A Rapid Single-view Radar Imaging Method with Window Functions

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Abstract – Monostatic rapid single-view radar imaging technology is a technique that employs single incidence angle and single frequency point information to implement rapid monostatic radar imaging within a small angular field. Owing to its analytical expression, this technique can substitute the traditional frequency-angle-scanning imaging in a small angular range, facilitating the rapid generation of highly realistic radar imaging data slices for complex targets and environments. This technology has been significantly applied in scatter hotspot diagnostics and target recognition. In order to achieve the windowing effect equivalent to that of frequency-angle-scanning imaging, and to enhance the scattering feature of monostatic imaging while controlling sidelobes, this paper derives analytic windowed imaging formulas for monostatic radar. It then obtains analytical expressions for various typical monostatic windowing rapid radar imaging scenarios. This enables the monostatic rapid imaging technology to maintain high efficiency in its analytical expressions while achieving the windowing effect equivalent to traditional imaging. The validity and correctness of the analytical formula and software implementation have been confirmed through 1D, 2D, and 3D imaging verifications. This technology can provide a vast amount of training data for modern radars.

Index Terms – Radar imaging, single-view, window functions.

I. INTRODUCTION

Radar imaging can reconstruct target geometry and material characteristics through echoes and has a wide range of applications in geographic exploration, ocean observation, disaster prediction and military reconnaissance [1-5], etc. High-resolution one-dimensional distance imaging is often utilized for determining detonation points, while two-dimensional radar imaging is typically used for target recognition and remote sensing data classification [6]. Three-dimensional radar imaging is a crucial basis for radar feature identification in the current field of autonomous driving. How to obtain three-dimensional radar imaging data of the target and the environment is one of the hot research topics in the field of autonomous driving [7].

Traditional radar imaging technology is built upon the Fourier transform relationship between the electromagnetic distribution of the antenna aperture and far-field scattering, typically employing synthetic aperture methods to enhance azimuth resolution. From one dimension to two, and then to three, the utilization rate of imaging information is increasingly high. Superior imaging technologies, such as two-dimensional and three-dimensional, are gradually becoming practical with the advancement of hardware technology. In the field of autonomous driving, due to the all-weather characteristics of millimeter-wave radar, research on target characteristics technology based on three-dimensional millimeter-wave radar imaging is gaining increasing attention.

Radar imaging technology, based on electromagnetic scattering characteristic theory modeling, is a vital means of obtaining data on the radar scattering characteristics of targets and their environments. It has already found applications in military target identification and civilian remote sensing. Monostatic millimeter-wave radar imaging, originating from electromagnetic simulation technology, can significantly reduce the bandwidth and angular sampling expenses required for single-station imaging, enhancing the efficiency of obtaining...
imaging slice data. This technology was first applied to time-domain simulation of radar target scattering characteristics [8], rapidly obtaining time-domain echoes through the convolution of the analytical expression of target partition element time-domain responses with the signal. Subsequently, this technology was used in two-dimensional Inverse Synthetic Aperture Radar (ISAR) imaging [9, 10], and in recent years, applied to automatic target recognition [11–13] and urban remote sensing electromagnetic feature extraction [14].

The windowing operation is a standard procedure within radar imaging algorithms. Imaging represents the comprehensive broadband and angular information from the radar within the time and/or spatial domains. By applying a non-uniform window function prior to the Fourier transform, the spectral leakage effect of the Fast Fourier Transform (FFT) is mitigated. Concurrently, the windowing process is akin to executing a convolution operation in the time or spatial domains, which results in the broadening of the main lobe, thus reducing resolution. Implementing a non-uniform window prior to the Fourier transformation in the imaging of aperture data can effectively lower the sidelobe levels in individual cell imaging and enhance the visibility of target characteristics [15][16]. Monostatic rapid imaging is mainly used for feature extraction and strong scattering diagnosis of targets. The literature [17] attempted to introduce a window function into the rapid single-view imaging algorithm, but it directly truncated the sinc function in time domain, which could not reproduce the conventional process of obtaining radar images through Fourier transformation after window function weighting.

II. SINGLE–VIEW RADAR IMAGING METHOD

For ease of understanding, this paper describes the imaging principle of single-view millimeter-wave radar using the most representative two-dimensional imaging as an example. The definition of single-view millimeter-wave imaging radar still adheres to the traditional frequency-angle-scanning radar imaging, that is, the received field value of the radar and the radar image constitute a Fourier transform pair [13].

\[
\Gamma(l,d) = \int \int E_s e^{-j2\pi(f_l+f_d)d} df_l df_d. \tag{1}
\]

Here, \( \Gamma(l,d) \) represents the image or target function in the image domain coordinates \((l,d)\), \( f_d \) is the spatial frequency in the radar line of sight direction, and it satisfies \( f_d = 2f/c \), so \( k_d = \pi f_d \) is the propagation constant of the electromagnetic wave. \( f_l \) is the spatial frequency in the direction orthogonal to the line of sight \( l \), and \( k_l = \pi f_l \). \( \hat{l} \) can be the direction indicated by azimuth \( \phi \) or elevation angle \( \hat{\theta} \). \( E_s \) is the received radar echo electric field value.

The process of sweep-frequency scattering radar imaging is to use FFT to solve the above formula (1). This process requires obtaining the target radar echo field value \( E_s \). In commonly used monostatic radars, it is necessary to implement monostatic wideband small-angle scanning in simulation modeling, especially for two-dimensional and three-dimensional imaging. To cover the entire target or scene, the number of scanning samples can reach the order of \( 10^4 \) and \( 10^6 \), respectively. Document [9] first gave the monostatic radar distance imaging formula, greatly improving the imaging efficiency. In order to keep the formula consistent with the windowed imaging formula below, the following simple derivation is given using the previous definition: Assume that any complex target is described by a geometric model represented by triangle collection \( \{ S_i, i \in [0, N-1] \} \). The expression of the far-field scattering field illuminated by a plane wave is as follows [19]:

\[
E_s = e^{-jkr} \frac{1}{4\pi r} \sum_{i=0}^{N-1} f_i(x,y)L_i. \tag{2}
\]

Here, \( f_i(x,y) \equiv (\hat{k}_i \times E_l + \hat{k}_r \times E_r) \times \hat{n}' + \hat{k}_s \times [(E_l + E_r) \times \hat{n}'] \) represents the polarization term, where \( \hat{k}_i \) and \( \hat{k}_r \) denote the wave vectors of incident and reflected waves for any surface element with the normal vector \( \hat{n}' \), respectively, and \( k \) is the wave vector constant. \( E_0 \) is the amplitude of \( E_s \) at source point. \( r \) is the distance from observation point to the origin. The term \( I_i \equiv \int_S e^{-2j\hat{k}_i \cdot r'} d s' \) is the phase integral for a unit surface element.

\[
f_i(x,y) = \int \int e^{-jk(\kappa_x x' + \kappa_y y')} dx' dy'
\]

\[
\cdot e^{-jk\hat{k}_i \cdot r_0} \equiv W_i e^{-jk\hat{k}_i \cdot r_0}, \tag{3}
\]

where \( r_0 \) represents the position vector of a certain point on the surface element \( S_i \) which serves as a phase reference point. The term \( e^{-jk\hat{k}_i \cdot r_0} \) represents the phase shift of this surface element relative to the origin of the coordinate system. \( W_i \equiv f_i(x,y) \int_S e^{-jk(\kappa_x x' + \kappa_y y')} dx' dy' \) represents the scattering shape factor of this surface element. By substituting equation (2) into the imaging formula (1), when \( r \to \infty \), the variation of \( e^{-jkr}/(4\pi r) \) with \( k \) can be ignored.

\[
\Gamma(l,d) = \frac{e^{-jkr}}{4\pi r} \int \int e^{-j2\pi(f_l+f_d)d} df_l df_d. \tag{4}
\]

Under the usual small-angle imaging assumption, ignore the variation of \( k_\alpha \) with \( \theta \) or \( \phi \), \( k_\alpha = k_\alpha \hat{\theta} = k_\alpha \hat{k}_s + k_\alpha \hat{k}_l \approx k_\alpha \hat{k}_s + k_\alpha \hat{k}_l \), where \( k_0 \) represents the wave number corresponding to the center frequency and \( \alpha \) represents \( \theta \) or \( \phi \). Let \( D = -\hat{k}_d \cdot R_0 \) and \( L = -\hat{k}_l \cdot R_0 \), which represent the line of sight distance and horizontal distance from
each triangular reference point to the coordinate origin, respectively.

\[ \Gamma(l, d) = \frac{e^{-jkr}}{4\pi r} \sum_{l=0}^{N-1} \left( W_i e^{j2\pi f_0 l f d} \right) \int e^{j2\beta E_i} e^{-j2\pi f d} \, df \]

\[ = \frac{e^{-jkr}}{4\pi r} \sum_{l=0}^{N-1} \left( W_i e^{-2j\pi f d} d - D/2 \right) \cdot B_d \text{sinc}(\pi B_d (d - D/2)) \text{sinc}(\pi B_l (l - L/2)). \]

When calculating the far field, let the amplitude distance \( r \) be 1, and the phase take \( r = 0 \), so:

\[ \Gamma(l, d) = \frac{jkE_0}{4\pi} \sum_{l=0}^{N-1} W_i e^{-2j\pi f d (d - D/2)} \cdot B_d \text{sinc}(\pi B_d (d - D/2)) \text{sinc}(\pi B_l (l - L/2)). \]

This equation indicates that the target radar image or target function can be composed of the superposition of sinc functions centered on each triangular element reference point. The terms sinc(\( \pi B_d (d - D/2) \)) and sinc(\( \pi B_l (l - L/2) \)) are referred to as expansion functions. Because the derivation process assumes a small angle, it’s easy to understand: one-dimensional imaging only uses sinc(\( \pi B_d (d - D/2) \)) as an expansion function, while three-dimensional imaging requires an additional expansion function similar to sinc(\( \pi B_l (l - L/2) \)).

Figure 1 provides a comparison between the three-dimensional imaging of a single-view radar of an aircraft and the three-dimensional imaging of a frequency scanning angle radar. Combined with more experiments, it shows that for complex targets, the rapid single-view imaging, whether in terms of the position of strong scattering points or the intensity of calibrated strong scattering points, is the same as that of frequency scanning angle. However, the former has much higher computational efficiency and is very suitable for strong scatter diagnosis and target feature extraction.

**III. WINDOWED SINGLE-VIEW RADAR IMAGING TECHNIQUE**

According to the definition of windowed normalized radar imaging in Cartesian coordinates \((l, d)\) [18], we have:

\[ \Gamma(l, d) = \frac{1}{A} \int S V e^{j2\pi f_l (l + f_d d) f d} \, df \, df_d, \]

where \( S = \sqrt{4\pi R E_i / E_t} \) represents the target scattering function, \( R \) is the distance from the target center to the observation point (in the far field, it is infinite), and \( E_t, E_i \) represent the scattered field and the incident field at the observation point, respectively. \( V \) is the window function, and \( A = \int V \, df \, df_d \) is the amplitude normalization factor [18]. After changing equation (5) to one-dimensional and three-dimensional integrals, we get the windowed normalized one-dimensional and three-dimensional imaging. For small angle imaging, we have:

\[ f_d = \frac{2f}{c}, \quad f_l \equiv \frac{2f\alpha}{c}. \]

When \( V \) is a rectangular window, we have:

\[ A = \int \frac{4f}{c^2} \, df \, d\alpha = 2\Delta_{\alpha} \int \left( f_{\text{max}}^2 - f_{\text{min}}^2 \right) \frac{4f}{c^2} \, df \, d\alpha \]

\[ \Gamma(l, d) = \frac{1}{A} \int S V e^{j2\pi f_d l} \frac{4f}{c^2} \, df \, d\alpha \]

\[ = \frac{1}{B\Delta_{\alpha}} \int S V e^{j2\pi f d} \, df \, d\alpha. \]

For traditional frequency-angle-scanning imaging, the aforementioned equation can be calculated through FFT. For rapid single-view imaging, it needs to be transformed into an analytical expression. Referring to the derivation in section II, we can get the single-view radar imaging formula weighted by a rectangular window:

\[ \Gamma(l, d) = \frac{1}{2\sqrt{\pi} E_t} \sum_{l=0}^{N-1} (E_0 W_i) e^{-j2\pi f_d l} \cdot B_l \text{sinc}[\pi B_l l] B_d \text{sinc}[\pi B_d d], \]

where \( B_l \equiv 2f_0 \Delta_{\alpha} / c, B_d \equiv 2B / c, f_0 \) is the center frequency point of bandwidth \( B, B \equiv f_{\text{max}} - f_{\text{min}}, l = l - L/2, d = d - D/2, \) and \( L \) and \( D \) are, respectively, the projections of the triangular reference phase vectors that make up the target in the \( l \) and \( d \) directions.

After similar derivation, Table I gives the key parameters of \( \Gamma(l, d) \) corresponding to commonly used window functions. For uniformity, all windowed single-view radar imaging formulas in the table are written in the following mode:

\[ \Gamma(l, d) = \frac{1}{A} \frac{1}{2\sqrt{\pi} E_t} \sum_{l=0}^{N-1} (E_0 W_i) e^{-j2\pi f_d l} E_F l, \]

where the key variables \( E_F l \) and \( E_F d \) represent the expansion functions in the \( l \) and \( d \) directions, respectively, and \( 1/A \) represents the weighting coefficient, specifics are shown in Table I. \( \Phi \) represents real part of a variable, \( \text{erf} \) is error function. Formulas of \( E_F l \) can be obtained by replacing \( d \) to \( l \) in \( E_F d \).
Table 1: Key quantities for rapid single-view radar imaging with different window functions

<table>
<thead>
<tr>
<th>Type</th>
<th>$\frac{1}{A}$</th>
<th>$\text{EF}_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangle</td>
<td>$\frac{1}{B_d B_l}$</td>
<td>$B_d \text{sinc} \left( \frac{\pi B_d d}{\sigma} \right)$</td>
</tr>
<tr>
<td>Triangle</td>
<td>$\frac{1}{B_d B_l}$</td>
<td>$\frac{(1-\cos(\pi B_d d))}{\pi^2 B_d^2 d^2}$</td>
</tr>
<tr>
<td>Welch</td>
<td>$\frac{9}{3B_d B_l}$</td>
<td>$\frac{2\sin(\pi B_d d)-2\pi B_d \cos(\pi B_d d)}{\pi^3 B_d^2 d^4}$</td>
</tr>
<tr>
<td>Sin</td>
<td>$\frac{\pi^2}{3B_d B_l}$</td>
<td>$\frac{2B_d \cos(\pi B_d d)}{\pi (1-4B_d^2 d^2)}$</td>
</tr>
<tr>
<td>Hann</td>
<td>$\frac{4}{B_d B_l}$</td>
<td>$\frac{\sin(\pi B_d d)}{2 \pi d (1-B_d^2 d^2)}$</td>
</tr>
<tr>
<td>Hamming</td>
<td>$\frac{46}{25B_d B_l}$</td>
<td>$(\frac{2}{\pi B_d^2 d^2-\frac{25}{4}} \sin(\pi B_d d))$</td>
</tr>
<tr>
<td>Blackman</td>
<td>$\frac{9304}{3919B_d B_l}$</td>
<td>$\frac{(\frac{7}{2}B_d^2 d^2-915B_d^2 d^2+7838) \sin(\pi B_d d)}{4652 \pi d (B_d^2 d^2-5B_d^2 d^4+4)}$</td>
</tr>
<tr>
<td>Gauss</td>
<td>$\frac{1}{2\pi^2 \sigma^2 \text{erf} \left( \frac{1}{2\sqrt{2} \sigma} \right) B_d B_l}$</td>
<td>$\sqrt{\frac{\pi}{2} \sigma B_d e^{-2\pi^2 \sigma^2 B_d^2 d^2} \text{erf} \left( \frac{1}{2\sqrt{2} \sigma} + j \sqrt{2\pi \sigma B_d d} \right)}$</td>
</tr>
</tbody>
</table>

IV. NUMERICAL EXPERIMENTS

To verify the effect of windowed single-view radar imaging, the windowing effect is demonstrated below using the one-dimensional distance image of a dihedral angle and two- and three-dimensional radar imaging of a certain aircraft as examples. The results are shown in Figs. 2, 3, 4, respectively.

For Fig. 2, the radar resolution is 0.01 m, the incident pitch angle is 90 degrees, the azimuth angle is 0 degrees, and the center frequency is 13.5 GHz. As shown in Fig. 2 when a rectangular window is used, the secondary strong scatterers of the dihedral High Resolution Range Profile (HRRP) are almost drowned out by the sidelobes. After using a non-uniformly distributed window function, all the strong scatterers introduced by the dihedral due to multipath are clearly displayed because the sidelobes are suppressed. To clearly demonstrate the effect, the upper part of Fig. 2 only shows a comparison of the Blackman window and the Rectangular (RECT) window. As can be seen from the figure, the peak positions and peak amplitudes of the two completely overlap where there is no sidelobe obstruction. This validates the correctness of the weight coefficients $A$ and $\text{EF}$ in the normalization formula. The main lobe of the Blackman window result shown in the figure is widened, which aligns with the characteristics of windowing in imaging. The lower part of Fig. 2 shows a comparison between all the window functions derived in this paper and the rectangular window. The correctness of the derivation of the weighting coefficients is validated through the degree of overlap of the peak positions and peak amplitudes, as well as the characteristics of the main lobe widening of each window function. Besides, the co-polarized RCS of this dihedral with the same excitation is 46.98 dBsm, which is equivalent to that of the main beam amplitude in HRRP. It also verifies the correctness of the normalized rapid imaging formulations.

Fig. 2. Comparison of dihedral angle one-dimensional distance images with rectangular and non-rectangular windows.

Figures 3 and 4 give examples of two- and three-dimensional radar images, the radar resolutions are both 0.3 m, the incident pitch angles are 45 degrees, the azimuth angles are 45 degrees, and the center frequencies...
Fig. 3. Comparison of two-dimensional imaging with rectangular and non-rectangular windows (using the Blackman window as an example).

Fig. 4. Application of three-dimensional windowed fast imaging.

Fig. 5. Two-dimensional imaging comparison between (a) simulated and (b) measured images.
Fig. 6. Three-dimensional imaging of typical vehicles and pedestrians at typical frequency bands.

vided in this paper can help acquire three-dimensional imaging training data for autonomous driving technology.

Figure 6 shows the three-dimensional imaging results of two types of road elements, vehicles and pedestrians, respectively, under 24 GHz (bandwidth 250 MHz) and 77 GHz (bandwidth 1 GHz), demonstrating different radar features from different perspectives. By taking them as typical scenes of panoramic streets and adding various typical elements such as vegetation, street lights, bicycles, etc. (as shown in Fig. 7), setting one of the vehicles with an autonomous driving MIMO radar, and setting the driving lane to constitute a typical autonomous driving scenario, the method proposed in this paper is used. The typical 3D imaging results under a working frequency of 77 GHz (bandwidth 1 GHz) are shown in Fig. 7. The left, middle, and right columns respectively represent the imaging results of beams irradiating to the left, middle, and right directions.

As can be seen from the figure, the method proposed in this paper can provide three-dimensional imaging electromagnetic data under different radar working conditions for typical road elements and panoramic autonomous driving scenarios under different assumed conditions, providing massive training data for autonomous driving machine learning.

V. CONCLUSION

This paper derives and implements a windowed rapid single-view radar imaging technique, providing the analytical extension functions and normalized weighting values when applying typical window functions in single-view imaging. This technique retains the analytical form of rapid single-view radar imaging and, like traditional imaging techniques, can achieve window function filtering and sidelobe suppression effects. Moreover, its imaging efficiency is significantly higher than that of traditional frequency-angle-scanning imaging. It can provide a large number of simulation-based training samples for technologies such as target recognition, remote sensing, and autonomous driving.

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REFERENCES


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Dr. Cui’s research interests include metamaterials and computational electromagnetics. He proposed the concepts of digital coding and programmable metamaterials, and realized their first prototypes, based on which he founded the new direction of information metamaterials, bridging the physical world and digital world. He has written books on the subject, published over 600 journal articles, and holds more than 150 patents. His work has been widely reported by Nature News, MIT Technology Review, Scientific American, Discover, New Scientists, etc.

Dr. Cui is the Academician of Chinese Academy of Science, and IEEE Fellow. He has held editorial roles for several scientific journals and has delivered over 100 keynote speeches. In 2019-2021, he was ranked in the top 1% for the highly cited papers in the field of Physics by Clarivate Web of Science (Highly Cited Researcher).