Dynamic Modeling of Linear Actuator Using Fuzzy System to Approximate Magnetic Characteristics

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Abstract **─** This paper proposes a dynamic model of a linear actuator using a fuzzy system to approximate its magnetic subsystem. Magnetic characteristics of the linear actuators indicate a nonlinear behavior, making the whole system complex. Deriving an accurate and proper model, results in the implementation of different control methods in the simulation procedure. This research developed dynamic equations of linear actuator with closed type magnetic circuit. Due to high capabilities of fuzzy approximators in the modeling of nonlinear systems, they are employed to approximate the magnetic subsystem of the linear actuator. According to the results, the model described in this paper, shows significant improvements in comparison with the previous models. Moreover, the proposed model apart from the nominal area, could accurately predict the behavior of linear actuator for out of the nominal operation area. This matter is important in transient situations and short-term overloads. High accuracy and performance is obviously demonstrated by comparing experimental and simulation results both in static and dynamic features.

Index Terms **─** Dynamic modeling, flux linkage, fuzzy system, linear actuator, magnetic characteristics.

I. INTRODUCTION

Linear actuators are widely used in industrial applications due to their simple construction, high ruggedness, and low prime cost. Electromagnetic valves [1], fluid flow control in hydraulic systems [2] and magnetic suspension are some typical applications. Many contactors and relays working in switching (on-off) state have the same operation of linear actuators [3]. There are two types of linear actuators, known as on-off and proportional [4]. The former is simpler owing to its specific design and applicable structure.

On-off actuators are being used in electrical contactors and electromagnetic valves. Since position control in force operators with high accuracy is so common, the entire plunger trajectory in proportional actuator is controlled. Transfer function in proportional actuators is more linear than on-off actuators but the design in proportional actuators is much more complex. Also, proportional actuators require position sensor; thus, they are more expensive [4]. Converting an on-off actuator to a proportional one using power electronic converters and external sensors with operating range of below 10 mm has been investigated in [5]. Having extracted a precise model for linear actuator, various control methods could be applied and also improvement on proportional operation of on-off actuators would be obtained. Figure 1 shows the structure of linear actuators. Linear actuators are separated into two actuators: actuator with open type magnetic circuit, and actuator with closed type magnetic circuit [6]. Linear actuators with open magnetic circuit are described by their linear behavior; whereas, actuators with closed magnetic circuit show nonlinear behavior due to core saturation. Owing to more rugged mechanical and electrical structure and also larger force density, closed magnetic circuit actuators are more popular. Moreover, to increase the output force, those actuators are designed in a way that saturation occurs in rated current [7]. Therefore, nonlinear magnetic behavior of the linear actuators should be taken into account in their modeling.

One of the most important parts in the modeling of linear actuator with closed magnetic circuit is magnetic characteristics of motor. They are directly in correlation with electrical and mechanical subsystems. Moreover, nonlinear behavior of magnetic characteristics makes the whole system nonlinear and more complex.

To date, different models have been proposed to describe dynamic characteristics of linear actuators. Those models considered different aspects and effective parameters of the linear actuator behavior. In some researches, for linear actuator modeling, several numerical methods have been proposed such as FEM[1]. In order to achieve a mathematical model, inductance estimation, force levels, and eddy currents effect is investigated [8]-[13]. In [14], a nonlinear model including

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[1] Finite Element Method

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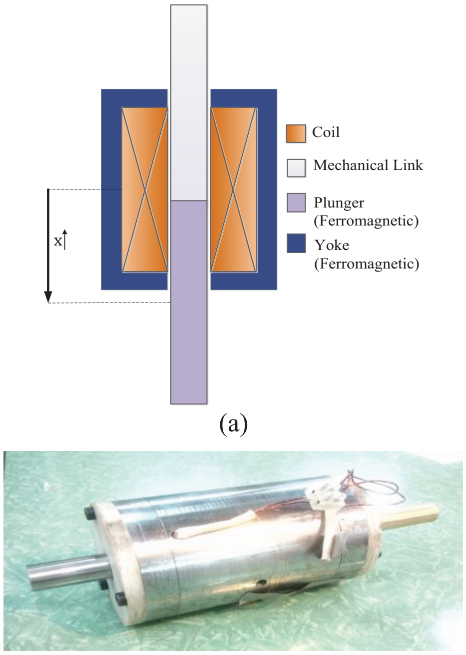
experimental detection of the system parameters is proposed to attain transient magnetic characteristics. Moreover, hysteresis and saturation features have been taken into account and have been focused on "system designing" in the modeling. In [15], core saturation curve is separated into linear and nonlinear parts and a model has been presented employing a first-order and second-order approximation. The experimental data are gained by employing a sinusoidal voltage to actuator winding.



(b)

Fig. 1. (a) Schematic of a linear actuator, and (b) a laboratory sample of actuator.

It should be noted that, in the modeling of linear actuator, in addition to the nominal operation area of the motor, a proper model must be sufficiently accurate for the currents out of nominal current. This matter is important in transient situations and short-term overloads that may be occurred in the motor. There is no remarkable study about this in previous researches. The proposed model in this paper, in addition to the nominal operating area, could accurately predict the behavior of actuator, out of nominal operation area. This has been achieved by using capabilities of the fuzzy systems.

Fuzzy theory has so many applications in various systems especially nonlinear ones. Describing or controlling of the systems where there is connoisseur person experience or input-output data, leads to appropriate results. In order to completely model the linear actuator, information of both fields is used in this paper. Having used fuzzy theory in this paper, a full description of linear actuator characteristic is presented which covers both nominal and over-current areas with adequate accuracy. A proper dynamic model is subsequently proposed according to system's dynamic equations. Eventually, comparison between simulation and experimental results, both in dynamic and static states, validates the proposed model. The results indicate desired accuracy both in dynamic and static states and also contain both nominal and over-current areas.

II. SYSTEM'S DYNAMIC EQUATIONS

According to the most researches, the equations of an electrical motor are divided into three parts: electrical, mechanical, and magnetic. In order to achieve an appropriate and accurate model, a block diagram consisting of electrical, magnetic, and mechanical blocks, is proposed in Fig. 2. Supply voltage and load force are inputs of the system, and plunger position is the output.

Electrical and magnetic subsystems connect with each other through λ and i; whereas, magnetic block is in relation with mechanical one through F_{mag} and x.

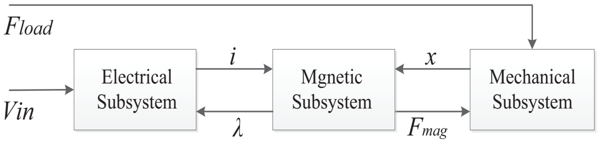


Fig. 2. Block diagram of linear actuator's model.

A. Mechanical analysis

Sub d, assuming that the actuator is in vertical position, dynamic equation of the mechanical subsystem is defined as:

$$-W_p - F_f + F_{mag} - F_{load} = m_p\ddot{x}. \tag{1}$$

In equation (1), W_p is the plunger weight, F_f is the friction force, F_{mag} is the magnetic force, and F_{load} is the load force. m_p is the mass of the plunger and x is the position of the plunger. The friction force value of the actuator is in proportion with the plunger velocity [16].

B. Electrical analysis

As shown in Fig. 1 (a), the linear actuator contains a winding fed by a voltage source. The total equation in electrical subsystem is:

$$V = Ri + N\frac{d\phi}{dt}. \tag{2}$$

In (2), V is the actuator's voltage, R is the winding resistance, N is the number of winding turns, and ϕ is the flux inside the winding. Considering $\lambda=N\varphi$, equation (2) would be rewritten as the following:

$$V = Ri + \frac{d\lambda}{dt}, \tag{3}$$

λ is the flux linkage that plays a fundamental role in the modeling. Its value is defined by magnetic analysis at any plunger position and anytime. V is the instantaneous voltage of winding, therefore when the actuator is modeled by power electronic components with *PWM*[2], it is modeled with $V=0$ at off periods, due to the presence of Freewheeling diode.

C. Magnetic analysis

According to the system's structure, the flux linkage value in linear magnetic systems is determined regarding the ampere-turn of the winding and equivalent magnetic reluctance (R_{eq}):

$$\frac{Ni}{R_{eq}} = \phi = \frac{\lambda}{N}, \tag{4}$$

$$\lambda = \frac{N^2 i}{R_{eq}}. \tag{5}$$

In (5), R_{eq} is the equivalent magnetic reluctance from the viewpoint of the winding. When the magnetic behavior of the system is linear, R_{eq} and therefore λ, are easily calculated via analyzing the linear magnetic circuits. However, it should be noted that the design must be conducted considering the saturation region in order to increase the output force and the effective density of force [7]. Thus, using equation (5) in the actuators with closed type magnetic circuit is not effective.

In the actuators with closed type magnetic circuit, the flux linkage value and therefore the inductance, apart from the plunger position, is dependent on the electrical operating point. This relation is nonlinear and is affected by the saturation. Accordingly, inductance could not be used only as a function of position. Thus, the flux linkage of the actuator should generally be considered as a function of x and i.

Figure 3 shows sets of λ-i curves related to the actuator for different positions of the plunger. In order to attain these characteristics, some experiments on the actuator are applied and sets of input-output data are obtained. Applying an adjustable alternating voltage (autotransformer) results in the measurement of the coil's current and voltage, and therefore λ values is obtained for different currents and positions. According to Fig. 3, the magnetic characteristics get nonlinear when $x<70$ *mm*. It means that for $x<70$ *mm*, the actuator behavior is nonlinear.

In the dynamic modeling of the actuator, magnetic characteristics (λ-x-i curves) are highly significant considering dynamic equations of current and voltage and magnetic force definition.

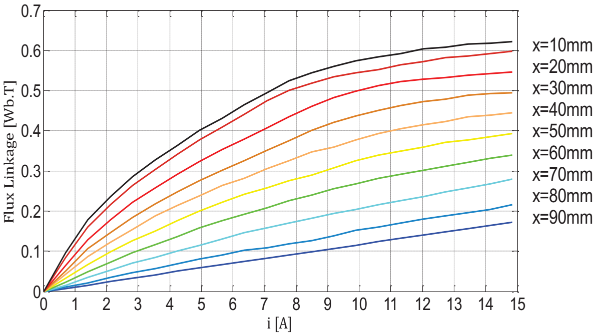


Fig. 3. Flux linkage characteristics of the actuator versus current for different positions of the plunger.

D. Calculation of magnetic force

Overall, there are different methods to calculate the magnetic force [17]-[21]. One of the most common methods is using the stored energy in the system. According to the theory of this method, magnitude and direction of the force in the conservative vector fields equals to the negative of the gradient of stored potential energy in those fields. Hence, if a magnetic body has a moving part, a force in the direction exerts on the moving part to decrease the magnetic potential energy stored in the body [6]. Thus, the force exerted on the moving part is expressed as:

$$F_{mag} = -\frac{\partial W_f'(i,x)}{\partial x}\Big|_{i=cons}, \tag{6}$$

where W_f' is co-energy and is defined by:

$$W_f'(i,x) = \int_0^i \lambda(i,x).di. \tag{7}$$

In linear magnetic systems, like actuators with open type magnetic circuit or systems that saturation effect is ignorable, the co-energy and the energy terms are equal. Moreover, λ has a linear relation with i and the output force is described as equation (8) after the inductance is defined:

$$F_{mag} = \frac{1}{2} i^2 \frac{dL(x)}{dx}. \tag{8}$$

When the magnetic characteristics is nonlinear and core saturation is not ignorable, force values should be calculated according to the (6) and (7), thus (8) is not usable.

III. FLUX LINKAGE DETERMINATION

As explained previously, the magnetic characteristic data of the linear actuators is obtained from experimental measurements. In order to complete the actuator modeling, data should be employed properly. The simplest method to use the data is lookup table. Applying the LUT decreases the simulation accuracy. In order to define the input intermediate values which is among the LUT inputs' values, linear

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[2] Pulse Width Modulation

interpolation method should be utilized. Since the derivatives of λ with respect to current and position are required in the simulation, the parameters are calculated discretely, thus it is different from the system nature. Based on the mentioned reasons, LUT has not been used in this research for modeling.

Another method to use the data of λ is to approximate λ-x-i characteristics employing polynomial functions [22]-[24]. The main advantage of this method is significant decrease in the calculations complexity. Nevertheless, the relatively much increase of the error in the extrapolation is the fundamental disadvantage. The current increase results in decrease in the flux linkage increase rate, whereas in the polynomial approximation, the estimated value of the flux linkage out of the relevant range might get large values. Since for the large values of ac current, there is restriction in the characteristic measurement, this issue is regarded as a significant disadvantage. By contrast, the actuator can easily operate in high DC currents. On the other hand, in the short-term over currents and transient situations, the amount of winding current could be increased in comparison with the nominal current. However, a proper model must predict the behavior of the system with the minimum error in different situations.

Fuzzy systems, regarded as general approximators, would be employed to estimate unknown nonlinear functions with any required accuracy. However, one of the fuzzy system features is that saturation occurs for the values out of the available data scope. This feature shows the primary advantage of fuzzy systems.

A. Flux linkage fuzzy approximator

Various engineering subjects are applied in fuzzy theory. A significant application is that a fuzzy system with a desired design is usually a general approximator [25]. It means, any system approximation with any accuracy could be performed via a proper design. Therefore, a desired fuzzy system was designed in order to model the magnetic characteristic of the system. Moreover, this fuzzy system estimates the flux linkage for different values of the plunger's position and the winding current. In order to design and model a fuzzy system, its features are required. These features may involve a connoisseur person experience, experimental results or a combination of both.

In order to use a fuzzy system, if-then rule is defined and equation (9) is described by [25]:

If x_1 is A_1 and x_2 is A_2 then y is B,

$$f(x_1,x_2)=\frac{\sum_{l=1}^{M} y^l.\mu_{A1}^l(x_1).\mu_{A2}^l(x_2)}{\sum_{l=1}^{M}\mu_{A1}^l(x_1).\mu_{A2}^l(x_2)}, \tag{9}$$

where the parameters μ_{A1}^l and μ_{A2}^l are membership functions of the input variables and would be triangular, pseudo-triangle, trapezium, and Gaussian. When triangular or trapezius functions are applied, the system is approximated to a piecewise linear curve. Equation (9) deals with a fuzzy system with Mamdani inference engine, individual fuzzifier, center average defuzzifier, and if-then fuzzy rule base. In this paper, Gaussian functions are used as input membership functions. Hence, the fuzzy system considering equation (9) is described as [25]:

$$f(x_1,x_2)=\frac{\sum_{l=1}^{M} y^l\left[a_1^l\exp(-(\frac{x_1-\bar{x}_1}{\sigma_1^l})^2).a_2^l\exp(-(\frac{x_2-\bar{x}_2}{\sigma_2^l})^2)\right]}{\sum_{l=1}^{M}\left[a_1^l\exp(-(\frac{x_1-\bar{x}_1}{\sigma_1^l})^2).a_2^l\exp(-(\frac{x_2-\bar{x}_2}{\sigma_2^l})^2)\right]}. \tag{10}$$

According to available data and the change range of each variable, a membership function is defined with respect to different states and different values of each variable. The output membership functions are defined in a same procedure. Finally, a fuzzy system is designed by defining a Mamdani inference engine and fuzzy rule base such as if-then rule.

B. Fuzzy system design

In order to extract the magnetic characteristics of the linear actuator, experimental measurements were conducted and λ values were calculated for different plunger positions with 10 mm steps and different effective currents with 0.5 Amp. steps. Thus, there are 11 and 22 states for the position and the current respectively, and the conclusion is 11×22=242 states for input-output pairs. Applying these pairs in equation (10) and considering some assumptions, a fuzzy system in order to model the actuator is obtained. According to the measured data, the following vectors and matrix are defined as:

$$\begin{cases}\bar{i}=[\bar{i}_k] \\ \bar{x}=[\bar{x}_j] \qquad k=1,2,...,22\ ,\ j=1,2,...,11\ , \\ \bar{\lambda}=[\bar{\lambda}_{kj}]\end{cases} \tag{11}$$

where vectors $\bar{i}$ and $\bar{x}$ are the points of input data and matrix $\bar{\lambda}$ is the output data.

It should be noted that current value of the actuator may outnumber the maximum tested value of the system. As mentioned before, this phenomenon would be occurred in short-term overloads or transient situations. Thus, one membership function for the values higher than the maximum current is defined. The value of this membership function is permanently "1" for the current values higher than the maximum actuator current. Defined membership function together with other membership functions of the actuator current is employed to complete the design and to avoid the flux linkage decrease with current increase.

Equation (10) is rewritten as:

$$\lambda(i,x)=\frac{\sum_{k=1}^{23}\sum_{j=1}^{11}\bar{\lambda}_{kj}.\exp(-(\frac{i-\bar{i}_k}{\sigma_i})^2).\exp(-(\frac{x-\bar{x}_j}{\sigma_x})^2)}{\sum_{k=1}^{23}\sum_{j=1}^{11}\exp(-(\frac{i-\bar{i}_k}{\sigma_i})^2).\exp(-(\frac{x-\bar{x}_j}{\sigma_x})^2)}. \qquad (12)$$

According to equation (12), different values of λ could be calculated for different currents and positions. σ_i and σ_x are smoothing parameters. The smaller the values of σ_i and σ_x get, the less difference the function values in sample points obtains, compared to real values. However, the function error increases for intermediate or out of range values, thus the function is not regarded as general. The function is smooth when the values of σ_i and σ_x are large enough.

In practice, through trial and error attempt and comparing the function characteristics with measured ones, proper values for σ_i and σ_x could be acquired. It should be remarked that proper values of σ_i and σ_x are different from each other due to the different change range of current and position. Figure 4 shows the membership function of input parameters.

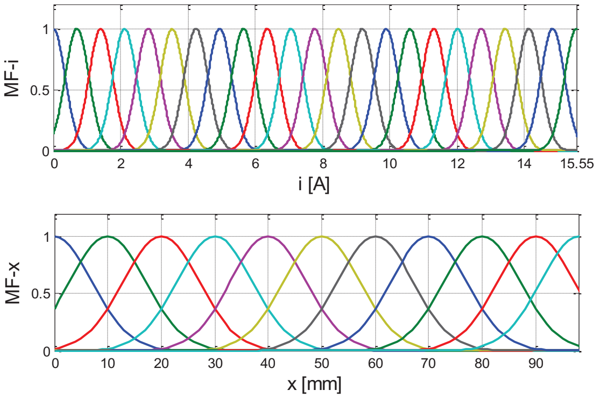

Fig. 4. Membership functions of the designed system as input parameters.

IV. RESULTS

The results of proposed model were thoroughly investigated in this section and were compared with the experimental data. Therefore, an experimental setup was designed and built to conduct different experiments and measure the system characteristics. Features of the system would be highlighted as a static features and dynamic behavior. Static features contain flux linkage values and actuator output force for different currents and different positions of the plunger. Moreover, due to the changes of the system's state variable along the time, actuator dynamic behavior could be analyzed with injecting step input. The designed setup in this research has the possibility of measuring magnetic force of the actuator for different positions and via adjustable current values. In addition, it's possible to inject a step voltage with different amplitudes and to measure plunger's current and position values.

Experimental setup includes a linear actuator, a power electronic convertor, a position sensor and other control and measurement circuits. Power electronic convertor is a bulk convertor which can inject regulated and controllable voltage into the actuator. Position sensor is an IR distance sensor and a *DSPIC30F4011* microcontroller is the main controller of system. Moreover, a sampling device (*USB47n-A, Advantech Automation*) is used to sample the required data. Figure 5 illustrates the experimental setup. The results of the actuator's static features and dynamic behavior are investigated in the following.

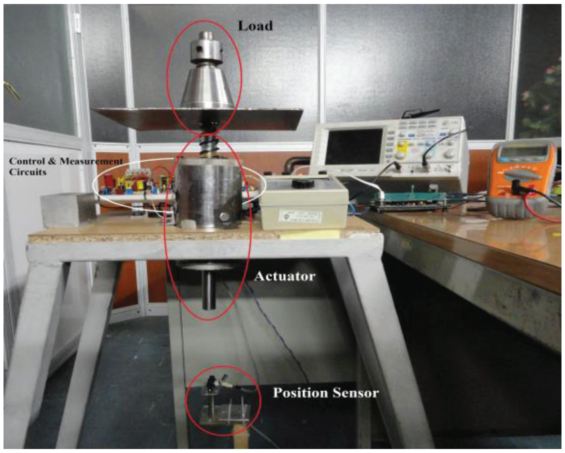

Fig. 5. Experimental setup to study the actuator.

A. Static features

In order to validate the designed system, first the flux linkage values and their change procedure, stemmed from the designed system, are compared with their real values. As shown in Fig. 6, the flux linkage values are calculated for large currents to ensure that the flux linkage value change is not inappropriate for the current increase. As mentioned before, for $x<70$ *mm*, the actuator behavior is nonlinear. The results indicate a closely match and less error of the approximator. Figure 7 shows the error value of the flux linkage estimation for different values of x. Figure 7 demonstrates that the error value is less than 2.5 percent and the total average of the approximation error is 1.22%.

The next step is to compare the magnetic force values concluded from designed system and equation (6), with experimental values. Figure 8 shows the comparison for some different positions. Within the measured data range of the flux linkage, the approximation is highly accurate and the estimation for out of the range, however, is satisfactory. According to (3), if the behavior of the magnetic characteristics is linear, the force value, when *x* is constant, will be in relation with the square of the current. However, as it's clear in Fig. 8, this issue would not occur due to saturation. The average values of force estimation error for different positions are illustrated in Table 1. According to Table 1, x=40 has the most error and x=60 has the least error. Mean value of the estimation error related to the output force is 2.17%.

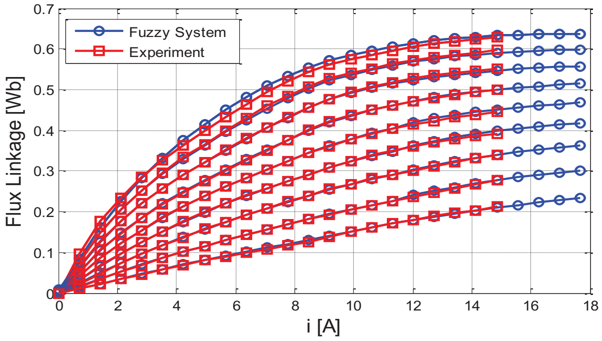

Fig. 6. Comparison of the flux linkage values for different currents and positions.

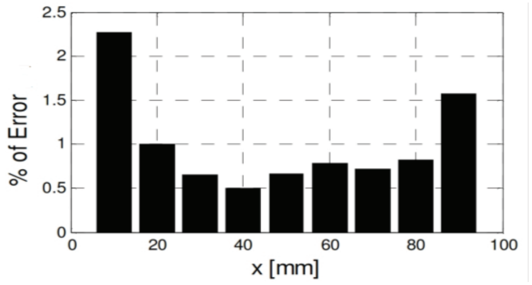

Fig. 7. Flux linkage error for different positions.

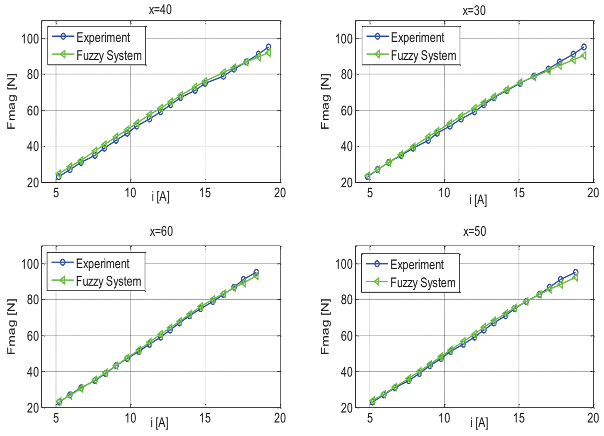

Fig. 8. Experimental and fuzzy approximator comparison for the output force.

Table 1: Estimation error of output force

Position [mm]	30	40	50	60
Error [%]	1.99	3.4	1.98	1.3

B. Dynamic behavior

In order to describe the dynamic behavior of the linear actuator in accordance with the equations of Section 3 and the designed fuzzy system, the simulation of the actuator is implemented. To determine magnetic force and also change rate values of the flux linkage versus position and current, the designed fuzzy system was applied. Dynamic behavior of the actuator for different loads was compared with experimental results. Injecting a step waveform as the input voltage to the actuator coil, time waveforms of the winding current and the plunger position for different loads were measured and recorded. Afterwards, dynamic behavior of the actuator was investigated and compared with the simulation results under equal conditions. Figure 9 shows two waveforms in relation to these comparisons for 35[N] and 55[N] loads. The current and position changes of the winding is determined in Fig. 9. Ripples related to position change for 55[N] load diagram originates from the noise in the position sensor. This noise has been increased when the current produced and increased. The comparison between the waveforms indicates desired accuracy of the modeling.

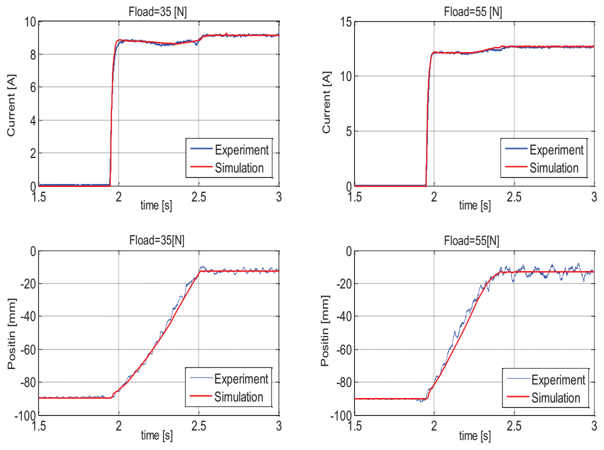

Fig. 9. Dynamic behavior of the linear actuator for two different loads.

V. CONCLUSION

Dynamic modeling of the linear actuator has been investigated in this paper. Having analyzed dynamic equations for mechanical, electrical and magnetic subsystems, the flux linkage was highlighted as a fundamental parameter making the system behavior nonlinear. Moreover, the flux linkage had a relatively high impact on mechanical, electrical and magnetic subsystems. Due to high capabilities of fuzzy systems to approximate nonlinear systems, a desired fuzzy system was designed to estimate the flux linkage of the linear actuator. Since behavior of the system is known and also according to the data concluded from the measurement of the actuator's magnetic characteristic, this fuzzy approximator was designed such as a system with Gaussian membership functions, individual fuzzifier, Mamdani inference engine and center average defuzzifier. In order to validate the designed fuzzy system, static data of the actuator was employed. These features include the values of the flux linkage and the magnetic force of the actuator. The comparison between the designed fuzzy system's estimation and experimental measurements indicates that the error of the flux linkage estimation is 1.22% and the error of the magnetic force estimation is 2.17%. This proves that the accuracy of the proposed system is desirable both in nominal area and out of it. In addition, dynamic behavior of the actuator and its simulation were compared to validate the proposed model. This comparison was drawn for 35[N] and 55[N] loads with different currents. Having injected a step voltage to the winding, the changes of the plunger's position and the winding current over a period of time were recorded and then compared. The comparison in experimental and simulation fields demonstrates a proper accuracy for the proposed dynamic modeling.

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