A Critical Examination of Receive and Transmit Scan Element Pattern for Phased Arrays

(*Invited Paper*)

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***Abstract* —** The measurement of Scan Element Pattern by exciting only the center element is evaluated, both for gain at broadside and for behavior versus scan (normalized at broadside). A large dipole array of 50 × 50 elements is used in a 64 bit computer test bed, for calculations including mutual impedance. These results are compared with those where all elements are properly excited. A simple rigorous derivation of SEP including impedance mismatch is presented.

***Index Terms* —** Scan Element Pattern, phased arrays, array measurements.

I. INTRODUCTION

Scan Element Pattern, (SEP), (formerly active element pattern[1]) was developed circa 1960 in [1–4], to provide phased array gain behavior versus scan angles. Its utility for decades has been to give insight and results on the scan performance of various elements and lattices. A common but incorrect measurement procedure terminates all elements in the array, with the excited center element connected to a gain measurement setup. It was recognized in the Lincoln Lab reports [1] that the impedance seen in the measurement was not the *scan impedance* (SI), (impedance seen when all elements are excited with the proper amplitude and phase), due to the passive mutual couplings. The textbook definition of gain was used in the derivation provided in [1] where the scan impedance mismatch loss was not included. Hannan included this mismatch, but his formulas were based on "intuitive reasoning" [3].

Clearly his SEP which is proportional to $\cos\theta$ is only an approximation [3, 5], which fails for large scan angles and for some types of elements at all angles. The excited center element procedure does not include the correct *scan impedance*, Z_s and it does not accurately yield the correct SEP. It is useful to examine closely how the 1960 results were obtained. In [1, 6], all mutual impedances were set to zero, and a zero order inversion of the impedance matrix was used, resulting in the array gain equal to N times the SEP. Hannan used superposition to produce the same result [3]. Superposition, as defined by Silver in [7], states that when currents are added the fields produced by the currents are added. There are N sets of currents; each set includes a current at the driven element and currents at all the other elements. In each current set a different element is driven. Unfortunately in any current set, none of the currents are what they would be if all elements were excited. Thus superposition is not useful: each current set produces incorrect voltages, and the sum of sets of incorrect voltages is also incorrect. Phased array books [8, 9] also use this incorrect formulation.

This paper determines the utility and inaccuracy of the excited center element procedure, herein called transmit SEP, and compares it with results from receive SEP. Both are simulated in computer programs.

First a rigorous derivation of scan element pattern is presented. It is similar to Lincoln Labs circa 1960 derivations, except that impedance mismatch is included.

II. DERIVATION OF SCAN ELEMENT PATTERN

Consider a linear or planar array with N elements. The *scan element pattern* is the gain per element at the peak of the scanned beam. All phase factors are considered zero, thus the array gain is written as:

$$G(\theta,\phi)=\frac{4\pi r^2\left|E(\theta,\phi)\cdot H(\theta,\phi)\right|}{P} \tag{1}$$

where P is the radiant power and in terms of electric field only,

$$G(\theta,\phi)=\frac{r^2E(\theta,\phi)\cdot E^*(\theta,\phi)}{30P}. \tag{2}$$

For any wire or patch antenna element, E is a constant times current times isolated pattern, divided by r. In particular for dipoles,

[1] This terminology is deprecated as "active" carries electron device connotation.

$$E(\theta,\phi) = \frac{60F(\theta,\phi)}{r}\sum^{N} I_n \,. \quad (3)$$

Here the sum is over the elements of the array, and the pattern function F is,

$$F(\theta,\phi) = \frac{\cos kh - \cos(kh\cos\theta)}{\sin kh\sqrt{1-\cos^2\theta\cos^2\phi}} \,. \quad (4)$$

The dipoles are along the x-axis and in the x-y plane. Thus the array axis is for $\phi = 0$. Dipole half-length is h, and k= $2\pi/\lambda$.

Power into the array, without matching or source impedance, is simply,

$$P = \sum^{N} I_n I_n^{*} R_{sn} \quad (5)$$

where R_{sn} is the *scan resistance* of the nth element. Gain equation now takes the form

$$G(\theta,\phi) = \frac{120F^2(\theta,\phi)\sum^{N} I_n \sum^{N} I_n^{*}}{\sum^{N} I_n I_n^{*} R_{sn}} \,. \quad (6)$$

Since the *scan element pattern* is gain per element, one gets

$$SEP = \frac{120F^2(\theta,\phi)\sum^{N} I_n \sum^{N} I_n^{*}}{N\sum^{N} I_n I_n^{*} R_{sn}} \,. \quad (7)$$

This can be written in terms of the isolated element gain g_{iso}, where

$$g_{iso}(\theta,\phi) = \frac{120F^2(\theta,\varphi)}{R_{iso}} \,. \quad (8)$$

Thus SEP becomes,

$$SEP(\theta,\phi) = \frac{R_{iso} g_{iso}(\theta,\phi)\sum^{N} I_n \sum^{N} I_n^{*}}{N\sum^{N} I_N I_N^{*} R_{sn}} \,. \quad (9)$$

For a large array most element impedances will be the same thus SEP reduces to,

$$SEP(\theta,\phi) = \frac{R_{iso} g_{iso}(\theta,\phi)\sum^{N} I_n \sum^{N} I_n^{*}}{NR_s \sum^{N} I_n I_n^{*}} \,. \quad (10)$$

For a large uniformly excited array the currents will also be nearly equal. Thus the infinite array SEP becomes,

$$SEP(\theta,\phi) = \frac{R_{iso} g_{iso}(\theta,\phi)}{R_s} \,. \quad (11)$$

Power transfer to, or from, each element is affected by the load impedance Z_l and matching impedance. For a complex load /match Z_l, and resistance R_l, the power transfer, compared to perfectly matched transfer, is,

$$\frac{P_l}{P_{noload}} = \frac{4R_s R_l}{(Z_s + Z_l)(Z_s + Z_l)^{*}} \,. \quad (12)$$

Again for large arrays the SEP reduces to

$$SEP(\theta,\phi) = \frac{4R_{iso} R_l g_{iso}(\theta,\phi)}{(Z_s + Z_l)(Z_s + Z_l)^{*}} \,. \quad (13)$$

The new factors represent power transfer with a reflection coefficient modified for complex load impedance,

$$\frac{4R_s R_l}{(Z_s + Z_l)(Z_s + Z_l)^{*}} = 1 - \left|\Gamma_{*}\right|^2 \quad (14)$$

where,

$$\Gamma_{*} = \frac{Z_s^{*} - Z_l}{Z_s + Z_l} \,. \quad (15)$$

Note that for all real values of load-match, these equations revert to the usual ones.

Going back to equation (10), an alternate from, appropriate for computer analysis is

$$SEP(\theta,\phi) = \frac{120R_{isom}F^2(\theta,\phi)\sum^{N} I_n \sum^{N} I_n^{*}}{R_s(\theta,\phi)\sum^{N} I_n I_n^{*}} \times \left[1 - \left|\Gamma_{*}(\theta,\phi)\right|^2\right]. \quad (16)$$

where R_s is the *scan resistance*.

The only approximation in (16) is that all element *scan resistances* are equal, which affect the power sum and the mismatch factor. For arrays that are not large, the mismatch factor can be calculated for each element, and then averaged. The effectiveness of this will be shown in Section 4.

III. MODELLING TRANSMIT SEP MEASUREMENT

To determine exactly how the transmit SEP measurement process behaves with angle and frequency, a planar array code using thin dipoles on a square lattice, is employed. This code treats a finite square array, up to 100 on elements each axis, thereby replicating the measurement procedure. As the dipoles are thin, and the maximum length is half-wave, the current distribution is very closely sinusoidal, thus Moment Methods solution is not necessary. All elements were terminated with a resistance, and the center element was excited. Solution of the mutual impedance matrix equation gives the complex currents for all the array elements. Mutual impedances and matrix inversion were calculated in double precision. Far field pattern was calculated by summing the element currents times the appropriate steering phases times the elements pattern. Power was calculated from Real (VI) for the driven element. Gain is simply $12E^2/P^*(1-|\Gamma|^2)$, where Γ is the reflection coefficient. Two matching impedances were used: one for an infinite array of excited dipoles, and the other for only the center element excited. Calculations were performed on an HP 64 bit UNIX workstation.[2]

Results are given for a 50 × 50 element array (2500 unknowns) in Fig.1. The values of SEP (gain per element) are normalized to zero dB at broadside as the absolute values are incorrect due to the passive mutual impedances. Absolute values are as discussed in Section 5. The broadside array impedance of 63 + j23 Ohms is used for matching. In comparison with the well-known infinite array results [9], the transmit SEP values are roughly 2 dB higher in the range of θ from 40 to 80 degrees for the E-plane. H-plane results are slightly higher. The departure from absolute gain is much worse than would be indicated by the modest change in the centre element impedance. The mutual coupling significantly reduces the current magnitudes, thus decreasing the gain per element. Significant oscillations in the E-plane SEP are due to edge effects, even for such a large array [10]. Note that in fig. 1 the E-plane SEP is higher than the H-plane, which is contrary to the infinite array Floquet results [9].

With the ground screen added, at a spacing equal to half the dipole spacing, the transmit SEP is as shown in Fig. 2. The broadside array impedance of 70 + j58 ohms is again used as match impedance. The E-plane result departs markedly from the infinite array result; the H-plane values are higher than the infinite array results to about 60 degrees, and lower for large angles. Note the large change in embedded impedance from the infinite array value of 153 + j32.

[2] CPU chips optimized for floating point operations tend to be several times as fast as PC chips optimized for integer handling, all for the same clock rate.

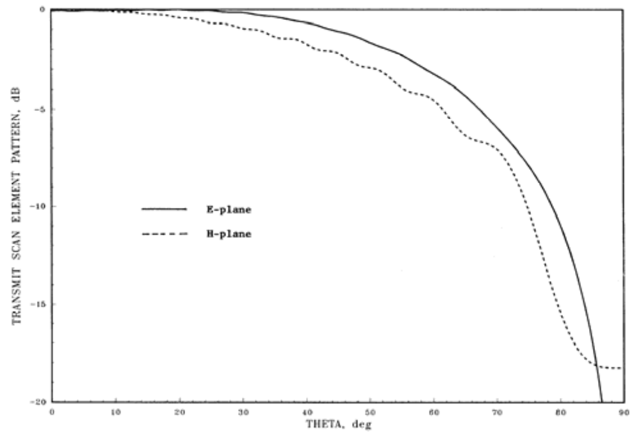

Fig. 1. A 50 × 50 dipole array, L = Dx = Dy = 0.5λ.

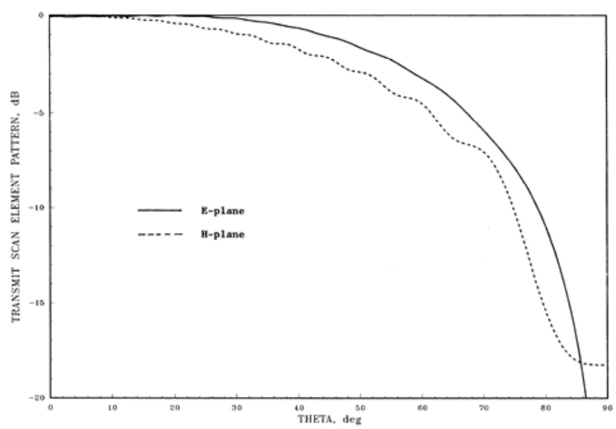

Fig. 2. A 50 × 50 dipole/screen array, L = Dx = Dy = 0.5λ, h = 0.25λ.

Not only is the absolute SEP at broadside is incorrect, but the scan performance, normalized to 0 dB at broadside, is not good predictor of array gain versus scan.

IV. MODELLING RECEIVE SEP MEASUREMENT

The same computer model was used to simulate the receive SEP measurement. All elements were excited by unit amplitude voltage and the proper scan phase. *Scan element pattern* was calculated again from $120E^2/P$ x $(1-|\Gamma|^2)$. Figure 3 shows SEP, in an absolute value.

A matched array would have an SEP of $2\pi A/N\lambda^2$, which is $\pi/2 = 1.97$ dB for the half wave case of Fig. 3. The E-plane curve is a fair fit (but slightly lower) to infinite array results out to about 70 degrees, but there are edge effect oscillations. The H-plane curve is slightly higher for large angles. Calculations of 20 x 20 and 30 x 30 arrays (not shown) indicate that as the array size is larger, the match at large θ becomes better.

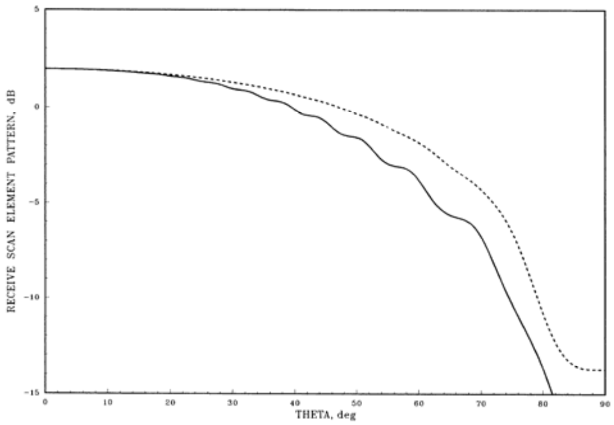

Fig. 3. A 50 × 50 dipole array, L = Dx = Dy = 0.5λ.

Figure 4 gives SEP with a ground screen, again spaced from the screen half the dipole spacing. The E-plane SEP contains large oscillations, building up as θ increases. These oscillations occur as the *scan impedance* of the center element oscillates about the infinite array value. An average curve through the oscillations matches well the infinite array data. H-plane data are roughly 1 dB high for angles larger than 50 deg.

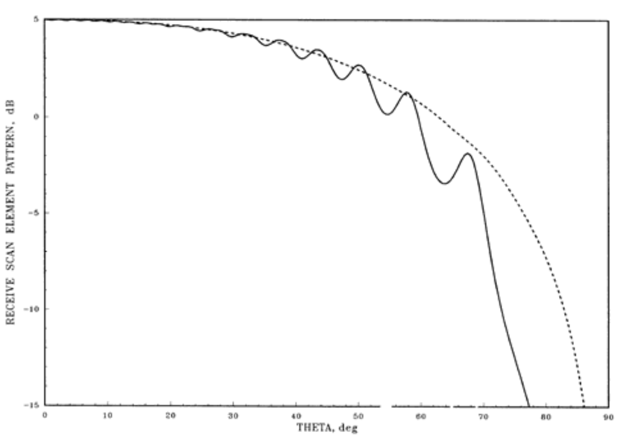

Fig. 4. A 50 × 50 dipole/screen array, L = Dx = Dy = 0.5λ, h = 0.25λ.

A better result comes from the average of the scan reflection coefficients of all elements. Figure 5 shows SEP for the half-wave dipole array, using the average reflection coefficient. The oscillations in Fig. 4 for the E-plane have been smoothed out and the H-plane SEP is higher, as it should be. Figure 6 is for the dipole with ground plane case with few oscillations. These two graphs compare well with the infinite array results of [9] but even for an array of 50 element wide, there are some edge effects at larger scan angles, thus Figs. 5 and 6 are slightly different from the infinite array results.

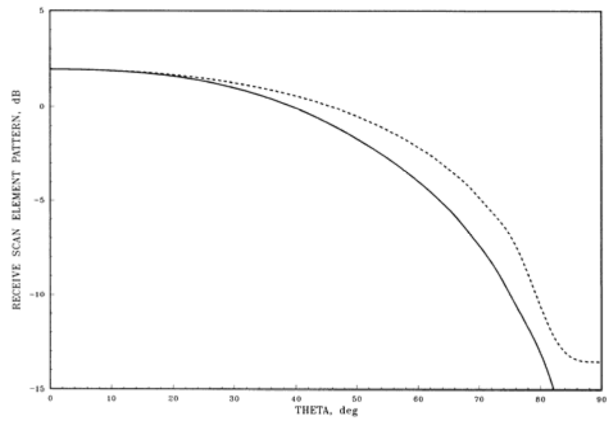

Fig. 5. A 50 × 50 dipole array, L = Dx = Dy = 0.5λ, average CGAM.

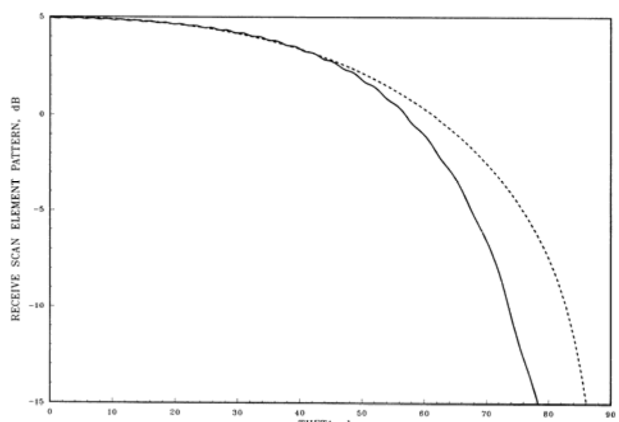

Fig. 6. A 50 × 50 dipole/screen array, L = Dx = Dy = 0.5λ, average CGAM.

V. COMPARISION OF BROADSIDE GAINS

The preceding transmit graphs were all normalized to 0 dB at broadside, to show scan behavior. Actual SEP, gain per element at broadside, is given in Table 1 for the 50 x 50 dipole array. For the (correct) receive case, with center element match, the SEP is $\pi/2$ =1.96 dB, as expected. The error in the (incorrect) transmit case is 6.2 dB. The passive mutual coupling reduces all currents but that of the center element, resulting in grossly incorrect gain.

Table 1. 50 x 50 Dipole Array at Broadside.

	SEP-T	SEP-R
Center Element Match	-4.26dB	+1.96 dB
Infinite Array Match	-5.00 dB	

The array over a ground plane gives the results in Table 2. Now the center element match case produces 4.97 dB as expected; the transmit case gives a gain of 11.4dB in error. When the infinite array impedances are used as a match, the results are nearly the same.

Table 2. 50 x 50 Dipole/Screen Array at Broadside.

	SEP-T	SEP-R
Center Element Match	-6.42 dB	+4.97 dB
Infinite Array Match	-8.71 dB	

VI. CONCLUSIONS

Measuring *scan element pattern* of an array with one element excited gives crude relative scan performance with significant errors, while the absolute values are grossly incorrect; due to only one element excitation leading to passive mutual coupling effects. SEP should be measured with the arrays in the receiving mode in a standard gain test facility. E-plane receive SEP may show oscillations for small arrays, which can be smoothed out by measuring or calculating SEP of several elements, and using averaging.

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Dr.Hansen is a Life Fellow of IEEE, a Fellow of IEE and was President of the Antennas and Propagation Society (1964 and 1980), and Director IEEE (1975). He was chair of U.S. Commission B of URSI (1967-69), and is a registered Professional Engineer in California and England. He is a member of the American Physical Society, Tau Beta Pi, Sigma Xi, Eta Kappa Nu, and Phi kappa Phi. He was awarded an honorary Doctor of Engineering degree by the University of Missouri-Rolla in 1975. The University of Illinois Electrical Engineering Department gave him a Distinguished Alumnus award in 1981, and the College of Engineering awarded a Distinguished Alumnus Service Medal in 1986. The IEEE AESS Barry Carlton best paper prize was awarded in 1991. He received the 2002 IEEE Electromagnetics Award. He was elected to the National Academy of Engineering in 1992.

Dr. Hansen has written over 100 papers on electromagnetics (most recently in 2006), has been Associate Editor of Microwave Journal (1960-95), was Associate Editor or Radio Science (1967-69), and Associate Editor of Microwave Engineer's Handbook(1970). His books are: Microwave Scanning Antennas(1964); Significant Phased Array Papers(1973); Geomentric Theory of Diffraction(1981); Moment Methods in Antennas and Scattering (1990); Phased Array Antennas (1998); and Electrically Small, Superdirective, and Superconducting Antennas(2006).