# Single Snapshot 2D-DOA Estimation in Impulsive Noise Environment using Linear Arrays

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***Abstract*** —This paper considers single snapshot two dimensional direction-of-arrival (2D-DOA) estimation in impulsive noise environment employing linear arrays. 2D-DOA estimation is realized in two steps. Firstly, the 2D-DOA estimation problem is decomposed into two independent one dimensional direction-of-arrival (1D-DOA) estimation problems. The 1D-DOA estimation is derived using the support vector regression based basis selection algorithm. Secondly, an over-complete dictionary is designed based on amplitude information of sources, and angle pairing is accomplished in perspective of basis selection. Validity and advantages of the proposed algorithm are shown through computer simulations.

***Index Terms*** **—** 2D-DOA, basis selection, linear array, single snapshot, support vector regression.

## I. INTRODUCTION

Direction-of-arrival estimation is to find the directions of sources impinging on antenna arrays [1]-[5]. Recently, a 1D-DOA estimation algorithm was proposed based on the sparse signal reconstruction [6], which renders several advantages over existing methods, including increased resolution, improved robustness against limited number of snapshots, and the capability to handle correlated sources.

Two-dimensional direction-of-arrival (2D-DOA) estimation is usually nontrivial. Although angle pairing can be accomplished by searching, such a method is computationally unattractive [7]. Based on the observation that the data matrices with the same set of eigenvectors can be diagonalized by the same similarity transform, two methods were introduced to realize angle pairing [8]. Some other methods based on eigen-structure of signals were also developed [9]. Recently, a 2D-DOA estimation algorithm has been proposed based on the support vector machine [10], whose performance is influenced by the training scenarios. When a limited number of snapshots are available, performance of the aforementioned algorithms will deteriorate. Thus, it is desirable to develop 2D-DOA estimation methods using a single snapshot. [11] and [12] presented two single snapshot 2D-DOA estimation methods, where nonuniformly spaced planar arrays were used. In [13], a uniform rectangular array is employed to realize single snapshot 2D-DOA estimation. All these algorithms are based on eigen-decomposition. Escot et.al. [14] proposed to accomplish 2D-DOA estimation by particle swarm optimization. However, it is known that evolutionary algorithms are unable to yield consistent solutions and usually suffer from high computational load. Furthermore, it has been shown that impulsive noise appears at wireless receivers in the form of impulsive bursts [15]. In this case, all second-order statistics based algorithms are unable to perform well.

In this paper, we address the problem of single snapshot 2D-DOA estimation in impulsive noise environment employing linear arrays. The rest of this paper is organized as follows. Section II

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briefly reviews 1D-DOA estimation in perspective of basis selection. Section III describes the proposed method in detail, and Section IV presents simulation results to show the validity and advantages of the proposed method. Section V concludes the work described in this paper.

## II. REVIEW OF 1D-DOA ESTIMATION IN PERSPECTIVE OF BASIS SELECTION

Generate an over-complete dictionary which consists of steering vectors from all possible directions of sources $\left\{\hat{\theta}_1,\ldots,\hat{\theta}_N\right\}$ i.e., $\overline{\mathbf{A}}=\left(\mathbf{a}(\hat{\theta}_1),\ldots,\mathbf{a}(\hat{\theta}_N)\right)$, where N denotes the number of spatial samplings. The 1D-DOA estimation problem is equivalent to solving $\bar{\mathbf{s}}$ of

$$\mathbf{x} = \overline{\mathbf{A}}\bar{\mathbf{s}} + \mathbf{n}, \tag{1}$$

where the $i$-th element of $\bar{\mathbf{s}}$ is nonzero if and only if a source comes from $\hat{\theta}_i$. $\mathbf{x}$ is the snapshot, and $\mathbf{n}$ denotes the noise. When the number of sensors, denoted by M, is much smaller than N, i.e., M<<N, most of entries in $\bar{\mathbf{s}}$ are zero. Solving $\bar{\mathbf{s}}$ from (1) can be formulated as a basis selection problem.

Under Gaussian noise assumption, the optimal $\bar{\mathbf{s}}$ in (1) can be found by solving the following optimization problem:

$$\min_{\bar{\mathbf{s}}} \mathrm{E}^{\mathrm{p}}(\bar{\mathbf{s}}), \tag{2a}$$

$$\text{subject to } \|\mathbf{x}-\mathbf{A}\bar{\mathbf{s}}\|_2^2 \le \varepsilon^2, \tag{2b}$$

where $\mathrm{E}^{\mathrm{p}}(\bar{\mathrm{s}})$ represents the diversity of $\bar{\mathrm{s}}$ which can be chosen according to some existing criteria [6]. There have been many algorithms to solve (2), one of which is the match pursuit [16]-[19].

## III. THE PROPOSED 2D-DOA ESTIMATION ALGORITHM

### A. Basis selection algorithm in impulsive noise environment

In this paper, we choose the $l_p$-norm as the diversity measurement [19]. In the impulsive noise environment, basis selection can be realized via solving the following problem:

$$\min_{\bar{\mathbf{s}}} \|\bar{\mathbf{s}}\|_{\mathrm{p}}^{\mathrm{p}},\ \mathrm{p} \le 1, \tag{3a}$$

$$\text{subject to } \left|\mathrm{x}_i - \mathbf{a}_i^{\mathrm{T}}\bar{\mathbf{s}}\right| < \varepsilon, \forall i = 1,\ldots,M, \tag{3b}$$

where $\mathbf{a}_i^{\mathrm{T}}$ denotes the $i$-th row of $\overline{\mathbf{A}}$, $\mathrm{x}_i$ denotes the ith element of $\mathbf{x}$, and $\varepsilon$ represents the impulsive noise. $\|\bar{\mathbf{s}}\|_{\mathrm{p}}$ denotes the $l_p$-norm of $\bar{\mathbf{s}}$ which is computed via $\|\bar{\mathbf{s}}\|_{\mathrm{p}} = \left(\sum_{i=1}^{N}|\bar{s}_i|^{\mathrm{p}}\right)^{1/\mathrm{p}}$.

Since direct solution of (3) is difficult, the affine scaling transformation [19] is applied to transform (3) into an equivalent problem

$$\min_{\mathbf{q}} \|\mathbf{q}\|_2^2, \tag{4a}$$

$$\text{subject to } \left|\mathrm{x}_i - \mathbf{b}_i^{\mathrm{T}}\mathbf{q}\right| < \varepsilon, \forall i = 1,\ldots,M, \tag{4b}$$

where $\mathbf{q} = \mathbf{W}^{-1}\bar{\mathbf{s}}$, $\mathbf{B} = \mathbf{AW}$, and $\mathbf{W} = \mathrm{diag}\left\{|\bar{s}_i|^{1-\mathrm{p}/2}\right\}$. $\mathbf{b}_i^{\mathrm{T}}$ denotes the $i$-th row of $\mathbf{B}$.

Considering $\mathrm{x}_i$ as the target for the input pattern $\mathbf{b}_i^{\mathrm{T}}$, (4) is identical to the optimization problem of SVR [20] formulated as

$$\min_{\mathbf{q},\boldsymbol{\epsilon},\boldsymbol{\epsilon}^*} \|\mathbf{q}\|_2^2 + \mathrm{C}\sum_{i=1}^{M}(\epsilon_i + \epsilon_i^*), \tag{5a}$$

$$\text{subject to } \begin{cases} \mathrm{x}_i - \mathbf{b}_i^{\mathrm{T}}\mathbf{q} \le \varepsilon + \epsilon_i \\ \mathbf{b}_i^{\mathrm{T}}\mathbf{q} - \mathrm{x}_i \le \varepsilon + \epsilon_i^* \\ \epsilon_i, \epsilon_i^* \ge 0 \end{cases} \tag{5b}$$

where $\epsilon_i, \epsilon_i^*$ are slack variables, and C>0 determines the trade-off between finding a sparse solution and retaining small residual error. The dual problem of (5) is given by

$$\min_{\boldsymbol{\alpha},\boldsymbol{\alpha}^*} -\sum_{i=1}^{M}\sum_{j=1}^{M}(\alpha_i-\alpha_i^*)(\alpha_j-\alpha_j^*)\langle \mathrm{b}_i,\mathrm{b}_j\rangle -\varepsilon\sum_{i=1}^{M}(\alpha_i+\alpha_i^*) + \sum_{i=1}^{M}\mathrm{x}_i(\alpha_i-\alpha_i^*), \tag{6a}$$

$$\text{subject to } \sum_{i=1}^{M}(\alpha_i-\alpha_i^*) = 0, \alpha_i, \alpha_i^* \in [0,\mathrm{C}], \tag{6b}$$

and $\mathbf{q}$ is given by

$$\mathbf{q} = \sum_{i=1}^{M}(\alpha_i-\alpha_i^*)\,\mathbf{b}_i, \tag{7}$$

which is called support vector expansion. By solving (6), $\mathbf{q}$ can be obtained, and $\bar{\mathbf{s}}$ can be calculated using $\bar{\mathbf{s}} = \mathbf{Wq}$.

For our problem, the input pattern $\mathbf{b}_i$ is unknown, we thereby propose the following iterative algorithm to solve (3):

*Step 1*: Initialize $\bar{\mathbf{s}}(0)$ using a randomly generated vector, $k$=0, $\mathbf{W}(0) = \mathrm{diag}\left\{|\bar{s}_i(0)|^{1-\mathrm{p}/2}\right\}$, and $\mathbf{B}(0) = \mathbf{AW}(0)$.

*Step 2*: Solve (4) using SVR and obtain $\mathbf{q}(k)$.

*Step 3*: $k = k + 1$, $\bar{\mathbf{s}}(k) = \mathbf{W}(k-1)\mathbf{q}(k-1)$, $\mathbf{W}(k) = \mathrm{diag}\left\{|\bar{s}_i(k)|^{1-\mathrm{p}/2}\right\}$, $\mathbf{B}(k) = \mathbf{AW}(k)$.

*Step 4*: If $\|\bar{\mathbf{s}}(k+1)-\bar{\mathbf{s}}(k)\|_2/\|\bar{\mathbf{s}}(k+1)\|_2 < \tau$, stop. Otherwise, go to *Step 2*.

In the aforementioned iterative algorithm, $k$ is the number of iteration steps, and $\tau$ is the convergence criterion, which is chosen to be 0.01 in this paper.

### B. Angle pairing using basis selection

*1) Over-complete dictionary with respect to directions of sources:* In this paper, three unparallel arrays A, B, and C are used. The unit direction vectors of the arrays are assumed to be $(1,0,0)$, $(\cos\theta_B, \sin\theta_B, 0)$, and $(\cos\theta_C\cos\varphi_C, \sin\theta_C\cos\varphi_c, \sin\varphi_C)$. Suppose that a narrow band source with azimuth angle $\theta$ and elevation angle $\varphi$ impinges array A, B and C with 1D-DOA $\vartheta_a, \vartheta_b$ and $\vartheta_c$, respectively. The following equations can be derived:

$$\cos\vartheta_a = \cos\theta\cos\varphi, \tag{8a}$$

$$\cos\vartheta_b = \cos\theta\cos\varphi\cos\theta_B + \sin\theta\cos\varphi\sin\theta_B, \tag{8b}$$

$$\cos\vartheta_c = \cos\theta\cos\varphi\cos\theta_C\cos\varphi_C + \sin\theta\cos\varphi\sin\theta_C\cos\varphi_C + \sin\varphi\sin\varphi_C. \tag{8c}$$

From (8), we may express $\cos\vartheta_c$ in terms of $\cos\vartheta_a$ and $\cos\vartheta_b$ as

$$\cos\vartheta_c = f(\vartheta_a, \vartheta_b) = \cos\vartheta_a\cos\varphi_C\sin(\theta_B - \theta_C)/\sin\theta_B + \cos\vartheta_b \cos\varphi_C\sin\theta_C/\sin\theta_B + \sqrt{1-(1+D)\cos^2\vartheta_a}\sin\varphi_C, \tag{9}$$

where $D = \left(\frac{\cos\vartheta_b}{\cos\vartheta_a} - \cos\theta_B\right)^2 / \sin^2\theta_B$. Therefore, the steering vector of array C can be expressed in terms of $\vartheta_a$ and $\vartheta_b$ as

$$\mathbf{a}(\vartheta_c) = \left(e^{j2\pi d_1^c\cos\vartheta_c/\lambda}, \dots, e^{j2\pi d_M^c\cos\vartheta_c/\lambda}\right)^T = \left(e^{j2\pi d_1^c f(\vartheta_a,\vartheta_b)/\lambda}, \dots, e^{j2\pi d_M^c f(\vartheta_a,\vartheta_b)/\lambda}\right)^T, \tag{10}$$

where $d_i^c$ denotes the distance between the origin and the *i*-th sensor of array C.

Denote the estimated two 1D-DOA with respect to array A and B as $\widehat{\boldsymbol{\vartheta}_a} = (\hat{\vartheta}_a^1, \dots, \hat{\vartheta}_a^{M_a})$ and $\widehat{\boldsymbol{\vartheta}_b} = (\hat{\vartheta}_b^1, \dots, \hat{\vartheta}_b^{M_b})$, where $M_a$ and $M_b$ are the number of 1D-DOA estimated with respect to array A and B, respectively. It is possible that some sources have identical 1D-DOA, thereby $M_a, M_b \le M$ holds. The equality holds only when $\hat{\vartheta}_a^i \ne \hat{\vartheta}_a^j$, $\hat{\vartheta}_b^i \ne \hat{\vartheta}_b^j$ are tenable for all $i \ne j$. Using (9) and (10), we may generate an over-complete dictionary with respect to array C in terms of $\vartheta_a$ and $\vartheta_b$ as

$$\bar{\mathbf{A}}(\vartheta_c) = \bar{\mathbf{A}}(f(\vartheta_a, \vartheta_b)) = [\bar{\mathbf{A}}_\Delta(\vartheta_a^1, \vartheta_b^1), \dots, \bar{\mathbf{A}}_\Delta(\vartheta_a^1, \vartheta_b^{M_b}), \bar{\mathbf{A}}_\Delta(\vartheta_a^2, \vartheta_b^1), \dots, \bar{\mathbf{A}}_\Delta(\vartheta_a^2, \vartheta_b^{M_b}), \dots, \bar{\mathbf{A}}_\Delta(\vartheta_a^{M_a}, \vartheta_b^1), \dots, \bar{\mathbf{A}}_\Delta(\vartheta_a^{M_a}, \vartheta_b^{M_b})], \tag{11}$$

where $\bar{\mathbf{A}}_\Delta(\vartheta_a^i, \vartheta_b^j)$ consists of steering vectors with respect to the neighboring region of $(\vartheta_a^i, \vartheta_b^j)$, i.e.,

$$\bar{\mathbf{A}}_\Delta(\vartheta_a^i, \vartheta_b^j) = [\mathbf{a}(\vartheta_a^i - \Delta_a, \vartheta_b^j - \Delta_b), \mathbf{a}(\vartheta_a^i - \Delta_a + \delta_a, \vartheta_b^j - \Delta_b + \delta_b), \mathbf{a}(\vartheta_a^i - \Delta_a + 2\delta_a, \vartheta_b^j - \Delta_b + 2\delta_b), \dots, \mathbf{a}(\vartheta_a^i + \Delta_a, \vartheta_b^j + \Delta_b)]. \tag{12}$$

In (12), $\Delta_a$ and $\Delta_b$ denote the neighboring region of $\vartheta_a^i$ and $\vartheta_b^j$, respectively. $\delta_a$ and $\delta_b$ denote the sampling interval of $\vartheta_a^i$ and $\vartheta_b^j$, respectively. By introducing neighboring region, potential error in the 1D-DOA estimation can be amended so that accurate 2D-DOA estimation can be achieved.

*2) Over-complete dictionary with respect to amplitudes of sources:* The over-complete dictionary $\bar{\mathbf{A}}(\vartheta_c)$ given by (12) contains all the possible angle pairings. The columns of $\bar{\mathbf{A}}(\vartheta_c)$ which match the snapshot of array C (denoted by $\mathbf{x}_c$) gives the correct angle pairing result. Therefore, the angle pairing problem can be formulated as the following inverse problem which aims to compute $\bar{\mathbf{s}}_c$:

$$\mathbf{x}_c = \bar{\mathbf{A}}\bar{\mathbf{s}}_c + \mathbf{n}_c, \tag{13}$$

where $\mathbf{n}_c$ denotes the additive noise on array C. However, solving (13) directly with basis selection cannot guarantee correct angle pairing result. It is noted that if there were two angle pairs satisfying $f(\vartheta_a^i, \vartheta_b^j) = f(\vartheta_a^k, \vartheta_b^l)$, $i \ne k, j \ne l$, incorrect angle pairing occurs. In order to avoid such a problem, additional constraint should be imposed on $\bar{\mathbf{s}}_c$ when solving (13).

It is observed that incorrect angle pairing probably results in significant difference between signal amplitudes estimated from (13) and those from the 1D-DOA estimation step. Thus, constraint can be imposed on the signal amplitude to guarantee correct angle pairing result.

Suppose that the estimated amplitudes of sources from 1D-DOA estimation are $\widehat{\mathbf{s}_a} = (\hat{s}_a^1, \dots, \hat{s}_a^{M_a})$ and $\widehat{\mathbf{s}_b} = (\hat{s}_b^1, \dots, \hat{s}_b^{M_b})$. It is assumed that $\widehat{\mathbf{s}_a}$ and $\widehat{\mathbf{s}_b}$ should not change significantly with respect to the three arrays. The following constraints can be imposed on $\bar{\mathbf{s}}_c$:

$$\|\mathbf{B}_a\bar{\mathbf{s}}_c - \widehat{\mathbf{s}_a}\|_2^2 \le \varepsilon_a^2, \tag{14a}$$

$$\|\mathbf{B}_b\bar{\mathbf{s}}_c - \widehat{\mathbf{s}_b}\|_2^2 \le \varepsilon_b^2, \tag{14b}$$

where the elements of $\mathbf{B}_a$ and $\mathbf{B}_b$ are given by

$$b_a(i,j) = \begin{cases} 1, \text{if } \bar{\mathbf{A}}_j(\vartheta_c) \in \bar{\mathbf{A}}_\Delta(\vartheta_a^i, \vartheta_b^1), \dots, \bar{\mathbf{A}}_\Delta(\vartheta_a^i, \vartheta_b^{M_b}) \\ 0, \text{otherwise} \end{cases},$$

$$b_b(i,j) = \begin{cases} 1, \text{if } \bar{\mathbf{A}}_j(\vartheta_c) \in \bar{\mathbf{A}}_\Delta(\vartheta_a^1, \vartheta_b^i), \ldots, \bar{\mathbf{A}}_\Delta(\vartheta_a^{M_a}, \vartheta_b^i) \\ 0, \text{otherwise} \end{cases}.$$

$\bar{\mathbf{A}}_j(\vartheta_c)$ denotes the j-th column of $\bar{\mathbf{A}}(\vartheta_c)$. With (14), angle pairing can be realized by finding a sparse solution $\bar{\mathbf{s}}_c$ from

$$\tilde{\mathbf{x}}_c = \tilde{\mathbf{A}}\bar{\mathbf{s}}_c + \tilde{\mathbf{n}}_c, \quad (15)$$

where $\tilde{\mathbf{A}} = [\bar{\mathbf{A}}(\vartheta_c) \quad \mathbf{B}_a \quad \mathbf{B}_b]^T$ and $\tilde{\mathbf{x}}_c = [\mathbf{x}_c \quad \widehat{\mathbf{s}_a} \quad \widehat{\mathbf{s}_b}]^T$.

### C. Discussions on the proposed single snapshot 2D-DOA algorithm

Conventional application of SVR requires a large number of training data to derive an accurate regression model [10], [21]. Then, the derived regression model is used for online testing. The computational complexity for training is usually large. Furthermore, if the real scenario is different from the trained ones, the performance of SVR will deteriorate. On the other hand, the proposed algorithm utilizes SVR as a solver to solve (4). Therefore, offline training and online testing are not required for the proposed algorithm.

For SVR, let $l$ be the number of training points, $N_S$ the number of support vectors, and $d_L$ the dimension of the input data. The complexity of SVR is $O(N_s^3 + N_s^2 l + N_s dLl)$ when $N_s/l << 1$ and $O(N_s^3 + N_s l + N_s d_L l)$ when $N_s/l \approx 1$ [22]. From (4), it is observed that for the proposed algorithm, the number of input patterns is L, and the dimension of the input pattern is $N$. Due to the property of the over-complete dictionary, $N > L$ holds, so that the computational complexity of the proposed algorithm is approximately given by $O(N_s NL)$. In order to reduce the computational complexity of the proposed algorithm, grid refining technique can be applied so that a smaller value of $N$ can be used. It should be mentioned that compared with applying SVR for training and testing, the proposed algorithm has much less computational load, because the number of training samples is usually much larger than $N$.

## IV. COMPUTER SIMULATIONS

Without loss of generality, we assume that three linear arrays lie in the same plane. The element spacing of each array is equal to half-wavelength with respect to the operating frequency. The azimuth angle of array B and C are assumed to be 30° and 90°, respectively. The number of sources is assumed to be 4. The angular sampling interval to generate the overcomplete dictionary $\bar{\mathbf{A}}(\vartheta_c)$ is 1°. The parameters for implementation of SVR are chosen as $\epsilon$=0.001 and C = 0.6, which are empirical values. Impulsive noise is generated as the mixture of a Gaussian process and a Bernoulli-Gaussian process [23]. The Gaussian process is with zero mean and variance $\sigma_1^2$. The impulsive bursts are generated by a Bernoulli-Gaussian process, where a Gaussian variable with zero mean and variance $\sigma_2^2$ and a Bernoulli variable with success probability $p$ are used. The Signal-to-Noise Ratio (SNR) is computed as $10\log_{10} 1/\left((1-p)\sigma_1^2 + p(\sigma_1^2+\sigma_2^2)\right)$ dB. In the simulations, $\sigma_2^2 = 100\sigma_1^2$ and $p$ = 0.1 are assumed.

### A. Sources with different 1D-DOAs

In the first simulation, the sources are assumed to be located at $(35^o, 47^o)$, $(47^o, 52^o)$, $(59^o, 56^o)$, $(66^o, 65^o)$ with unity power. The phase of each source is randomly distributed between 0 and $2\pi$. The number of sensors is assumed to be 10.

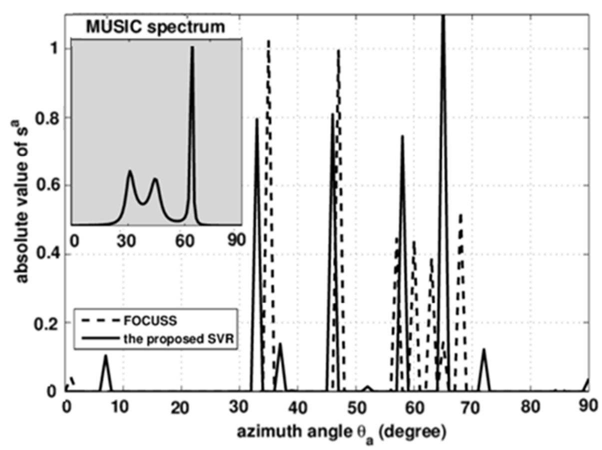


Fig. 1. The estimated spectrum for 1D-DOA estimation $\hat{\vartheta}_a$ with SNR=20 dB and L=10.

Figures 1 and 2 show the estimated spectra for 1D-DOA estimation $\hat{\vartheta}_a$and $\hat{\vartheta}_b$, respectively. It is observed from Figs. 1 and 2 that the proposed SVR based basis selection algorithm shows less spurious peaks than that of the FOCUSS algorithm. This is because the proposed SVR based algorithm is robust against the impulsive noise. Also, the MUSIC spectrum using a single snapshot [24] is plotted with the number of sensors in subarray equal to 5. It is observed that because the number of sensors is small, the single snapshot MUSIC algorithm is unable to precisely locate the four sources.

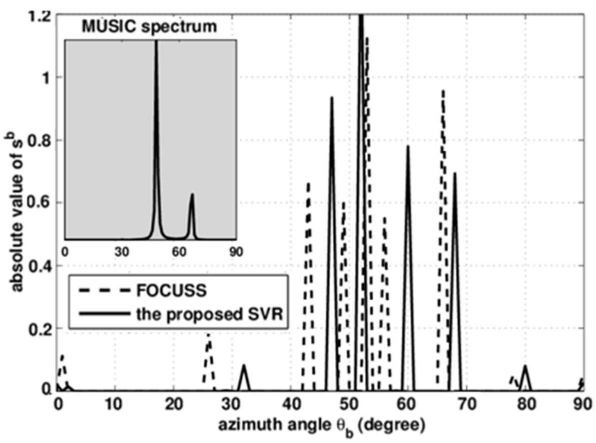


Fig. 2. The estimated spectrum for 1D-DOA estimation $\hat{\vartheta}_b$ with SNR=20 dB and L=10.

Figure 3 presents the 2D-DOA estimation results for 50 independent trials with SNR equal to 20 dB. From Fig. 3, we see that incorrect angle pairing does not occur during the 50 independent trials.

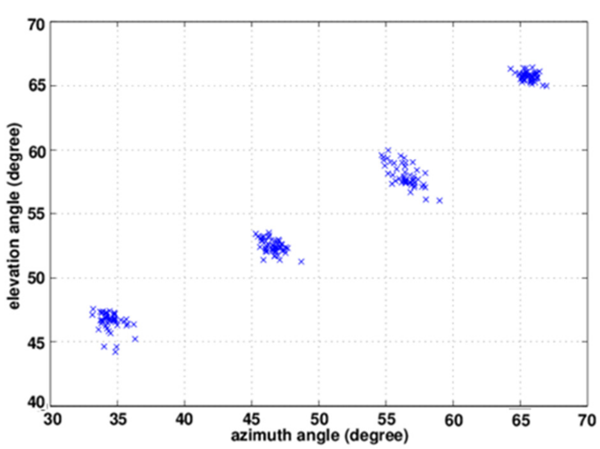


Fig. 3. 2D-DOA estimation results for 50 independent trials with SNR=20 dB and L=10.

Table 1 shows the Root-Mean-Square-Error (RMSE) of the proposed method with different SNR for 50 independent trials, and Table 2 shows the RMSE of the proposed method against the number of sensors. From Table 1 and Table 2, we see that the proposed algorithm is able to give satisfactory performance. As the value of SNR or L increases, the RMSE of estimation decreases.

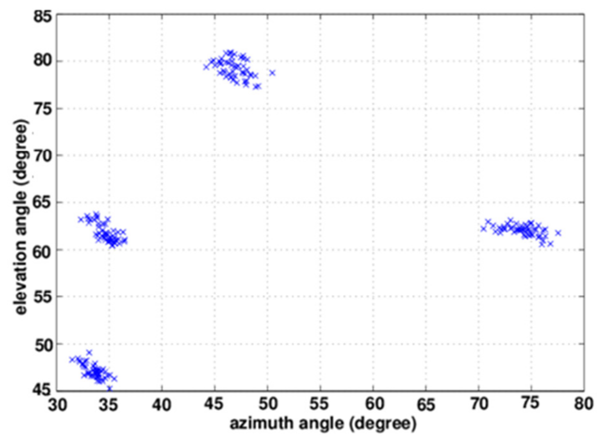


Fig. 4. 2D-DOA estimation results for 50 independent trials with SNR=20 dB and L=10.

Table 1: RMSE(degree) versus SNR for sources located at $(35^o, 47^o)$, $(47^o, 52^o)$, $(59^o, 56^o)$, $(66^o, 65^o)$.

| RMSE(degree) of 2D-DOA estimations (L=10) | | | | |
|---|---|---|---|---|
| SNR(dB) | $(35^o, 47^o)$ | $(47^o, 52^o)$ | $(59^o, 56^o)$ | $(66^o, 65^o)$ |
| 10 | 3.9530 | 2.2170 | 2.7130 | 2.9181 |
| 15 | 2.0245 | 1.3396 | 2.0770 | 1.3156 |
| 20 | 1.4917 | 0.9763 | 1.8636 | 1.0039 |
| 25 | 0.8356 | 0.7345 | 1.2893 | 0.7087 |
| 30 | 0.6461 | 0.6799 | 1.0668 | 0.7038 |
| 35 | 0.6080 | 0.6406 | 0.7332 | 0.6338 |
| 40 | 0.5849 | 0.6373 | 0.6429 | 0.5703 |

Table 2: RMSE(degree) versus L for sources located at $(35^o, 47^o)$, $(47^o, 52^o)$, $(59^o, 56^o)$, $(66^o, 65^o)$.

| RMSE(degree) of 2D-DOA estimations (SNR=20 dB) | | | | |
|---|---|---|---|---|
| L | $(35^o, 47^o)$ | $(47^o, 52^o)$ | $(59^o, 56^o)$ | $(66^o, 65^o)$ |
| 8 | 1.9921 | 1.8592 | 2.6749 | 1.8475 |
| 9 | 1.6292 | 1.3489 | 2.2948 | 1.4928 |
| 10 | 1.4917 | 0.9763 | 1.8636 | 1.0039 |
| 11 | 1.0192 | 0.8674 | 1.4059 | 0.8924 |
| 12 | 0.9019 | 0.8347 | 1.3873 | 0.7294 |

### B. Sources with the same 1D-DOA

In this simulation, we assume that the four sources are located at $(35^{o},47^{o})$, $(35^{o},62^{o})$, $(50^{o},80^{o})$, $(76^{o},62^{o})$ with unity power. In this case, two sources have identical azimuth angle, and the other two sources have identical elevation angle.

Figure 4 shows the 2D-DOA estimation results for 50 independent trials with SNR = 20 dB and L = 10. Tables 3 and 4 show the RMSE of the proposed method with different SNR and L, respectively. When some sources have identical 1D-DOA, the number of derived 1D-DOA estimations is smaller than that of sources.

In this simulation, only three DOA are estimated in the 1D-DOA estimation step. However, using the proposed angle pairing method, the sources with the same 1D-DOA automatically split. As shown in Fig. 4, the proposed algorithm does not give incorrect angle pairing results for 50 independent trials. Tables 3 and 4 show that the performance of the proposed algorithm in this case is a little bit poorer than that in the previous simulation, but it is still satisfactory.

Table 3: RMSE (degree) versus SNR for sources located at $(35^{o},47^{o})$, $(35^{o},62^{o})$, $(50^{o},80^{o})$, $(76^{o},62^{o})$.

| RMSE(degree) of 2D-DOA estimations (L=10) | | | | |
|---|---|---|---|---|
| SNR(dB) | $(35^{o},47^{o})$ | $(35^{o},62^{o})$ | $(50^{o},80^{o})$ | $(76^{o},62^{o})$ |
| 10 | 3.6711 | 2.5581 | 4.0508 | 2.6989 |
| 15 | 2.6146 | 1.9299 | 3.0435 | 2.3649 |
| 20 | 1.5336 | 1.2835 | 2.1237 | 1.9011 |
| 25 | 1.1902 | 1.0214 | 1.8015 | 1.5975 |
| 30 | 1.0508 | 0.9582 | 1.1362 | 1.1212 |
| 35 | 0.9112 | 0.9011 | 0.9571 | 0.9821 |
| 40 | 0.8489 | 0.7530 | 0.8960 | 0.9155 |

Table 4: RMSE (degree) versus L for sources located at $(35^{o},47^{o})$, $(35^{o},62^{o})$, $(50^{o},80^{o})$, $(76^{o},62^{o})$.

| RMSE(degree) of 2D-DOA estimations (SNR=20 dB) | | | | |
|---|---|---|---|---|
| L | $(35^{o},47^{o})$ | $(35^{o},62^{o})$ | $(50^{o},80^{o})$ | $(76^{o},62^{o})$ |
| 8 | 2.5935 | 2.9737 | 2.9207 | 2.1537 |
| 9 | 2.1788 | 2.6361 | 2.3228 | 2.0449 |
| 10 | 1.5336 | 1.2835 | 2.1237 | 1.9011 |
| 11 | 1.0564 | 1.0190 | 1.8211 | 1.4085 |
| 12 | 0.9382 | 0.9015 | 1.5020 | 1.0349 |

## V. CONCLUSIONS

In this paper, a new method has been described to address the problem of 2D-DOA estimation using a single snapshot in impulsive noise environment. Three unparallel linear arrays are employed. The 2D-DOA estimation problem is decomposed into two 1D-DOA estimation problems which are solved by the proposed SVR based basis selection algorithm. To realize angle pairing, an over-complete dictionary is designed using estimated amplitudes of sources. Computer simulation shows that the proposed algorithm is able to realize single snapshot 2D-DOA estimation in impulsive noise environment with satisfactory accuracy. The proposed method is especially useful for 2D-DOA estimation using a limited number of snapshots in the presence of impulsive noise. Future work is to extend the proposed method to other array structures.

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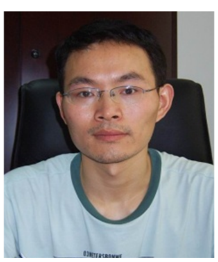

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