An Adaptive Sparse Array Beamforming Algorithm Based on Approximate L_0-norm and Logarithmic Cost

Haixu Wang and YingSong Li*

College of Information and Communication Engineering
Harbin Engineering University, Harbin, 150001, China
*liyingsong@ieee.org

Abstract **─** This paper introduces a constrained normalized adaptive sparse array beamforming algorithm based on approximate L_0-norm and logarithmic cost (L0-CNLMLS). The proposed algorithm can control the sparsity of the array by introducing an approximate function of L_0-norm. In addition, the introduction of logarithmic cost improves the stability of the algorithm as well as the convergence rate of the algorithm. The sparsity of the array can be controlled when adjusting related parameter in the proposed algorithm. Simulation results show the better performance of L0-CNLMLS compared with some conventional algorithms.

Index Terms **─** Approximate L_0-norm, constrained adaptive beamforming, logarithmic cost function, sparse sensor arrays.

I. INTRODUCTION

The sparse antenna signal processing technology has a wide application in modern signal processing. In practical applications, the communication system may be restricted by conditions such as energy, which may require the system to reduce the number of equipment. The sparse array signal processing technology is proposed to address this problem. In recent years, a main method of sparse array technology is to use sparse algorithms which can achieve the same performance and use less actual elements. Aiming at the application of beamforming, a sparse adaptive beamforming algorithm is proposed.

The adaptive digital beamforming technology is widely used due to its good characteristics. The early classic beamforming algorithm is the linearly constrained minimum-variance (LCMV) algorithm proposed in [1], which can form the ideal beam in the case of fixed interference signals and desired signals. Subsequently, people successfully achieve the adaptive realization of LCMV algorithm which called constrained normalized least mean square (CNLMS) algorithm [2]. These adaptive beamforming algorithms can solve the optimization problem for any desired signal and interference direction.

With the introduction of compressed sensing technology in [3] and the corresponding algorithms in [4-5], people start relative study about sparse adaptive algorithms. The zero attracting LMS algorithm generates a zero attractor according to a combination of L_1-norm penalty and the quadratic LMS cost function in the iteration. A series of related algorithms [6-7] provide an idea to study sparse adaptive algorithms.

Based on the ability of these previously mentioned algorithms to induce sparsity, we can expand these methods' application by introducing linear constraints. Inspired by the sparse adaptive algorithm [8-10], for beamforming applications, the L_1-norm CNLMS algorithm proposed in [11] and its weighted version successfully introduce L_1-norm penalty into beamforming algorithm. However, it has a disadvantage that the sparsity cannot be adjusted. In addition, many beamforming algorithms based on the adaptive algorithm LMS have been proposed and applied to various aspects of signal processing for specific conditions [12-13]. In this paper, we propose an adaptive sparse array beamforming algorithm based on approximate L_0-norm and logarithmic cost (L0-CNLMLS). The proposed algorithm can control the sparsity of array as well as improve the stability.

In compressed sensing theory, the sparse signal reconstruction ability of L_0-norm is much better than L_1-norm. However, due to the non-convex nature of L_0-norm and the optimization problem of L_0-norm is NP-hard problem, most algorithms use L_1-norm to solve the problem of sparse signal reconstruction. In recent years, more and more approximate methods for the L_0-norm have been proposed and widely used. In the algorithm proposed in this paper, an approximate function of L_0-norm is used for calculation and derivation. Using the steepest descent iteration method, the update expression is successfully obtained. Besides this, a novel of new convergence factor is used and makes the algorithm get a better convergence rate.

In addition, we introduce a kind of logarithmic cost function [14] based on the original algorithm. When the algorithm uses a logarithmic cost function, its convergence rate can be better than that of the classical algorithm like LMS. The logarithmic cost function

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makes the algorithm compromise between convergence speed and steady-state performance. In the case of ensuring a certain convergence speed, the stability of the algorithm is also improved. Furthermore, the introduction of a logarithmic cost function does not add too much computational complexity.

II. THE PROPOSED L0-CNLMLS ALGORITHM

In this section, the derivation steps of L0-CNLMLS is shown in detail. Since the logarithmic cost shows good characteristics for the disturbance on the error, we add the logarithmic cost function into the mean square error. Then the linearly constrained minimization problem can be expressed as follows:

$$\min_{\mathbf{w}} E\left[|e_k|^2 - \frac{1}{\alpha}\log\left(1+\alpha e_k^2\right) \right] \text{s.t.} \begin{cases} \mathbf{C}^{\mathrm{H}}\mathbf{w}_k = z \\ \|\mathbf{w}_k\|_0 = p \end{cases}, \quad (1)$$

where e_k and $\mathbf{w}_k$ is iteration error and the vector of coefficients in the algorithm. $\mathbf{C}$ is an $N \times L$ constraint matrix which contains the array orientation information and z is the corresponding constraint vector containing L (number of constraints) elements. p is the parameter of L_0-norm to adjust the sparsity of the algorithm.

Considering that L_0-norm minimization is a Non-Polynomial (NP) hard problem, an approximate function is carried out to simplify the complexity of computation. A popular approximation is the Geman-McClure function, the expression can be written as:

$$F_\beta(x) = 1 - \frac{1}{1+\beta|x|}. \quad (2)$$

The derivative form of the Geman-McClure function which will be used in the following derivation is given by:

$$f_\beta(x) = \frac{\beta\,\mathrm{sign}(x)}{(1+\beta|x|)^2}, \quad (3)$$

where sign(x) denotes the basic signum function. By employing the Lagrange multiplier λ, the constraints can be included into the objective function. And then we can get the cost function with L_0-norm penalty as follows:

$$J(\mathbf{w}) = E\left[|e_k|^2 - \frac{1}{\alpha}\log\left(1+\alpha e_k^2\right) \right] - \lambda_1^{\mathrm{H}}\left(z - \mathbf{C}^{\mathrm{H}}\mathbf{w}\right) - \lambda_2\left(p - \|\mathbf{w}_k\|_0\right). \quad (4)$$

According to the steepest descent method, the solution for $J(\mathbf{w})$ can be given as:

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \frac{\mu_0}{2}\hat{\nabla}_{\mathrm{w}} J(\mathbf{w}). \quad (5)$$

Symbol $\hat{\nabla}$ denotes the gradient operator and μ_0 is a fixed value which controls the misadjustment. Differentiate and calculate the cost function which contains L_0-norm penalty, the gradient vector can be obtained as (6):

$$\hat{\nabla}_{\mathrm{w}} J(\mathbf{w}) = -2g(e_k^*)\mathbf{x}_k + \mathbf{C}\lambda_1 + \lambda_2 f_\beta(\mathbf{w}), \quad (6)$$

where function $g(e_k^*) = \dfrac{\alpha(e_k^*)^3}{1+\alpha(e_k^*)^2}$. Using the prior knowledge $\mathbf{C}^{\mathrm{H}}\mathbf{w}_k = z$, we can get the solution of Lagrangian multiplier λ_1 giving by:

$$\lambda_1 = \left(\mathbf{C}^{\mathrm{H}}\mathbf{C}\right)^{-1}\mathbf{C}^{\mathrm{H}}\left(2g(e_k^*)\mathbf{x}_k - \lambda_2 f_\beta(\mathbf{w})\right). \quad (7)$$

The value $\mathbf{x}_k$ denotes the input signal vector in the k^{th} iteration. By defining the L_0-norm error function as $e_{L_0}(k) = p - f_\beta^{\mathrm{H}}\left(\mathbf{w}_k\right)\mathbf{w}_k$ and rearranging the terms of formula, the solution of λ_2 can be obtained as follows:

$$\lambda_2 = \frac{1}{N}[-\frac{2}{\mu}e_{L_0}(k) + f_\beta^{\mathrm{H}}\left(\mathbf{w}_k\right)(2g(e_k^*)\mathbf{x}_k - \mathbf{C}\lambda_1)], \quad (8)$$

where $N = f_\beta^{\mathrm{H}}(x) f_\beta(x)$. After organizing the formulas, the solutions for λ_2 and λ_1 can be obtained. The final update equation of L0-CNLMLS algorithm can be expressed as follows:

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \mu_k \frac{\alpha(e_k^*)^3}{1+\alpha(e_k^*)^2}\mathbf{Q} + \mathbf{f}_{L_0}(k), \quad (9)$$

where

$$\begin{cases} \mathbf{P} = \mathbf{I}_{N\times N} - \mathbf{C}\left(\mathbf{C}^{\mathrm{H}}\mathbf{C}\right)^{-1}\mathbf{C}^{\mathrm{H}}, \\ a = f_\beta^{\mathrm{H}}\left(\mathbf{w}_k\right)\mathbf{P}\mathbf{x}_k, \\ b = f_\beta^{\mathrm{H}}\left(\mathbf{w}_k\right)\mathbf{P} f_\beta\left(\mathbf{w}_k\right), \\ \mathbf{Q} = \mathbf{P}\left(\mathbf{x}_k - \dfrac{a f_\beta\left(\mathbf{w}_k\right)}{b}\right), \\ \mathbf{f}_{L_0}(k) = \left(p - f_\beta^{\mathrm{H}}\left(\mathbf{w}_k\right)\mathbf{w}_k\right)\left(\dfrac{\mathbf{P} f_\beta\left(\mathbf{w}_k\right)}{b}\right), \\ \mu_k = \dfrac{\mu_0\left(e_k - \mathbf{f}_{L_0}^{\mathrm{H}}(k)\mathbf{x}_k\right)}{e_k\mathbf{Q}^{\mathrm{H}}\mathbf{x}_k + \gamma}. \end{cases} \quad (10)$$

The variable μ_k is a new convergence factor carried out to minimize the instantaneous posteriori squared error at instant k [15]. Bases on the above derivation process, we can get:

$$\frac{\partial\left[e_{ap}(k)e_{ap}^*(k)\right]}{\partial\mu_k^*} = 0, \quad (11)$$

where $e_{ap}(k) = e_k\left(1 - \mu_k\mathbf{x}_k^{\mathrm{H}}\mathbf{Q}\right) - \mathbf{f}_{L_0}(k)\mathbf{x}_k$. According to the prior knowledge and solve the above formula (11), we can get the expression of the new convergence factor:

$$\mu_k = \frac{\mu_0\left[e_k - \mathbf{f}_{L_0}^{\mathrm{H}}(k)\mathbf{x}_k\right]}{e_k\mathbf{Q}^{\mathrm{H}}\mathbf{x}_k + \gamma}. \quad (12)$$

The parameter γ is a small positive constant to ensure the correctness of the calculation. By replacing μ_0 with μ_k, the above final iterative formula can be obtained.

III. SIMULATION RESULTS

In this section, several simulations are carried out to show the better performance of the proposed L0-CNLMLS algorithm. Simulation results and comparisons of L0-CNLMLS with similar algorithms (shrinkage L1-norm constrained LMS algorithm (SL1-CLMS) [16], L0-norm feature LMS (L0-F-LMS) [17] and L0-CNLMS) are illustrated to demonstrate the improvement of the proposed algorithm. Interferers and the signal of interest (SOI) used in the experiments are narrowband QPSK signals. The experiments are conducted under two different sparsities.

In the first simulation the parameter of the approximate L_0-norm function is set to be 15 and the parameter α is set to be 2. The initial adaptation step size of L0-CNLMLS and L0-CNLMS is fixed at 0.004 and 0.003, respectively. For L0-F-LMS, the adaptation step size is set to be 5×10^{-9}. The sparse parameter p is 0.3. The iteration number is 6000 in this simulation.

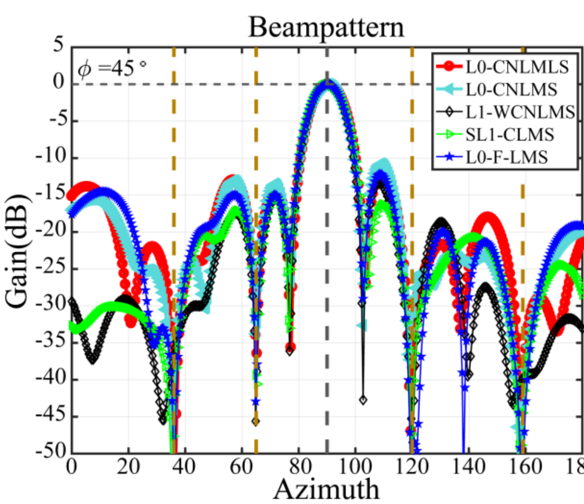

Fig. 1. Beampatterns for L0-CNLMLS algorithm and similar algorithms.

Figure 1 shows the beampatterns for L0-CNLMLS algorithm and some other algorithms. Results show that the proposed algorithm and the other algorithms all can form main lobe and nulls. In the terms of side lobe, the average height of the five algorithms is almost the same.

Figure 2 shows the thinned triangular array and the change of coefficient vector for L_0-norm to L0-CNLMLS algorithm and the other two algorithms which use L_0-norm. The array used in the simulations is a triangular array considered as the senor for P-band signals which has particularly advantage in satellite detection. The triangular array contains 117 array elements. The white points represent the array elements which are closed while the black points denote normal working antenna arrays. From the L_0-norm in Fig. 2, the L0-CNLMLS algorithm converges after 5000 iterations. It is found that the L0-CNLMLS algorithm converges faster than the L0-CNLMS algorithm and L0-F-LMS algorithm.

According to the numbers of the points, the sparsity of the array can be calculated as 30.7% which is very close to the preset sparse parameter value of 30%. The L0-CNLMS algorithm and L0-F-LMS algorithm achieve a sparsity of 31.6%. It is obvious that the L0-CNLMLS algorithm can control the sparsity much better than the other algorithms under the same conditions.

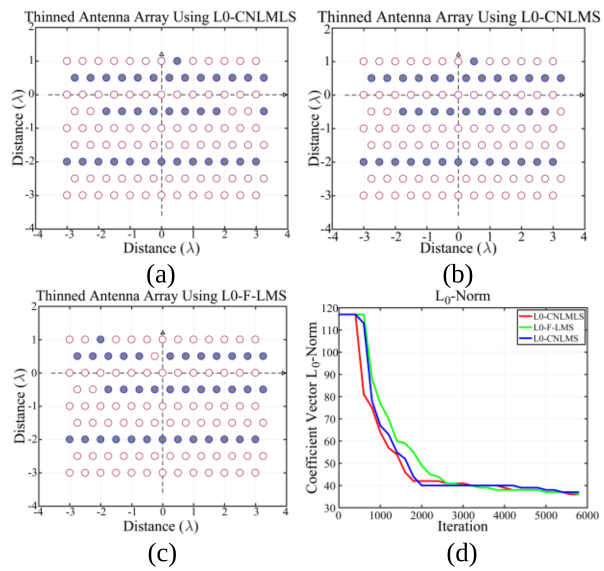

Fig. 2. Thinned triangular array for the L0-CNLMLS and other two algorithms: (a) L0-CNLMLS; (b) L0-CNLMS; (c) L0-F-LMS; (d) convergence of the used algorithms.

In experiment 2, the sparse parameter p set to be 0.7 with the other parameters unchanged. Figure 3 and Fig. 4 show the respective beampatterns and thinned triangular array of the proposed algorithm and other algorithms.

According to the thinned antenna array in Fig. 4, the sparsity of the array in this simulation can be obtained as 70.1%. For large sparse ratio, the L0-CNLMLS and L0-CNLMS are almost the same in the ability to control the sparsity of the array while the L0-F-LMS shows a worse performance by calculation. Simulation results prove that the sparsity of the array can be controlled by the adjustment of corresponding parameter.

Figure 4 (d) shows the L_0-norm of the coefficient vector. The better convergence ability of proposed algorithm can be seen. The L0-CNLMLS algorithm converges after about 2300 iterations, the L0-CNLMS and L0-F-LMS converged slight slowly. Table 1 shows the comparison of the two experiments. The proposed algorithm achieves sparsities of 30.7%, 70.1% which equal to the prescribed parameter p of 0.3, 0.7.

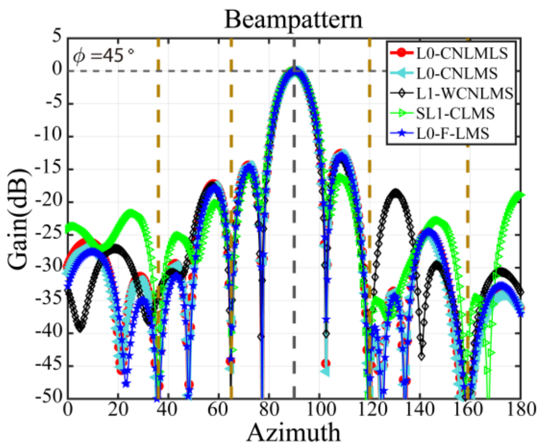

Fig. 3. Beampatterns for L0-CNLMLS algorithm and the other algorithms under large sparsity ratio.

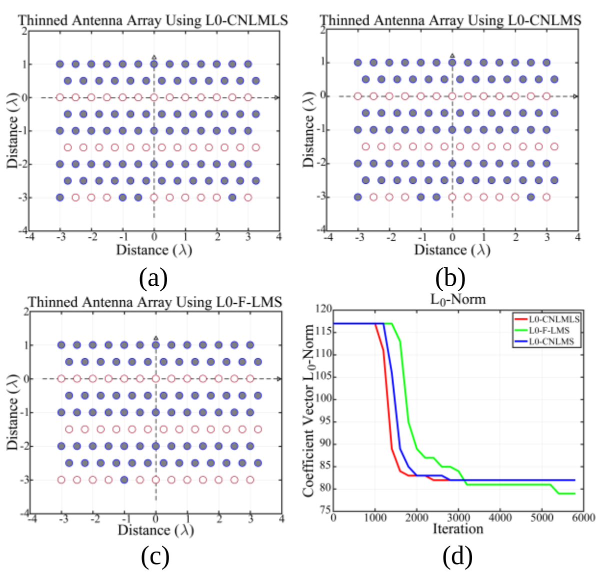

Fig. 4. Thinned triangular array in experiment 2: (a) using L0-CNLMLS; (b) L0-CNLMS; (c) L0-F-LMS; (d) convergence of the used algorithms with p=0.7.

Table 1: Comparison with different sparsity ratio

Experiment	Parameter	Result
I	0.3	0.307
II	0.7	0.701

In addition, we found that the logarithmic cost can improve the stability of proposed algorithm. Figure 5 is two different results in simulation I. The L0-CNLMS achieves different sparsities of 31.6% and 32.4% under the same parameters, while the L0-CNLMLS shows better stability in controlling the sparsity than L0-F-LMS algorithm. As for SL1-CLMS algorithm, the algorithm uses too much different array elements in different sparsities. Compared with algorithm using L_1-norm, the L0-CNLMLS algorithm achieves the ability to precisely control the sparsity of the array. The logarithmic cost ensures a good agreement between the sparse parameter and simulation results, which shows the better performance of proposed algorithm.

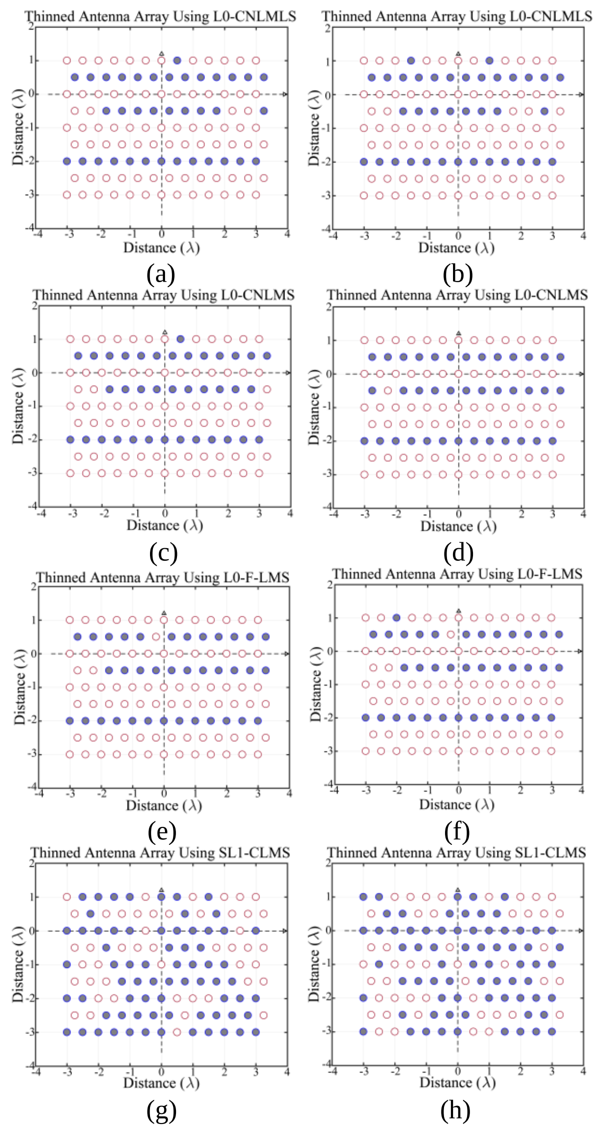

Fig. 5. Two different results in simulation I under same conditions. (a), (b) The antenna array thinned by L0-CNLMLS with sparsity of 30.7%. (c), (d) The antenna array thinned by L0-CNLMS with sparsity of 31.6%, 32.4%, respectively. (e), (f) The antenna array thinned by L0-F-LMS with sparsity of 30.7%, 31.6%, respectively. (g), (h) Different results of antenna array using SL1-CLMS.

IV. CONVERGENCE ANALYSIS

In this section, the convergence analysis of L0-CNLMLS is carried out. Defining the priori error in the k^{th} iteration as:

$$e_k = \mathbf{x}_k^{\mathrm{H}}\mathbf{w}_o + \mathbf{n}_k - \mathbf{x}_k^{\mathrm{H}}\mathbf{w}_k = \mathbf{n}_k - \mathbf{x}_k^{\mathrm{H}}\Delta\mathbf{w}_k, \qquad (13)$$

where $\Delta\mathbf{w}_k = \mathbf{w}_k - \mathbf{w}_o$, $\mathbf{w}_o$ is considered as the optimal coefficient vector and $\mathbf{n}_k$ denotes the noise in the k^{th} iteration. Defining $\varepsilon_k = \mathbf{Q}^{\mathrm{H}}\mathbf{x}_k$, the final iteration equation of proposed algorithm can be written as:

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \frac{\mu_0}{\varepsilon_k}\left[g(e_k) - \mathbf{f}_{L_0}^{\mathrm{H}}(k)\mathbf{x}_k\right]\mathbf{Q} + \mathbf{f}_{L_0}(k). \qquad (14)$$

Combining the element $\mathbf{f}_{L_0}(k)$ and inserting e_k into (14), the equation can be expressed as:

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \left[\mathbf{I} - \frac{\mu_0}{\varepsilon_k}\mathbf{Q}\mathbf{x}_k^{\mathrm{H}}\right]\mathbf{f}_{L_0}(k) + \frac{\mu_0}{\varepsilon_k}g(e_k^*)\mathbf{Q}. \qquad (15)$$

When the algorithm is converged, the constraint conditions $f_\beta^{\mathrm{H}}(\mathbf{w}_k)\mathbf{w}_{k+1} = p$ should be satisfied. Defining $f_\beta(\mathbf{w}_k) = \boldsymbol{f}_k$. According to equation (10), we can get:

$$\mathbf{f}_{L_0}(k) = \left(\boldsymbol{f}_k^{\mathrm{H}}\mathbf{w}_o - \boldsymbol{f}_k^{\mathrm{H}}\mathbf{w}_k\right)\left(\frac{\mathbf{P}\boldsymbol{f}_k}{b}\right) = -\mathbf{A}\Delta\mathbf{w}_k, \qquad (16)$$

where $\mathbf{A} = \dfrac{\mathbf{P}\boldsymbol{f}_k\boldsymbol{f}_k^{\mathrm{H}}}{b}$ is an idempotent matrix. The conclusion $\mathrm{tr}[\mathbf{A}] = 1$ can be easily obtained shows that there is only one non-zero eigenvalue among all the eigenvalues of matrix $\mathbf{A}$. Therefore, the coefficient error form of (15) is:

$$\begin{aligned}\Delta\mathbf{w}_{k+1} &= \Delta\mathbf{w}_k + [\mathbf{I} - \frac{\mu_0}{\varepsilon_k}\mathbf{Q}\mathbf{x}_k^{\mathrm{H}}]\left(-\mathbf{A}\Delta\mathbf{w}_k\right) \\ &\quad + \frac{\mu_0}{\varepsilon_k}g(e_k^*)\mathbf{Q} \\ &= \left[\mathbf{I} - \mu_0\mathbf{B}\right]\Delta\mathbf{w}_k - [\mathbf{I} - \mu_0 g'(e_k^*)\mathbf{B}]\mathbf{A}\Delta\mathbf{w}_k \\ &\quad + \frac{\mu_0}{\varepsilon_k}g'(e_k^*)n_k^*\mathbf{Q}\end{aligned}, \qquad (17)$$

where $\mathbf{B} = \dfrac{\mathbf{Q}\mathbf{x}_k^{\mathrm{H}}}{\varepsilon_k}$ is also an idempotent matrix with a non-zero eigenvalue of 1. From simulation results $g'(e_k^*) = \dfrac{\alpha(e_k^*)^2}{1+\alpha(e_k^*)^2}$ can be obtained as a constant very close to 1. Then, take expectation on both sides of (17), we will have:

$$E\left[\Delta\mathbf{w}_{k+1}\right] = E\left\{[\mathbf{I} - \mu_0\mathbf{B}][\mathbf{I} - \mathbf{A}]\Delta\mathbf{w}_k\right\} + E\left[\frac{\mu_0}{\varepsilon_k}n_k^*\mathbf{Q}\right]. \qquad (18)$$

According to [15], $\Delta\mathbf{w}_k$ is statistic independence with n_k, $\mathbf{x}_k$ and $\boldsymbol{f}_k$. Under the truth of the expectation of n_k is 0, (18) can be written as:

$$\begin{aligned}&E[\Delta\mathbf{w}_{k+1}] = [\mathbf{I} - \mu_0\mathbf{B}][\mathbf{I} - \mathbf{A}]E[\Delta\mathbf{w}_k] = \\ &[\mathbf{I} - \mu_0\mathbf{B}][\mathbf{I} - \mathbf{A} - \mu_0\mathbf{B} - \mu_0\mathbf{AB}]^k[\mathbf{I} - \mathbf{A}]E[\Delta\mathbf{w}_0].\end{aligned} \qquad (19)$$

In the case of $\mathbf{AB} = 0$, we can get the final convergence domain as:

$$0 < \mu_0 < 1. \qquad (20)$$

Actually, the convergence domain is a little larger according to [14] because the introduction of logarithmic cost can influence convergence performance. The selection of step-size for L0-CNLMLS is always far below the upper bound to gain better performance in actual application.

V. CONCLUSION

In this paper, an adaptive sparse array beamforming algorithm based on approximate L_0-norm and logarithmic cost (L0-CNLMLS) is proposed and analyzed. The L0-CNLMLS algorithm uses GMF function to be the approximate function of the L_0-norm penalty and avoid the NP-hard problem. The introduction of L_0-norm allows the algorithm to control the sparseness of the array. The use of logarithmic cost function improves stability while ensuring a certain convergence speed of the algorithm.

Simulation results show that the proposed algorithm exhibits better performance and convergence speed compared with some sparse beamforming algorithm in recent years under different sparsities. In addition, the L0-CNLMLS algorithm can control the sparsity of antenna array more precisely for small sparse ratio, so as to improve the stability performance. In the future, the algorithms in [18-21] will be considered to construct new sparsity beamforming algorithms.

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