Improving the Convergence of the Wave Iterative Method by Filtering Techniques

H. Hrizi[1], L. Latrach[1], N. Sboui[1], A. Gharsallah[1], A. Gharbi[1], and H. Baudrand[2]

[1] Electronics Laboratory, Department of Physics, Faculty of Sciences in Tunis, 2092 El Manar Tunisia
noureddine.sboui@fst.rnu.tn

[2] Electronics Laboratory EN SEEIHT in Toulouse France

Abstract **—** Among the numerical methods used in the electromagnetic modeling of high frequency electronic circuits, we include the wave concept iterative method. In this paper, we propose to improve the convergence speed of this method when modeling a complex structure. This method requires a maximum number of iterations, noted “Nmax”, to achieve the convergence to the optimal value. Our goal is to reduce the number of iterations calculated by this method to the value “Nmin” in order to reduce the computing time and to improve the convergence speed. This is done by adding a new algorithm based on filtering techniques.

Index Terms **—** Adaptive and autoregressive Filtering, LMS algorithm, rapid convergence, WCIP method.

I. INTRODUCTION

The wave concept iterative method (WCIP) has shown efficiency in solving problems of electromagnetism and analysis of radio frequency circuits (RF) [1-4]. Although this method is absolutely convergent, the number of iterations is relatively high and it needs much time for multilayer or complex structures requiring a fine mesh as demonstrated in [5-7], the numerical complexity is related to the number of cells describing the circuit. For example, for structures of 512x512 cells, it takes 24 minutes to perform 1000 iterations. This result is calculated by a machine having a microprocessor Intel(R) Pentium(R) Dual Core CPU 2x2.16GHz and 3 GB of RAM. In this case of complex structures, the WCIP method takes much time to give good results. To avoid this problem, the techniques of adaptive filtering are an effective means to ensure a rapid convergence to the optimal value with a minimal error. Adaptive filtering is a powerful tool in signal processing, digital communications, and automatic control [8-10]. This tool has been applied in various fields such as system identification, prediction, inverse modeling, and the interference cancellation. We use the adaptive algorithm least mean square (LMS) because it is the simplest one in terms of calculation. In addition, it is the most efficient algorithm in terms of minimization criterion of mean squared error [11-12]. To improve the classical WCIP method, we use a new algorithm based on adaptive and autoregressive filtering. We aim at reducing the number of iterations in this method; hence, we reduce the computing time and we improve the convergence speed of the method.

II. THEORETICAL STUDY

A. Summary of the WCIP method

The WCIP method is developed in detail in [1-7]. It is an integral method based on the wave concept and it is used in solving problems of electromagnetic modeling. It is noted "WCIP" because it treats the waves instead of electromagnetic fields. It is called iterative because it establishes a recurrent relation between incident and reflected waves. This method is different from

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the other integral methods (method of moments, Galerkin method ...) because it does not use the scalar product or the matrix inversion. Thus, the method defines two operators; one is defined in the space domain and the other in the spectral domain. The fast Fourier mode transformation (FMT) and the reverse transformation FMT^{-1} insure the transition from one area to another. Applied in guided spaces, it allows us to define the impedance seen by the source of a waveguide. We use this method to study the electromagnetic modeling of a frequency selective surface (FSS) having a complex structure as in Fig. 1.

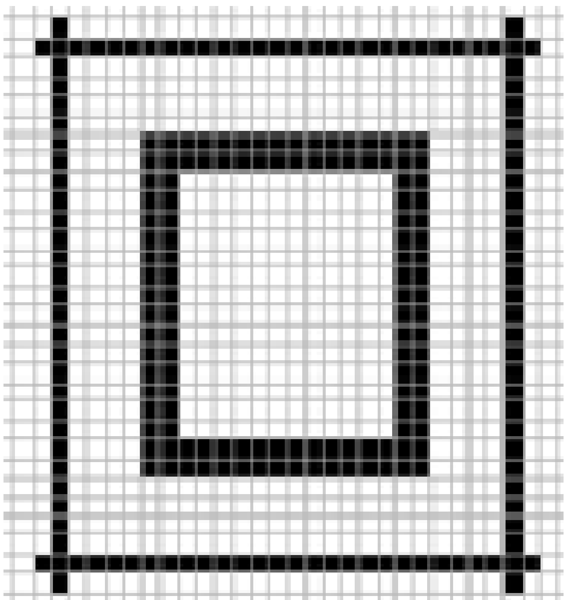

Fig. 1. Unit cell of an FSS structure.

The convergence of the WCIP method is reached after a maximum number of iterations called "Nmax". In our study, we will try to minimize the number of iterations calculated by this method to the value "Nmin" in order to have a fast and better convergence with a minimum calculation time. The remaining iterations until “Nmax” will be treated by a new adaptive filtering algorithm which provides a rapid convergence towards the best result with minimum error. This reduces the computation time and improves the convergence speed of this method. It is important to clarify the definition of the term "convergence speed" that will be the time to run the number of iterations required to converge "close enough" to the optimal result.

B. Functional blocks of the new algorithm

The new proposed algorithm is composed of two functional blocks as illustrated by Fig. 2 below. The first block is an autoregressive (AR) filter and the second is an adaptive filtering block.

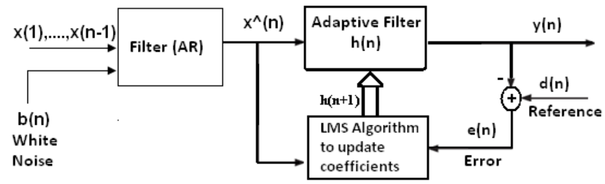

Fig. 2. Functional block of the new algorithm.

The AR filter block is used to model the input signal. This modeling is necessary to predict and regenerate the missing samples in an uncompleted input signal having length equal to “Nmin”. The prediction of $\hat{x}(n)$ at the instant “n” is from the signal at previous time [x(n-1), x(n-2), ...] in addition to the value of the white Gaussian noise at the same time “n”. Thus the prediction and estimation of samples $\hat{x}(n+1)$, $\hat{x}(n+2)$, $\hat{x}(n+3)$ ... can generate the following sequence samples from $\hat{x}(N\min+1)$ to $\hat{x}(N\max)$. We must choose the optimal order of the AR model that gives the best prediction of the input signal. It remains to estimate the coefficients of the AR filter, this is obtained from the equations of "Yule-Walker” for an AR filter; this uniquely defines the coefficients of the AR filter that are the most suitable for modeling the input signal. The iterations from “1” to “Nmin” are calculated by the classical WCIP method. As in the next equation, the prediction of the signal samples in the following iterations from “Nmin+1” to “Nmax” is realized by the AR filter using a Gaussian white noise $b(n)$:

$$\hat{x}(n) = \sum_{i=1}^{m} a(i)x(n-i) + b(n) , \tag{1}$$

where $\hat{x}(n)$ is an estimation of $x(n)$ in the iteration” n” and “a(1), a(2),…….., a(m)” are the coefficients of the AR filter in the following transfer function, the value “m” is the order of this filter :

$$H(z) = \frac{1}{(1+\sum_{k=1}^{k=m} a(k)z^{-k})} . \tag{2}$$

Concerning the second block, it is an adaptive filter whose coefficients are changing in function of external signals. A filter is called adaptive if its coefficients are modified and updated in each new sample of the input signal. As in Fig. 2, we have an input x(n), the desired response (reference) d(n) and the error e(n) which is the difference between d(n) and the filter output y(n). The error e(n) serves to control (adjust) the values of filter coefficients. To estimate y(n) from x(n) the adaptive filter uses the programmable coefficients h(n) but to estimate the next sample y(n+1) the filter uses the new coefficients h(n+1) which will be calculated by an adaptive algorithm as we show below. We use the adaptive filter to ensure a rapid convergence to the optimal value with a minimum square error.

In our study, we choose the LMS (least-mean-square) adaptive algorithm developed by Widrow and Hoff in 1959. This algorithm is certainly the most popular adaptive algorithm that exists due to its simplicity [11-12]. As in Fig. 2, the LMS algorithm updates the coefficients $h(n)$ of the adaptive filter transfer function in every new iteration as in the following relation:

$$h(n+1) = h(n) + \mu X(n)e(n) . \tag{3}$$

The coefficients $h(n) = [h_0(n), h_1(n), ...h_{L-1}(n)]^T$ are defined in the iteration "n" and the coefficients $h(n+1)$ are defined in the iteration "$n+1$". The input vector is: $X(n) = [x(n), x(n-1), ..., x(n-L+1)]^T$, L is the order of the adaptive filter, "μ" is the adaptation step. The error value "$e(n) = d(n) - y(n)$" is relative to a reference signal $d(n)$. As below, the value $y(n)$ is the output in the iteration "n":

$$y(n) = h^T(n)X(n) . \tag{4}$$

The adaptive filter coefficients "$h(n) = [h_0(n), h_1(n), ...h_{L-1}(n)]^T$" are changed in each new iteration until they become stable and equal to "$\mathrm{h_{opt}}$". That's how we define the stability and the convergence of the adaptive filter; it is represented by the next equation:

$$\lim_{n\to\infty} E\{\mathrm{h(n)}\} = \mathrm{h_{opt}} . \tag{5}$$

We aim at finding the condition of the stability and the convergence of the adaptive filter so we have to minimize the following function:

$$\mathrm{J(n)} = \left\|\mathrm{h(n+1) - h(n)}\right\|^2 . \tag{6}$$

We consider the following constraint:

$$\mathrm{h}^{\mathrm{T}}(n+1)X(n) = d(n) . \tag{7}$$

The solution of the problem is obtained by the multiplier λ of Lagrange. In fact, we want to minimize in reference to h(n+1) as in the following equations:

$$\mathrm{J(n)} = \left\|\mathrm{h(n+1) - h(n)}\right\|^2 + \lambda[d(n) - \mathrm{h}^{\mathrm{T}}(n+1)X(n)] , \tag{8}$$

$$\frac{\partial J(n)}{\partial h(n+1)} = 2[h(n+1) - h(n)] - \lambda X(n) = 0_{Lx1} . \tag{9}$$

We obtain the next result:

$$h(n+1) = h(n) + \frac{\lambda}{2} X(n) . \tag{10}$$

The constraint becomes as in the next equation:

$$d(n) = h^T(n+1)X(n) = h^T(n)X(n) + \frac{\lambda}{2} X^T(n)X(n) . \tag{11}$$

Also, we have:

$$d(n) = e(n) + y(n) = e(n) + h^T(n)X(n) . \tag{12}$$

Thus, we find the following expression of λ:

$$\lambda = \frac{2e(n)}{X^T(n)X(n} . \tag{13}$$

Finally, we obtain the equation of the LMS algorithm:

$$\mathrm{h(n+1) = h(n) + \frac{\lambda}{2} X(n) = h(n) + \frac{1}{X^T(n)X(n}X(n)e(n)} . \tag{14}$$

In fact, to have the best control of the filter coefficients updating, we introduce a positive factor α , $(0 < \alpha < 2)$:

$$h(n+1) = h(n) + \frac{\alpha}{X^T(n)X(n} X(n)e(n) . \qquad (15)$$

In comparison with equation (3), we obtain the following result:

$$\frac{\alpha}{X^T(n)X(n} = \frac{\alpha}{\sum_{l=0}^{l=L-1} x^2(n-l)} \approx \frac{\alpha}{L\sigma_x^2} = \mu . \ (16)$$

This value μ is called the adaptation step of the LMS algorithm. We consider the condition of α $(0 < \alpha < 2)$. It gives the next relation:

$$0 < \mu < \frac{2}{L\sigma_x^2} . \qquad (17)$$

This condition ensures the LMS algorithm convergence and the adaptive filter stability. So the best choice of the adaptation step μ provides the stability and the convergence of the LMS algorithm to the optimal results with minimal error as in equation (17). This choice of the adaptation step μ depends on the power σ_x^2 of the input signal and the adaptive filter order L. Thus, the performance of the LMS algorithm depends on three factors: the adaptation step μ, the power of the input signal, and the order L of the adaptive filter.

As shown in Fig. 3, the idea is to add to the classical WCIP algorithm a new algorithm describing an AR filter and an adaptive filter based on the LMS algorithm. Thus, the algorithm of the new method will be noted as an adaptive wave concept iterative process (A-WCIP). We introduce an input sequence that has a length equal to “Nmin”, the iterations of this sequence are calculated by the WCIP algorithm.

The new “A-WCIP” algorithm predicts the result of the remaining iterations until achieving the convergence to the optimal value with “Nmax” iterations.

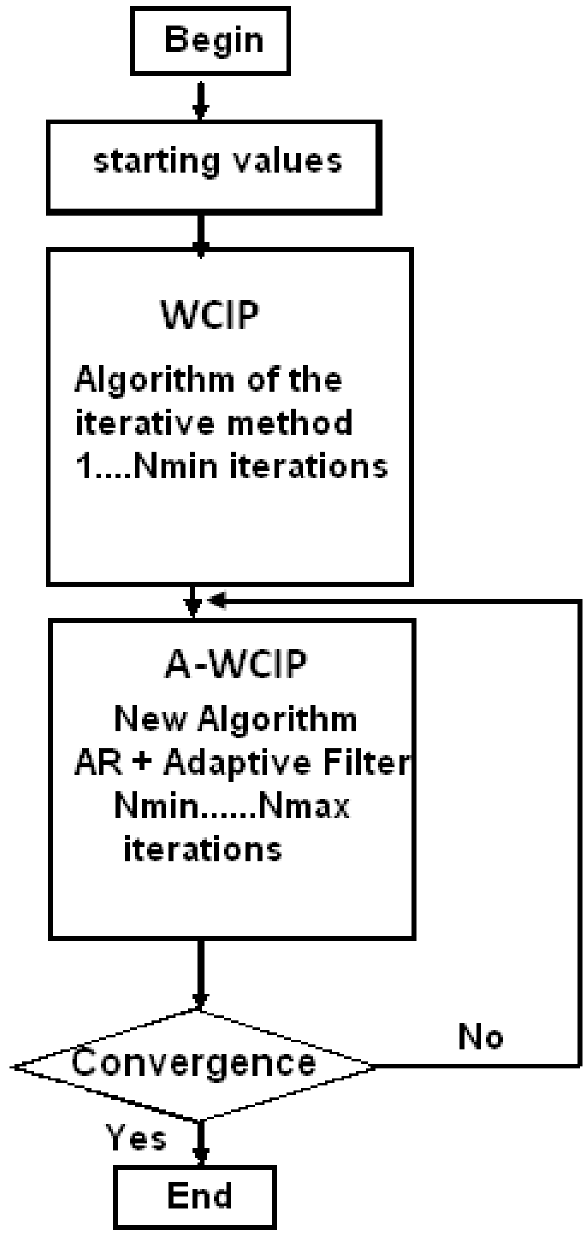

Fig. 3. The new “A-WCIP” approach.

The best conditioning of our system is done first by a good selection of the optimal order “m” of the AR filter. Then, it is done by a good choice of the adaptation step “ μ ” of the LMS algorithm, which depends on the input energy σ_x^2 and the order “ L ”of the adaptive filter. We conclude that the conditioning of this system is mainly based on the nature of the input signal. This is an important point of our approach because it ensures that the system is adapted to all types of input signals that vary from one frequency to another, especially that we are testing a wide frequency range. This provides stability and convergence of our system, whatever the conditions of the input signal.

Therefore, with the new algorithm, we improve the convergence of the classical iterative method which will calculate only a limited number of iterations equal to "Nmin". The number "Nmax" of iterations needed to reach the convergence will be achieved by the new A-WCIP algorithm which is a very rapid algorithm. Thus, an important gain in computing time will be accomplished by this new approach.

III. SIMULATIONS AND RESULTS

The input signal $x(n)$ in the last theoretical part will be designated in the next part by the coefficients of the diffraction matrix S_{11} or S_{21}.

A. Convergence improvement

In Figs. 4 and 5, the coefficients S_{11} and S_{21} are represented as a function of the number of iterations by both methods (“WCIP” and “A-WCIP”) in order prove that the new method gives also good results.

First, the classical WCIP method uses only the WCIP algorithm to calculate “Nmax=200” iterations. We conclude that the maximum number of iterations necessary to achieve the convergence is equal to 200 iterations. This number is relatively high because the electronic circuit studied has a complex structure. So in this case to have good results, the wave iterative method needs a big number of iterations (Nmax=200 iterations). Thus, this method takes much time to obtain the optimal result. So to reduce the computing time necessary to have a good result, we need to reduce the number of iterations calculated by the WCIP algorithm to the value “Nmin”. The remaining iterations are calculated by the new “A-WCIP” algorithm, which does not take much time to reach the convergence.

Then, we use the new “A-WCIP” method to calculate the same maximum number of iterations (Nmax=200 iterations). This new method is composed of two algorithms: the classical WCIP algorithm to which we add the new filtering algorithm. In the new “A-WCIP” approach, the number of iterations calculated by the classical WCIP algorithm is reduced to “Nmin=50” iterations. The maximum number of iterations is achieved by the new filtering algorithm (Nmax=200). The adaptive filter takes the output of the AR filter as input. As a result, the final output of the global new system “A-WCIP” must converge to the optimal values of S_{11} and S_{21} with minimum errors.

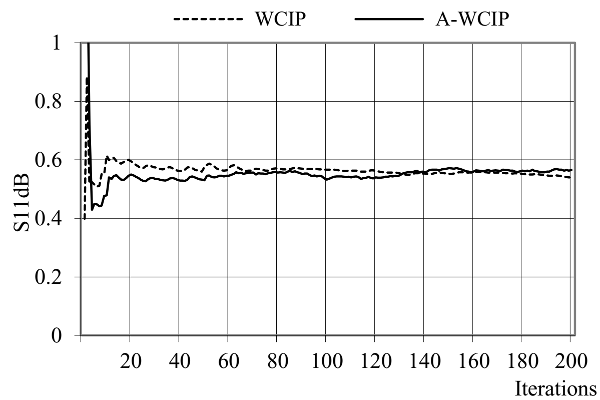

Fig. 4. Variation of S_{11} by both methods.

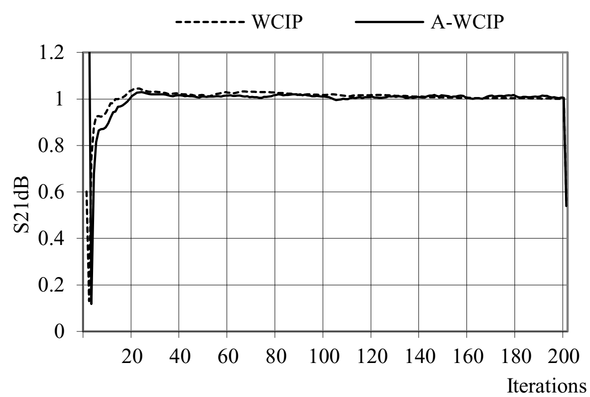

Fig. 5. Variation of S_{21} by both methods.

Here, we compare the results given by the two methods in order to prove that the new “A-WCIP” method provides good results. Of course, the new approach is faster than the classical one because it uses the WCIP algorithm to calculate “Nmin=50” iterations and it uses the filtering algorithm to calculate the remaining iterations until reaching the number “Nmax=200”. However, the classical method uses only the WCIP algorithm to calculate 200 iterations which takes much time.

B. Comparison in terms of computing time

In the next paragraph, we use the classical WCIP method to calculate 1000 iterations (Nmax=1000) and we use the new “A-WCIP” method to calculate the same number of iterations (Nmax=1000, Nmin=400). In Table 1, we observe an important gain in convergence time when calculating S_{11} and S_{21} values after 1000 iterations by the two methods. This gain of time is provided by the new "A-WCIP" method because we add a rapid adaptive filtering algorithm "LMS" in this method. The mesh used in this structure is 512x512 cells. We use a machine having a microprocessor Intel(R) Pentium(R) Dual Core CPU 2x2.16GHz and 3GB of RAM.

Table 1: Comparison of time between the two methods

The used method	**Computing time (mn)**
"A-WCIP"	10 mn
"WCIP"	24 mn
Gain of Time	58,33%

C. Comparison in terms of the average error

In Table 2, we represent the values of the average error calculated on the reflection and transmission coefficients S_{11} and S_{21}. The band of frequency is from 10GHz to 15GHz. The error is calculated when using the new "A-WCIP" filtering method in comparison with the classical WCIP method. We choose two different values of "Nmin" (25 and 50). The number "Nmin" represents the minimum number of iterations calculated by the WCIP algorithm in the new "A-WCIP" method. The maximum number of iterations is equal to 200 iterations. We find that the average error in comparison with the classical WCIP method is limited. This proves the effectiveness and robustness of our approach. Finally, we ensure the convergence to an optimum result very close to the desired value with a minimum average error in each frequency.

Table 2: Comparison in terms of the average error

Nmax	**Nmin**	**Average error S_{11} (dB)**	**Average error S_{21} (dB)**
200	25	2,782	0,893
200	50	1,646	0,399

D. Variation of S_{11} and S_{21} calculated by the new method in function of frequency

In Figs. 6 and 7, we represent the variation of the coefficients S_{11} and S_{21} in function of frequency. These coefficients are calculated by our new "A-WCIP" method. These results are compared with those calculated by the classical "WCIP" method in order to prove that our results are close to the best and optimal results. The maximum number of iterations calculated by the two methods is equal to 200 iterations (Nmax=200). In the new method, we test two values of "Nmin" (25 and 50 iterations) and we obtain good results in comparison with the WCIP method.

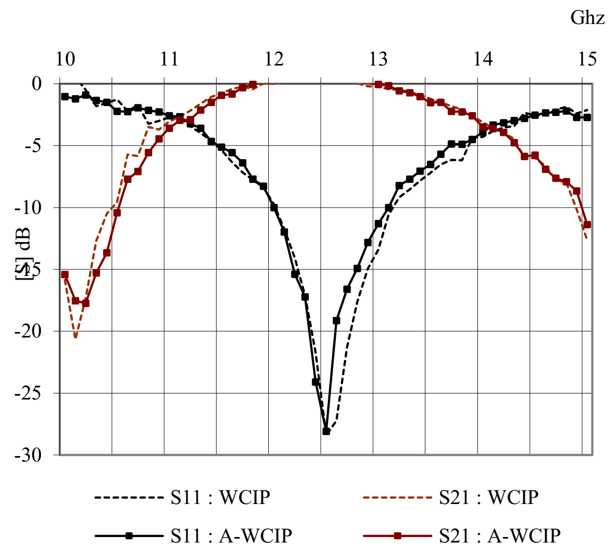

Fig. 6. S_{21} and S_{11}variations in function of frequency with Nmax= 200, Nmin= 25 iterations.

Thus in our new A-WCIP method, the WCIP algorithm is used to calculate only 25 or 50 iterations and the following iterations are calculated by the adaptive filtering algorithm until achieving 200 iterations. In the classical WCIP method, we use only the WCIP algorithm to calculate all the 200 iterations. If we compare the two methods, we conclude that the number of iterations in the classical WCIP algorithm is reduced from 200 to 25 iterations in the new approach. Thus, we achieve our principal goal which is the reduction of the number of iterations in the WCIP algorithm. That is why we obtain a good reduction of computing time in the new A-WCIP method. Finally, we obtain a fast convergence with minimum average error.

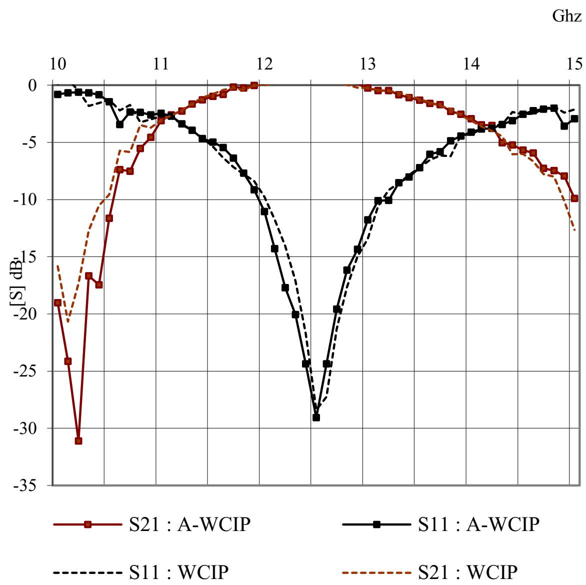

Fig. 7. S_{21} and S_{11}variations in function of frequency with Nmax= 200, Nmin= 50 iterations.

IV. CONCLUSION

In our study, the convergence speed of the classical WCIP algorithm has been improved. In the new "A-WCIP" method, the WCIP algorithm has to calculate only a minimum number "Nmin" of iterations that can be reduced from 200 to 25 iterations. The remaining iterations after "Nmin" are treated by the new filtering algorithm to achieve the convergence to the optimal value after "Nmax=200" iterations. Thus, we have a very fast convergence in comparison with the classical WCIP method in which the WCIP algorithm calculates all the 200 iterations. Finally, a very significant reduction in computing time has been obtained. Thus, we ensure a rapid convergence with a limited average error hence the efficiency and robustness of our new approach.

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