

Robust Adaptive Beamforming based on Automatic Variable Loading in Array Antenna

Bin Yang¹, Wenxing Li¹, Yuanyuan Li², and Yunlong Mao³

¹ College of Information and Communication Engineering
Harbin Engineering University, Harbin, Heilongjiang 150001, China
yb_alonline@126.com, liwenxing@hrbeu.edu.cn

² The 54th Research Institute of China Electronics Technology Group Corporation
Shijiazhuang, Hebei 050081 China
lyy008@hotmail.com

³ School of Electronics and Information
Jiangsu University of Science and Technology, Zhenjiang, Jiangsu 212003, China
maoyunlong0511@just.edu.cn

Abstract — Diagonal loading technology is widely used in array antenna beamforming because of its simple method, low computational complexity and the ability to improve the robustness of beamformer. On this basis, this paper proposes a robust adaptive beamforming method based on automatic variable loading technology. The automatic variable loading matrix (AVLM) of the method is composed of two parts. The non-uniform loading matrix dominants when the input signal-to-noise ratio (SNR) is small, effectively control the influence of noise disturbance without affecting the ability of array antenna to suppress interference. The variable diagonal loading matrix dominants when the input SNR is high to improve the output performance of array antenna. Simulated results show that compared to other methods, the proposed method has better output performance for both low and high input SNR cases.

Index Terms — Array antenna, loading, robust adaptive beamforming.

I. INTRODUCTION

Adaptive beamforming technology is an important topic in array signal processing. It is widely used in radar, sonar, array antenna, wireless communication, medical imaging and other fields [1-3]. In numerous adaptive beamforming methods, Capon beamforming method is used extensively for its effective suppression of interference and noise and good output performance in ideal environment [4]. However, in practical application, various error environments have led to the serious degradation in output performance of traditional Capon beamforming method. How to improve the robustness of beamformer has become a hot topic in

recent years.

To increase robustness of array antenna, many beamformers based on interference plus noise covariance matrix (INCM) reconstruction are proposed [5-7]. This kind of method has good performance, but these methods are computationally complex and rely heavily on array manifold information. To make the method beamformer simple and easy to implement, we mainly study the beamformer based on loading technology in this paper. Carlson proposed a diagonal loading method (LSMI) [8], which is robust towards the mismatch of the steering vector (SV) of desired signal and the influence of the low number of snapshots, and it is easy to implement without increasing computation. The output performance of conventional LSMI method varies with the selection of diagonal loading factor, but there is no certain method to determine the value of optimal loading factor. For diagonal loading technology, the selection of optimal loading factor is still an unsolved problem, which worth further research and discussions [9]. A beamforming method based on worst case performance optimization was proposed in [10]. In this method, the upper limit of error is set between imaginary SV and the SV. By constraining the response of the beamformer when the error of the desired signal steering vector reaches the upper limit, the worst-case performance can be optimized.

Compared to these fixed diagonal loading beamforming methods, more variable loading beamforming methods have been widely studied [11-13]. Zhuang proposed a variable loading method, which can improve the robustness of the array antenna by preventing the weight vector from converging to the noise subspace and setting the loading factor in a special way [11]. Li proposed a diagonal loading method which

makes loading factor change with the input SNR, and corrects the steering vector of the desired signal, so as to improve the robustness of the array antenna [12]. In reference [13], according to the interval of diagonal loading value, an adaptive diagonal loading technology with diagonal loading factor varying with input signal power is proposed to further improve the robustness of array antenna. However, above methods have limited robustness improvement for beamformer, especially when the input SNR is large. The output performance of Capon beamformer decreases sharply, because of the cancellation of desired signals when array antenna receives data with small snapshots.

In view of the above problems, this paper proposes a novel robust adaptive beamforming method based on automatic variable loading technology (AVL-RAB), which constructs the non-uniform loading matrix and variable diagonal loading matrix without increasing the complexity of calculation firstly. In order to better integrate the above matrices, we construct a mixed factor to make non-uniform loading matrix play a leading role when SNR is low. In this way, the beamformer can restrain the influence of noise disturbance, and try to keep the ability to suppress interference. When the SNR is high, variable diagonal loading matrix play an important role. Although the ability of array antenna to suppress interference is reduced, the desired signal cancellation is avoided to ensure the output performance of the array antenna. To further improve the robustness of the array antenna, the method corrects SV of the desired signal in a way similar to reference [14]. Numerical results demonstrate the superior performance of the proposed beamformer relative to other existing beamformers.

II. SIGNAL MODEL AND DIAGONAL LOADING

A. Signal model of array antenna

Consider a uniform linear array (ULA), which is composed of N omnidirectional antennas spaced by half a wavelength, receiving uncorrelated far-field narrowband signals. The sample data of array antenna at the k th snapshot is modeled as:

$$\mathbf{X}(k) = \mathbf{A}\mathbf{S}(k) + \mathbf{N}(k), \quad (1)$$

Where $\mathbf{X}(k) = [x_1(k) \ x_2(k) \ \cdots \ x_N(k)]^T$ is a $N \times 1$ data vector, $(\bullet)^T$ indicates transpose of the matrix, $\mathbf{S}(k) = [s_0(k) \ s_1(k) \ \cdots \ s_M(k)]^T$ is a mixed signal vector containing M narrow band interference and a desired signal. $\mathbf{N}(k)$ is assumed to be the additive spatially Gaussian white noise with zero mean and variance σ_n^2 . $\mathbf{A} = [\mathbf{a}(\theta_0) \ \mathbf{a}(\theta_1) \ \cdots \ \mathbf{a}(\theta_M)]$ is array manifold matrix, $\mathbf{a}(\theta_p) = [1 \ e^{j\beta_p} \ \cdots \ e^{j(N-1)\beta_p}]^T$ denotes the SV of θ_p . $\beta_p = 2\pi d \sin(\theta_p)/\lambda$ is the phase difference.

The output of this array antenna is given as:

$$y(k) = \mathbf{w}^H \mathbf{x}(k), \quad (2)$$

where $\mathbf{w} = [w_1, w_2, \dots, w_N]^T$ is the weight vector of the array antenna, $(\bullet)^H$ is the conjugate transpose of matrix. The minimum variance distortionless response (MVDR) beamformer is obtained by minimizing the variance of the interference and noise at the output while constraining the target response to be unity, hence can be formulated as:

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R}_{i+n} \mathbf{w} \quad \text{s.t.} \quad \mathbf{w}^H \mathbf{a}(\theta_0) = 1, \quad (3)$$

where $\mathbf{a}(\theta_0)$ is the desired signal steering vector, \mathbf{R}_{i+n} is the interference plus noise covariance matrix (INCM) matrix. In practice, \mathbf{R}_{i+n} is unavailable, so replace it with the following data sample covariance matrix (SCM):

$$\hat{\mathbf{R}} = \frac{1}{K} \sum_{k=1}^K \mathbf{x}(k) \mathbf{x}^H(k), \quad (4)$$

where K is the number of snapshots. Therefore, by solving the above problems, the weighted vector of the beamformer can be obtained as:

$$\mathbf{w}_{\text{opt}} = \frac{\hat{\mathbf{R}}^{-1} \mathbf{a}(\theta_0)}{\mathbf{a}^H(\theta_0) \hat{\mathbf{R}}^{-1} \mathbf{a}(\theta_0)}. \quad (5)$$

In the ideal cases, Capon beamformer has good output performance. However, since the desired signal is contained in the training data, the standard Capon beamforming method is more sensitive to the steering vector error of the desired signal. When beamformer suffers from large input SNR, small snapshots and steering vector mismatches, the performance of beamformer decreases sharply.

B. Diagonal loading method

Diagonal loading method can solve the problem of noise disturbance effect well. The principle is as follows:

$$\min_{\mathbf{w}} \mathbf{w}^H (\hat{\mathbf{R}} + \lambda \mathbf{I}) \mathbf{w} \quad \text{s.t.} \quad \mathbf{w}^H \mathbf{a}(\theta_0) = 1, \quad (6)$$

where λ is the diagonal loading factor. \mathbf{I} is the identity matrix. According to the formula (5), it can be concluded that:

$$\mathbf{w}_{DL} = \frac{(\hat{\mathbf{R}} + \lambda \mathbf{I})^{-1} \mathbf{a}(\theta_0)}{\mathbf{a}^H(\theta_0) (\hat{\mathbf{R}} + \lambda \mathbf{I})^{-1} \mathbf{a}(\theta_0)}. \quad (7)$$

The beampattern can be expressed as:

$$\begin{aligned} \mathbf{G}(\mathbf{w}_{DL}, \theta) &= \mathbf{w}_{DL}^H \mathbf{a}(\theta) \\ &= \frac{\mu}{\lambda_N^2} \left[\mathbf{a}^H(\theta_0) \mathbf{a}(\theta) - \sum_{i=1}^{N-1} \frac{\lambda_i - \lambda_N}{\lambda_i + \lambda} \mathbf{a}^H(\theta_0) \mathbf{v}_i \mathbf{v}_i^H \mathbf{a}(\theta) \right] \\ &= \mathbf{G}(\mathbf{a}(\theta_0), \theta) - \sum_{i=1}^M \frac{\lambda_i - \lambda_N}{\lambda_i + \lambda} \mathbf{a}^H(\theta_0) \mathbf{v}_i \mathbf{G}(\mathbf{v}_i, \theta) \\ &\quad - \sum_{i=M+1}^{N-1} \frac{\lambda_i - \lambda_N}{\lambda_i + \lambda} \mathbf{a}^H(\theta_0) \mathbf{v}_i \mathbf{G}(\mathbf{v}_i, \theta), \end{aligned} \quad (8)$$

where $\mu = 1/\mathbf{a}^H(\theta_0)(\hat{\mathbf{R}} + \lambda\mathbf{I})^{-1}\mathbf{a}(\theta_0)$. λ_i and \mathbf{v}_i are eigenvalues and eigenvectors of $\hat{\mathbf{R}}$, respectively, $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_N$. The three terms on the right side of formula (8) are respectively the static array response, the weighted sum of interference beam response and the weighted sum of noise beam response. It can be seen that for the diagonal loading technology, the increase of the loading factor is conducive to reducing the impact of noise on the beam quality, while the large loading factor will reduce the interference suppression ability of the array antenna.

III. THE PROPOSED METHOD

In this section, a robust adaptive beamforming method based on automatic variable loading in array antenna is proposed.

A. The construction of non-uniform loading matrix

In this section, the SCM is preprocessed by forward and backward spatial smoothing technique. Define a transformation matrix \mathbf{J} :

$$\mathbf{J} = \begin{bmatrix} 0 & \dots & 0 & 1 \\ 0 & \dots & 1 & 0 \\ \vdots & \ddots & \vdots & 0 \\ 1 & \dots & 0 & 0 \end{bmatrix}, \quad (9)$$

A new covariance matrix $\bar{\mathbf{R}}$ is obtained by using forward and backward spatial smoothing technique. This technique can be considered as regularizing the unstructured SCM into a more structured one, which leads to a higher convergence rate. The proof, which uses this reconstructed SCM to improve the performance of the beamformer, is shown in [15]. $\bar{\mathbf{R}}$ can be expressed as:

$$\bar{\mathbf{R}} = \frac{\hat{\mathbf{R}} + \hat{\mathbf{J}}\hat{\mathbf{R}}^*\hat{\mathbf{J}}}{2}, \quad (10)$$

where $(\bullet)^*$ is conjugate operation of matrix.

According to formula (3), a constraint is imposed on Capon beamformer to make the weighting vector approximately orthogonal to the noise space. It can be expressed as:

$$\min_{\mathbf{w}} \mathbf{w}^H \bar{\mathbf{R}} \mathbf{w} \text{ subject to } \mathbf{w}^H \tilde{\mathbf{a}}(\theta_0) = 1, \mathbf{w}^H \bar{\mathbf{R}}^{-m} \mathbf{w} \leq T, \quad (11)$$

where T is a minimum value, $\tilde{\mathbf{a}}(\theta_0)$ is assumed desired signal SV. The above formula can be resolved into:

$$\min_{\mathbf{w}} \mathbf{w}^H (\bar{\mathbf{R}} + \gamma \bar{\mathbf{R}}^{-m}) \mathbf{w} \text{ subject to } \mathbf{w}^H \tilde{\mathbf{a}}(\theta_0) = 1. \quad (12)$$

The non-uniform loading matrix is as follows:

$$\lambda_{\mathbf{u}} = \gamma \bar{\mathbf{R}}^{-m}. \quad (13)$$

Through the non-uniform diagonal loading matrix, the weighted vector can be avoided to converge to the noise space, which is conducive to the array antenna to

suppress small eigenvalue disturbance and ensure the interference suppression ability of the array antenna.

B. The construction of variable diagonal loading factor

In this section, the variable diagonal loading factor (VDLF) is constructed. Process the $\bar{\mathbf{R}}$ in formula (10) via eigen-decomposition:

$$\begin{aligned} \bar{\mathbf{R}} &= \bar{\mathbf{U}} \bar{\mathbf{\Lambda}} \bar{\mathbf{U}}^H \\ &= \sum_{i=1}^N \bar{\lambda}_i \bar{\mathbf{v}}_i \bar{\mathbf{v}}_i^H, \end{aligned} \quad (14)$$

where $\bar{\mathbf{U}} = [\bar{\mathbf{v}}_1, \bar{\mathbf{v}}_2, \dots, \bar{\mathbf{v}}_N]$. $\bar{\mathbf{\Lambda}} = \text{diag}(\bar{\lambda}_1, \bar{\lambda}_2, \dots, \bar{\lambda}_N)$ is a diagonal matrix, $\bar{\lambda}_1 \geq \bar{\lambda}_2 \geq \dots \geq \bar{\lambda}_N$. The desired signal SV falls in the signal subspace formed by large eigenvalues. Because the difference between the assumed desired signal SV $\tilde{\mathbf{a}}(\theta_0)$ and the real value is little. Project $\tilde{\mathbf{a}}(\theta_0)$ to each eigenvector to get the following result:

$$p(i) = |\bar{\mathbf{v}}_i^H \tilde{\mathbf{a}}(\theta_0)|^2, \quad i = 1, 2, \dots, N, \quad (15)$$

when $p(i)$ is the maximum, the eigenvalue of the corresponding eigenvector is $\hat{\lambda}_{1N1}$. It is the eigenvalue of the desired signal. The small eigenvalues are added and averaged to estimate the noise power:

$$\sigma_n^2 = \frac{\sum_{i=M+2}^N \bar{\lambda}_i}{N - M - 1}. \quad (16)$$

The VDLF can be set as:

$$\lambda_v = \frac{\hat{\lambda}_{1N1} - \sigma_n^2}{N \sigma_n^2}. \quad (17)$$

C. Construction of automatic variable loading matrix

In this section, we construct the mixed factor, which can change the parameters according to the change of the input SNR and has obvious allocation ability whether SNR is large or small. The mixed factor can be constructed as follows:

$$\alpha = \frac{\hat{\lambda}_{1N1}}{\hat{\lambda}_{1N1} + N \sigma_n^2}, \quad (18)$$

$$\kappa = 1 - \alpha = \frac{N \sigma_n^2}{\hat{\lambda}_{1N1} + N \sigma_n^2}, \quad (19)$$

where α and κ are the mixed factors constructed in this paper, which can effectively reflect the low input SNR and the high input SNR.

Based on the above research, the automatic variable loading matrix can be constructed as follows:

$$\begin{aligned} \lambda_{\mathbf{m}} &= \alpha \lambda_{\mathbf{u}} + \kappa \lambda_v \mathbf{I} \\ &= \alpha \gamma \bar{\mathbf{R}}^{-m} + \kappa \frac{\hat{\lambda}_{1N1}}{\sigma_n^2} \mathbf{I}. \end{aligned} \quad (20)$$

D. Desired signal SV estimation

This section estimates the SV of the desired signal with the method similar to that in reference [14]. We assume that the Θ is the angular sector in which the desired signal is located. Define the correlation matrix of the SV:

$$\mathbf{G} = \sum_i^N \bar{\mathbf{a}}(\theta_i) \bar{\mathbf{a}}^H(\theta_i) \Theta / N. \quad (21)$$

Process the \mathbf{G} via eigen-decomposition. The eigenvectors corresponding to the first L large eigenvalues are extracted as orthogonal matrices $\mathbf{U} = [\bar{\mathbf{v}}_1, \bar{\mathbf{v}}_2 \cdots \bar{\mathbf{v}}_L]$. So the actual SV of the desired signal can be estimated as:

$$\hat{\mathbf{a}}(\theta_0) = \mathbf{U} \mathbf{y}, \quad (22)$$

where \mathbf{y} is defined as a rotating vector. By maximizing the output power of the desired signal, take \mathbf{y} into the norm constraint. The optimization problem can be expressed as:

$$\min_{\mathbf{y}} \mathbf{y}^H \mathbf{U}^H \bar{\mathbf{R}}^{-1} \mathbf{U} \mathbf{y} \text{ s.t. } \mathbf{y}^H \mathbf{y} = N. \quad (23)$$

The problem (23) can be solved by Lagrange multiplier methodology. We can get:

$$\mathbf{U}^H \bar{\mathbf{R}}^{-1} \mathbf{U} \mathbf{y} = \mu \mathbf{y}. \quad (24)$$

Define \mathbf{y}_U as the eigenvector corresponding to the minimum eigenvalue of matrix $\mathbf{U}^H \bar{\mathbf{R}}^{-1} \mathbf{U}$ and $\mathbf{y}_U^H \mathbf{y}_U = N$. Then, the estimated SV of the desired signal can be obtained by substituting this solution into (22):

$$\hat{\mathbf{a}}(\theta_0) = \frac{\sqrt{N}}{\|\mathbf{y}_U\|} \mathbf{U} \mathbf{y}_U. \quad (25)$$

E. Calculation of weighted vector

The mixed loading matrix of formula (20) and the desired signal steering vector estimated by formula (25) are introduced into formula (12):

$$\min_{\mathbf{w}} \mathbf{w}^H (\bar{\mathbf{R}} + \alpha \gamma \bar{\mathbf{R}}^{-m} + \kappa \lambda_v \mathbf{I}) \mathbf{w} \text{ s.t. } \mathbf{w}^H \hat{\mathbf{a}}(\theta_0) = 1. \quad (26)$$

By solving the above equation, it can be concluded that:

$$\mathbf{w}_{al} = (\bar{\mathbf{R}} + \alpha \gamma \bar{\mathbf{R}}^{-m} + \kappa \lambda_v \mathbf{I})^{-1} \hat{\mathbf{a}}(\theta_0). \quad (27)$$

Substitute formula (14) into the above formula, we can get:

$$\mathbf{w}_{al} = \sum_{i=1}^N \frac{(\bar{\mathbf{v}}_i^H \hat{\mathbf{a}}(\theta_0))}{\tilde{\lambda}_i + (\alpha \gamma / \tilde{\lambda}_i^m) + \kappa \lambda_v} \bar{\mathbf{v}}_i. \quad (28)$$

In order to further suppress the influence of small eigenvalue disturbance, we take the noise power estimation as the threshold of small eigenvalue, which is defined as $\tilde{\lambda}_i = \max(\bar{\lambda}_i, \sigma_n^2)$, $i = 1, 2, \dots, N$. Through the formula (28), we can find that when $m = 0$, the non-uniform loading technology becomes uniform diagonal loading technology. When m becomes larger, the non-

uniform loading corresponding to large eigenvalue and the influence on the interference suppression will be smaller. Larger m will result with smaller non-uniform loading which corresponds to large eigenvalue, hence leading to smaller influences on interference suppression of array antenna. However, if m is overlarge, the beam sidelobe will be enhanced. In this paper, we define $m = 2$, $\gamma = 10\sigma_n^{2+m}$. Thus, the weight vector can be expressed as:

$$\mathbf{w}_{al} = \sum_{i=1}^N \frac{(\bar{\mathbf{v}}_i^H \hat{\mathbf{a}}(\theta_0))}{\tilde{\lambda}_i + (\alpha \gamma / \tilde{\lambda}_i^2) + \kappa \lambda_v} \bar{\mathbf{v}}_i. \quad (29)$$

The main computational complexity of the AVL-RAB is the eigen-decomposition operation. Its overall computational complexity is of $\mathcal{O}(N^3)$. Compared to the methods using optimization algorithms to estimate diagonal loading value, the computational complexity is relatively low. Table 1 shows the computational complexity of several methods.

Table 1: Comparison of computational complexity

Beamformer	Computational Complexity
INCM-RAB [5]	$\mathcal{O}(N^{3.5})$
INCM-NVM [6]	$\mathcal{O}(MN^3)$
LSMI [8] LC-RAB [11] LCHP-RAB [12] ADL-SMI [13] AVDL-RAB	$\mathcal{O}(N^3)$

IV. SIMULATIONS AND COMPARISONS

Consider a ULA with 10 antennas spaced half-wavelength. The desired signal direction is 0° . Two sidelobe interferences impinge on the ULA from -30° and 50° with interference-to-noise ratio (INR) 30dB. The signal and interference are statistically independent, and the added noise is Gaussian white noise. The snapshots of received data is 100. All experimental results are from 100 independent Monte Carlo experiments. The AVL-RAB in this paper is compared to IPNM-NVM [6], LSMI [8] with the loading factor $\gamma = 10\sigma_n^2$, LC-RAB [11], LCHP-RAB [12], ADL-SMI [13].

A. The simulation of VHDL and mixed factor

In this simulation, the snapshots number is 100 and the input SNR changes from -10dB to 30dB uniformly. Fig. 1. shows the VDLF versus the input SNR. From the simulation results, the proposed VDLF can estimate the input SNR of the received signal, and the estimation result is accurate. This factor can make the array antenna avoid the desired signal cancellation when the input SNR is large and snapshots are small, so as to ensure the

output performance of the array antenna. Figure 2 shows the curve of mixed factor changing with input SNR. It can reasonably allocate the proportion of two loading matrices according to the value of SNR.

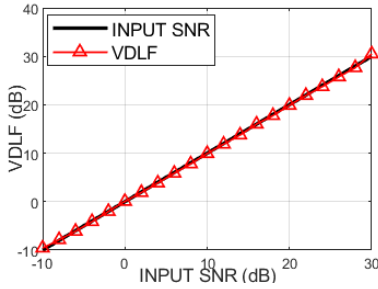


Fig. 1. VDLF versus the input SNR.

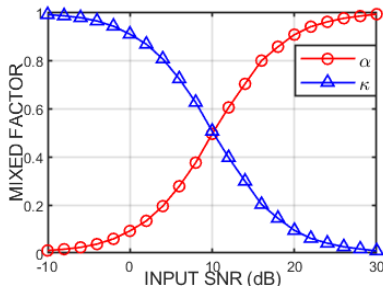


Fig. 2. Mixed factors versus the input SNR.

B. Ideal condition

In this simulation, the performance comparison of the above methods is made. Figure 3 shows the curves of the output signal to interference and noise ratio (SINR) with the input SNR when the number of snapshots is 100. The input SNR changes from -10dB to 30dB uniformly. Figure 4 shows the curves of the output SINR of each method changing with the number of snapshots. The input SNR is 15dB and the range of snapshots number changes from 20 to 100.

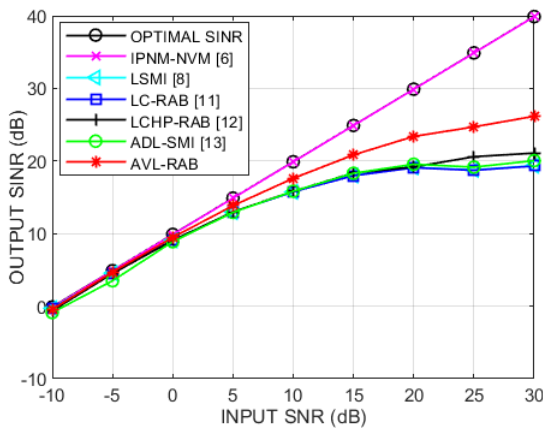


Fig. 3. Output SINR versus the SNR in ideal condition.

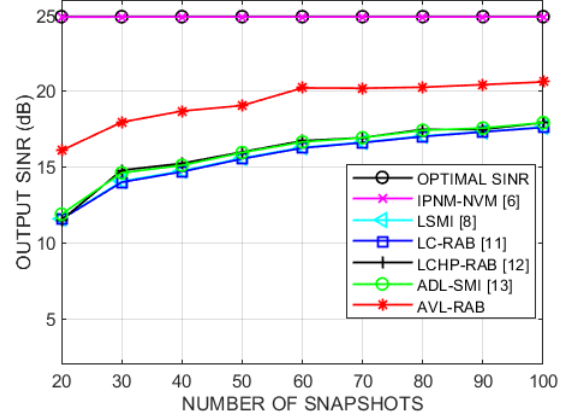


Fig. 4. Output SINR versus the number of snapshots in ideal condition.

From Fig. 3 and Fig. 4, the IPNM-NVM has excellent performance, but it is very complex and needs more prior information. For the methods based on loading technology, the proposed method has higher output SINR and fast convergence speed. Thus, the proposed method outperforms other similar methods in ideal condition.

C. Desired signal steering vector mismatch

In this simulation, the look direction error of the desired signal is randomly distributed in $[-5^\circ, 5^\circ]$. The true steering vector is formed by five signal paths and is given by $\mathbf{a} = \mathbf{a}(\theta_0) + \sum_{i=1}^4 e^{j\phi_i} \mathbf{a}_i(\theta_i)$, where $\mathbf{a}_i(\theta_i)$ corresponds to the coherently scattered paths. θ_i is random value in $[-5^\circ, 5^\circ]$, ϕ_i is the phase of the independent path and randomly distributed in $[0, 2\pi]$. Other simulation conditions remain unchanged, the performance of each method is simulated.

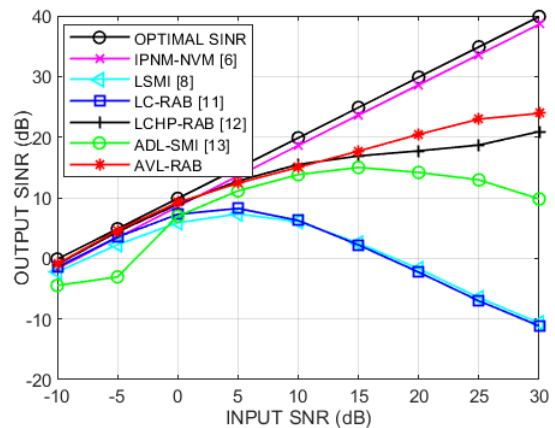


Fig. 5. Output SINR versus the SNR with desired signal SV mismatches.

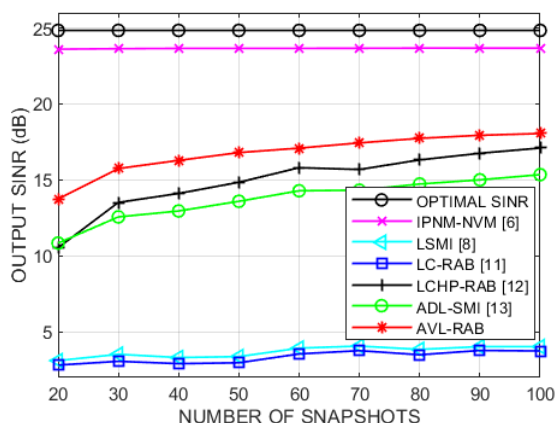


Fig. 6. Output SINR versus the number of snapshots with desired signal SV mismatches.

From above simulation results, the output SINR of all methods decreases significantly in this condition. The beamformer based on INCM reconstruction still perform well. The output performance of the method in this paper is better than other existing similar methods.

V. CONCLUSION

In this paper, a novel robust adaptive beamforming method based on automatic variable loading technology is proposed for array antenna. The method constructs the automatic variable loading matrix by mixing the non-uniform loading matrix and variable diagonal loading matrix so as to ensure that the array antenna has better output performance whether SNR is large or small. The computational complexity of the matrix is relatively low. Simulation results show that the proposed method has better robustness in the error environment of small number of data snapshots and mismatch of SV. The proposed method outperform other existing similar methods obviously.

ACKNOWLEDGMENT

This paper is supported by the Natural Science Foundation of Jiangsu province of China (BK20190956).

REFERENCES

- [1] W. Shi, Y. Li, S. Luo, and Ieee, "Adaptive antenna array beamforming based on norm penalized NLMS algorithm," *IEEE Antennas and Propagation Society International Symposium*, pp. 2207-2208, 2018.
- [2] W. Shi and Y. Li, "A p-norm-like constraint LMS algorithm for sparse adaptive beamforming," *Applied Computational Electromagnetics Society Journal*, vol. 34, no. 12, pp. 1797-1803, Dec. 2019.
- [3] S. Mohammadzadeh and O. Kukrer, "Robust

adaptive beamforming based on covariance matrix and new steering vector estimation," *Signal Image and Video Processing*, 2019.

- [4] J. W. Jia, W. Jin, S. Zhou, and M. Yao, "Robust adaptive beamforming based on a new steering vector estimation algorithm," *Signal Process*, vol. 93, no. 9, pp. 2539-2542, Sep. 2013.
- [5] Y. Gu and A. Leshem, "Robust adaptive beamforming based on interference covariance matrix reconstruction and steering vector estimation," *IEEE Transactions on Signal Processing*, vol. 60, no. 7, pp. 3881-3885, 2012.
- [6] X. Yuan and L. Gan, "Robust adaptive beamforming via a novel subspace method for interference covariance matrix reconstruction," *Signal Processing*, 2017.
- [7] P. Zhang, Z. Yang, G. Jing, and T. Ma, "Adaptive beamforming via desired signal robust removal for interference-plus-noise covariance matrix reconstruction," *Circuits Systems and Signal Processing*, vol. 40, no. 1, pp. 401-417, Jan. 2021.
- [8] B. D. Carlson, "Covariance matrix estimation errors and diagonal loading in adaptive arrays," *IEEE Transactions on Aerospace Electronic Systems*, vol. 24, no. 4, pp. 397-401, 1988.
- [9] Y. L. Chen and J. H. Lee, "Finite data performance analysis of Mvdr antenna array beamformers with diagonal loading," vol. 134, pp. 475-507, 2013.
- [10] S. A. Vorobyov, A. B. Gershman, and Z. Q. Luo, "Robust adaptive beamforming using worst-case performance optimization: A solution to the signal mismatch problem," *IEEE Transactions on Signal Processing*, vol. 51, no. 2, pp. 313-324, 2003.
- [11] J. Zhuang, Q. Ye, Q. Tan, and A. H. Ali, "Low-complexity variable loading for robust adaptive beamforming," *Electronics Letters*, vol. 52, no. 5, pp. 338-340, 2016.
- [12] W. Li, X. Mao, Z. Zhai, and Y. Li, "A low complexity high performance robust adaptive beamforming," *Applied Computational Electromagnetics Society Journal*, vol. 32, no. 5, pp. 441-448, 2017.
- [13] J. Gao, J. Zhen, Y. Lv, and B. Guo, "Beamforming technique based on adaptive diagonal loading in wireless access networks," *Ad Hoc Networks*, vol. 107, p. 102249, 2020.
- [14] W. Zhang, J. Wang, and S. Wu, "Robust Capon beamforming against large DOA mismatch," *Signal Processing*, 2013.
- [15] L. Zhang, W. Liu, and L. Yu, "Performance analysis for finite sample MVDR beamformer with forward backward processing," *IEEE Transactions on Signal Processing*, vol. 59, no. 5, pp. 2427-2431, 2011.