Application of the Multi-element Grid in EMC Uncertainty Simulation

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Abstract – Uncertainty analysis is a research hotspot in the field of electromagnetic compatibility (EMC) simulation. The stochastic collocation method (SCM) is considered particularly suitable for uncertainty analysis in the EMC field because it is characterized by a high level of computational efficiency and accuracy while requiring no replacement solver. However, the post-processing process of the SCM is too complex, which will seriously limit its application in many industrial environments such as real-time simulation analysis. Multi-element grid (MEG) is a novel uncertainty analysis method recently for successful application in another area. It is proved that its calculation accuracy is same as the SCM, and its post-processing process is facile. This paper introduces the MEG to the EMC field and makes a detailed comparison between it and the SCM in performance, aiming to apply uncertainty analysis to solve more practical EMC engineering problems.

Index Terms – Electromagnetic compatibility, uncertainty analysis, stochastic collocation method, multi-element grid.

I. INTRODUCTION

Uncertainty analysis theory has been developed and perfected in the field of computational fluid dynamics (CFD) [1] and introduced into the field of electromagnetic compatibility (EMC) simulation in previous years. Gradually, it has become a hot research issue in the field of EMC simulation.

EMC simulation has its unique characteristics. One is that the time of simulation is usually long due to the frequent use of the finite element method. Therefore, the uncertainty analysis methods with slow convergence speed are not competitive, such as the Monte Carlo method (MCM) [2, 3]. The reason is that too much time cost is unacceptable. However, the calculation accuracy of MCM is the highest; so its simulation results are usually used as standard data in theoretical research to verify the accuracy of other uncertainty analysis methods, and this paper is no exception.

Another characteristic of EMC simulation is that its calculation needs the help of commercial simulation software in most cases. On this premise, the uncertainty analysis method which needs to change the previous solver cannot be used normally, such as the perturbation method [4], the stochastic Galerkin method [5, 6], and the stochastic testing method [7]. At the same time, there are still some uncertainty analysis methods, such as the moment method [8] and the stochastic reduced order models [9]. There are no restrictions in terms of application, but their accuracy is not ideal. Thus, these methods are also difficult to promote.

The stochastic collocation method (SCM) is an efficient uncertainty analysis method based on generalized polynomial chaos theory [10–12]. It has the advantages of high calculation accuracy, high calculation efficiency, and no need to change the solver. From this point of view, the SCM is an ideal uncertainty analysis method in the EMC field at this stage. However, the post-processing process of the SCM is extremely complex, resulting in the following problems in engineering application. First, in the process of real-time simulation, very long post-processing time will affect the response speed of subsequent control operations. Second, the SCM has high algorithm complexity and needs to store a large number of sampling points information. From the perspective of algorithm implementation and storage, it is not easy to write into the one-chip computer; so it is difficult to realize industrial application.

The multi-element grid (MEG) is a novel uncertainty analysis method, which also has the characteristics of high computational efficiency and no need to change the solver [13]. It has been applied successfully in the control area, and its outstanding advantage is that the post-processing process is simple. This paper aims to use
this novel method to solve typical EMC simulation problems and provide a better uncertainty analysis method for EMC field.

The remainder of this paper is organized as follows. In Section II, the framework of the SCM is briefly introduced. Section III presents the application of the MEG in EMC simulation in detail. Section IV validates the algorithm’s accuracy by using crosstalk simulation example. Shielding effect simulation example of anechoic chamber is shown in Section V. The prospect of further application of the MEG is discussed in Section VI. Section VII summarizes this paper.

II. OUTLINE OF THE SCM

In the actual electromagnetic environment, randomness and cognitive limitations are inevitable; so it is impossible to realize accurate simulation completely using deterministic parameter models. It is more appropriate to use a random variable model to describe random events in practical engineering, shown as follows:

\[ \xi = \{ \xi_1, \xi_2, \ldots, \xi_n \}, \]

where \( \xi \) represents a random variable with probability density, and \( \xi \) is the set of all random variables.

When the probability density function of the random variable is known, the orthogonal polynomial corresponding to the random variable can be obtained by three-term recurrence formula [10]

\[ \pi_{r+1}(\xi) = (\xi - \alpha_r) \pi_r(\xi) - \beta_r \pi_{r-1}(\xi), \quad r = 0, 1, \ldots, \]

\[ \pi_0(\xi) = 1, \]

where \( \pi_r(\xi) \) is the orthogonal polynomial of one-dimensional random variable \( \xi_r \). The intermediate variables can be calculated by the following formula:

\[ \alpha_r = \frac{\langle \xi \pi_r, \pi_r \rangle}{\langle \pi_r, \pi_r \rangle}, \]

\[ \beta_r = \frac{\langle \pi_r, \pi_r \rangle}{\langle \pi_{r-1}, \pi_{r-1} \rangle}, \]

\[ \beta_0 = \frac{\pi_0(\xi)}{\pi_1(\xi)} \]

The internal product calculation formula is

\[ \langle x(\xi), y(\xi) \rangle = \int x(\xi)y(\xi) \, \text{pdf}(\xi) \, d\xi, \]

where pdf(\( \xi \)) represents the probability density function of the random variable \( \xi \).

For the SCM, the collocation points corresponding to the one-dimensional random variable are the zero points of orthogonal polynomial in formula (2). In the multidimensional case, the collocation points are the tensor product of one-dimensional collocation points [11].

In SCM, the uncertainty analysis result is the sum of orthogonal polynomials

\[ U(\xi) = \sum_{r=0}^{M} c_r \pi_r(\xi), \]

where \( c_r \) is the constant to be solved.

A single deterministic EMC simulation is implemented at each collocation point \( q_i \) to obtain the corresponding result \( U_{\text{EMC}}(q_i) \). The fitting of formula (7) is realized based on the least square method to obtain the constant \( c_r \). It can be seen that the result of the SCM is in the form of a random variable polynomial. Finally, the statistical results such as expectation, standard deviation, and worst-case estimation are obtained by sampling the random variables in formula (7).

Obviously, in the whole calculation process, only deterministic simulation is required at each collocation point; thus, the SCM has the advantage of no need to change the original solver. At the same time, the generalized polynomial chaos theory ensures the fast convergence speed of the SCM. So that the required collocation points are far lower than the collocation points required by the MCM, which ensures that the SCM has high computational efficiency [11, 12].

It is worth noting that the SCM can only obtain statistical results after fitting and sampling calculation. Therefore, the implementation of the SCM requires a long post-processing process, which will have an adverse impact on calculation efficiency and algorithm promotion.

III. IMPLEMENTATION OF THE MEG IN EMC SIMULATION

In MEG, it is also necessary to construct the orthogonal polynomial under the three-term recurrence formula, and it is also necessary to select the zero points of the orthogonal polynomial to ensure the fastest convergence speed. At the same time, in the case of multidimensional random variables, the way to select the zero points is still in the form of a tensor product. In other words, under the same random variables model, the collocation points of the SCM are exactly the same as those required by the MEG.

It is more convenient for the MEG to give uncertainty statistic results. After a single deterministic EMC simulation at each selected zero point, the expectation result can be expressed as follows:

\[ E(U) = \sum_{i=1}^{M} w_i \times U_{\text{EMC}}(q_i). \]

Similarly \( U_{\text{EMC}}(q_i) \) represents the result of a single simulation at the selected point \( q_i \). As the selected configuration points are exactly the same as the SCM, the total number \( M \) is also consistent with formula (7). \( w_i \) expresses the weight proportion of \( U_{\text{EMC}}(q_i) \), and its calculation is the core idea of the MEG algorithm. When the random variables model is one-dimensional, the weight calculation formula is as follows:

\[ w_i = \int L(\xi, p_i) \text{pdf}(\xi) \, d\xi, \]

where \( L(\xi, p_i) \) represents the Lagrange interpolation polynomial constructed by each selected point and its
single simulation results
\[ L(\xi, q_i) = \prod_{k \neq i}^{\text{d}} \frac{\xi - q_k}{q_i - q_k}. \] (10)

If the random variables model is multidimensional, the weight can be directly obtained by multiplying the one-dimensional weight in the form of a tensor product. For example, if there are only two random variables, and their one-dimensional weights are only three, namely \( w_1, w_2, \) and \( w_3 \). The results in tensor product form are as follows:
\[ \begin{bmatrix} w_1w_1 & w_1w_2 & w_2w_1 & w_2w_2 & w_2w_3 & w_3w_1 & w_3w_2 & w_3w_3 \end{bmatrix}. \] (11)

Similarly, the variance results can be calculated in the same way
\[ \sigma(U) = \sum_{i=1}^{N_d} w_i \times [U_{\text{EMC}}(\xi) - E(U)]^2. \] (12)

It can be seen that the MEG also has the advantage of no need to change the original solver and has the same computational efficiency as the SCM. Meanwhile, the MEG is more concise in the post-processing stage of uncertainty analysis.

IV. SIMULATION EXAMPLE OF PARALLEL CABLE CROSSTALK

This section presents a benchmark calculating example in [11] to verify the accuracy of the MEG. It is a crosstalk simulation example with three uncertain parameters as shown in Figure 1.

The first uncertain parameter is the voltage source value, which satisfies the following formula:
\[ E_m(\xi_1) = 1 + 0.1 \times \xi_1. \] (13)

Its probability density function is supposed as follows:
\[ \text{pdf}(\xi_1) = \begin{cases} \frac{1}{\sqrt{2\pi}} \sin\left(\frac{\pi}{2} \xi_1\right) + (1 - \frac{1}{\sqrt{\pi}}), & 0 \leq \xi_1 \leq 1 \\ 0, & \text{other values} \end{cases}. \] (14)

Obviously, this probability density function satisfies the following conditions:
\[ \begin{cases} \text{pdf}(\xi_1) \geq 0 \\ \int_{-\infty}^{\infty} \text{pdf}(\xi_1) d\xi_1 = 1 \end{cases}. \] (15)

The other two uncertainty parameters are the heights of two cables, and both of them have a uniform distribution. Their random variable models are shown as follows:
\[ \begin{cases} h_1(\xi_2) = 0.045 + 0.005 \times \xi_2 \\ h_2(\xi_3) = 0.035 + 0.005 \times \xi_3 \end{cases}. \] (16)

where \( \xi_2 \) and \( \xi_3 \) are the random variables with the uniform distribution \([-1, 1]\).

The longitudinal distance along the paper between the two cables is 0.05 m, and the frequency range of crosstalk results is from 1 to 100 MHz. The other detailed information of the model is completely consistent with [11]. For MEG, the third-order chaotic polynomial of random variable \( \xi_1 \) is \( \xi_1^3 - 1.4360\xi_1^2 + 0.5513\xi_1 - 0.0465 \), the collocation points are \{0.8546, 0.4642, 0.1172\}. The third-order chaotic polynomials of random variables \( \xi_2 \) and \( \xi_3 \) are \( \xi_2^3 - \frac{3}{8} \xi_2 \), and both collocation points are \{\( \frac{\sqrt{15}}{5}, 0, -\frac{\sqrt{15}}{5} \}\}. Finally, calculate formulas (9)–(12), respectively.

Figures 2 and 3 show expectation results and standard deviation results of the MEG in calculating the crosstalk voltage, respectively. As a comparison, the simulation results of the MCM and the SCM are also given.

Taking the calculation results of the MCM as standard data, the feature selective validation (FSV) method [14] is used to evaluate the differences between the calculation results of other uncertainty analysis methods, so as to judge the accuracy of the algorithm. The FSV value between the MCM and the MEG in expectation results is 0.0092, and that between the MCM and the SCM is 0.0211. It presents that the accuracy of the MEG is slightly better than the SCM, and both of them are at an “Excellent” level. Meanwhile, the FSV value between the MCM and the MEG in standard deviation results is 0.0232, and that between the MCM and the SCM is 0.0873. Similarly, the same conclusion as the expectation results can be obtained.

Fig. 2. Expectation results of the crosstalk voltage from 1 to 100 MHz.
and material parameter uncertainty. The height $h_3$ of the absorber cone is uniform distribution of [0.33,0.37] m. The relative dielectric constant $\varepsilon_r$ of the shielding material is uniform distribution of [0.96,1], and the conductivity $\sigma$ of the shielding material is uniform distribution of [0.53,0.57] S/m. The random variable models are shown as follows:

$$
\begin{align*}
\left\{ 
\begin{array}{l}
\xi_1 = 0.35 + 0.02 \times \xi_1 \\
\xi_2 = 0.98 + 0.02 \times \xi_2 \\
\xi_3 = 0.55 + 0.02 \times \xi_3 
\end{array}
\right.
\end{align*}
$$

where $\xi_1$, $\xi_2$, and $\xi_3$ are consistent with the meaning of symbols in formulas (13) and (15).

There is a biconical antenna in the center of the darkroom, which emits 240 MHz spherical wave. The measurement position is 0.5 m away from the right wall, and the simulation output is the electric field intensity. In order to better show the change of results, it is expressed in decibels

$$
E_{\text{final}} = 20 \times \log_{10}(E_{\text{norm}}).
$$

Figure 5 shows the distribution of electric field intensity value at the test point without considering the parameter uncertainty. It can be seen that the variation range of electric field intensity is close to 40 dBV/m, and its change range is large. Therefore, it is more meaningful to pay attention to the maximum and minimum values when considering parameter uncertainty.

Considering the parameter uncertainty in formula (16), the MEG and the SCM are used for simulation, and their results are presented in Table 2. The results are the mean value of maximum $M_{\text{max}}$, the standard deviation of maximum $\sigma_{\text{max}}$, the mean value of minimum $M_{\text{min}}$, and the standard deviation of minimum $\sigma_{\text{min}}$ at the position to be measured. It is shown that the accuracy of the MEG is the same as the SCM. The mean equivalent area method (MEAM) is an effectiveness evaluation method for EMC uncertainty simulation results. The MEAM value of the MEG and the SCM in maximum results is 0.9035, and
that in minimum results is 0.9195. It is also proved that the uncertainty analysis results of the MEG and the SCM are very similar. More details about the MEAM can be found in [15].

The number of deterministic simulations required by the MEG and the SCM is 27, and the simulation time is 2.08 h. For post-processing time, the MEG takes 0.34 s and the SCM takes 7.23 min. Therefore, the conclusion is exactly the same as that in Section IV.

It is worth noting that the uncertainty analysis results of the MCM are not given in this section; the reason is that its estimated simulation time cost is unacceptable. When the number of deterministic simulations required by the MCM is still 12,000, the required simulation time is 38.5 days. More importantly, convergence may not be guaranteed under this simulation time. The MCM, the SCM, and the MEG are non-embeded uncertainty analysis methods, and the single EMC simulation process can be seen as a black box. Therefore, this paper proposes a third example, and the formula is as follows:

\[
\text{Ana}(\xi) = \begin{pmatrix} 
\frac{h_{1}(\xi_{1})}{1[\text{m}]} 
\end{pmatrix}^{2} + e_{1} + 2 \times \frac{\sigma_{1}(\xi_{3})}{1[\text{S}/\text{m}]}. \tag{19}
\]

Through the analytical value of formula (18), the accuracy of the SCM and the MEG is verified based on the analytical calculation results of the MCM. It is worth noting that each variable in formula (18) has eliminated the unit; so the abscissa in Figure 6 is unitless. Figure 6 gives the probability density results. According to probability theory, the closer the common area of the two curves is to 1, the more similar the results of the uncertainty analysis. When using the calculation results of the MCM as standard data, it can be clearly seen that both the SCM and the MEG are valid and accurate.

Furthermore, the SCM has been proved many times to have high calculation accuracy; so the calculation accuracy verification of the MEG in shielding effect simulation example in Figure 4 is guaranteed.

VI. COMPARISON OF THE MEG AND THE SCM IN INDUSTRIAL APPLICATION

First, the required storage space is compared. In crosstalk simulation example, storage space required by the MEG is 100 (Frequency number) × 27 (Number of matching points formed by tensor product) × 16 (Number of bytes required for real storage) + 9 (Weight values calculated in advance) × 16 (Number of bytes required for real storage) = 42.33 KB. Storage space required by the SCM is 100 (Frequency number) × 3200 (Number of collocation points) × 36 (9 calculated values of multidimensional Lagrange interpolation + 27 matching points formed by tensor product) × 16 (Number of bytes required for real storage) = 175.27 MB. In shielding effect simulation example, storage space required by the MEG is 2 (Maximum and minimum) × 27 (Number of matching points formed by tensor product) × 16 (Number of bytes required for real storage) + 9 (Weight values calculated in advance) × 16 (Number of bytes required for real storage) = 0.98 KB. Storage space required by the SCM is 2 (Maximum and minimum) × 1600 (Number of collocation points) × 36 (9 calculated values of multidimensional Lagrange interpolation + 27 matching points formed by tensor product) × 16 (Number of bytes required for real storage) = 1.76 MB.

Obviously, the MEG needs less storage space, and it is easier to use one-chip computer to store information, so as to further realize industrial application.
Second, in order to generate uniformly distributed collocation points satisfying $\xi_2$ and $\xi_3$, the post-processing stage of the SCM needs to use the rand function, which can be completed by one-chip computer under specific conditions. To obtain collocation points satisfying $\xi_1$, a more complex function is needed, and even the cumulative probability density equation needs to be solved by inverse solution. In this case, one-chip computer is obviously unable to complete.

Therefore, the MEG is easier to realize industrial application than the SCM in terms of algorithm implementation and storage implementation.

VII. CONCLUSION

In this paper, a novel uncertainty analysis method called MEG is introduced to solve the problem of EMC simulation. It verifies that the MEG is as good as the SCM in both calculation accuracy and calculation efficiency by two typical EMC calculating example. Finally, through quantitative calculation, it is verified that the MEG is easier to be realized in one-chip computer, so as to complete industrial application scenarios such as online real-time prediction.

REFERENCES


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