Analysis of Square Coaxial Line Family

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Abstract **─** In this paper, the Equivalent Electrodes Method (EEM) has been proposed for the analysis of square coaxial lines family. Lines with single and two layer perfect or imperfect medium have been analyzed. The capacitance per unit length of these lines has been calculated. The results obtained by EEM have been compared with those reported in the literature, obtained by other methods, and those obtained by using software package COMSOL Multiphysics. Also, with the aim of comparison of the results, capacitance measurements based on a high resolution CDC (Capacitance to Digital Converter) have been realized. All results obtained have been found to be in very good agreement.

Index Terms **─** Capacitance per unit length, equivalent electrodes method, measurements, square coaxial line.

I. INTRODUCTION

The application of square coaxial lines is very important in transmitting RF energy, in antenna and microwave circuit design (particularly in satellite beam-forming networks in the lower microwave bands), and in equalizers, filters, branch line couplers and coaxial-to-stripline transformers. For calculating of the capacitance per unit length in the TEM mode of wave propagation in coaxial lines, quasistatic analysis can be used. The analysis of circular coaxial lines is simple and leads to exact analytical solutions. However, such a solution for square coaxial lines cannot be found. For determining of the capacitance per unit length of these lines, many numerical and analytical methods have been used. The main method in this analysis is the conformal mapping method [1], but it only gives close analytical solutions in a narrow range of geometries of square coaxial lines. Other methods which are commonly used are the numerical inversion of the Schwarz-Christoffel transformation [2], quasianalytical method of multipole theory [3], finite element method [4,5], finite-difference method [6], and other methods.

The aim of this paper is to apply the Equivalent Electrodes Method (EEM) in the analysis of square coaxial lines family. Coaxial lines, concentric and eccentric, with single layer perfect dielectric medium have been most common analyzed in the existing literature. Concerning this, the EEM has been firstly applied to the analysis of these lines. The results obtained have been compared with those found in the literature and those obtained by using the COMSOL software package (Finite Element Method). Very good agreement of the results confirms the applicability and accuracy of the EEM in solving these problems. Further, the proposed procedure and the results obtained by using EEM have been used for solving more complex problems, such as problems with two-layer perfect or imperfect medium. Additionally, an appropriate measurement system is designed and some of the obtained results are compared with measurement results.

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II. THE EQUIVALENT ELECTRODES METHOD APPLICATION

EEM is a very simple method for solving non-dynamic electromagnetic fields and other potential fields of theoretical physics. The mathematical form of this method is similar to the Method of Moments (MoM) [7] and Boundary Element Method (BEM) [8,9] form, but essentially EEM has different physical basics from these methods. Furthermore, the EEM does not require any integration during the calculation procedure and the process of matrix filling differs from MoM and BEM. EEM has been successfully used in the static and quasistatic analysis of electromagnetic fields and for transmission line analysis [10-16]. Through this, EEM has been compared with other analytical and numerical methods, and very good agreement of the results obtained has been found.

In this paper, the application of the EEM will be explained on general example; i.e., on a double eccentric square coaxial line, Fig. 1 (a).

This line contains two square electrodes on the electric potentials φ_1 and φ_2. The inner electrode is shifted of the longitudinal axis of symmetry in both horizontal and vertical directions.

By applying EEM, both electrodes have been replaced with two finite systems of Equivalent Electrodes (EE), placed on the surface of the electrodes, q_i', i=1,2,…,N_1 for one side of the inner electrode, and Q_j', j=1,2,…,M_1 for one side of the outer electrode.

The total number of EE is $N_u + M_u$, where $N_u = 4N_1$ is the EE number on the inner electrode and $M_u = 4M_1$ is the EE number on the outer electrode. These equivalent electrodes have the same radius, potential and charge as the part of the real electrode they represent (in this case, thin flat strip conductor).

In our example, each thin flat strip conductor with a width Δx has been replaced with a cylindrical EE with a circular cross-section as presented in Fig. 1 (b). The equivalent radius of these EE can be calculated using conformal mapping, Joukowsky transform [10], as $a_{en} = \Delta x_1/4$ and $b_{en} = \Delta x_2/4$, where $\Delta x_1 = a/N_1$ and $\Delta x_2 = b/M_1$ for the inner and outer electrode, respectively [12].

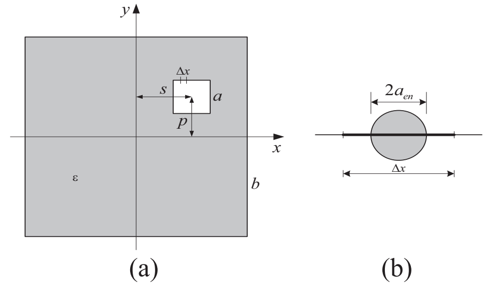

Fig. 1. (a) Double eccentric square coaxial line, and (b) thin flat strip conductor replaced with cylindrical EE.

These two systems of EE create the electric potential [12,13]:

$$\varphi = \varphi_0 + \sum_{i=1}^{N_u} q_i' G(\boldsymbol{r}, \boldsymbol{r}_n) + \sum_{j=1}^{M_u} Q_j' G(\boldsymbol{r}, \boldsymbol{r}_m), \tag{1}$$

where

$$G(\boldsymbol{r}, \boldsymbol{r}') = -\frac{1}{2\pi\varepsilon} \ln|\boldsymbol{r} - \boldsymbol{r}'|, \tag{2}$$

is the Green's function for the potential of uniform single line charge, $\boldsymbol{r}$ is the field point position vector, $\boldsymbol{r}'$ is the position vector of the electrical middle of the EE, ε is the electrical permittivity of dielectric medium and φ_0 is the unknown potential constant. So, the total number of unknowns which should be determined is $N_u + M_u + 1$. They can be obtained as the solution of the system of $N_u + M_u + 1$ linear equations.

By satisfying the boundary condition for the electric potential on the surface of the inner and the outer electrode, $N_u + M_u$ equations can be obtained:

$\varphi_1 = \varphi\big|_{\substack{x=x_n \\ y=y_n}}$, n=1,2,…,N_u;

$$\begin{aligned} \varphi_1 = \varphi_0 &- \sum_{i=1}^{N_u} \frac{q_i'}{4\pi\varepsilon} \ln\left[\left| r_n^2 \right| + a_{en}^2 \delta_{in} \right] - \\ &- \sum_{j=1}^{M_u} \frac{Q_j'}{4\pi\varepsilon} \ln\left[\left| r_m^2 \right| \right], \end{aligned} \tag{3}$$

and

$\varphi_2 = \varphi\big|_{\substack{x=x_m \\ y=y_m}}$, m=1,2,…,M_u;

$$\begin{aligned} \varphi_2 = \varphi_0 &- \sum_{i=1}^{N_u} \frac{q_i'}{4\pi\varepsilon} \ln\left[\left| r_n^2 \right| \right] - \\ &- \sum_{j=1}^{M_u} \frac{Q_j'}{4\pi\varepsilon} \ln\left[\left| r_m^2 \right| + b_{en}^2 \delta_{jm} \right], \end{aligned} \tag{4}$$

where

$$r_n^2 = (x - x_n)^2 + (y - y_n)^2 ,$$
$$r_m^2 = (x - x_m)^2 + (y - y_m)^2 .$$

x_n and y_n are positions of the EE on the inner electrode, x_m and y_m are positions of the EE on the outer electrode and δ is the Kronecker symbol (equal to 1 when $i = n$ and $j = m$, otherwise is equal to 0).

Additional equation which completes the system of equations is:

$$\sum_{i=1}^{N_u} q_i' + \sum_{j=1}^{M_u} Q_j' = 0 . \quad (5)$$

After calculation of the unknown charges, the capacitance per unit length can be easily calculated as:

$$C' = \frac{1}{\varphi_1 - \varphi_2} \sum_{i=1}^{N_u} q_i' . \quad (6)$$

III. MEASUREMENT SETUP

In order to measure the unknown capacitance an appropriate system is designed, which is based on a high resolution CDC (Capacitance-to-Digital Converter) AD7746 [17]. This converter can measure up to ± 4.096 pF changing capacitance, while it can accept up to 17pF common-mode, not changing capacitance, which can be balanced by a programmable on-chip, digital-to-capacitance converter. This digital-to-capacitance converted value can be thought of as a programmable negative capacitance value which can be added to the input pin to minimize base capacitance. The AD7746 has high resolution down to 4aF, which is 21 effective numbers of bits, high linearity ±0.01%, and high accuracy ±4fF. It can communicate with a microcontroller using TWI (Two Wire Interface), I^2C (Inter-Integrated Circuit) compatible serial interface.

Besides the CDC, the system also contains a microcontroller and PC application. The main task of the microcontroller is to set the configuration of AD7746, while PC application presents the results of capacitance-to-digital conversion. The block diagram of the whole system for the capacitance measurement is presented in Fig. 2. The main part of the system from the control point of view is the microcontroller. It communicates with both the AD7746 and the PC. It sets the control registers of the AD7746 in order to enable capacitance-to-digital conversion and tells the AD7746 when to do it, after which it reads the conversion's results, processes them and sends to the PC. On the PC side, there is an application installed which communicates with microcontroller through USART (Universal Synchronous/Asynchronous Receiver/Transmitter) interface. The application receives data, processes them and presents to the end user of the system. The hardware part of the system that includes microcontroller and AD7746 and which is designed for the purpose of this experiment, is shown on the Fig. 3.

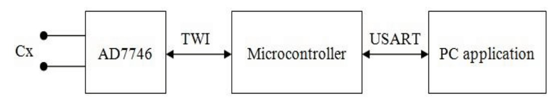

Fig. 2. Block diagram of the measurement system.

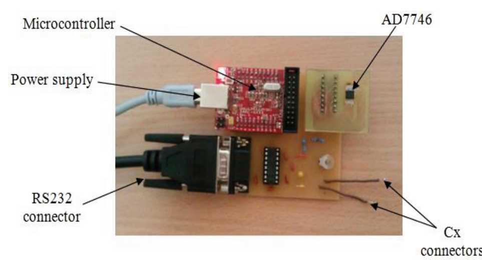

Fig. 3. Hardware part of the measurement system.

The PC application communicates with microcontroller in order to get the value of measured capacitance and to present it to the end user. It is a real time application that also graphically presents changes in measured capacitance, and which is specifically designed for the purpose of this experiment.

Two simple models of the square coaxial line have been made. Both electrodes of the first model are made of aluminum foil and dielectric between them is air. The second model represents the square coaxial line with two layer dielectric. One electrode is made of aluminum foil. For the preparation of second electrode, Printed Circuit Board (PCB) with copper on one side of the board has been used. Four identical pieces have been made using this PCB. Copper edges of these pieces have been soldered together in order to create a square electrode with a 2 mm pertinax layer around it. By placing the first electrode around the second electrode condenser with two layer dielectric medium has been created. First layer is pertinax on the inner electrode and the second layer is the air between electrodes.

The capacitance per unit length of the condenser model can be calculated as:

$$C' = C_x / h, \quad (7)$$

where C_x is measured capacitance and h is height of the condenser model.

IV. NUMERICAL AND MEASUREMENTS RESUTS

In this section, EEM has been applied for analysis of square coaxial lines with single layer perfect medium. Then, the results obtained have been used for analysis of square coaxial lines with two layer perfect and imperfect medium. Also, some of the results which are related to the single and two layer perfect medium have been compared with results obtained by measurements.

A. Concentric square coaxial line

Firstly, the EEM has been applied to the calculation of the capacitance per unit length of the concentric square coaxial line from Fig. 1, when $p = s = 0$. The accuracy of the results obtained using EEM depends on the total number of EE. The obtained result will converge, and accuracy should be higher by increasing this number. In other words, the width of the electrode parts Δx_1 and Δx_2, which one EE represents, decreases with an increase in the total number of EE. The convergence of the capacitance per unit length, when $w = a/b = 0.5$ has been presented in Table 1.

Table 1: Convergence of the *C'*/*ε* when *w*=0.5

Total Number of EE	*C'*/*ε*	Total Number of EE	*C'*/*ε*
72	10.5482	4800	10.2420
120	10.4507	5400	10.2411
240	10.3585	6000	10.2404
360	10.3220	7200	10.2394
480	10.3023	8400	10.2387
600	10.2899	9600	10.2381
840	10.2751	10800	10.2377
1200	10.2635	12000	10.2373
1800	10.2542	13200	10.2370
2400	10.2494	14400	10.2368
3000	10.2465	15600	10.2366
3600	10.2445	16800	10.2364
4200	10.2430	18000	10.2363

Mathematica Wolfram software has been used to write a program code that calculates the capacitance per unit length by applying EEM. This program has been executed on a PC with Intel(R) Core(TM) i5-3210M CPU at 2.50 GHz and with 6 GB RAM memory in order to test its run time. Obtained results show that run time vary with the total number of EE, Table 2, between several second and more than 10 minutes (obtained with maximal number of EE). It can be noticed from the results presented in Table 2 that the relative difference between results obtained with 3600 or 6000 EE and the result obtained with 18000 EE is less than 0.1%. Therefore, optimal number of EE can be taken so that the run time does not last more than several minutes.

Table 2: CPU run time variation with EE number

Total Number of EE	*C'*/*ε* EEM	*t* [s] EEM
1200	10.2635	3
3600	10.2445	27
6000	10.2404	75
12000	10.2373	305
18000	10.2363	704
Number of Mesh Elements	*C'*/*ε* COMSOL	*t* [s] COMSOL
7712	10.2388	1
30848	10.2360	6
123392	10.2348	51
278316	10.2344	308

According to the results presented in Table 1 and Table 2, the total number of EE in other calculations has been set between 9600 and 15200 depending on the geometry of the line, setting the width of the electrode part Δx to be constant. In this case, since $\Delta x = \Delta x_1 = \Delta x_2$ and $w = 0.5$ then $N_u / M_u = 0.5$. Due to the symmetry in all four quadrants of the coordinate system, the total number of equations is four times smaller than the total number of EE.

Lo and Lee [18] gave the capacitance per unit length of this line with approximate expression (8):

$$\frac{C'}{\varepsilon_0} = \frac{8(0.279 + 0.721w)}{1 - w}; \quad 0.25 \le w \le 0.5. \quad (8)$$

The results for the capacitance per unit length of the square coaxial line obtained using EEM have been presented in Table 3. In addition, a comparison of the results obtained using COMSOL, expression (8) and other methods given in the literature have been presented in this table. Good agreement between the results is evident.

According to the results for the capacitance per unit length, in the case of imperfect medium, the conductance per unit length can be determined as:

$$G' = C'\sigma/\varepsilon\,, \tag{9}$$

where σ is the conductivity of the imperfect medium.

Table 3: C'/ε for different w

w		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
EEM		2.841	4.135	5.634	7.564	10.237	14.240	20.912	34.260	74.342
COMSOL		2.844	4.141	5.642	7.573	10.249	14.255	20.932	34.283	74.340
[3]	Cockcroft	3.126	4.238	5.668	7.575	10.244	14.246	20.921	34.272	74.357
	Bowan	2.842	4.138	5.638	7.567	10.241	14.246	20.921	34.272	-
	Green	2.853	4.144	5.634	7.565	10.306	14.635	22.697	43.432	224.40
	Ivanov	2.848	4.151	5.638	7.611	10.197	14.280	-	-	-
	Costamagna	2.842	4.138	5.638	7.567	10.241	14.253	20.921	34.272	74.357
	Riblet	2.849	4.138	5.638	7.569	10.244	14.246	20.921	34.272	74.357
	Zheng	2.843	4.140	5.637	7.569	10.247	14.302	20.909	34.272	74.357
[1]		2.847	4.135	5.633	7.561	10.234	14.235	20.902	34.235	74.235
Exp. (8)		3.121	4.232	5.660	7.565	10.232	14.232	20.899	34.232	74.232

B. Single eccentric square coaxial line

The results for the capacitance per unit length of a single eccentric square coaxial line ($p \neq 0$ or $s \neq 0$, Fig. 1), when $w = 0.2$ and for different ratios s/b, have been presented in Table 4. The results have been obtained using EEM, COMSOL and measurement and they are in good agreement.

Table 4: C'/ε when $w = 0.2$ and $\varepsilon_2/\varepsilon_1 = 5.5$

	C'/ε		
s/b	EEM	COMSOL	Measured
0	4.13831	4.1407	4.14236
0.05	4.16326	4.1656	4.17268
0.10	4.24311	4.2454	4.23985
0.15	4.39528	4.3973	4.41768
0.20	4.65936	4.6610	4.66325
0.25	5.12778	5.1285	5.10983
0.30	6.06468	6.0624	6.10354
0.35	8.69264	8.6735	8.73456

The results for the capacitance per unit length for a single eccentric square coaxial line for different w and $s/b = 0.1; 0.2; 0.3$, obtained using EEM, have been presented in Table 5.

This type of line is analyzed by YuBo, et al. [2] for the $w = 0.4$ and a 40% eccentricity in the vertical direction, $0.4(b-a)/2$, and the capacitance per unit length has been calculated to be $8.19773\varepsilon_0$. By using EEM for this line, the capacitance per unit length has been calculated as $8.20778\varepsilon_0$.

Table 5: C'/ε of single eccentric coaxial line

	C'/ε		
w	$s/b = 0.1$	$s/b = 0.2$	$s/b = 0.3$
0.1	2.88931	3.06549	3.54787
0.15	3.55174	3.8285	4.65727
0.2	4.24247	4.65852	6.06283
0.25	4.99693	5.61388	8.08914
0.3	5.84673	6.76584	11.6288
0.35	6.82799	8.22542	20.9898
0.4	7.9876	10.1974	-
0.45	9.3919	13.1346	-
0.5	11.142	18.3439	-
0.55	13.4054	32.1948	-
0.6	16.4909	-	-
0.65	21.0625	-	-
0.7	28.9636	-	-
0.75	48.8447	-	-

C. Double eccentric square coaxial line

The numerical and measurement results for he capacitance per unit length of a double eccentric square coaxial line (and, Fig. 1), when $w = 0.2$ and for different ratios $k = s/b$ and p/b have been presented in Table 6.

This type of line is analyzed by YuBo, et al. [2] for the ratio $w = 0.4$ and a 40% eccentricity in the diagonal direction and the capacitance per unit length has been calculated to be $8.81411\varepsilon_0$. By using EEM for this line, the capacitance per unit length has been calculated as $8.82621\varepsilon_0$.

Table 6: C'/ε when w=0.2

C'/ε–Numerical Results (EEM)							
p/b	$k = 0.05$	$k = 0.1$	$k = 0.15$	$k = 0.2$	$k = 0.25$	$k = 0.3$	$k = 0.35$
0.05	4.18801	4.26727	4.41841	4.68099	5.14744	6.08188	8.70691
0.1	4.26727	4.34469	4.49267	4.75067	5.21096	6.13762	8.75333
0.15	4.41841	4.49267	4.63519	4.88509	5.33431	6.24665	8.84473
0.2	4.68099	4.75067	4.88509	5.12273	5.55468	6.44376	9.01183
0.25	5.14744	5.21096	5.33431	5.55468	5.96116	6.81383	9.33134
0.3	6.08188	6.13762	6.24665	6.44376	6.81383	7.60981	10.0385
0.35	8.70691	8.75333	8.84473	9.01183	9.33134	10.0385	12.2968
C'/ε–Measurement Results							
p/b	$k = 0.05$	$k = 0.1$	$k = 0.15$	$k = 0.2$	$k = 0.25$	$k = 0.3$	$k = 0.35$
0.05	4.17962	4.34569	4.32568	4.70125	5.09561	6.15983	8.65214
0.1	4.34569	4.25697	4.62347	4.76589	5.24589	6.29871	8.86998
0.15	4.32568	4.62347	4.74257	4.93654	5.21458	6.36955	8.65542
0.2	4.70125	4.76589	4.93654	5.35847	5.45214	6.33224	9.11245
0.25	5.09561	5.24589	5.21458	5.45214	6.25477	7.85264	9.34569
0.3	6.15983	6.29871	6.36955	6.33224	7.85264	7.96547	9.89654
0.35	8.65214	8.86998	8.65542	9.11245	9.34569	9.89654	12.45697

The variation in the capacitance per unit length has been presented in Fig. 4.

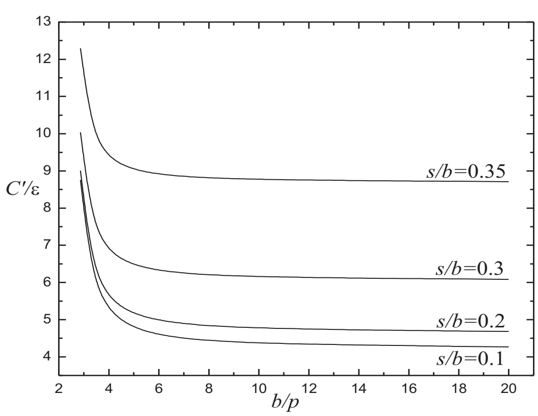

Fig. 4. Variation of C'/ε for a double eccentric square coaxial line.

D. Square coaxial lines with two layer medium

As stated in the introductory section of this paper, the results for the capacitance per unit length presented in previous sub-headings can be used for the determination of characteristic parameters of square coaxial lines with perfect or imperfect two-layer medium, Fig. 5.

For the line from Fig. 5 with two-layer perfect dielectric medium, when $\sigma_1 = \sigma_2 = 0$, the capacitance per unit length can be determined as it is presented in [7] by using expression (10):

$$\frac{1}{C'} = \frac{2}{(\varepsilon_1 + \varepsilon_2) g'_{13}} + \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2}\left(\frac{1}{\varepsilon_2 g'_{23}} - \frac{1}{\varepsilon_1 g'_{12}}\right), \quad (10)$$

where g'_{13} is a coefficient of the proportionality of the line when $\varepsilon_1 = \varepsilon_2$ and g'_{12} and g'_{23} are coefficients of the proportionality of the lines which are formed by the existing electrode and the electrode's shield coinciding with separation surface, Fig. 6 [11].

For the line in Fig. 5 with a two-layer imperfect dielectric medium, when $\sigma_1, \sigma_2 \neq 0$, the admittance per unit length can be determined as presented in [7] by using the expression (11):

$$\frac{1}{\underline{Y'}} = \frac{2}{(\underline{\sigma}_1 + \underline{\sigma}_2) g'_{13}} + \frac{\underline{\sigma}_1 - \underline{\sigma}_2}{\underline{\sigma}_1 + \underline{\sigma}_2}\left(\frac{1}{\underline{\sigma}_2 g'_{23}} - \frac{1}{\underline{\sigma}_1 g'_{12}}\right), (11)$$

where $\underline{\sigma}_1 = \sigma_1 + \mathrm{j}\omega\varepsilon_1$ and $\underline{\sigma}_2 = \sigma_2 + \mathrm{j}\omega\varepsilon_2$. The complex effective conductivity can be calculated as:

$$\underline{\sigma}_{\mathrm{e}} = \frac{\underline{Y}'}{g'_{13}} = \frac{G'_{\mathrm{e}} + \mathrm{j}\omega C'_{\mathrm{e}}}{g'_{13}} = \sigma_{\mathrm{e}} + \mathrm{j}\omega\varepsilon_{\mathrm{e}}, \qquad (12)$$

where σ_{e} and ε_{e} are the effective permittivity and effective conductivity, respectively.

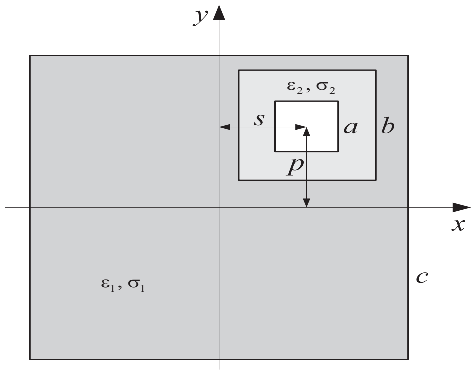

Fig. 5. Square coaxial line with two-layer medium.

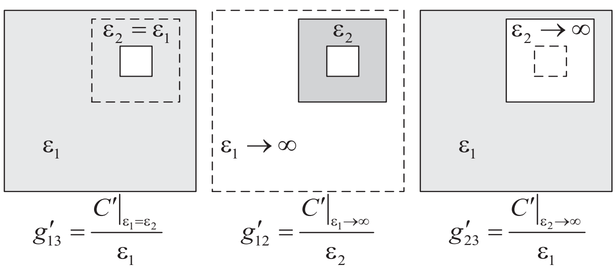

Fig. 6. Calculation of the coefficients of the proportionality.

Previously presented results for C'/ε (sub-sections A, B and C) have been used as values of coefficients of the proportionality g'_{12}, g'_{23} and g'_{13} in expressions (10), (11) and (12).

The variations in the results for the effective permittivity and effective conductivity for the line from Fig. 5 with imperfect two-layer medium obtained using EEM have been presented in Figs. 7 and 8.

The comparison of the measurement and numerical results for square coaxial line with two-layer perfect medium has been presented in Table 7.

The comparison of results for the capacitance per unit length and the complex effective conductivity for the line in Fig. 5 obtained using EEM and COMSOL, has been presented in Tables 8 and 9.

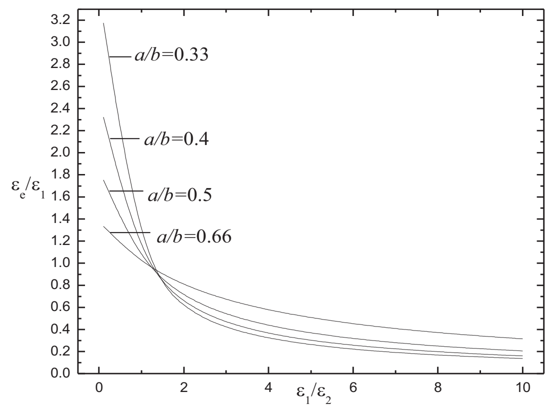

Fig. 7. Variation of $\varepsilon_{\mathrm{e}}/\varepsilon_1$ for different ratios a/b and $\varepsilon_1/\varepsilon_2$ when $s/c = p/c = 0.1$, $a/c = 0.2$, and $\sigma_1/\sigma_2 = 5$.

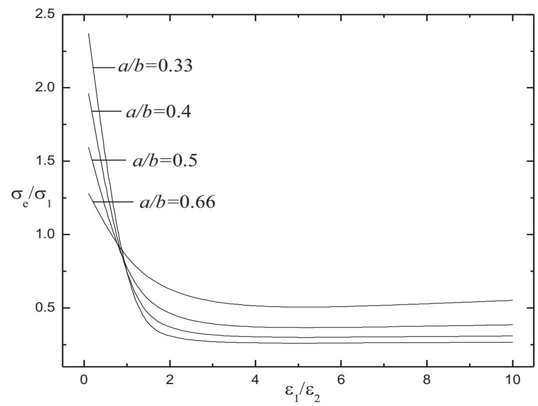

Fig. 8. Variation of $\sigma_{\mathrm{e}}/\sigma_1$ for different ratios a/b and $\varepsilon_1/\varepsilon_2$ when $s/c = p/c = 0.1$, $a/c = 0.2$, and $\sigma_1/\sigma_2 = 5$.

Table 7: C'/ε_1 when $w = 0.2$

p/c	C'/ε_1 Measured	C'/ε_1 EEM
0	6.66365	6.51268
0.05	6.73895	6.58094
0.1	6.88954	6.80707
0.15	7.49191	7.27374
0.2	8.39545	8.22093
0.25	9.67548	10.5723

Table 8: C'/ε when $a/b = 0.5$, $a/c = 0.2$, $\varepsilon_1/\varepsilon_2 = 5$ and $l = s/c$

EEM and Expression (10)						
p/c	l=0	l=0.05	l=0.1	l=0.15	l=0.2	l=0.25
0	1.56468	1.56791	1.57789	1.59547	1.62169	1.65379
0.05	1.56791	1.5703	1.58015	1.59751	1.62461	1.65653
0.1	1.57789	1.58015	1.58983	1.60692	1.63374	1.66520
0.15	1.59547	1.59751	1.60692	1.62365	1.65019	1.68126
0.2	1.62169	1.62461	1.63374	1.65019	1.67554	1.70726
0.25	1.65379	1.65653	1.66520	1.68126	1.70726	1.74493
COMSOL						
p/c	l=0	l=0.05	l=0.1	l=0.15	l=0.2	l=0.25
0	1.568526	1.571311	1.581271	1.597932	1.624109	1.660482
0.05	1.571311	1.574405	1.583967	1.600861	1.62654	1.663086
0.1	1.581271	1.583967	1.593682	1.610272	1.635559	1.671230
0.15	1.597932	1.600861	1.610272	1.626023	1.650649	1.68589
0.2	1.624109	1.62654	1.635559	1.650649	1.674832	1.708755
0.25	1.660482	1.663086	1.671230	1.685890	1.708755	1.742107

Table 9: $\underline{\sigma}_e/\sigma_1$ when $a/b = 0.5$, $a/c = 0.2$, $\varepsilon_1/\varepsilon_2 = 5$, $\sigma_1/\sigma_2 = 5$, $\omega/2\pi = 50$ Hz and $l = s/c$

EEM and Expression (12)						
p/c	l=0	l=0.05	l=0.1	l=0.15	l=0.2	l=0.25
0	0.378+j1049.75	0.376+j1045.60	0.372+j1032.41	0.363+j1007.7	0.349+j966.08	0.323+j895.88
0.05	0.376+j1045.60	0.375+j1041.53	0.370+j1028.60	0.361+j1004.33	0.347+j963.34	0.322+j893.93
0.1	0.372+j1032.41	0.370+j1028.60	0.366+j1016.46	0.358+j993.55	0.344+j954.53	0.319+j887.66
0.15	0.363+j1007.70	0.361+j1004.33	0.358+j993.55	0.350+j973.02	0.337+j937.58	0.315+j875.49
0.2	0.344+j966.08	0.347+j963.34	0.344+j954.53	0.337+j937.58	0.327+j907.75	0.307+j853.76
0.25	0.323+j895.88	0.322+j893.94	0.319+j887.66	0.315+j875.49	0.307+j853.76	0.293+j813.10
COMSOL						
p/c	l=0	l=0.05	l=0.1	l=0.15	l=0.2	l=0.25
0	0.380+j1055.99	0.378+j1051.55	0.373+j1038.31	0.364+j1012.99	0.349+j972.00	0.324+j902.49
0.05	0.378+j1051.55	0.376+j1047.40	0.372+j1034.21	0.363+j1009.54	0.348+j968.27	0.324+j900.45
0.1	0.373+j1038.31	0.372+j1034.21	0.367+j1021.50	0.359+j998.19	0.345+j958.89	0.321+j893.83
0.15	0.364+j1012.99	0.363+j1009.54	0.359+j998.19	0.351+j977.50	0.338+j941.61	0.317+j880.83
0.2	0.349+j972.00	0.348+j968.27	0.345+j958.89	0.338+j941.61	0.327+j910.68	0.308+j857.39
0.25	0.324+j902.49	0.324+j900.45	0.321+j893.83	0.317+j880.84	0.308+j857.390	0.293+j814.40

V. CONCLUSION

In this paper, EEM has been used for the analysis of the square coaxial lines, concentric and eccentric, with single and two-layer perfect and imperfect medium.

Application of the method is very simple, using just simple mathematical operations without numerical integrations. Short calculation time is an additional quality of the proposed method. Fast convergence of the results is evident.

All results obtained using EEM have been compared with those obtained using COMSOL and some results with those found in the literature. They have been found to be in very good agreement. By increasing the number of EE, EEM gives more accurate results compared to other methods.

Additionally, in order to confirm the numerical results, appropriate system based on the high resolution capacitance-to-digital converter is designed. Some of the results obtained have been compared to the measurement results and good agreement has been found. In the case of two-layer medium, small differences between results exist because of the simplicity of the physical model used in measurements.

The eccentricity of the lines has been analyzed in details. So, the paper contains many new results in this area which cannot be found in the existing literature.

According to this, one can conclude that EEM can be successfully used for the analysis of all types of lines with square cross section.

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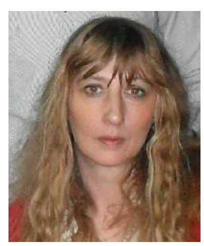

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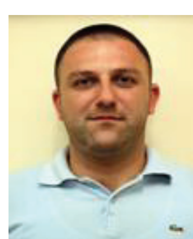

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