilon}$.

The relative permittivity and permeability tensors of the anisotropic material referred to the coordinate axes ξ, η, z are assumed to be biaxial and written as [17-19]:

$$\overline{\overline{\varepsilon}}_r = \begin{bmatrix} \varepsilon_{r1} & 0 & 0 \\ 0 & \varepsilon_{r2} & 0 \\ 0 & 0 & \varepsilon_{r3} \end{bmatrix}, \quad \overline{\overline{\mu}}_r = \begin{bmatrix} \mu_{r1} & 0 & 0 \\ 0 & \mu_{r2} & 0 \\ 0 & 0 & \mu_{r3} \end{bmatrix}. \tag{4}$$

The Maxwell's equations for an anisotropic region can be written as:

$$\nabla \times \mathbf{E} = -jkZ\overline{\overline{\mu}}_r \cdot \mathbf{H}, \tag{5}$$

$$\nabla \times \mathbf{H} = j(k/Z)\overline{\overline{\varepsilon}}_r \cdot \mathbf{E}, \tag{6}$$

Submitted On: June 8, 2015
Accepted On: September 8, 2015

where Z is the free space wave impedance, and a bold symbol referring to a vector sign. The electric and magnetic field components within the anisotropic elliptic cylinder can be written using equation (5) and (6) as:

$$H_u^{II} = \frac{j}{k\,Z\,\mu_{r1} h}\frac{\partial E_z^{II}}{\partial v}, \quad (7)$$

$$H_v^{II} = -\frac{j}{k\,Z\,\mu_{r2} h}\frac{\partial E_z^{II}}{\partial u}, \quad (8)$$

$$E_z^{II} = \frac{jZ}{k\,\varepsilon_{r3} h^2}\left[\frac{\partial}{\partial v}\left(hH_u^{II}\right) - \frac{\partial}{\partial u}\left(hH_v^{II}\right)\right]. \quad (9)$$

Substituting for H_u^{II}, H_v^{II} in (9) from equations (7) and (8), we obtain:

$$\frac{\partial^2 E_z^{II}}{\partial v^2} + \frac{\mu_{r1}}{\mu_{r2}}\frac{\partial^2 E_z^{II}}{\partial u^2} + c^2\mu_{r1}\varepsilon_{r3}(\cosh^2 u - \cos^2 v)E_z^{II} = 0. \quad (10)$$

If E_z^{II} is written as the product $g_1(v)g_2(u)$, then by using the method of separation of variables, equation (10) can be separated into the two differential equations:

$$\frac{\partial^2 g_1(v)}{\partial v^2} - (c_{13}^2\cos^2 v - b^2)\, g_1(v) = 0, \quad (11)$$

$$\frac{\partial^2 g_2(u)}{\partial u^2} + \left\{c_{23}^2\cosh^2 u - b^2\frac{\mu_{r2}}{\mu_{r1}}\right\} g_2(u) = 0, \quad (12)$$

satisfied by $Sq_n(c_{13},\cos v)$, $Rq_p^{(1)}(c_{23},\cosh u)$, respectively, with $p = m\sqrt{\mu_{r2}/\mu_{r1}}$, $c_{13} = F\omega\sqrt{\mu_{r1}\varepsilon_{r3}}$, $c_{23} = F\omega\sqrt{\mu_{r2}\varepsilon_{r3}}$, $h = F\sqrt{(\cosh^2 u - \cos^2 v}$, and b^2 being the separation constant.

The electric field inside the biaxial anisotropic dielectric coating (region II) for $\xi_c < \xi < \xi_1$ can be written in terms of Mathieu functions as:

$$E_z^{II} = \sum_{q,m}^{\infty}[A_{qm}R_{qp}^{(1)}(c_{23},\xi) + B_{qm}R_{qp}^{(2)}(c_{23},\xi)]S_{qm}(c_{13},\eta), \quad (13)$$

where A_{qm} and B_{qm} are the unknown transmitted field expansion coefficients, $R_{qp}^{(1)}$ and $R_{qp}^{(2)}$ are the radial Mathieu functions of the first and second kind, respectively. The magnetic field components in regions (I) and (II) are derived using Maxwell's equations and written as:

$$H_v^I = \frac{-j}{\omega\mu h}\left\{\sum_{q,m}^{\infty} C_{qm}R_{qm}'^{(4)}(c_0,\xi)S_{qm}(c\;,\eta)\right\}, \quad (14)$$

$$H_v^{II} = \frac{-j}{\omega\mu_{r2} h}\left\{\sum_{q,m}^{\infty}[A_{qm}R_{qp}'^{(1)}(c_{23},\xi) + B_{qm}R_{qp}'^{(2)}(c_{23},\xi)]S_{qm}(c_{13},\eta)\right\}. \quad (15)$$

The prime in equations (14) and (15) refers to the derivative with respect to u.

The electric field E_z should be continuous across the dielectric layer at $\xi = \xi_1$, this leads to:

$$[A_{qn}R_{qp}^{(1)}(c_{23},\xi_1) + B_{qn}R_{qp}^{(2)}(c_{23},\xi_1)]N_{qn}(c_{13}) = \sum_{m}^{\infty} C_{qm}R_{qm}^{(4)}(c_0,\xi_1)M_{qnm}(c_{13},c_0), \quad (16)$$

where $N_{qn}(c_{13})$ and $M_{qnm}(c_{13},c_0)$ are defined as:

$$N_{qn}(c_{13}) = \int_0^{2\pi}[S_{qn}(c_{13},\eta)]^2 dv, \quad (17)$$

$$M_{qnm}(c_{13},c) = \int_0^{2\pi} S_{qn}(c_{13},\eta)S_{qm}(c,\eta)dv. \quad (18)$$

The tangential magnetic field components at $\xi = \xi_1$ should also be continuous and require that:

$$[A_{qn}R_{qp}'^{(1)}(c_{23},\xi_1) + B_{qn}R_{qp}'^{(2)}(c_{23},\xi_1)]N_{qn}(c_{13}) = \mu_{r2}\sum_{m}^{\infty} C_{qm}R_{qm}'^{(4)}(c_0,\xi_1)M_{qnm}(c_{13},c). \quad (19)$$

The tangential electric field on the conducting surface ($\xi = \xi_c$) must be zero except at the slot surface. This leads to:

$$\sum_{q,m}^{\infty}[A_{qm}R_{qp}^{(1)}(c_{23},\xi_c) + B_{qm}R_{qp}^{(2)}(c_{23},\xi_c)]S_{qm}(c_{13},\eta) = \begin{cases} F(v) & v_1 < v < v_2 \\ 0, & \text{elsewhere.} \end{cases} \quad (20)$$

Multiplying both sides of (20) by $S_{qn}(c_{13},\eta)$ and integrating over $0 < v < 2\pi$, we obtain:

$$[A_{qn}R_{qp}^{(1)}(c_{23},\xi_c) + B_{qn}R_{qp}^{(2)}(c_{23},\xi_c)]N_{qn}(c_{13}) = F_{qn} = \int_{v_1}^{v_2} F(v)S_{qn}(c_{13},\eta)dv. \quad (21)$$

The integral in equation (21) may be computed numerically by expressing the field at the slot surface as [6,7]:

$$F(v) = E_0\cos[\pi(v_0 - v)/(2\alpha)], \quad (22)$$

$$v_0 = (v_1 + v_2)/2, \quad (23)$$

$$\alpha = (v_2 - v_1)/2. \quad (24)$$

The angular Mathieu functions are written in terms of Fourier series as:

$$S_{en}(c_{13},\eta) = \sum_k D_e^k(c_{13},n)\cos(kv), \quad (25)$$

$$S_{on}(c_{13},\eta) = \sum_k D_o^k(c_{13},n)\sin(kv). \quad (26)$$

Substituting equations (25)-(26) into equation (21), F_{en} and F_{on} can finally be written as:

$$F_{en} = E_o\sum_k D_e^k(c_{13},n)\int_{v_1}^{v_2}\cos\left[\pi(v - v_0)/(2\alpha)\right]\cos(kv)dv, \quad (27)$$

$$F_{on} = E_o \sum_k D_o^k(c_{13}, n) \int_{\nu_1}^{\nu_2} \cos\left[\pi(\nu - \nu_0)/(2\alpha)\right] \sin(k\nu) d\nu. \quad (28)$$

Solving for B_{en} by equation (21) and incorporating the result in equations (16) and (19) by eliminating A_{en}, we obtain the radiated field coefficients C_{en}. Similar procedure may be followed to obtain the odd coefficients C_{on}.

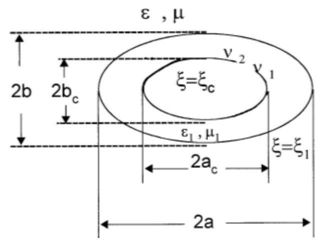

Fig. 1. Slotted antenna on elliptic cylinder with biaxial anisotropic material coating.

III. NUMERICAL RESULTS

After computing the radiated field coefficients C_{qn}, the far-field radiation pattern, antenna gain, and the aperture conductance may be evaluated. The slotted antenna far field can be written as:

$$E_z^I(\rho, \varphi) = \sqrt{\frac{j}{k\rho}} e^{-jk\rho} \left[\sum_{q,n}^{\infty} j^n C_{qn} S_{qn}(c\ , \cos\varphi) \right], \quad (29)$$

where ρ and φ represent the polar coordinates in the cylindrical coordinate system. The antenna gain is expressed as [6,7]:

$$G(\varphi) = \frac{1}{Z\ k\rho} \left[\left| \sum_{q,n}^{\infty} j^n C_{en} S_{en}(c\ , \cos\varphi) \right|^2 \right]. \quad (30)$$

The accuracy of the obtained numerical results is checked against slotted circular and elliptic antennas coated with conventional dielectric material [6,8]. Figure 2 shows the radiation pattern numerical results (gain versus φ) obtained for a conventional dielectric coating material represented by solid line ($\varepsilon_r = 2$ and $\mu_r = 1$), while biaxial anisotropic coating represented by dotted μ_{r2}= 4 and circled μ_{r2}= 9 lines. The slot location is at $\nu_0 = 90^o$ where $\alpha = 2.8657^o$. The parameters for the biaxial medium are $\varepsilon_{r1} = 1.0$, $\varepsilon_{r2} = 1.0$, $\varepsilon_{r3} = 2.0$, $\mu_{r1} = 1.0$, $\mu_{r3} = 1.0$, while for cylinder geometries are ka$_c$ = 1.0, kb$_c$ = 0.6, ka = 1.13, kb = 0.8. It can be seen the gain of the antenna is slightly dropped at the location of the antenna by increasing the value μ_{r2} from 4 to 9 while the gain is increased for the range -100° to 0° and 150° to 250°.

Figure 3 is similar to Fig. 2 except for circular antenna with ka$_c$ = 1.0, kb$_c$ = 1.0, ka = 1.6, kb = 1.6. The gain is not affected by increasing the value μ_{r2} at the location of the antenna while it is significantly decreased at the side lobes locations.

Figure 4 shows the again for an elliptic antenna by fixing the value μ_{r2} at 4.0 while changing the value of ε_{r3} from 2.0 to 4.0. Increasing the value of $\varepsilon_{r3} = 2.0$ has little on the antenna except for minor increase at the side lobe locations. Figure 5 is similar to 4 except for circular antenna it behaves like elliptical antenna.

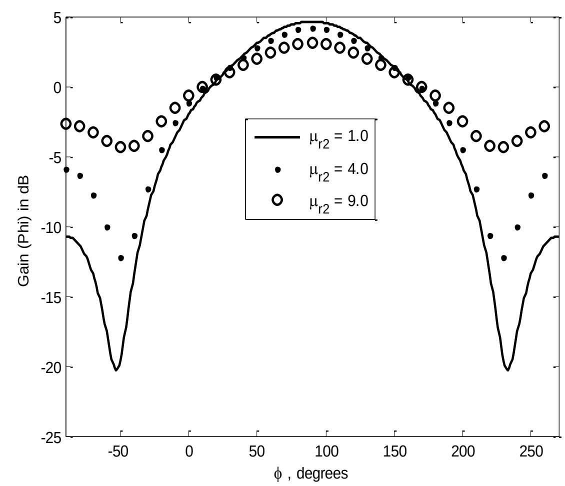

Fig. 2. Radiation pattern of slotted elliptic antenna coated with biaxial anisotropic material and different values of μ_{r2} ($\varepsilon_{r1} = 1.0$, $\varepsilon_{r2} = 1.0$, $\varepsilon_{r3} = 2.0$, $\mu_{r1} = 1.0$, $\mu_{r3} = 1.0$, ka$_c$ = 1.0, kb$_c$ = 0.6, ka = 1.13, kb = 0.8).

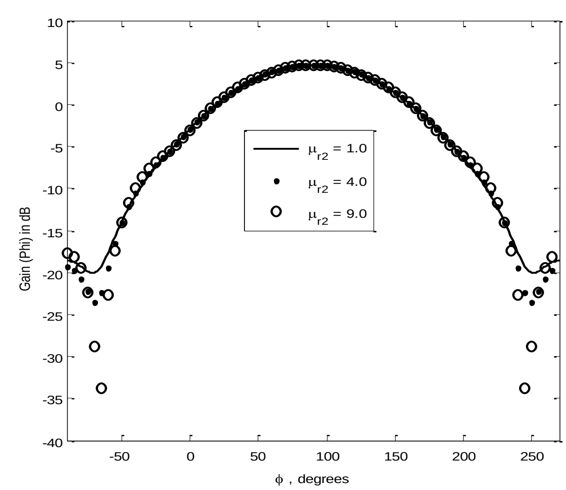

Fig. 3. Radiation pattern of slotted circular antenna coated with biaxial anisotropic material and different values of μ_{r2} ($\varepsilon_{r1} = 1.0$, $\varepsilon_{r2} = 1.0$, $\varepsilon_{r3} = 2.0$, $\mu_{r1} = 1.0$, $\mu_{r3} = 1.0$, ka$_c$ = 1.0, kb$_c$ = 1.0, ka = 1.6, kb = 1.6).

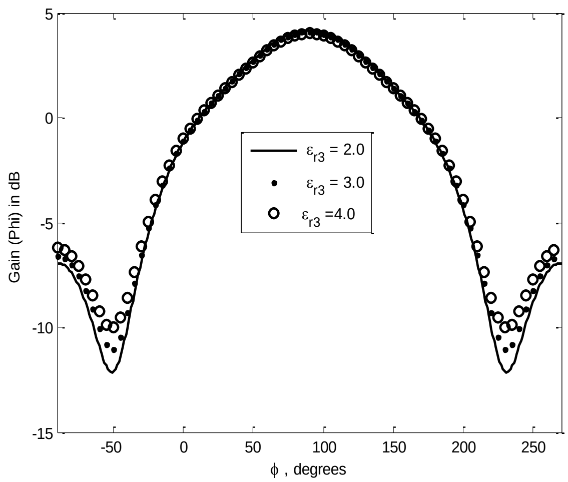

Fig. 4. Radiation pattern of slotted elliptic antenna coated with biaxial anisotropic material and different values of ε_{r3} ($\varepsilon_{r1} = 1.0$, $\varepsilon_{r2} = 1.0$, $\mu_{r1} = 1.0$, $\mu_{r2} = 4.0$, $\mu_{r3} = 1.0$, ka_c = 1.0, kb_c = 0.5, ka = 1.1, kb = 0.7).

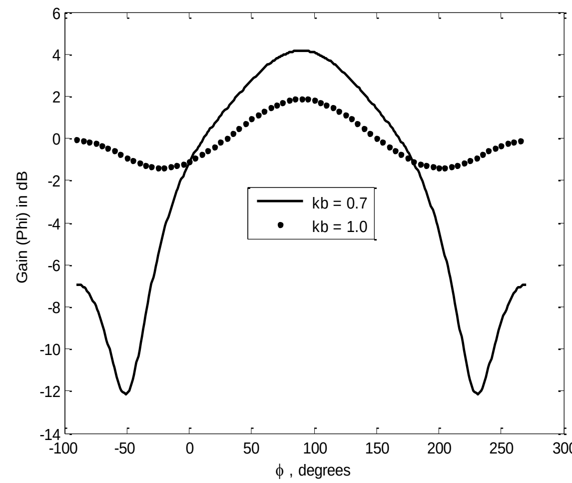

Fig. 6. Radiation pattern of slotted elliptic antenna coated with biaxial anisotropic material and different values of kb ($\varepsilon_{r1} = 1.0$, $\varepsilon_{r2} = 1.0$, $\varepsilon_{r3} = 2.0$, $\mu_{r1} = 1.0$, $\mu_{r2} = 4.0$, $\mu_{r3} = 1.0$, ka_c = 1.0, kb_c = 0.5).

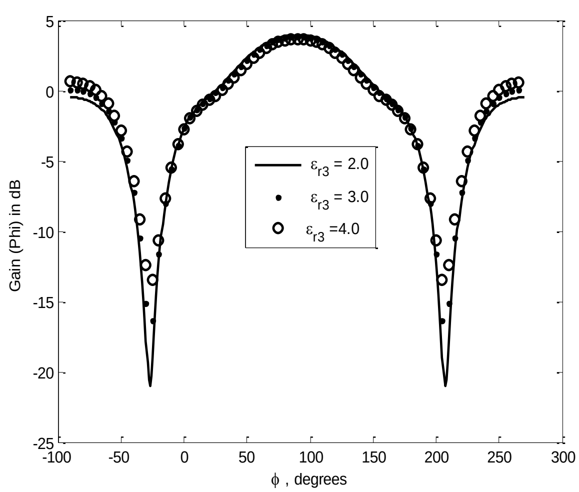

Fig. 5. Radiation pattern of slotted circular antenna coated with biaxial anisotropic material and different values of ε_{r3} ($\varepsilon_{r1} = 1.0$, $\varepsilon_{r2} = 1.0$, $\mu_{r1} = 1.0$, $\mu_{r2} = 4.0$, $\mu_{r3} = 1.0$, ka_c = 1.0, kb_c = 1.0, ka = 2.0, kb = 2.0).

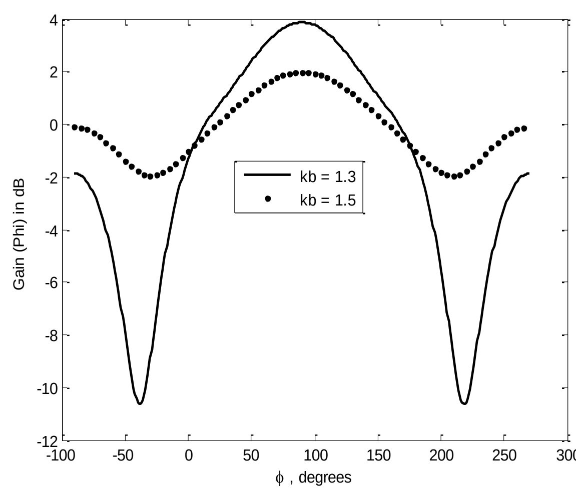

Fig. 7. Radiation pattern of slotted circular antenna coated with biaxial anisotropic material and different values of kb ($\varepsilon_{r1} = 1.0$, $\varepsilon_{r2} = 1.0$, $\varepsilon_{r3} = 2.0$, $\mu_{r1} = 1.0$, $\mu_{r2} = 4.0$, $\mu_{r3} = 1.0$, ka_c = 1.0).

In Fig. 6 we plotted the gain versus the anisotropic dielectric material thickness for the cases kb = 0.7 and kb = 1.0. It can be noticed for these cases, the side lobes decrease and the antenna bandwidth increase. Figure 7 is similar to Fig. 6 except for circular antenna for thickness kb = 1.3 and kb = 1.5.

IV. CONCLUSIONS

The radiation characteristics of an axially slotted circular, for comparison, or elliptical antenna coated by biaxial anisotropic material were investigated using series solution. It was shown that the presence of biaxial anisotropic material coating with specific characteristic values and thickness lead to reducing the side lobes and enhancing the bandwidth of the antenna.

ACKNOWLEDGMENT

The author would like to acknowledge the support by the University of Sharjah, United Arab Emirates.

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A-K. Hamid received the B.Sc. degree in Electrical Engineering from West Virginia Institute of Technology and University, West Virginia, U.S.A. in 1985, M.Sc. and Ph.D. degrees from the University of Manitoba, Winnipeg, Manitoba, Canada in 1988 and 1991, respectively, both in Electrical Engineering. From 1991-1993, he was with Quantic Laboratories Inc., Winnipeg, Manitoba, Canada, developing two and three dimensional electromagnetic field solvers using boundary integral method. From 1994-2000 he was with the Faculty of Electrical Engineering at King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia. Since Sep. 2000, he is with the Dept. of Electrical and Computer Engineering at the University of Sharjah, Sharjah, United Arab Emirates. His research interest includes EM wave scattering from two and three dimensional bodies, FDTD simulation of cellular phones, inverse scattering using neural networks and investigating thermal PV system experimentally.