A SURFACE INTEGRAL EQUATION FORMULATION FOR LOW CONTRAST SCATTERERS BASED ON RADIATION CURRENTS

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Abstract *- A new surface integral equation formulation based on radiation currents is presented. Numerical problems have been observed when treating low contrast scatterers with surface integral equation formulations based on the total equivalent currents. These problems result because the total equivalent currents are large compared with the radiation currents for low contrast materials. The surface integral equation formulation presented here avoids this problem by solving directly for the radiation currents.*

I. INTRODUCTION

This paper presents a new surface integral equation formulation for determining the electromagnetic field scattered by objects with low contrast. Low contrast materials are characterized by a relative permittivity and relative permeability close to unity. Low contrast materials include compressed gases and commonly used plastic foams.

When used in conjunction with a numerical integral equation solution, the conventional surface integral equation formulations can result in a poor estimate of the field scattered by a low contrast object. The poor performance of the standard formulations is due to their being cast in terms of the total equivalent current on the boundary surface of the scattering body. Only a portion of the total equivalent current gives rise to the scattered field. When this portion is small (as in the case of low contrast scatterers), errors in the radar cross section (RCS) calculation can occur. The new integral equation formulation is cast only in terms of the portion of the total current that gives rise to the scattered field; hence, the new formulation avoids this problem.

Here the new integral equation formulation is implemented in a moment method (MM) program for two-dimensional objects. Results of the numerical method are verified by comparison with series solution results for circular cylindrical objects.

II. THE SURFACE INTEGRAL EQUATIONS

Consider a closed dielectric scatterer in free space with surface A and illuminated by source currents in the free space region. For a common choice of the combination constants a set of combined field integral equations [1,2] for this scatterer may be written succinctly as

$$-\left[\mathbf{E}^{+}_{tan}(\mathbf{J})+\mathbf{E}^{+}_{tan}(\mathbf{M})\right]\frac{1}{\eta_o}-\hat{\mathbf{n}}\times\left[\mathbf{H}^{+}(\mathbf{J})+\mathbf{H}^{+}(\mathbf{M})\right]= \mathbf{E}^{+}_{tan}(\mathbf{J}^i,\mathbf{M}^i)/\eta_o+\hat{\mathbf{n}}\times\mathbf{H}^{+}(\mathbf{J}^i,\mathbf{M}^i) \quad (1)$$

just inside A

$$-\left[\mathbf{E}^{-}_{tan}(\mathbf{J})+\mathbf{E}^{-}_{tan}(\mathbf{M})\right]\frac{1}{\eta_o}+\hat{\mathbf{n}}\times\left[\mathbf{H}^{-}(\mathbf{J})+\mathbf{H}^{-}(\mathbf{M})\right]=0. \quad (2)$$

just outside A

The following definitions are made with respect to (1) and (2):

- A - Dielectric/free space boundary
- **J**,**M** - Equivalent electric and magnetic surface currents on A
- $\mathbf{J}^i,\mathbf{M}^i$ - Source currents of the incident wave
- $\hat{\mathbf{n}}$ - Outward surface normal on A
- $\mathbf{E}^{+},\mathbf{H}^{+}$ - Electric and magnetic fields calculated in an infinite homogeneous free space region
- $\mathbf{E}^{-},\mathbf{H}^{-}$ - Electric and magnetic fields calculated in an infinite homogeneous region with constitutive parameters of the dielectric
- η_o - Characteristic impedance of free space

The tangential portion of the electric field on A is determined using the expression

$$\mathbf{E}_{tan}=-\hat{\mathbf{n}}\times\hat{\mathbf{n}}\times\mathbf{E} \quad (3)$$

Equation (1) corresponds to the exterior equivalent situation where the scatterer is replaced with free space and the equivalent surface currents radiate the scattered field outside the boundary of the scatterer and the negative of the incident fields inside the scatterer. Equation (2) corresponds to the interior equivalent situation where the free space region is replaced with dielectric and the negative of the equivalent surface currents radiate zero fields in the exterior region and the same fields as in the original scattering problem in the interior region.

The total equivalent surface currents can be partitioned into scattering (radiation) currents and incident currents as follows:

$$\mathbf{J} = \mathbf{J}^s + \mathbf{J}^o \qquad \mathbf{M} = \mathbf{M}^s + \mathbf{M}^o \tag{4}$$

where

$$\mathbf{J}^o = \hat{\mathbf{n}} \times \mathbf{H}^+(\mathbf{J}^i, \mathbf{M}^i) \tag{5}$$

and

$$\mathbf{M}^o = -\hat{\mathbf{n}} \times \mathbf{E}^+(\mathbf{J}^i, \mathbf{M}^i) \; . \tag{6}$$

The incident currents $\mathbf{J}^o$ and $\mathbf{M}^o$ are the total equivalent surface currents for a free space or "phantom" dielectric scatterer. Thus, the incident currents must radiate zero scattered field in the exterior region of the exterior equivalent problem. Clearly, for an actual dielectric scatterer, the scattering currents alone produce the total scattered field in the exterior equivalent problem.

Substituting (4) into (1) and making use of the linearity of the electric and magnetic field operators to manipulating the result yields:

$$-\left[\mathbf{E}^+_{tan}(\mathbf{J}^s) + \mathbf{E}^+_{tan}(\mathbf{M}^s)\right]\frac{1}{\eta_o} - \hat{\mathbf{n}} \times \left[\mathbf{H}^+(\mathbf{J}^s) + \mathbf{H}^+(\mathbf{M}^s)\right] = \left[\mathbf{E}^+_{tan}(\mathbf{J}^i, \mathbf{M}^i) + \mathbf{E}^+_{tan}(\mathbf{J}^o, \mathbf{M}^o)\right]\frac{1}{\eta_o} + \hat{\mathbf{n}} \times \left[\mathbf{H}^+(\mathbf{J}^i, \mathbf{M}^i) + \mathbf{H}^+(\mathbf{J}^o, \mathbf{M}^o)\right] \tag{7}$$

just inside A

Because the "incident currents" produce the negative of the incident fields in the interior region of the exterior equivalent problem (7) can be reduced to

$$-\left[\mathbf{E}^+_{tan}(\mathbf{J}^s) + \mathbf{E}^+_{tan}(\mathbf{M}^s)\right]\frac{1}{\eta_o} - \hat{\mathbf{n}} \times \left[\mathbf{H}^+(\mathbf{J}^s) + \mathbf{H}^+(\mathbf{M}^s)\right] = 0 \, . \tag{8}$$

just inside A

Equation (8) is a statement of the fact that the scattering currents must radiate zero fields in the interior of the exterior scattering problem. Substituting (4) into (2) yields:

$$-\left[\mathbf{E}^-_{tan}(\mathbf{J}^s) + \mathbf{E}^-_{tan}(\mathbf{M}^s)\right]\frac{1}{\eta_o} + \hat{\mathbf{n}} \times \left[\mathbf{H}^-(\mathbf{J}^s) + \mathbf{H}^-(\mathbf{M}^s)\right] = \left[\mathbf{E}^-_{tan}(\mathbf{J}^o) + \mathbf{E}^-_{tan}(\mathbf{M}^o)\right]\frac{1}{\eta_o} - \hat{\mathbf{n}} \times \left[\mathbf{H}^-(\mathbf{J}^o) + \mathbf{H}^-(\mathbf{M}^o)\right] \tag{9}$$

just outside A

Equations (8) and (9) are a set of coupled integral equations which can be solved for the scattering currents directly. In this new integral equation formulation the excitation is moved from the exterior region to the interior region. The excitation in the new formulation is dependent on the material parameters of the scatterer. For a "phantom" dielectric scatterer, the right hand side of (9) (the excitation) is zero so that the scattering currents are identically zero.

The radiation current integral equation formulation presented here is developed from the conventional combined field integral equation (CFIE) formulation. A similar radiation current integral equation formulation can also be obtained for the PMCHW [2,3] formulation by following the same steps. The radiation current integral equation formulation is also easily extended to the multiple dielectric scatterer case.

III. NUMERICAL RESULTS

To demonstrate the radiation current integral equation formulation it was implemented in a two-dimensional moment method program [4]. For this program (8) and (9) were specialized to the transverse magnetic (TM) and transverse electric (TE) polarization cases. The 2-D MM program used a piecewise linear approximation to the contour of the scatterer and pulse current expansion with point matching. Segments of the piecewise linear representation of the scatterer are referred to as zones.

Because the program originally used the conventional CFIE there was no need to change the calculation of the MM impedance matrix when implementing the new formulation (note that the operator forms on the left sides of (1) and (2) are identical to those on the left sides of (8) and (9)). Only the calculation of the excitation required modification. Accurate calculation of the excitation is critical to the success of the method. On linear zones the incident currents $\mathbf{J}^o$ and $\mathbf{M}^o$ have constant magnitude and linear phase for an incident plane wave. If the excitation in (9) is calculated using the existing impedance elements (which assume constant magnitude and constant phase) it was observed that the radiation current based formulation yields no better results than the conventional formulation when treating low contrast scatterers. This observation is true for both the TE and TM cases. To achieve accurate results, the phase variation of the incident currents on each zone must be incorporated in the calculation of the excitation. The best approach is to derive new impedance elements which incorporate the linear phase variation of the incident currents. An alternative numerical approximation is to subdivide each linear zone into an odd number of subzones to more accurately model the phase variation of the incident currents. In this approach the field at the match point of the center subzone of each zone is calculated using the existing impedance elements (not the ones in the MM matrix but ones for each subzone calculated with the same routines). The use of an odd number of subzones is required so that match point of the center subzone will be the same as the match point of the entire zone. The former approach is more accurate and requires less computation. The latter approach, however, has the advantage of not being limited to plane wave excitation. For expedience, the latter approach was taken here with each zone being divided into eleven subzones for the calculation of the excitation.

Figure 1 compares the bistatic RCS of a low contrast dielectric cylinder calculated using the conventional formulation with those calculated using the radiation current formulation. Also plotted in Fig. 1 are series solution results. The radius of the cylinder is one free space wavelength, λ_o. The dielectric constant of the cylinder is 1.01. The MM program is written in FORTRAN and uses single precision floating point numbers. For both numerical solutions the cylinder was divided into 100 linear zones. The plane wave was incident from $\phi = 180^\circ$. In Fig. 1 the RCS, σ, is normalized by λ_o. Figure 1a is for a TM polarized incident wave and Fig. 1b is for a TE polarized incident wave. Figure 1 demonstrates the error in the calculated RCS which can occur when treating low contrast scatterers using the conventional formulation.

Figure 2 compares the magnitude of the normalized electric and magnetic scattering current for the case of Fig. 1b (TE polarization). Series solution results are also given. Figure 2a plots the electric current and Fig. 2b plots the magnetic current. The electric current is normalized by the incident magnetic field and the magnetic current is normalized by the incident electric field. Because the conventional formulation calculates the total equivalent currents, the scattered currents in Fig. 2 for the conventional formulation were determined by subtracting the incident currents from the calculated total currents. The radiation current formulation determines the scattered currents directly. Figure 2 illustrates that use of the conventional formulation results in errors in the scattering currents for low contrast scatterers. As expected these errors occur at places on the scatterer where the scattering current is small.

IV. CONCLUSIONS

A radiation current based surface integral equation formulation for low contrast scatterers was presented. Numerical results demonstrate the utility and validity of the new formulation.

Although its main application appears to be in treating low contrast scatterers, the radiation current formulation is also useful for accurately determining the equivalent currents on general material bodies in regions of the surface where the radiation currents are small. Note that unlike the most commonly used low-contrast formulation, the Müller formulation [3], the radiation current formulation works for high contrast scatterers as well as low contrast scatterers.

Implementation of the formulation in existing moment method programs is relatively straightforward since the moment matrix is unchanged. Only the excitation vector must be changed.

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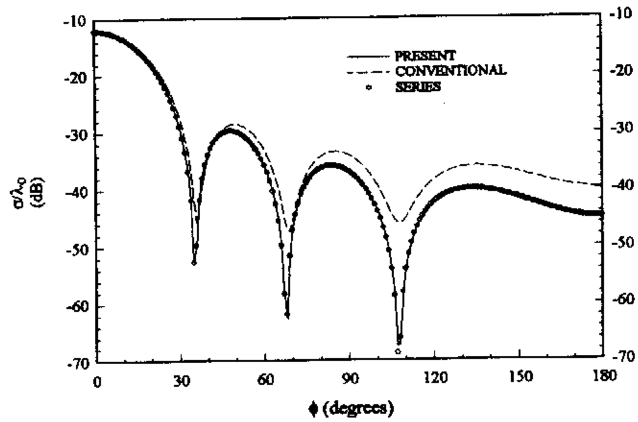
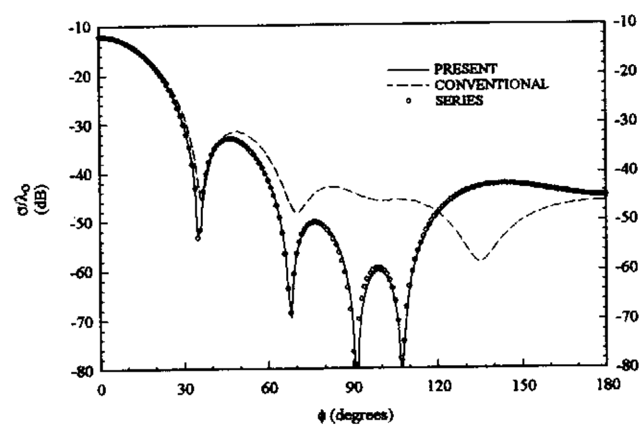
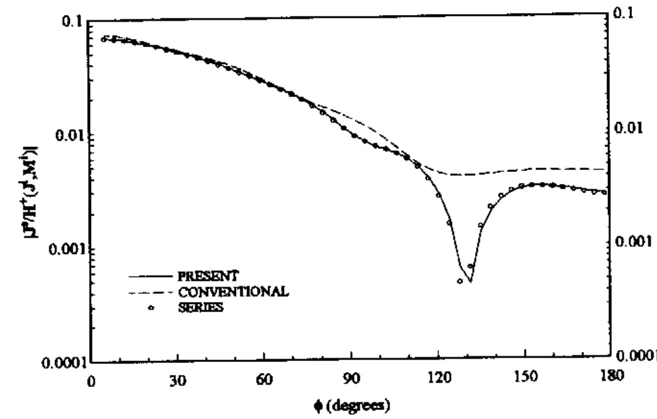


(a) TM polarization

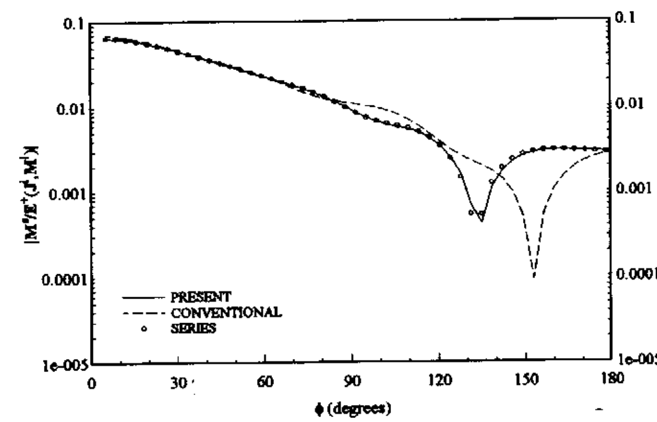


(b) TE polarization

Figure 1. Bistatic RCS of a dielectric cylinder with a radius of one free-space wavelength and a dielectric constant of 1.01. The plane wave is incident from $\phi = 180°$.



(a) Electric current



(b) Magnetic current

Figure 2. Normalized scattering currents on of a dielectric cylinder with a radius of one free-space wavelength and a dielectric constant of 1.01. The TE polarized plane wave is incident from $\phi = 180°$.