Use of the Simultaneous Diagonalization Technique in the $A\bar{x} = \lambda B\bar{x}$ Eigenproblem Applied to the Computation of the Characteristic Modes

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Abstract- Characteristic modes developed by Garbacz, Harrington and Mautz have long been used in the analysis of radiation and scattering from conducting bodies and apertures. For their computation, it is necessary to solve an eigensystem of the form $\mathbf{A}\bar{x} = \lambda\mathbf{B}\bar{x}$. If the matrices $(\mathbf{A}, \mathbf{B})$ are Hermitian and $\mathbf{B}$ is positive definite, the generalized eigenvalue problem can be accurately solved using the simultaneous diagonalization technique (SDT). Because of numerical approximations and rounding sometimes it may happen that the matrices properties deteriorate and the SDT procedure becomes inapplicable. In this work a new technique, developed recently by Higham and Cheng is proposed as a method to solve these deteriorate cases. It is applied to the computation of the characteristic modes for some scattering problems. Results are analyzed and discussed.

1. Introduction

The generalized eigenvalue problem:

$$\mathbf{A}\bar{x} = \lambda\mathbf{B}\bar{x} \tag{1}$$

is often encountered in computational electromagnetics [1]. The solution of (1) depends on properties of the matrices $\mathbf{A}$ and $\mathbf{B}$ resulting from the discretization of the field equations. If these matrices are not singular, then (1) can be reduced to the solution of the standard eigenproblem:

$$\mathbf{H}\bar{x} = \lambda\bar{x} \;\; with \;\; \mathbf{H} = \mathbf{B}^{-1}\mathbf{A} \tag{2}$$

However, if $\mathbf{A}$ and $\mathbf{B}$ are Hermitian and $\mathbf{B}$ is positive definite, the problem (1) can be solved with a higher degree of accuracy applying the *simultaneous diagonalization technique* (SDT) to the matrices $(\mathbf{A}, \mathbf{B})$ than the direct calculation of the inverse matrix $\mathbf{B}^{-1}$ [2]. Although the underlined properties are sometimes held of the matrices issued in electromagnetic problems, unfortunately the discretization performed by numerical algorithms on the electromagnetic field equations often causes the loss of these properties. As a consequence, the SDT becomes inapplicable. Recently, Higham and Cheng have faced the above problem from a theoretical point of view and they have proposed a technique that allows to use the SDT even in presence of discretization errors. This method is based on the concept of *nearest definite pair* [3]. In this work, we have applied Higham-Cheng results to evaluation of the characteristic modes for the scattering from conducting bodies. The paper is organized as follows: section II presents the basics on the theory of characteristic modes for conducting bodies. Section III and IV illustrate the SDT procedure for the resolution of the positive definite Hermitian generalized eigenproblem and its extension. In section V we show the numerical results. Finally, in section VI, the conclusions.

2. Theory of Characteristic Modes for Conducting Bodies

The characteristic modes have been intensively used in the analysis of radiation and scattering from conducting bodies and apertures [4]-[13]. They are numerical entire basis functions that in principle can be computed for any object shape. Since these eigenfunctions include the behavior of unknown current density flowing on the metallic bodies and apertures, only a small number of them are required for a good reconstruction of it. So, if available, characteristic modes would lead to a scattering or radiation problem treatable with a classical Method of Moment even in presence of a large number of objects [10]-[13]. In the case of scattering or radiation from conducting bodies, these modes are basically solutions of a generalized eigenproblem involving the impedance operator $\mathcal{Z}$, which relates the surface current $\bar{J}$ on a conducting body to the tangential component of the incident electric field $\bar{E}^{Inc}_{Tan}$ on it, i.e. [5]:

$$\mathcal{Z}\bar{J} = \bar{E}^{Inc}_{Tan} \tag{3}$$

One method to solve a functional equation of the form (3) is to obtain a modal solution in terms of eigenfunctions of $\mathcal{Z}$. For this purpose, we consider the operational eigenproblem:

$$\mathcal{Z}\bar{J}_k = \nu_k \mathcal{M}\bar{J}_k \tag{4}$$

in which $\mathcal{M}$ is a suitable weight operator and $\nu_k = 1 + j\lambda_k$. The operator $\mathcal{Z}$ can be uniquely represented in the form:

$$\mathcal{Z} = \mathcal{R} + j\mathcal{X} \tag{5}$$

where the operators $\mathcal{R}$ and $\mathcal{X}$ are defined as:

$$\mathcal{R} = \frac{\mathcal{Z} + \mathcal{Z}^*}{2} \tag{6}$$

$$\mathcal{X} = \frac{\mathcal{Z} - \mathcal{Z}^*}{2j} \tag{7}$$

Note that $\mathcal{R}$ and $\mathcal{X}$ are real selfadjoint operators and furthermore $\mathcal{R}$ is positive definite [5]. The choice $\mathcal{M} = \mathcal{R}$ reduces the complex operatorial eigenproblem (4) to a real operatorial eigenproblem:

$$\mathcal{X}\bar{J}_k = \lambda_k \mathcal{R}\bar{J}_k \tag{8}$$

The eigenfunctions $\bar{J}_k$ have been named *characteristic modes* [5]. They can be numerically evaluated through the reduction of the operator equation (8) to a matrix equation using the Method of Moment [5],[10]. For this aim the *k-th* modal current density on the metallic body is expanded in a set of N suitable subsectional basis functions $\bar{B}_n$:

$$\bar{J}_k = \sum_{n=1}^{N} I_n \bar{B}_n \tag{9}$$

where the I_n are unknown complex constants. Substitution of (9) into (8) and application of the Galerkin method leads to a matrix equation of the form:

$$\mathbf{X}\bar{I}_k = \lambda_k \mathbf{R}\bar{I}_k \tag{10}$$

where $\bar{I}_k$ is the column vector of the unknown coefficients I_n. Equation (10) is a generalized eigensystem of the form (1) in which the eigenvalues λ_k and the eigenvectors $\bar{I}_k$ approximate the eigenvalues and the eigenfunctions of the equation (8). It follows by $\mathcal{X}$ and $\mathcal{R}$ properties that the matrices $\mathbf{X}$ and $\mathbf{R}$ are expected Hermitian and furthermore $\mathbf{R}$ is expected positive definite. Consequently, in principle it is possible to use the SDT for solving the eigensystem (1). But because of numerical approximation, the $\mathbf{R}$ matrix often becomes indefinite and the SDT algorithm becomes inapplicable. In the following, it will be shown that the use of the nearest definite pair technique permits to employ the SDT giving a significant improvement in the evaluation of the characteristic modes over the direct inversion technique.

3. Resolution of the Positive Definite Generalized Eigenproblem with the Simultaneous Diagonalization Technique

The generalized eigenproblem where the matrices $\mathbf{A}$ and $\mathbf{B}$ are Hermitian and in which one is positive definite plays an important role in matrix theory. If we assume, without loss of generality, that $\mathbf{B}$ is positive definite, the problem (1) can be accurately solved by means of a method named *simultaneous diagonalization technique* [2] as follows: Let $\mathbf{U}_1$ be an unitary matrix whose columns are an orthonormal set of eigenvectors for $\mathbf{B}$. Premultiplying and postmultiplying (1) by $\mathbf{U}_1^*$ and $\mathbf{U}_1$, where the star denotes the complex conjugate transpose operation, and taking into account that:

$$\mathbf{U}_1^*\mathbf{U}_1 = \mathbf{U}_1\mathbf{U}_1^* = diag(1) \tag{11}$$

where $diag(1)$ is the identity matrix, we obtain:

$$\dot{\mathbf{A}}\bar{y} = \lambda\dot{\mathbf{B}}\bar{y} \tag{12}$$

in which the matrices $\dot{\mathbf{A}}$, $\dot{\mathbf{B}}$ and the vector $\bar{y}$ are of the form:

$$\begin{aligned} \dot{\mathbf{A}} &= \mathbf{U}_1^*\mathbf{A}\mathbf{U}_1 \\ \bar{y} &= \mathbf{U}_1^*\bar{x} \\ \dot{\mathbf{B}} &= \mathbf{U}_1^*\mathbf{B}\mathbf{U}_1 = diag(\mu) \end{aligned} \tag{13}$$

and in which $diag(\mu)$ is the diagonal matrix formed by eigenvalues of the $\mathbf{B}$ matrix. Next, we introduce the nonsingular transformation $\mathbf{H}$:

$$\mathbf{H} = diag(\mu^{-\frac{1}{2}}) \tag{14}$$

Using $\mathbf{H}$ in (12), we obtain:

$$\ddot{\mathbf{A}}\bar{z} = \lambda\ddot{\mathbf{B}}\bar{z} \tag{15}$$

where $\ddot{\mathbf{A}}$, $\bar{z}$ and $\ddot{\mathbf{B}}$ are:

$$\begin{aligned} \ddot{\mathbf{A}} &= \mathbf{H}^*\dot{\mathbf{A}}\mathbf{H} \\ \bar{z} &= \mathbf{H}^{-1}\bar{y} \\ \ddot{\mathbf{B}} &= \mathbf{H}^*\dot{\mathbf{B}}\mathbf{H} = diag(1) \end{aligned} \tag{16}$$

Finally, if we construct an unitary matrix $\mathbf{U}_2$ whose columns are an orthonormal set of eigenvectors for $\ddot{\mathbf{A}}$, the matrix transformation $\mathbf{T} = \mathbf{U}_1\mathbf{H}\mathbf{U}_2$ permits simultaneously to reduce the matrices $\mathbf{A}$, $\mathbf{B}$ to diagonal form:

$$\left.\begin{aligned} \mathbf{T}^*\mathbf{A}\mathbf{T} &= diag(\lambda) \\ \mathbf{T}^*\mathbf{B}\mathbf{T} &= diag(1) \end{aligned}\right\} \tag{17}$$

resolving the eigenproblem (1). An efficient implementation of the above procedure that utilizes both the Cholesky factorization and the symmetric QR algorithm is described in [14].

4. Extension of the Simultaneous Diagonalization Technique

The application of the procedure discussed in the previous section is not limited to the treatment of Hermitian pairs $(\mathbf{A}, \mathbf{B})$ with $\mathbf{B}$ positive definite, but sometimes it can be extended to the Hermitian pairs in which the $\mathbf{B}$ matrix is indefinite. For this aim, the concept of definiteness has been restated referring directly to the matrix pair $(\mathbf{A}, \mathbf{B})$ rather than to a single matrix $\mathbf{B}$. More in details, pair $(\mathbf{A}, \mathbf{B})$ having *Crawford number* γ [15], denoted as:

$$\gamma(\mathbf{A}, \mathbf{B}) = \min_{\|\bar{z}\|_2=1} \sqrt{(\bar{z}^*\mathbf{A}\bar{z})^2 + (\bar{z}^*\mathbf{B}\bar{z})^2} \tag{18}$$

strictly positive has been named as *definite* and, when $\gamma = 0$ the pair has been named as *indefinite*. Also, it is possible evaluate the definiteness of a Hermitian pair $(\mathbf{A}, \mathbf{B})$ drawing its *Field of Values* $F(\mathbf{A} + j\mathbf{B})$ in the complex plane. The field of values is defined as the set of all the values assumed by the Rayleigh quotients of the pair [3]:

$$F(\mathbf{A} + j\mathbf{B}) = \frac{z^*(\mathbf{A} + j\mathbf{B})z}{z^*z} \quad z \neq 0 \tag{19}$$

The pair will be *definite* if and only if zero does not lie in $F(\mathbf{A} + j\mathbf{B})$, otherwise it will be *indefinite*. A solution of (1) in which $\mathbf{B}$ is indefinite, but the Crawford number γ of the pair $(\mathbf{A}, \mathbf{B})$ is positive, has been given in [15] by means of the *Stewart's theorem* [16]. This theorem ensured the existence of an angle $\theta \in [0, 2\pi)$ that permits to define, starting from a given pair $(\mathbf{A}, \mathbf{B})$ with $\gamma > 0$, a new pair $(\mathbf{A}_\theta, \mathbf{B}_\theta)$ in which $\mathbf{B}_\theta$ is positive definite, as follows:

$$\left.\begin{aligned} \mathbf{A}_\theta &= \mathbf{A}\cos\theta + \mathbf{B}\sin\theta \\ \mathbf{B}_\theta &= -\mathbf{A}\sin\theta + \mathbf{B}\cos\theta \end{aligned}\right\} \tag{20}$$

For this new pair is now applicable the SDT. The eigenvalues λ of the old pair $(\mathbf{A}, \mathbf{B})$ and the eigenvalues λ_θ of the new pair $(\mathbf{A}_\theta, \mathbf{B}_\theta)$ are simply related by:

$$\lambda = \frac{\lambda_\theta\cos\theta - \sin\theta}{\lambda_\theta\sin\theta + \cos\theta} \tag{21}$$

However, it may happen that the discretization errors may affect the matrices in a way that a pair $(\mathbf{A}, \mathbf{B})$ theoretically expected to be definite becomes indefinite in practice. As a consequence the SDT cannot be applied. For this last case it can be useful evaluate the nearest definite pair $(\mathbf{A} + \Delta\mathbf{A}, \mathbf{B} + \Delta\mathbf{B})$ and attempt to resolve the problem (1) for this one. Else, assigned the indefinite Hermitian pair $(\mathbf{A}, \mathbf{B})$, we must find the nearest Hermitian definite pair

$(\mathbf{A}+\Delta\mathbf{A}, \mathbf{B}+\Delta\mathbf{B})$, having a specified Crawford number $\gamma = \delta > 0$ according to the distance [3],[17]:

$$d_\delta(\mathbf{A},\mathbf{B}) = min\{||[\Delta\mathbf{A}\ \Delta\mathbf{B}]||_2 : \gamma(\mathbf{A}+\Delta\mathbf{A}, \mathbf{B}+\Delta\mathbf{B}) = \delta\} \quad (22)$$

and the angle θ by means of which the nearest pair can be simultaneously diagonalized. This problem has been recently solved by Higham and Cheng by means of the following theorem [3]:

Higham-Cheng's Theorem: Let $\mathbf{A}, \mathbf{B} \in \mathcal{C}^{n\times n}$ be Hermitian and let $\mathbf{C} = \mathbf{A} + j\mathbf{B}$ and $\mathbf{A}_\phi = \mathbf{A}cos(\phi) + \mathbf{B}sin(\phi)$. Let $\min_{(0\leq\phi\leq 2\pi)} \lambda_{max}(\mathbf{A}_\phi)$ be attained at the angle θ and let $\mathbf{A}_\theta$ have the spectral decomposition:

$$\mathbf{A}_\theta = \mathbf{Q}diag(\mu_k)\mathbf{Q}^* \quad with \quad \mu_n \leq ... \leq \mu_1 \quad (23)$$

If $0 \in F(\mathbf{C})$ then:

$$d_\delta(\mathbf{A},\mathbf{B}) = \delta + \mu_1 \quad (24)$$

If $0 \notin F(\mathbf{C})$ then:

$$d_\delta(\mathbf{A},\mathbf{B}) = max(\delta + \mu_1, 0) \quad (25)$$

In both cases, two sets of optimal perturbations in (22) are:

$$\left.\begin{array}{l} \Delta\mathbf{A}_1 = \mathbf{Q}diag(min(-\delta+\mu_k, 0))\mathbf{Q}^*\cos\theta \\ \Delta\mathbf{B}_1 = \mathbf{Q}diag(min(-\delta+\mu_k, 0))\mathbf{Q}^*\sin\theta \end{array}\right\} \quad (26)$$

and

$$\left.\begin{array}{l} \Delta\mathbf{A}_2 = -d_\delta(\mathbf{A},\mathbf{B})diag(1)\cos\theta \\ \Delta\mathbf{B}_2 = -d_\delta(\mathbf{A},\mathbf{B})diag(1)\sin\theta \end{array}\right\} \quad (27)$$

A detailed demonstration of this theorem can be found in [3]. The most important consequence of the above results is that the SDT can be generalized to the Hermitian pairs $(\mathbf{A},\mathbf{B})$ having $\mathbf{B}$ matrix indefinite.

5. Numerical Results

In order to check the validity and accuracy of the foregoing procedure in the computation of characteristic modes for conducting bodies, numerical results for some cases are carried out. Firstly, characteristic modes have been computed for the TM_z scattering from a perfectly conducting circular cylinder and from an elliptic one, since for these cases characteristic modes are known in analytical form [12]. In Fig.(1) are represented the geometries for both problems. In Fig.(2) is reported the plot of the smaller eigenvalues of the $\mathbf{R}$ matrix obtained discretizing the EFIE for the scattering due to a plane wave impinging on a metallic circular cylinder with radius $\frac{\lambda}{2}$. Pulse basis functions and delta function testing functions are used to create the matrix $\mathbf{R}$ [1]. Since only the properties of the matrices above are considered here for the computation of characteristic modes, it is not necessary to specify the direction of the incident field. The depicted eigenvalues are scaled to show the relative value only. This plot clearly shows that the $\mathbf{R}$ matrix is indefinite, even if it is expected definite because the properties of the operator $\mathcal{R}$. In Fig.(3) is plotted the field of values for the pair $(\mathbf{X},\mathbf{R})$. Because the origin of the complex plane is not contained in $F(\mathbf{X}+j\mathbf{R})$ this pair is definite. In Figs.(4) and (5) are reported the plot of the smaller eigenvalues of the $\mathbf{R}$ ma-

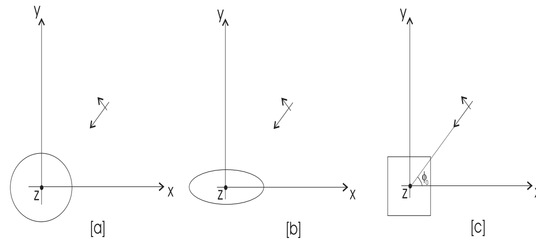

Figure 1: Geometry for scattering from circular, elliptic and square metallic cylinders.

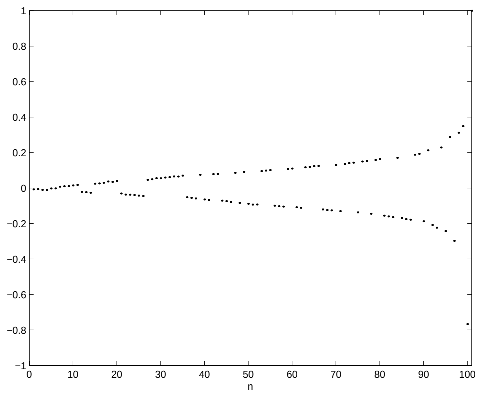

Figure 2: Plot of the smaller normalized eigenvalues for the TM_z scattering from a circular cylinder of radius $a = \frac{\lambda}{2}$ (Problem Size 128×128).

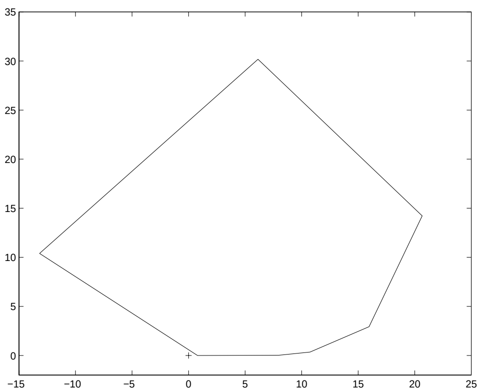

Figure 3: Field of Values for pair related to computation of characteristic modes for circular cylinder.

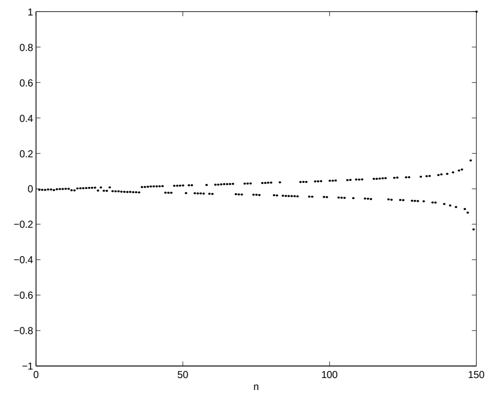

Figure 4: Plot of the smaller normalized eigenvalues for the TM_z scattering from an elliptic cylinder with semi major axis $a = \lambda$ and semi minor axis $b = \frac{\lambda}{4}$ (Problem Size 180×180).

trix and the field of values $F(\mathbf{X} + j\mathbf{R})$ for the pair obtained considering the scattering from a metallic elliptic cylinder with semi major axis $a = \lambda$ and semi minor axis $b = \frac{\lambda}{4}$ by an incident plane wave. For this scattering problem we have obtained both an indefinite matrix $\mathbf{B}$ and an indefinite pair $(\mathbf{X}, \mathbf{R})$ as clearly depicted in these figures. In Table (1) are shown the analytical eigenvalues versus the numerical ones for the previously analyzed scattering problems. Numerical eigenvalues have been obtained using both the Higham-Cheng procedure and the direct inversion technique. Excellent agreement is shown comparing the exact eigenvalues and the numerical ones computed by means of the former procedure while the eigenvalues obtained using the direct inversion technique are very inaccurate. As last case, consider the evaluation of the characteristic modes for the scattering from a metallic square cylinder of side $a = \lambda$. In Figs.(1) and (6) are reported the geometry for the problem and its field of values, respectively. As it is shown in Fig.(6), the matrix pair $(\mathbf{X}, \mathbf{R})$ for this case is definite because the field of values does not contain the origin of the complex plane. In Fig. (7) is shown the obtained current distribution on the cylinder by a plane wave impinging at an angle $\phi_0 = 0$. The continuous line indicates the current computed by means of the standard MoM while crosses indicate the solution obtained using characteristic modes computed using the Higham-Cheng procedure. A very good agreement is clearly observed.

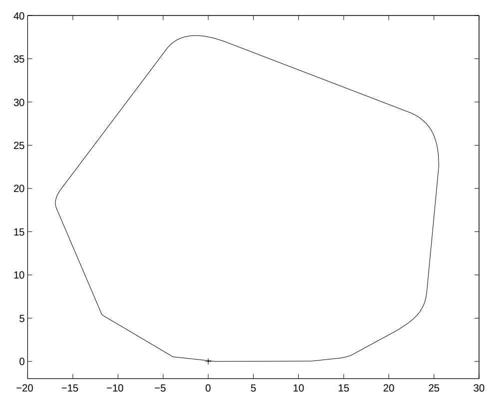

Figure 5: Field of Values for pair related to computation of characteristic modes for elliptic cylinder.

6. Conclusions

A technique developed by Higham and Cheng for the treatment of the Hermitian generalized eigenproblem based on the concept of nearest definite pair has been proposed for the computation of the characteristic modes. Numerical results for some scattering problems are presented. In all the cases, the outlined procedure, even if acting on very ill conditioned matrices, provides very accurate numerical results. Finally, we point out that the application of the method is not limited to the presented cases being fruitfully applicable to a wide number of electromagnetic problems which posses the discussed properties [18].

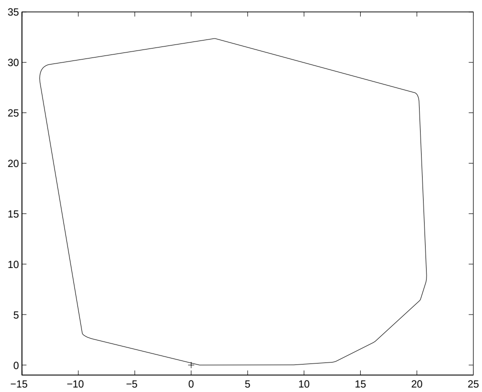

Figure 6: Field of Values for pair related to computation of characteristic modes for square cylinder (Problem Size 165×165).

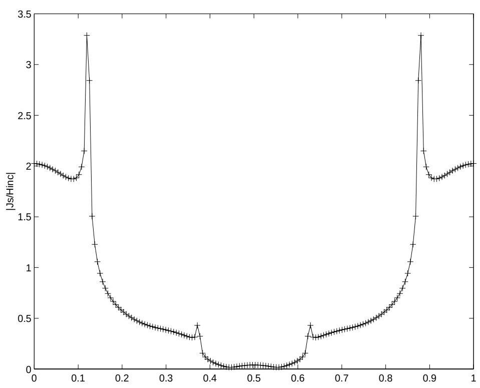

Figure 7: Plot of the current density on the metallic square cylinder. The continuous line indicate solution computed by means of the MoM. Crosses indicate solution obtained using characteristic modes.

Exact Solution (*circular cylinder*) cond($\mathbf{X}$)=1.29e+01 cond($\mathbf{R}$)=5.56e+17	**Higham-Cheng Solution**	**Direct Inversion**
-0.12609	-0.12609	-35.6120-32.1893j
-0.12609	-0.12609	-35.6120+32.1893j
1.07929	1.07921	49.1555
0.20579	0.20572	14.8938+76.7077j
0.20579	0.20572	14.8938-76.7077j
Exact Solution (*elliptic cylinder*) cond($\mathbf{X}$)=3.22e+01 cond($\mathbf{R}$)=3.54e+18	**Higham-Cheng Solution**	**Direct Inversion**
-0.958287	-0.961337	-3.34497 +1.71189j
-0.143256	-0.143658	-0.987728-1.30952j
0.139654	0.137123	1.305403
0.454565	0.451928	-0.987728+1.30952j
0.831470	0.832260	-2.58841

Table 1: Analytical vs. numerical eigenvalues.

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