Numerical Modeling of Reconfigurable RF MEMS-based Structures Involving the Combination of Electrical and Mechanical Force

Kohei Kawano Shafrida Sahrani Takashi Mori Michiko Kuroda
kuroda@cc.teu.ac.jp
School of Computer Science, Tokyo University of Technology, Hachioji, Tokyo 192-0982, Japan

Manos M. Tentzeris
School of ECE, Georgia Institute of Technology, Atlanta, GA 30332-250, USA

Abstract

MEMS are minimized electromechanical devices and systems that are realized using integrated micro fabrication methods. And the technology is growing rapidly in RF field, because of the advantages over p-i-n diode or FET switches. The main application areas of MEMS devices in the future are Information Technology, Bioelectromagnetic, Medical Science. For the accurate design of RF MEMS structures, effective computationally modeling of their transient and steady state behaviors including the accurate analysis of their time-dependent moving boundaries is essential. This is because an accurate knowledge of the electromagnetic field (EM) evolution around a moving or rotating body is very important for the realization of new optical devices or microwave devices, such as the RF-MEMS structures used in phase-shifters, couplers, filters, tuners or antennas. The technique proposed in this paper to model MEMS structures is based on the finite-difference time-domain (FDTD) method with an adaptive implementation of grid generation. Here, this simulation method is applied to the analysis of a two-dimensional MEMS variable capacitor with non-uniform motions, such as accelerated motions. The acceleration of the MEMS capacitor is derived under the equilibrium between the spring force and electrical force. Using this acceleration, the motion characteristic for each time step is derived. The numerical results that express the relationship between the acceleration of the plates and the spring constant and the mass of the plates are shown and the transient effect is accurately modeled.

1. Introduction

As compared with PIN diode and Transistor switches, RF MEMS have many excellent advantages such as high isolation and low power consumption, and, as a result, MEMS technology is growing rapidly in RF field [1]-[3]. An addition, RF MEMS have many application areas, such as switches, antennas or tuners. For the modeling and optimization of microwave devices, as in the case of the RF MEMS structures used in phase-shifters, couplers, filters, tuners or antennas, an accurate knowledge of the electromagnetic field distribution around a moving or rotating body is required. But due to the limitations of the conventional numerical techniques for the time changing boundaries, it is tedious to solve these problems numerically for the electromagnetic fields [4]. Computational techniques for moving boundary problems have been pursued mainly in heat and fluid flow area [5]-[8]. In this paper, we propose a new numerical approach for the analysis of this type of problems that alleviates the limitations of the conventional time-domain techniques. This method is expanded to analyze MEMS devices with moving parts with the FDTD method for EM fields [9]-[10]. Employing a transformation in the time factor, it is possible to apply the grid generation technique of [11] to the time-domain analysis of a moving object. With such a grid, the FDTD method [12] can be solved very easily on a "static" (time-invariant) rectangular mesh regardless of the shape and the motion of the physical region, something that makes it an especially good tool to analyze structures of arbitrary shape and motion.

In this paper, this simulation method is applied to the analysis of a two-dimensional MEMS variable capacitor with accelerated motion. The acceleration of the plates are derived from the equilibrium between the spring force and electrical force. Using this acceleration, the relation between the mass, the spring constant and the oscillation of the plates are shown. This acceleration is useful in determining the switching time of the MEMS device. For the validation of this method, the computational results of the transient capacitance are compared with the theoretical results.

2. Two-Dimensional Variable Capacitor with Acceleration Effect

The dynamic behavior of the MEMS structure is shown in Fig. 1. The top plate is suspended by a spring.

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Under the combined effect of mechanical and electrical force, the top plate moves until the equilibrium between the electrostatic and mechanical forces is reached. F_m means spring force and F_e means electrostatic force, that is defined as the gradient of the stored energies and these forces are expressed in the following equations, respectively,

$$F_m = mx''+bx'+kx\,, \tag{1}$$

$$F_e = \frac{1}{2}\frac{\partial C}{\partial x}V^2\,, \tag{2}$$

where m is the mass of the plate, b is the mechanical resistance, k is the spring constant, and V is the bias voltage. From the equilibrium between the spring force and the electrostatic force, the following equation is derived,

$$mx'' = \frac{1}{2}\frac{\partial C}{\partial x}V^2 - bx' - kx\,. \tag{3}$$

From eq. (3), the acceleration x'' is obtained.

The geometry to be considered here is shown in Fig. 2. Under the combined effect of mechanical and electrical force, the two fingers are assumed to move with different velocities for the x-direction. For the two-dimensional TM-propagation case, as shown in Fig. 2, there are only Ex, Ey, and Hz nonzero components with a time variation given by the following equations,

$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\right), \tag{4}$$

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial y} - J_x\right), \tag{5}$$

$$\frac{\partial E_y}{\partial t} = -\frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial x} + J_y\right), \tag{6}$$

where ε, μ are the constitutive parameters of the respective medium. The configurations of the physical and of the computational regions are shown in Fig. 2.

Employing the transformation with the time factor, the partial differential equation in the physical region (x, y, t) is related to the computational region (ξ, η, τ) as follows:

$$x = x(\xi, \eta, \tau)\,, \tag{7}$$

$$y = y(\xi, \eta, \tau)\,, \tag{8}$$

$$t = t(\xi, \eta, \tau)\,. \tag{9}$$

The inverse transformation is given by

$$\xi = \xi(x, y, t)\,, \tag{10}$$

$$\eta = \eta(x, y, t)\,, \tag{11}$$

$$\tau = \tau(x, y, t)\,. \tag{12}$$

According to the transformation, the first derivatives are transformed as follows,

$$\begin{bmatrix} \partial/\partial x \\ \partial/\partial y \\ \partial/\partial t \end{bmatrix} = K \begin{bmatrix} \partial/\partial \xi \\ \partial/\partial \eta \\ \partial/\partial \tau \end{bmatrix}. \tag{13}$$

The inverse transformation is given by,

$$\begin{bmatrix} \partial/\partial \xi \\ \partial/\partial \eta \\ \partial/\partial \tau \end{bmatrix} = L \begin{bmatrix} \partial/\partial x \\ \partial/\partial y \\ \partial/\partial t \end{bmatrix} \tag{14}$$

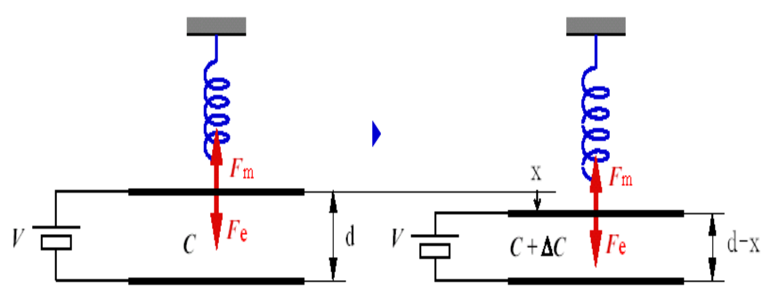


Fig. 1. Functional model of MEMS capacitor

where the matrices K and L are given by

$$K = \begin{bmatrix} \partial\xi/\partial x & \partial\eta/\partial x & \partial\tau/\partial x \\ \partial\xi/\partial y & \partial\eta/\partial y & \partial\tau/\partial y \\ \partial\xi/\partial t & \partial\eta/\partial t & \partial\tau/\partial t \end{bmatrix} \tag{15}$$

and

$$L = K^{-1} = \begin{bmatrix} \partial x/\partial\xi & \partial y/\partial\xi & \partial t/\partial\xi \\ \partial x/\partial\eta & \partial y/\partial\eta & \partial t/\partial\eta \\ \partial x/\partial\tau & \partial y/\partial\tau & \partial t/\partial\tau \end{bmatrix}. \tag{16}$$

By this transformation, there is a unique correspondence between the computational region and the physical region. The transformed region can be easily solved in the rectangular computational region by FD-TD method.

Under the combined effect of mechanical and electrical force, the plates are assumed to move for x-direction with velocities v and u, and the acceleration a_v, a_u, respectively. Using a coordinate transformation technique, written from eq. (7) to eq. (16), the time-changing physical region (x,y,t)
can evolve to a time-invariant computational domain. For the geometry of Fig. 2, the transform equations between the physical and the computational regions are chosen as:

$$\xi = \frac{x - h_n(t)}{h_{n+1}(t) - h_n(t)}, \tag{17}$$

$$\eta = \frac{y - y_m(t)}{y_{m+1}(t) - y_m(t)}, \tag{18}$$

$$\tau = t, \tag{19}$$

where n=1,2,3, m=1,2,3, and $h_1(t), h_2(t), h_3(t), h_4(t)$ are written in the following form assuming that the plate accelerations and velocities remain time changing values for the whole time of their motion,

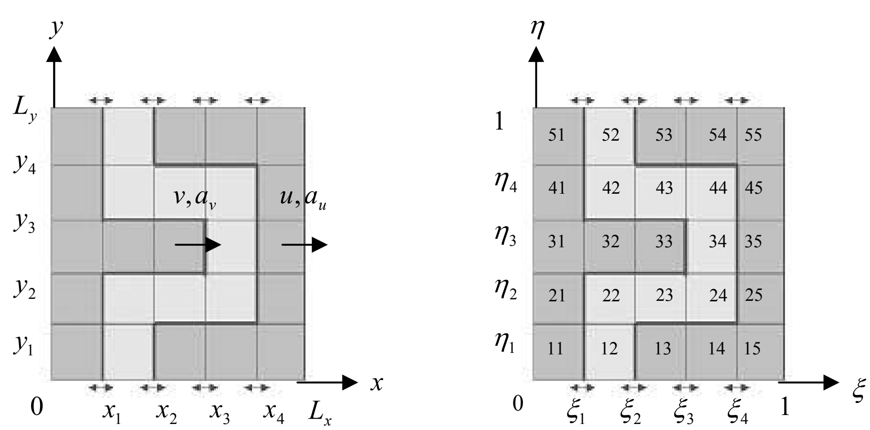
$$h_1(t) = x_1 + vt + \frac{1}{2}a_v t^2, \tag{20}$$

$$h_2(t) = x_2 + ut + \frac{1}{2}a_u t^2, \tag{21}$$

$$h_3(t) = x_3 + vt + \frac{1}{2}a_v t^2, \tag{22}$$

$$h_4(t) = x_4 + ut + \frac{1}{2}a_u t^2. \tag{23}$$

The functions $h_1(t)$, $h_2(t)$, $h_3(t)$, $h_4(t)$ describe the movement along the x axis, and allow for the realization of a rectangular grid with stationary boundary conditions. The partial time-derivatives in the transformed domain (ξ,η,τ) can be expressed in terms of the partial derivatives of the original domain (x,y,t) using eqs. (17)-(23). The FDTD technique can provide the time-domain solution of the rectangular (ξ,η,τ) grid. The stability criterion in this case is chosen as $c\Delta t \le \delta/\sqrt{2}$, where $\delta = \Delta x_0 = \Delta y_0$, assuming the grid is uniformly discretized in both directions. In general, δ is a space increment for x and y direction when the grid increment is minimum (minimum cell size).



(a) (b)

Fig. 2. (a) Physical region and (b) computational region.

3. Numerical Results

To validate this approach, numerical results are calculated for a two-dimensional variable capacitor with the movement of the finger only to the *x*-direction. The grid includes 200 x 200 cells, input frequency is f = 20 GHz and $L_x = L_y = L_z = L = 5\lambda$, $\Delta x = \Delta y = L/200$, and $\Delta t = 3.125 \times 10^{-13}$ (sec). The initial plate separation is $L/5$ and the grid is terminated with Mur's absorbing boundary conditions. Here, the left plate is assumed to move due to the coupling of the electrostatic and the

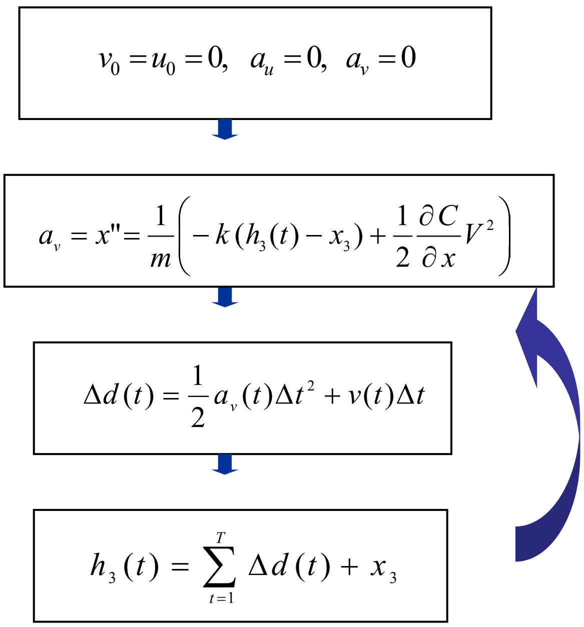


Fig. 3. Computational Algorithm.

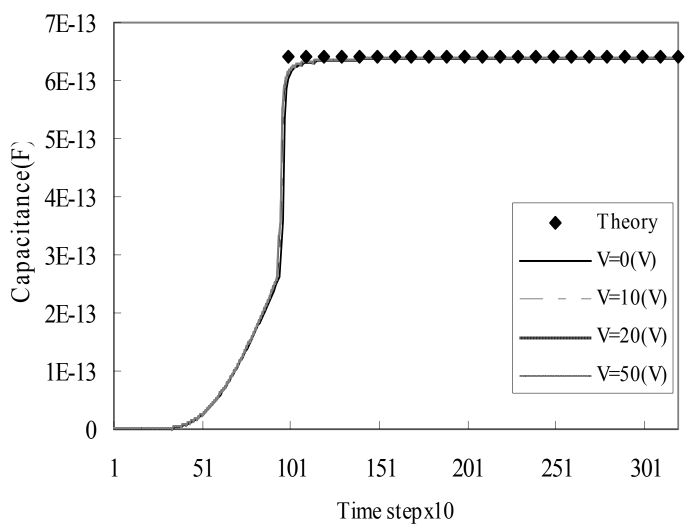
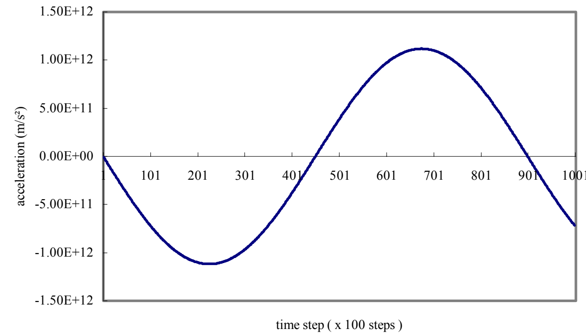


Fig. 4. Time dependence of the capacitance for bias voltage in the range of V=0 (V) to 50 (V) are compared with the theoretical result.

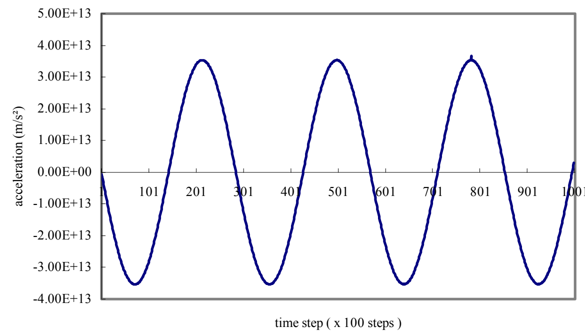
mechanical forces. Fig. 3 displays the computational algorithm used in this calculation. Initial values of the velocity are assumed $u_0=0$, $v_0=0$, and $a_u=0$. From eq.(3), the acceleration value is derived. Inputting this acceleration value into the transform function $h_1(t)$, $h_3(t)$, the new capacitance C and displacement Δd, transformation function $h_1(t)$, $h_3(t)$, and acceleration a_v are obtained. Then from the capacitance and displacement, the new acceleration is obtained. Iterating this algorithm, it is easy to obtain the capacitance, acceleration and displacement controlled by the coupling of spring and electrostatic force.

Numerical results are given in Fig. 4, Fig. 5, and Fig. 6. Fig. 4 shows the computational results of the time dependence of the capacitance for bias voltage values in the range of $V=0(V)$ to $V=50(V)$, for a motion lasting 3010 time steps. The stationary values, when the velocity is zero, are displayed as a reference, and show that stationary values have good agreement with the theoretical values. Fig. 5 displays the time dependence of the acceleration for each mechanical resonant frequency values in the range of $\omega=\sqrt{k/m}=10^{17/2}$ to $\omega=\sqrt{k/m}=10^{19/2}$, when the plate moves away from the bottom one. The mechanical resonant frequency in Fig. 5(c) is $\sqrt{10}$ times of the resonant frequency in Fig. 5(b) and 10 times of the resonant frequency in Fig. 5(a). According to these values, the ratio of the resonant frequency is shown accurately in each figure. In Fig. 5, when the mechanical resonant frequency is low, the amplitude remains almost constant value. In Fig. 6, time dependence of the acceleration, the velocity and the displacement are shown, where $\omega=10^{17/2}$, $V=10(V)$. It is effectively demonstrated that when the value of the acceleration is zero, the velocity value takes on a stationary value and, conversely, when the value of the velocity is zero, the value of the acceleration takes on a stationary value.

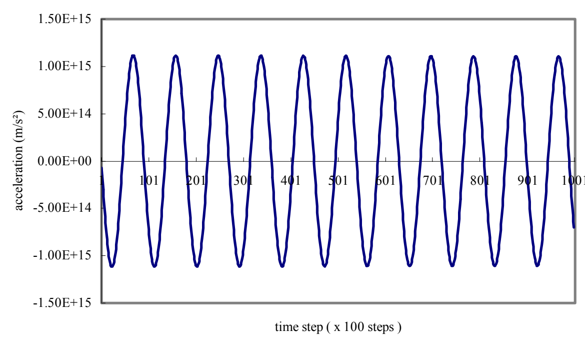
Since the typical MEMS device has low mass around 10^{-10} kg and spring constant around 5-30 N/m, the mechanical resonant frequency is 30-100 kHz and bias voltage is 10-30 V [13]. In this paper, the values of the mechanical resonant frequency are in the order of 10^7, but if a longer computation time is taken, it is possible to obtain results around the order of 10^5. The results derived in this paper for acceleration and mechanical resonant frequency are very important for determing the switching time.



(a)

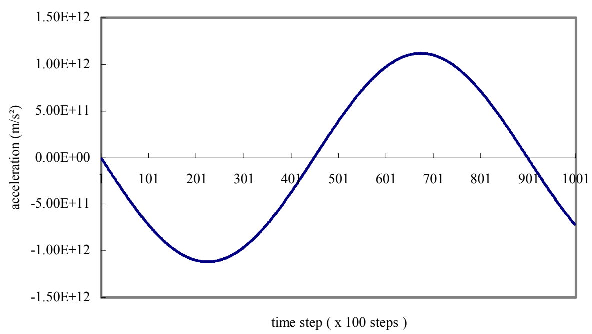


(b)

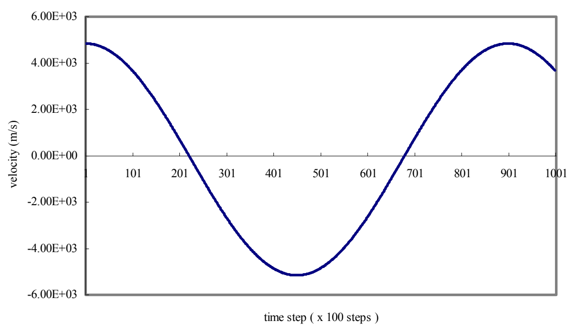


(c)

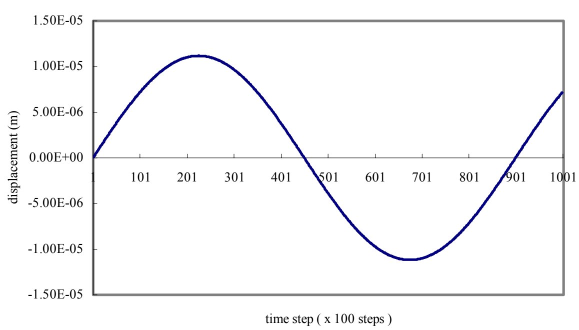
Fig. 5. Time dependence of the acceleration for each resonant frequency (a) $\omega=10^{17/2}$, (b) $\omega=10^{9}$, and (c) $\omega=10^{19/2}$ for $V=10(V)$.



(a) acceleration



(b) velocity



(c) displacement

Fig. 6. Time dependence of the (a) acceleration, (b) velocity, and (c) displacement $\omega = 10^{17/2}$ for $V = 10(V)$.

Conclusions

A novel time-domain modeling technique that has the capability to accurately simulate the transient effect of variable capacitors with accelerated motion controlled by the coupling of the electrostatic and mechanical forces is proposed. This technique is a combination of the FDTD method and the body fitted grid generation technique. The key point of this approach is the enhancement of a space and a time transformation factor that leads to the development of a time-invariant numerical grid. Using this technique, the numerical results of the relation between the acceleration, the velocity and the displacement of the motion are shown for a MEMS capacitor that demonstrate its unique computational advantages in the modeling of microwave devices with moving boundaries.

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Kohei Kawano: He received the B. E. degree and the M. E. in Information Networks from Tokyo University of Technology, Tokyo, Japan. Now he is a research student at Tokyo University of Technology. His research interests are FDTD method and grid generation method.

Shafrida Sahrani: She received the B.E. degree in Information Networks from Tokyo University of Technology, Tokyo Japan. Now she is a graduate student.

Takashi Mori: He received the B.E. degree in Information Networks from Tokyo University of Technology, Tokyo Japan. Now he is a graduate student.

Michiko Kuroda Professor Michiko Kuroda received the B. E. degree in Electrical Engineering from Shizuoka University, Shizuoka, Japan and the M. E. and D. E. in Electrical Engineering from Waseda University, Tokyo, Japan. She was a visiting researcher at the ElectroScience Laboratory, the Ohio State University, OH from 1986 to 1987. She subsequently became an Associate Professor at Tokyo University of Technology. Since 1998, she has been a professor at the department of Information Networks in 2002. Now she is a professor of the school of Computer Science. Since 2003, she has been a member of the steering committee of Tokyo University of Technology. Since 1994, she has been a Correspondent of URSI (International Union of Radio Science) in Japan. Since 2004, she has been a scientific advisor of Guidance on the Ministry of Public Management, Home Affairs, post and Telecommunications. Since 2004, she has been_ an advisory committee member of the Saitama Prefecture in Japan and a board of director of ACES. . Her research interests are electromagnetic theory, and numerical methods including FDTD method and grid generation method. She proposed this novel numerical method for the analysis of the moving boundary problems and its application for the analysis of MEMS devices. She is a member of ACES and the IEEE Antennas and Propagation and Microwave Theory and Techniques Societies and OSA, IEICE.

Manos M. Tentzeris Professor Manos M. Tentzeris received the Diploma Degree in Electrical and Computer Engineering from the National Technical University of Athens ("Magna Cum Laude") in Greece and the M.S. and Ph.D. degrees in Electrical Engineering and Computer Science from the University of Michigan, Ann Arbor, MI and he is currently an Associate Professor with School of ECE, Georgia Tech, Atlanta, GA. He has published more than 200 papers in refereed Journals and Conference Proceedings and 8 book chapters and he is in the process of writing 3 books. Dr. Tentzeris has helped develop academic programs in Highly Integrated/Multilayer Packaging for RF and Wireless Applications, Microwave MEM's, SOP-integrated antennas and Adaptive Numerical Electromagnetics (FDTD, MultiResolution Algorithms) and heads the ATHENA research group (15 researchers). He is the Georgia Tech NSF-Packaging Research Center Associate Director for RF Research and the RF Alliance Leader. He is also the leader of the RFID Research Group of the Georgia Electronic Design Center (GEDC) of the State of Georgia. He was the recipient of the 2006 IEEE MTT Outstanding Young Engineer Award, the 2004 IEEE Transactions on Advanced Packaging Commendable Paper Award, the 2003 NASA Godfrey "Art" Anzic Collaborative Distinguished Publication Award for his activities in the area of finite-ground low-loss low-crosstalk coplanar waveguides, the 2003 IBC International Educator of the Year Award, the 2003 IEEE CPMT Outstanding Young Engineer Award for his work on 3D multilayer integrated RF modules, the 2002 International Conference on Microwave and Millimeter-Wave Technology Best Paper Award (Beijing, CHINA) for his work on Compact/SOP-integrated RF components for low-cost high-performance wireless front-ends, the 2002

Georgia Tech-ECE Outstanding Junior Faculty Award, the 2001 ACES Conference Best Paper Award and the 2000 NSF CAREER Award for his work on the development of MRTD technique that allows for the system-level simulation of RF integrated modules and the 1997 Best Paper Award of the International Hybrid Microelectronics and Packaging Society for the development of design rules for low-crosstalk finite-ground embedded transmission lines. He was also the 1999 Technical Program Co-Chair of the 54th ARFTG Conference, Atlanta, GA and the Chair of the 2005 IEEE CEM-TD Workshop and he is the Vice-Chair of the RF Technical Committee (TC16) of the IEEE CPMT Society. He has organized various sessions and workshops on RF/Wireless Packaging and Integration in IEEE ECTC, IMS and APS Symposia in all of which he is a member of the Technical Program Committee in the area of "Components and RF". He is the Associate Editor of IEEE Transactions on Advanced Packaging. Dr.Tentzeris was a Visiting Professor with the Technical University of Munich, Germany for the summer of 2002, where he introduced a course in the area of High-Frequency Packaging. He has given more than 40 invited talks in the same area to various universities and companies in Europe, Asia and America. He is a Senior Member of IEEE, a member of URSI-Commission D, an Associate Member of EUMA and a member of the Technical Chamber of Greece.