# Calculation of Electric Field Created by Transmission Lines, by 3D-FE Method Using Complex Electric Scalar Potential


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***Abstract* - This paper presents a methodology for tridimensional analysis of the electric field produced by transmission lines. It utilizes the complex electric scalar potential, which makes it possible to consider variation of module and phase in the voltages of supply lines. The Finite Element Method (FEM) is applied to solve the differential equation that describes the phenomenon in the domain of this study. Finally, a comparison is established between the results obtained applying the proposed methodology and the values reached through the classical method of charge simulation.**


## I. Introduction

With a voltage level increase for transmission of large quantities of electrical energy, the effect of electrostatic field becomes an important factor in transmission lines (TL's) design.

This problem has been receiving increasing attention in recent years, from electrical power companies throughout the world. The origin of this concern is due, not only to the constant increase in the voltage levels in transmission systems, but also to the steady growth of urban concentrations, resulting in an increasing number of residential areas with TL's.

It is a well-known fact that the electric field produced by TL's at ground level, acting on a person, can cause nasty sensations, such as sparkling in the skin, attraction of the hair, organic and physiological alterations, when exposed for a prolonged period to a field of high intensity.

There exists the further risk of people touching objects with a high degree of isolation in relation to the ground (such as vehicles, fences, antennas and others) and being in contact with electric currents that can reach alarming levels, as a result of the electrostatic energy stored up by these objects when exposed to electric fields.

Although the gravity of some effects may still be debatable, the influence of the electrostatic field on men and the environment may become critical with the advent of high voltage and extra-high voltage TL's.

There is also the problem of the interference that these systems (TL's) can cause in other nearby installations, such as pipelines for the transport of fluids (oil ducts, gas ducts, aquaducts, etc.), railways and communication systems, etc. For this reason, a study of the electromagnetic compatibility (EMC) between these installations and the environment in which they are inserted, has become a matter of fundamental importance. One way of evaluating the EMC of these installations (in terms of disturbances of electrical origin) is through a knowledge of the values and distribution in the electrical field produced by them. For this purpose, the availability of precise and versatile calculation tools becomes necessary in order to guarantee the quality of the results.

The great majority of softwares for calculating the electric field of TL's are based on the axial symmetry that characterizes the fields in this installations, thus permitting a bidimensional analysis [1,6]. However, there are several different situations in which such a consideration could not apply, without giving incorrect results, or due to the very complexity of the geometry (for example, crossing among TL's).

The proposed methodology allows for a tridimensional analysis, making possible the study of a series of practical and interesting situations, such as considering various TL's, with some laid out assymetrically in relation to others [2]. Another important aspect concerns the utilization of the complex electric scalar potential, that permits to calculate the field for a sinusoidal applied voltage, still making possible a temporal analysis of the phenomenon throughout a cicle alternation voltage.

## II. Mathematical Formulation

The Maxwell's equations used are:

$$\operatorname{curl}\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \qquad (1)$$

$$\operatorname{curl}\vec{E} + \frac{\partial \vec{B}}{\partial t} = 0 \qquad (2)$$

$$\operatorname{div}\vec{B} = 0 \qquad (3)$$

where $\vec{H}$ is the magnetic field (A/m), $\vec{E}$ the electric field (V/m), $\vec{B}$ the magnetic flux density (Tesla), $\vec{D}$ the electric flux density ($C/m^2$), $\vec{J}$ the current density ($A/m^2$) and t the time (s).

The constitutive relations, concerning isotropic and linear materials are:

$$\vec{D} = \varepsilon\vec{E} \tag{4}$$

$$\vec{J} = \sigma\vec{E} \tag{5}$$

where $\varepsilon$ is the electric permissivity (F/m) and $\sigma$ the electric conductivity $(\Omega m)^{-1}$.

Utilizing equations (2) and (3) and knowing that (curl grad) is always zero [2], yields:

$$\vec{E} = -\text{grad}\,V - \frac{\partial\vec{A}}{\partial t} \tag{6}$$

But considering that in the case of the TL's, the electric field originated mainly due to the potential to which the conductors are submitted and, assuming that the variation of the values are sufficiently slow that the effect of the potential vector $\vec{A}$ can be ignored ($\partial\vec{A}/\partial t \cong 0$), we can define the electric field as:

$$\vec{E} = -\text{grad}\,V \tag{7}$$

However, the electric scalar potential V is not sufficient to represent the total characteristics of the electric field produced by TL's, for the voltages in the line supply are sinusoidal, showing variations in module and angle of phase [2].

Therefore is necessary to utilize the complex electric scalar potential, defined how:

$$\dot{V} = V_0\, e^{j(\omega t+\alpha)} \tag{8}$$

where $V_0$ is the voltage to which the line conductors are submitted (V), $\alpha$ is the phase angle (rad), $\omega$ is the angular frequency (rad/s) and t the instant of time considered (s).

Applying the divergent operator in the equation (1) and using equations (4) and (5), we get the equation that describes the phenomenon in the domain of this study:

$$\text{div}\,[(\sigma + j\omega\varepsilon)\text{grad}\dot{V}] = 0 \tag{9}$$

In this problem the following boundary conditions are considered:

- Dirichlet boundary condition, where the value of complex electric scalar potential is specified.

$$\dot{V} = \dot{V}_0 \tag{10}$$

- Neumann boundary condition, where the normal derivative of complex electric scalar potential is specified.

$$\frac{\partial\dot{V}}{\partial n} = 0 \tag{11}$$

Since an analytic solution to equation (9) is difficult, numerical techniques are utilized to solve it. Applying the FEM to solve this equation, subject to contour conditions (10) and (11), for each element in the mesh, a matrix, called the matrix of elementary contributions is obtained. In our case, the elementary contribution matrix is complex and symmetrical. Its generic term is given by:

$$\dot{g}_{ij} = \int_{\Omega_e} \left[(\sigma + j\omega\varepsilon)\,\text{grad}\,N_i{}^t\,\text{grad}\,N_j\right] d\Omega_e \tag{12}$$

where i and j are lines and columns positions in the matrix, $N_i$ and $N_j$ are functions of nodal interpolation and $\Omega_e$ represents the domain of the finite element being considered.

The sum of all the elementary matrixes will form a global matrix system, where all the elements of the mesh are considered. This could be represented by:

$$\sum_{i=1}^{nno} \dot{g}_{ij}\dot{V}_j = 0 \qquad i = 1, nno \tag{13}$$

where $\dot{V}_j$ is the value of complex electric scalar potential at node j and nno is the total number nodes in the mesh.

The resolution of this matrix system gives the value of complex electric scalar potential at the nodes of the mesh.

For its resolution the method of conjugated orthogonally conjugated gradients (COCG) is utilized [5]. Once the system is solved, the electric field can be obtained by (7).

## III. Results

In the following section, results obtained for three situations will be shown, in order to demonstrate the validity of the methodology utilized.

### *A. Analysis of the 1050 kV three-phase transmission line*

Here, a curve on the lateral profile of the electric field is shown. It was obtained by FEM, for a three-phase transmission line of 1050 kV (Fig. 3.1.2) and will be compared with the result given by the classical method of charge simulation and also with the values measured.

The picture below (Fig. 3.1.1), shows the view in the plan of the domain of study.

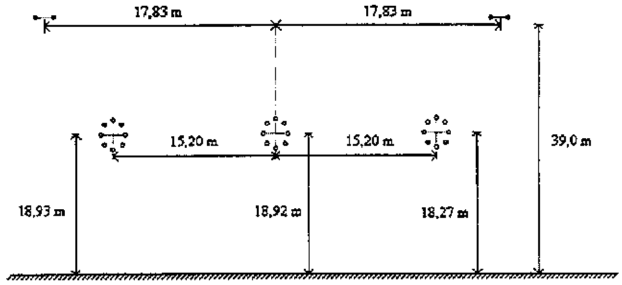


Fig. 3.1.1 Domain of study.

In this case, the domain of the study was separated into 10586 elements. 79 iterations were needed to obtain convergence and a 4 minute and 28 second calculation time in a station Sun SparkStation 2.

Table I, below, shows the main characteristics of the TL being studied [1].

Table I - Main features of 1050 kV transmission line.

| | |
|---|---|
| Voltage (kV) | 1050 |
| Conductors per phase | 8 |
| Diameter of conductors (m) | 0,03307 |
| Diameter of lighting conductors (m) | 0,01016 |
| Distance between phases (m) | 15,20 |
| Height of phases (m) | 18,93; 18,92; 18,27 |
| Distance between lightning conductors (m) | 35,66 |
| Height of lightning conductors (m) | 39,0 |

It was adopted for ABC phase sequence calculations from left to right and $t = 0$.

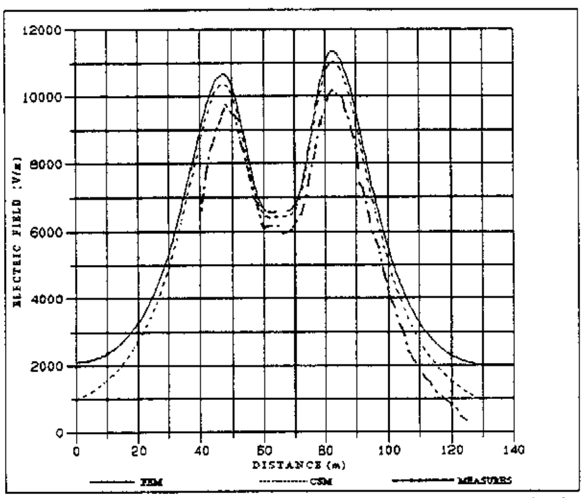


Fig. 3.1.2 Electric field calculated by FEM, CSM and measured values.

It is observed in Fig 3.1.2 that the results supplied by FEM are satisfactory. In Fig 3.1.3 equipotential lines are shown with the application of FEM. They provide a notion of potential distribution within the domain of this study and also point to regions where the electric field is or is not uniform.

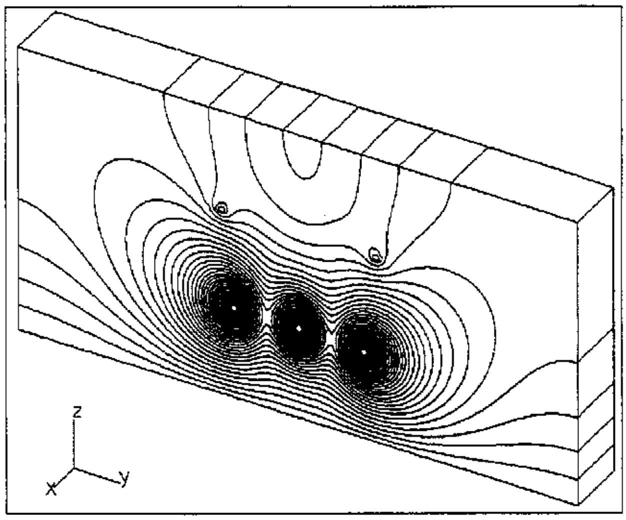


Fig. 3.1.3 Equipotential lines.

*B. 1050 kV Line in the presence of material conductors*

In this case, we consider the existence of two metallic sheds, under phases of 1050 kV transmission line. The figure that follows shows the domain of this study.

P1 and P2 are lightning conductors and A, B and C are phase conductors.

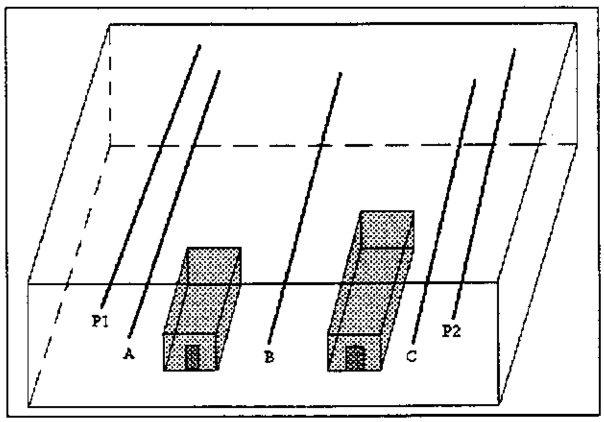


Fig. 3.2.1 Domain of study.

Fig. 3.2.2, presents a view of the equipotential lines, obtained for this study.

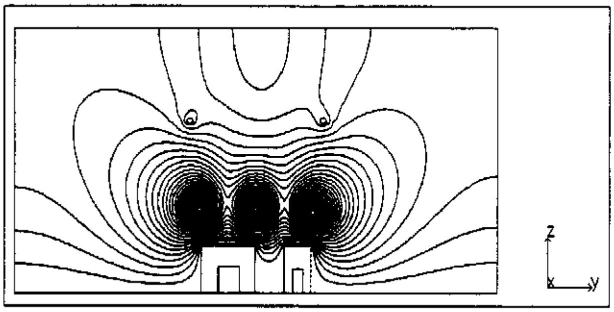


Fig. 3.2.2 Equipotential lines.

The influence of the metal sheds on the potential distribution is observed, and consequently the distribution of values in the electric field.

Due to the continuity of the tangential component of the electric field, the appearance of electric current density is observed in the inner superficial part of the conductor materials.

### *C. Hypothetical case of crossing between two TL's*

Here we imagine the crossing (at 90°) between two TL's, one of 138 kV and the other of 500 kV. In the literature, no similar case is registered, which justifies the difficulty and even the impossibility of making such an analysis, with methods that were then available. For this case two curves on the lateral profile of the electric field were obtained, one on the y axis, that is, on the central phase of the 138 kV line (Fig. 3.3.2) and the other on the x axis, on the central phase of the 500 kV line (Fig. 3.3.3). The equipotential lines is shown in Fig. 3.3.4.

In the picture below (Fig. 3.3.1) is observed the approach of the domain of the considered study.

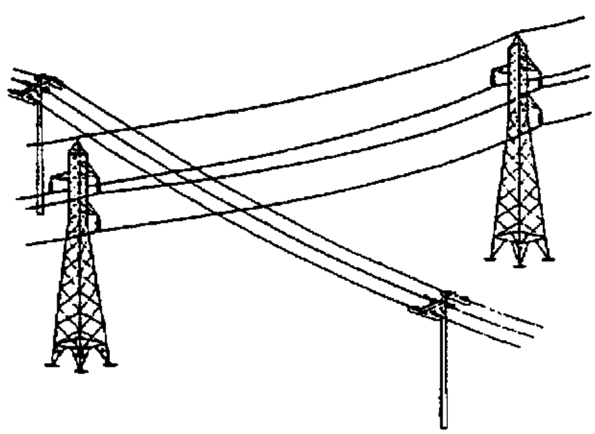

Fig. 3.3.1 Approach of the domain of study.

In this case, the domain of study was separated into 39,600 elements. 35 iterations were needed in order to obtain convergence, as well as a calculation time of 5 minutes and 23 seconds in a station Sun SparkStation 2. Tables II and III shown below, present the characteristics of each one of the lines considered.

Table II - Principal features of 138 kV transmission line.

| | |
|---|---|
| Voltage (kV) | 138 |
| Conductors per phase | 1 |
| Diameter of conductors (m) | 0,03195 |
| Distance between phases (m) | 7,0 |
| Height of phases (m) | 10,0 |

Table III - Principal features of 500 kV transmission line.

| | |
|---|---|
| Voltage (kV) | 500 |
| Conductors per phase | 2 |
| Diameter of conductors (m) | 0,03195 |
| Spacing between subconductors (m) | 0,40 |
| Distance between phases (m) | 15,0 |
| Height of phases (m) | 18,0 |

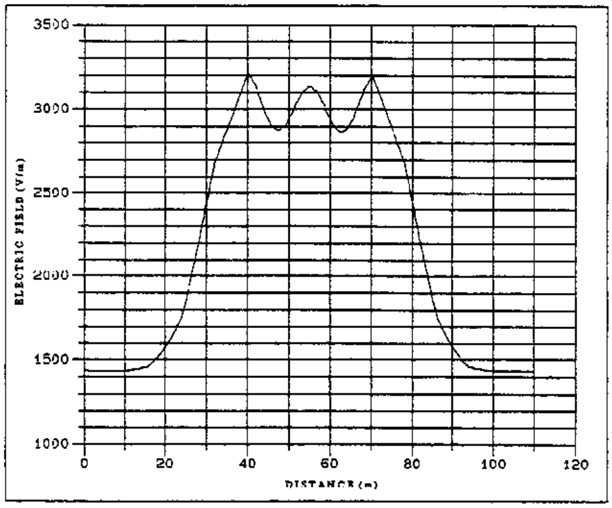


Fig. 3.3.2 Curve on the lateral profile of electric field on the y axis.

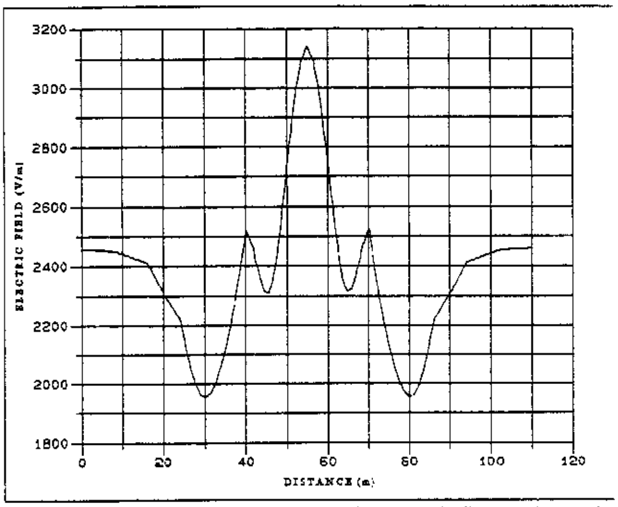

Fig. 3.3.3 Curve on the lateral profile of the electric field on the x axis.

In this case, values were not measured for comparative purposes. Nevertheless, analyzing the two lines separately, it can be observed that the results obtained are coherent.

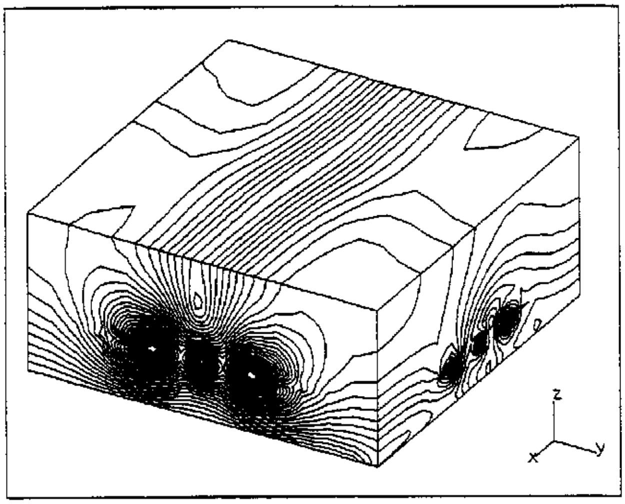


Fig. 3.3.4 Equipotential lines in crossing between two transmission lines.

## IV. Conclusions

In this study, a mathematical model was presented to analyze a tridimensional electric field generated by TL's.

Values of the electric field are presented for two situations: one tri-phase transmission line of 1050 kV and the other a hypothetical case of crossing between two TL's.

Potential distribution on 1050 kV line was also shown, considering the presence of metallic objects along the line.

FEM has proved to be highly flexible, inasmuch as it easily permits the analysis of a large number of interesting practical situations, such as, the evaluation of values and of electric field distribution when there is a crossing of two or more TL's, the possibility of considering in calculations both the presence of conductor and/or multidielectrics materials, as well as irregularities of the land.

Therefore it can be demonstrated that the methodology utilized makes possible the analysis of more realistic situations, and consequently leads to obtaining more correct results.

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