Using the Best Uniform Approximation with Compression for Efficient Computation of Monostatic Scattering

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Abstract ─ The best uniform approximation method hybridized with Singular Value Decomposition (SVD) is proposed to reduce the time requirement for computation of monostatic Radar Cross Section (RCS). In contrast to our previous work, the traditional best uniform approximation technique is applied to compute the key excitation vectors instead of electric current vectors. Reduction of the number of multiple excitation vectors can lead to significantly reduced computation time. Furthermore, with low-rank property, the key excitation vectors could be further compressed by SVD, resulting in a more efficient method. Numerical results demonstrate that the proposed method is efficient for monostatic RCS calculation with high accuracy.

Index Terms ─ Best uniform approximation, monostatic scattering, radar cross-section, Singular Value Decomposition (SVD).

I. INTRODUCTION

Analysis of electromagnetic wave scattering from electrically large objects using conventional Method of Moment (MoM) [1,2] requires a mass of computation time and storage, because the Electrical Field Integral Equation (EFIE) matrix associated with the resulting linear systems is large, dense, and ill-conditioned. To solve integral equations by applying traditional MoM, the computation complexity for the iterative solver is $O(kN^2)$ and the memory requirement is $O(N^2)$, where N refers to the number of unknowns and k refers to the number of iterative steps. Obviously, it is impractical to use a personal computer to solve equations with more than 10,000 unknowns. This difficulty can be overcome by using the multi-level fast multi-pole algorithm (MLFMA) [2,3] while accelerating the operation of matrix-vector product. The computation complexity can be reduced to $O(N\log N)$ and the memory requirement to $O(N\log N)$. Moreover, preconditioning techniques [4-8] can speed up convergence of iterative solvers by improving spectral properties of the EFIE matrix. However, it is still time- and memory-consuming for calculation of monostatic RCS since it requires repeated solution of EFIE at each incident direction and frequency.

The Model-Based Parameter Estimation (MBPE) technique is presented by Miller and Burke to accurately compute the wide band response with a few direct calculations [9,10]. In this technique, the electric current or field is expanded as a rational function. The coefficients of the rational function are obtained using either frequency/angular data or the related derivative data [22]. In [11,12], an adaptive sampling method to obtain the optimal samples for monostatic RCS calculation with wide angular band. The adaptive technique is employed to generate new sampling points automatically by using a coarse-to-fine hierarchy.

As an alternative technique, the best uniform approximation [13] has been introduced. The best uniform approximation are important in approximation theory since the roots of the Chebyshev polynomials of the first kind, which are also called Chebyshev nodes, are used as nodes in polynomial interpolation. The resulting

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interpolation polynomial minimizes the problem of Runge's phenomenon and provides an approximation that is close to the polynomial of best approximation to a continuous function under the maximum norm. In [13], the best uniform approximation is proposed to optimally select the most informative angles in monostatic RCS curve, resulting in an efficient computation of monostatic scattering.

In [14], it is reported that multiple excitation vectors or right hand side vectors can be compressed by use of the low-rank property. Accordingly, the optimal excitation vectors corresponding to the selected angles can be compressed by Singular Value Decomposition (SVD). In this paper, the combination of SVD and best uniform approximation is applied to efficient computation of monostatic RCS. The numerical simulations demonstrate that this framework can reduce the computation time significantly.

The remainder of this paper is organized as follows. Section II demonstrates the basic theory and formulations of integral equations. The theory of the best uniform approximation and using SVD to achieve compression will be discussed in section III. Numerical experiments of several geometries are presented to demonstrate the efficiency of this proposed method in Section IV.

II. EFFICIENT COMPUTATION OF MONOSTATIC RCS BY BEST UNIFORM APPROXIMATION

A. Theory of moment method

For electromagnetic scattering from a Perfect Electrical Conductor (PEC), the Combined Field Integral Equation (CFIE) which consists of EFIE and MFIE is widely used for closed structures [1]. The CFIE formulation of electromagnetic wave scattering problems using planar Rao-Wilton-Glisson (RWG) basis functions for surface modeling is presented in [15,16]. Once the resulting linear systems from the CFIE formulation after Galerkin's test are solved by numerical matrix equation solvers, the CFIE matrix equation can be symbolically rewritten as:

$$\mathbf{Ax}=\mathbf{b}. \tag{1}$$

Here, **A** refers to the impedance matrix. **x** is the column vector containing the unknown coefficients of the surface current expansion with RWG basis functions and it can be used to calculate the scattered field and RCS. **b** devotes the right hand side which generated by the incident wave.

To solve the above matrix equation by an iterative method, the matrix-vector products are required at each iteration step. Physically, a matrix-vector product corresponds to one cycle of iterations between the basis functions. The basic idea of the Fast Multi-pole Method (FMM) is to convert the interaction of element-to-element to the interaction of group-to-group. Here a group includes the elements residing in a spatial box. The mathematical foundation of the FMM is the addition theorem for the scalar Green's function in free space. Using the FMM, the matrix-vector product **Ax** can be written as:

$$\mathbf{Ax}=\mathbf{A}_N\mathbf{x}+\mathbf{A}_F\mathbf{x}. \tag{2}$$

Here $\mathbf{A}_N$ is the near part of **A** and $\mathbf{A}_F$ is the far part of **A**. In the FMM, the calculation of matrix elements in $\mathbf{A}_N$ remains the same as in the MoM procedure. However, those elements in $\mathbf{A}_F$ are not explicitly computed and stored. Hence, they are not numerically available in the FMM. It has been shown that the operation complexity of FMM to perform Ax is $O(N^{1.5})$. If the FMM is implemented in multilevel, the total cost can be reduced further to $O(N\log N)$ [2,3]. However, it is still time consuming for the computation of a monostatic RCS since it requires repeated solution of CFIE for each incident direction. As a result, new methods are required to circumvent this difficulty.

B. Accelerated by best uniform approximation

In order to compute the monostatic RCS of arbitrary geometry, MoM solver and MLFMA have to be applied angle by angle over a given angular band. To accelerate it by the best uniform approximation, the specific algorithm is as follows:

For a given angular band $[\theta_m, \theta_n]$, let:

$$k=\frac{2\theta-(\theta_m+\theta_n)}{\theta_n-\theta_m}. \tag{3}$$

Accordingly, the surface current vector can be written as:

$$\mathbf{I}(k)=\mathbf{I}\left[\frac{k(\theta_n-\theta_m)+(\theta_m+\theta_n)}{2}\right], \tag{4}$$

where k belongs to [-1, 1].

Assume that $T_l(k)$ (l=1,2,...,n) as the l-order Chebyshev polynomials and it is defined as:

$$T_0(k)=1, \tag{5a}$$

$$T_1(k)=k, \quad (5b)$$

$$T_{l+1}(k)=2kT_l(k)-T_{l-1}(k),\ 2\le l\le n. \quad (5c)$$

So the Chebyshev Approximation of $\mathbf{I}(k)$ can be expressed as:

$$\mathbf{I}(k)\approx\sum_{l=0}^{n-1}\mathbf{c}_l T_l(k)-\frac{\mathbf{c}_0}{2}. \quad (6)$$

Suggest k_i (i=1,2,...,n) as the i-th zreo point of $T_n(k)$, so,

$$k_i=\cos(\frac{i-0.5}{n}\pi),\ i=1,2,...,n, \quad (7)$$

$$\mathbf{c}_l=\frac{2}{n}\sum_{i=1}^{n}\mathbf{I}(\theta_i)T_l(k_i). \quad (8)$$

Here, θ_i is called the Chebyshev samples in [θ_m, θ_n]. Where:

$$\theta_i=\frac{k_i(\theta_n-\theta_m)+(\theta_m+\theta_n)}{2},\ i=1,2,...,n. \quad (9)$$

Above all, we summarize the algorithm of the best uniform approximation about the target surface current, which is presented as follows.

The best uniform approximation algorithm

Step 1: Calculate the zero points k_i of $T_n(k)$.

Step 2: Transform the zero points to Chebyshev samples. That is transform k_i to θ_i using formulation (9).

Step 3: Using MoM and MLFMA to compute current vector $\mathbf{I}(\theta_i)$. Then apply the formula of (8) to get the coefficients $\mathbf{c}_l$.

Step 4: Using the formula of (6) to calculate the approximate current throughout the whole angular band [θ_m, θ_n].

Step 5: Using the approximated current vector to compute the monostatic RCS.

III. BEST UNIFORM APPROXIMATION WITH SVD COMPRESSION

Theoretically, the combination of MoM and MLFMA is able to accurately analyze the scattering of any geometry. Improved by the best uniform approximation, the computation of a monostatic RCS can be accelerated greatly. However, in some cases, the number of coefficients of the interpolation polynomials is so large as to compromise the efficient calculation of monostatic scattering. This process can be computationally prohibitive for electrically large objects. In order to alleviate this difficulty, a singular value decomposition based method is proposed and discussed in this section.

Firstly, a brief review of compression of right hand sides is given. The computation of monostatic RCS can be considered as linear equations with multiple right hand sides:

$$\mathbf{AX}=\mathbf{B}, \quad (10)$$

where $\mathbf{A}$ is the impedance matrix, $\mathbf{X}$ is the multiple complex coefficient vector of RWG basis and $\mathbf{B}$ is the multiple right hand side generated by the incident wave. In addition,

$$\mathbf{X}=[\mathbf{x}(\theta_1), \mathbf{x}(\theta_2), \ldots, \mathbf{x}(\theta_n)], \quad (11)$$

$$\mathbf{B}=[\mathbf{b}(\theta_1), \mathbf{b}(\theta_2), \ldots, \mathbf{b}(\theta_n)], \quad (12)$$

where θ_i is the i^{th} incident angle. Using traditional singular value decomposition, the matrix $\mathbf{B}$ can be described in the form of an eigenvalue and eigenvector:

$$\mathbf{B}=\mathbf{U\Sigma V}^{\mathrm{H}}. \quad (13)$$

The superscript 'H' denotes the conjugate transpose. If the dimension of $\mathbf{B}$ is $N\times M$, the dimension of matrices $\mathbf{U}$、$\mathbf{\Sigma}$ and $\mathbf{V}$ are $N\times M$, $M\times M$, $M\times M$, respectively. N is the number of unknowns. $\mathbf{\Sigma}$ is a diagonal matrix including all the eigenvalues of $\mathbf{B}$ while $\mathbf{U}$ and $\mathbf{V}$ contain all the eigenvectors of $\mathbf{B}$. When $\mathbf{B}$ is the multiple right hand sides in the linear system connecting with the SIE used for monostatic RCS, the matrix $\mathbf{B}$ is low-rank and can be approximately described as a low-rank SVD form:

$$\mathbf{B}=\mathbf{U}_k\mathbf{\Sigma}_k\mathbf{V}_k^{\mathrm{H}}, \quad (14)$$

where the dimension of matrices $\mathbf{U}_k$、$\mathbf{\Sigma}_k$ and $\mathbf{V}_k$ are $N\times k$, $k\times k$, $M\times k$, respectively. Only the k largest eigenvalues and corresponding eigenvectors are reserved. Substituting (14) into (10), the linear equations can be rewritten as:

$$\mathbf{X}\approx(\mathbf{A}^{-1}\mathbf{U}_k)\mathbf{\Sigma}_k\mathbf{V}_k^{\mathrm{H}}. \quad (15)$$

Here, $\mathbf{A}^{-1}\mathbf{U}_k$ can be computed by any iterative solver. If using a direct solver to compute the inversion of matrix $\mathbf{A}$ [17], the proposed method will become useless. Therefore, the number of repeated solutions of $\mathbf{Ax}=\mathbf{b}$ is k for SVD method. Using traditional method, the number is M. Generally, k is much smaller than M which leads to an efficient method for computation of monostatic RCS over a wide angular band.

Using the best uniform approximation, we can write the induced current into the sum of the samples shown in (14). We rewrite this formulation in matrix form:

$$\mathbf{X}=\mathbf{A}^{-1}\mathbf{B}_s\mathbf{C}, \quad (16)$$

where $\mathbf{X}$ is a matrix containing all induced currents over the whole angular band. $\mathbf{C}$ is the

coefficient matrix of the non-uniform interpolation method with the dimension of $s \times M$. $\mathbf{B}_s$ contains all the key samples of the excitation vectors. According to formulation (16), the required number of repeated solution of $\mathbf{Ax}=\mathbf{b}$ is s, where s is the number of required terms of Chebyshev Polynomials. Using singular value decomposition for matrix $\mathbf{B}_s$:

$$\mathbf{B}_s=\mathbf{U}_{sk}\mathbf{\Sigma}_{sk}\mathbf{V}_{sk}^{\mathrm{H}}. \quad (17)$$

Then,

$$\mathbf{X}=(\mathbf{A}^{-1}\mathbf{U}_{sk})\mathbf{\Sigma}_{sk}\mathbf{V}_{sk}^{\mathrm{H}}\cdot\mathbf{C}. \quad (18)$$

As a result, the required number of repeated solution is reduced to sk.

Above all, we can summarize the algorithm of the compressed best uniform approximation, which is presented as follows.

The compressed best uniform approximation algorithm

Step 1: Calculate the zero points k_i of $T_n(k)$.

Step 2: Transform the zero points to Chebyshev samples. That is transform k_i to θ_i using formulation (9).

Step 3: Computing the right hand sides $\mathbf{V}(\theta_i)$ using MoM formulation. Apply the formula of (8) to get the coefficients $\mathbf{c}_l$. Then put all the Chebyshev basis into matrix $\mathbf{B}_s$.

Step 4: Transfer $\mathbf{B}_s=[\mathbf{B}_{s1}, \mathbf{B}_{s2}, ..., \mathbf{B}_{sn}]$ into the form of $\mathbf{U}_k\mathbf{\Sigma}_k\mathbf{V}_k^{\mathrm{H}}$ by singular value decomposition, where k is the rank of $\mathbf{B}_s$.

Step 5: Computing the value $\mathbf{A}^{-1}\mathbf{U}_{sk}$. Then using formulation (18) to get the value of current vector $\mathbf{I}$.

Step 6: Using the formula of (6) to calculate the approximate current throughout the whole angular band $[\theta_m, \theta_n]$.

Step 7: Using the approximated current vector to compute the monostatic RCS.

Considering the accuracy of the proposed method, the error control is very important in SVD. The Convergence Error (CE) is defined here to control the error of SVD. That is, some rows of matrix $\mathbf{U}$ and some columns of $\mathbf{V}$ should be truncated in SVD when the corresponding eigenvalue over the largest eigenvalue is smaller than CE. In order to reduce the numerical error, CE is required to be sufficiently small. However, large CE is needed for efficiency. The value of CE is chosen as $CE=10^{-2}$ in this paper, and this will be discussed in the next session.

IV. NUMERICAL RESULTS

In this section, a number of numerical results are presented to demonstrate the accuracy and efficiency of the proposed method for fast calculation of monostatic RCS over a wide angular band. The Flexible General Minimal Residual (FGMRES) [18,19,20] algorithm is applied to solve linear systems. The dimension size of the Krylov subspace is set to be 30 for outer iteration and the dimension is set to be 10 for inner iteration. The tolerance of the inner iteration is 0.1 in this paper. All experiments are conducted on a Quad-Core AMD Opteron (tm) with 4.00 GB local memory and run at 2.31 GHz in single precision. The iteration process is terminated when the 2-norm residual error is reduced by 10^{-3}, and the limit of the maximum number of iterations is set as 1000.

Four geometries are applied to illustrate the performance of our proposed method. They consist of a NASA Almond with 1815 unknowns [23], a Reentry Vehicle (RV) with 26566 unknowns [21], a Cube with 49260 unknowns and the VFY-218 model [3] with 40725 unknowns. The NASA Almond is used for testing the value of CE, while the last three geometries is used for testing the accuracy and efficiency of the proposed method.

As shown in Fig. 1, the NASA Almond is one of the most popular geometry in electromagnetics and its structure is define in [23]. The RV has a blunt nose as well as a deeply recessed rear cavity that are expected to be significant sources of backscatter. The three-dimensional shape of the RV is illustrated in Fig. 2 (unit: wavelength). As the third geometry, the length of the Cube is 1 m. The VFY-218 is a well-known model in the field of electromagnetic scattering and its geometry is shown in Fig. 3. The VFY-218 is 15.5 m from nose to tail, 8.9 m from one wing to another, and 4.1 m from top to bottom.

In our numerical experiments, the geometries are illuminated by a plane wave with the incident pitch angles range from 0 to 180 deg. The frequency is 1.0 GHz for the RV, 300 MHz for Cube and 300 MHz for VFY-218. For all cases the azimuth angle is 0 deg. Firstly, the factor 'CE' must be determined. In Fig. 3, the NASA almond is used for testing the value of CE since it has a

small number of unknowns. The results of the monostatic RCS of by four different CE are compared with the reference result (the direct method). The frequency is 3.0 GHz for the almond geometry. The reference result is the RCS curve computed with repeated solution without compression at each angle. Other curves are computed by our proposed method. We select a part of the curve where the difference is much bigger with the incident pitch angles range from 20 to 100 deg. From this figure, when CE is set to be 0.1 or 0.05, although the decrease of computation time meets the requirement, the RCS curve is not accurate enough. When CE is set to be 0.01 and 0.001, the proposed method will be able to perform a good result. The time of the comparison is listed in Table 1. As is shown, the CPU time is 338s when CE is set to be 0.001, and 232s when CE is set to be 0.01. Therefore, CE=0.001 leads to larger computation time than 0.01. If using the proposed method without compression, which means CE is set to be 0, it would spend 5 times longer than CE is set to be 0.01 for the high-rank. That is, there is a tradeoff between accuracy and efficiency. In this paper, the value of CE is set to be 0.01 to keep the RCS curve accurate enough.

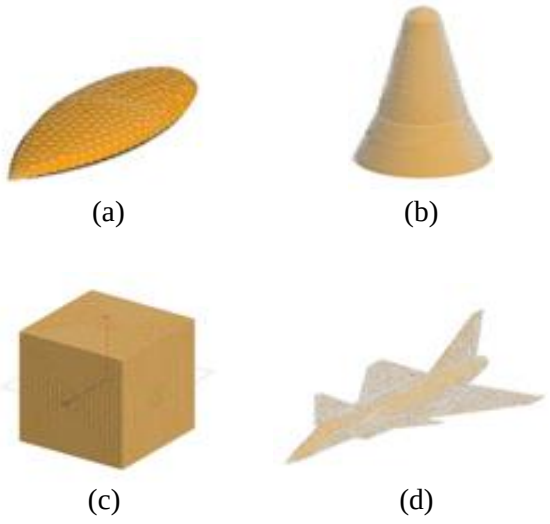

Fig. 1. Four geometries used in this paper: (a) NASA Almond, (b) Reentry Vehicle, (c) Cube, and (d) VFY-218.

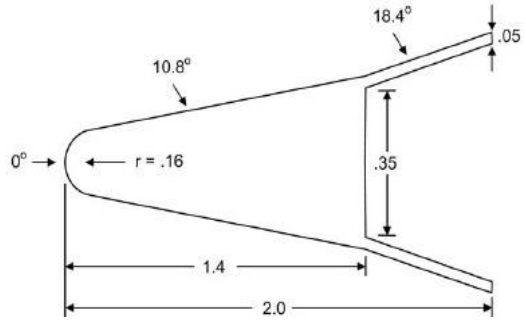

Fig. 2. Reentry vehicle model (unit: wavelength).

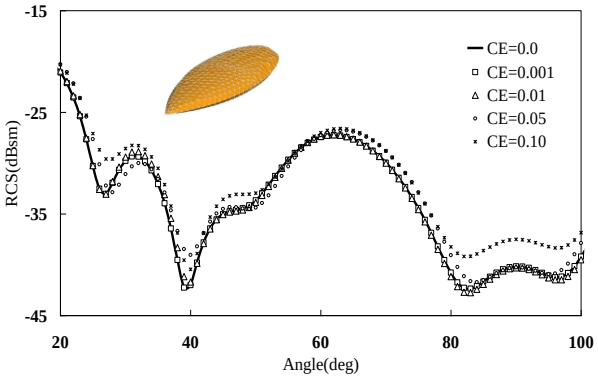

Fig. 3. Comparison of the results by proposed method with different CE.

Table 1: Computation time of monostatic RCS with different CE (time: second)

CE	0	0.001	0.01	0.05	0.1
Time	1386	338	232	152	78

As is shown in Figs. 4-6, the monostatic RCS curve of RV, Cube, and VFY-218 which computed by the proposed method is compared with the curve computed by the reference result repeatedly. "Reference" refers to the results computed by the MLFMA without interpolation, while "proposed method" denotes results computed by proposed method in section III. The CE is set to be 0.01 and the SVD is used to compress the multiple vectors generated by the best uniform approximation. It is obvious that the results matched very well. Consequently, the proposed method is accurate since there is no significant difference between the RCS result obtained by the reference result and the proposed

method. As is shown in Table 2, when compared with the reference results, the proposed method provides great advantage on total computation time since the number of right hand side is small. For reference results, the total computation time is 351497s, 53497s and 594576s, respectively, and the number of linear equation solutions is 721 since the space for angle sweep is 0.25 degree. It is time consuming for repeated solution of linear systems. In order to demonstrate the efficiency, it is applied for comparison. When SVD compression is used, the multiple vectors for these three geometries can be compressed. Consequently, the CPU time is reduced to 12004s, 1672s and 95838s. By using the proposed method, it saved time much than 29 times for RV, 31 times for Cube and 6 times for VFY-218, which is mentioned before while contrasting with the direct method. For different geometries, the saving time is different due to their complexity of the structures and the frequency.

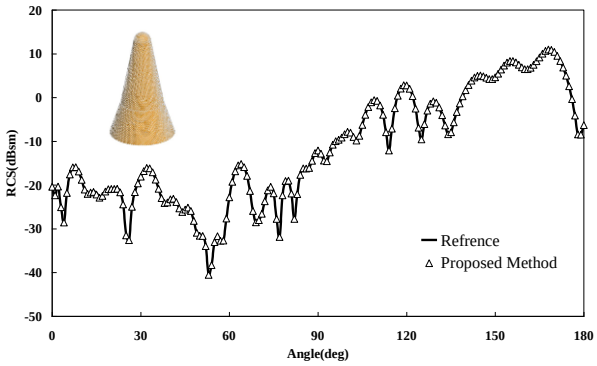

Fig. 4. The monostatic RCS results for RV.

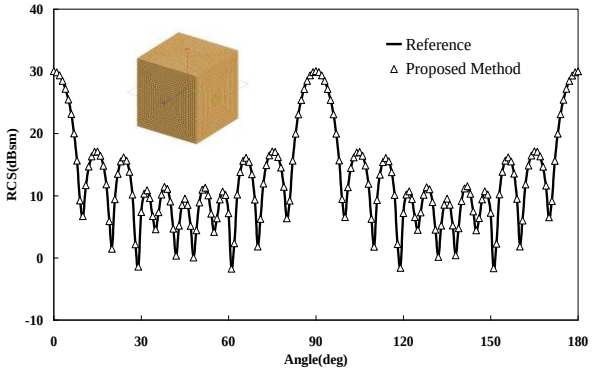

Fig. 5. The monostatic RCS results for Cube.

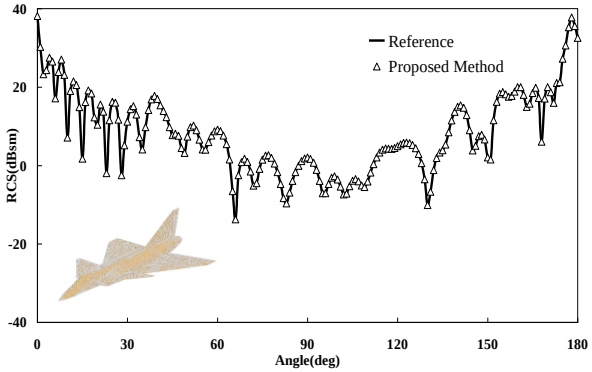

Fig. 6. The monostatic RCS results for VFY-218.

From Table 2, it is concluded that when compared with traditional method, the solution times for the linear system are reduced greatly and total computation time can be reduced significantly with this proposed method. As a result, our proposed method can be considered as an accurate and efficient method.

Table 2: Computation time of monostatic RCS with different methods

Geometry	Unknown	f	θ (deg)	φ (deg)	Computation Time (s)	
					Repeated Solution	Proposed Method
RV	26566	1.0 GHz	0~180	0	351497	12005
Cube	49260	300 MHz	0~180	0	53498	1672
VFY-218	40725	300 MHz	0~180	0	594577	95838

V. CONCLUSIONS

In this paper, combining singular value decomposition with the best uniform approximation has been proposed for efficient analysis of monostatic scattering. Unlike interpolation of the electric current, the best uniform approximation algorithm is used to approximate the multiple right hand sides on a set of non-uniform sampling angles and SVD is employed to reduce the consumption time automatically. The most informative angles may be selected by this procedure. Moreover, applying SVD to compute the eigenvectors of the selected vectors leads to reduced times for the iterative solutions of linear systems. Numerical experiments demonstrate that the proposed method is more efficient when compared with the traditional direct method.

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REFERENCES

[1] R. F. Harrington, "Field computation by moment methods," *Malabar, Fla.: R. E. Krieger*, 1968.

[2] W. C. Chew, J. M. Jin, E. Michielssen, and J. M. Song, "Fast and efficient algorithms in computational electromagnetics," *Boston, MA: Artech House*, 2001.

[3] J. M. Song, C. C. Lu, and W. C. Chew, "Multilevel fast multipole algorithm for electromagnetic scattering by large complex objects," *IEEE Trans. Antennas Propagation*, vol. 45, no. 10, pp. 1488-1493, October 1997.

[4] R. S. Chen, E. K. N. Yung, C. H. Chan, and D. G. Fang, "Application of SSOR preconditioned conjugate gradient algorithm to edge FEM for 3-dimensional full wave electromagnetic boundary value problems," *IEEE Trans. Microwave Theory Technology*, vol. 50, pp. 1165-1172, 2002.

[5] R. S. Chen, X. W. Ping, and E. K. N. Yung, "Application of diagonally perturbed incomplete factorization preconditioned CG algorithms for edge FEM analysis of helemholtz equations," *IEEE Trans. Antennas Propag*., vol. 54, pp. 1064-1068, 2006.

[6] J. Lee, J. Zhang, and C. C. Lu, "Sparse inverse preconditioning of multilevel fast multipole algorithm for hybrid integral equations in electromagnetics," *IEEE Trans. Antennas Propag*., vol. 52, pp. 2277-2287, 2004.

[7] P. L. Rui and R. S. Chen, "An efficient sparse approximate inverse preconditioning for FMM implementation," *Microwave Optical Technology Letter*, vol. 49, pp. 1746-1750, 2007.

[8] P. L. Rui, R. S. Chen, D. X. Wang, and E. K. N. Yung, "Spectral two-step preconditioning of multilevel fast multipole algorithm for the fast monostatic RCS calculation," *IEEE Trans. Antennas Propagation*, vol. 55, pp. 2571-2577, 2007.

[9] E. K. Miller and G. J. Burke, "Using model-based parameter estimation to increase the physical interpretability and numerical efficiency of computational electromagnetics," *Computer Physics Communications*, vol. 68, pp. 43-75, 1991.

[10] E. K. Miller, "Model-based parameter estimation in electromagnetics-part I: background and theoretical development," *IEEE Antennas Propag. Mag*., vol. 40, no. 1, pp. 42-52, February 1998.

[11] Z. W. Liu, R. S. Chen, and J. Q. Chen, "Adaptive sampling cubic-spline interpolation method for efficient calculation of monostatic RCS," *Microwave Optical Technology Letter*, vol. 50, no. 3, pp. 751-755, 2008.

[12] Z. W. Liu, D. Z. Ding, Z. H. Fan, and R. S. Chen, "Adaptive sampling bicubic spline interpolation method for fast computation of monstatic RCS," *Microwave Optical Technology Letter*, vol. 50, no. 7, pp. 1851-1857, 2008.

[13] C. Tian, Y. J. Xie, Y. Y. Wang, and Y. H. Jiang, "Fast solutions of wide-band RCS pattern of objects using MLFMA with the best uniform approximation," *Journal of Electronics & Information Technology*, vol. 31, no. 11, pp. 2772-2775, November 2009.

[14] Z. Peng, M. B. Stephanson, and J. F. Lee, "Fast computation of angular responses of large-scale three-dimensional electromagnetic wave scattering," *IEEE Trans. Antennas Propagation*, vol. 58, no. 9, pp. 3004-3012, September 2010.

[15] S. M. Rao, D. R. Wilton, and A. W. Glisson, "Electromagnetic scattering by surfaces of arbitrary shape," *IEEE Trans. Antennas Propagation*, vol. 30, no. 3, pp. 409-418, 1982.

[16] G. K. Carvajal, D. J. Duque, and A. J. Zozaya, "RCS estimation of 3D metallic targets using the moment method and rao-wilton-glisson basis functions," *Applied Computational Electromagnetic Society Journal*, vol. 24, no. 5, pp. 487-492, October 2009.

[17] J. Shaeffer, "Direct solve of electrically large integral equations for problem sizes to 1 M unknowns," *IEEE Trans. Antennas Propagation*, vol. 56, no. 8, pp. 2306-2313, August 2008.

[18] C. Rorres and H. Anton, "Application of linear algebra." *New York: John Wiley and Sons*, 1984.

[19] Y. Saad and M. Schultz, "GMRES: a generalized minimal residual algorithm for solving nonsymmetric linear systems," *SIAM J. Sci. Stat. Comput*., vol. 7, no. 3, pp. 856-869, 1986.

[20] V. Simoncini and D. B. Szyld, "Flexible inner-outer krylov subspace methods," *SIAM J. Numer. Anal*., vol. 40, no. 6, pp. 2219-2239, 2003.

[21] W. C. Gibson, "The method of moments in electromagnetics," *Boca Raton: Chapman & Hall/CRC*, 2008.

[22] X. Y. Zhang and X. Q. Sheng, "Fast RCS computation in both spatial and frequency domains using adaptive MBPE technique," *ICMTCE*, 2009.

[23] A. C. Woo, H. T. G. Wang, M. J. Schuh, and M. L. Sanders, "EM programmer's notebook-benchmark radar targets for the validation of computational electromagnetics programs," *IEEE Antennas and Propagation Magazine*, vol. 35, no. 1, pp. 84-89, February 1993.

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