# Experimental Assessment of a Non-Redundant Approach to Minimize Data in a Spherical NF-FF Transformation for Offset Mounted Elongated AUTs

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***Abstract* –** This work concerns the experimental validation of a near-field (NF) spherical scanning for an offset mounted long antenna under test (AUT), that requires a non-redundant (NR), namely minimum, amount of NF data. We address the issue of decreasing the number of voltage samples required to execute the traditional NF-far-field transformation (NF-FFT) technique in a non-centered case scenario, which would generally need a considerably higher amount of input data as compared to the onset case. In particular, by exploiting the theory of the NR sampling representations of electromagnetic field and adopting a rounded cylinder model of the antenna, the number of required samples is exactly the same as the minimum one involved in the onset scenario. Experimental results, which prove the goodness and efficacy of the approach, are presented.

***Index Terms* –** Antenna measurements, NF-FF transformations, non-redundant sampling representations of the electromagnetic field, offset mounting, spherical scan.

## I. INTRODUCTION

Antenna characterization plays a key role in validating the performances of the antenna under test (AUT) in its realistic working conditions, since it ensures compliance with the design specifications and operational standards relevant to the intended application. The main objective of the measurement process is to determine the AUT far-field (FF) radiation pattern, which can be achieved using different methods, which exhibit distinct advantages and drawbacks in terms of accuracy, measurement setup complexity, and spatial constraints.

Conventional FF measurement techniques allow us to directly acquire the AUT FF parameters of interest but come with notable limits. Chief among these is the FF distance condition, which imposes a minimum separation between the measurement probe and the AUT, potentially leading to impractically large indoor measurement setups. These limitations have driven the spread of near-field (NF) measurement techniques over the past decades [1–11]. NF methods offer, among many others, a significant advantage by removing the need to satisfy the FF distance condition, enabling measurements to be performed at much shorter ranges than the FF ones in an anechoic chamber, a climate and electromagnetic controlled location [6]. NF techniques are classified into spherical, cylindrical, and planar NF measurements. In particular, spherical NF (SNF) methods [12–26] allow us to obtain the AUT complete FF pattern without introducing the errors resulting from the scanning area truncation but can lead to long measurement times with high related costs, due to the great number of necessary NF samples, particularly when electrically large AUTs are considered. This led to the development of innovative NF techniques aimed at reducing the massive amount of needed samples, such as non-redundant (NR) sampling representations of the electromagnetic (EM) field [27, 28], adaptive sampling [23], or compressed sensing [25, 26]. In particular, NR representations have emerged as exceedingly powerful tools, owing to their distinctive advantages. Indeed, they allow us to exploit all and only the available independent information of the problem, leading to a considerable reduction of required NF data. By properly exploiting the spatial bandlimitation of EM fields [29, 30], NR NF far-field transformation (NF-FFT) techniques with spherical scan have been developed [16, 20–22]. They necessitate the measurement of a minimum amount of samples, practically equal to the number of degrees of freedom (NDFs) of the field [30, 31], to accurately recover the voltage revealed by a non-directive probe [20–22], which exhibits a spatial bandwidth almost equal to the antenna field. Then, the NR samples, which are non-uniformly spaced, are efficiently interpolated to recover the massive NF data at the points, uniformly spaced, imposed by the traditional SNF sampling theory,

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by employing a 2-D optimal sampling interpolation (OSI) expansion. Finally, the AUT far-field is obtained by executing the standard spherical NF-FFT technique. These NR NF-FFT techniques outperform the standard spherical one, not only by requiring drastically less samples, thus entailing shorter acquisition times, but also enabling to exactly determine the highest-order spherical harmonic involved in the spherical wave expansion (SWE) of the field [16].

Unluckily, mechanical constraints or specific antenna mounting configurations often inevitably prevent the alignment of the antenna center with that of the measurement sphere (offset mounting). In such offset setups, the classical spherical NF-FFT technique requires collecting a drastically higher number of samples, due to the minimum sphere rule [15], thus increasing the acquisition time and related costs. To address this issue, Foged et al. [32, 33] proposed a modified formulation based on the computation of the SWE of the field with respect to the (offset) antenna center rather than to the one of measurement sphere. A similar approach was developed in [34]. While these techniques help in reducing the number of expansion coefficients and sampling points, they entail a huge computational effort, since they require us to perform a full matrix inversion. This problem was later mitigated by the iterative method proposed in [35]. In any case, these techniques require a number of samples slightly greater than in the onset mounting.

The goal of this paper is to give an experimental assessment of NR spherical scanning developed in [36] for an elongated AUT, mounted in offset and modeled by a rounded cylinder (see Fig. 1). This approach enables us to accurately recover the antenna far-field by using a minimum number of samples, equal to that required for the NR onset case, as previously demonstrated in [37] for an offset mounted long AUT modeled by a prolate spheroid, and in [38, 39] for quasi planar antennas offset mounted and modeled by an oblate spheroid and a double bowl. The experimental proofs have been performed via the roll-over-azimuth spherical NF measurement facility present in the Antenna Characterization Laboratory of the UNIversity of SAlerno (UNISA). The showcased results have demonstrated the efficacy of the technique from a practical viewpoint.

## II. NR REPRESENTATION ON THE SCANNING SPHERE

In this section, the development of an NR representation of the voltage acquired by a non-directive probe on a scan sphere $M$ having radius $d$ in the NF region of a non-centered elongated AUT is presented. The problem geometry is displayed in Fig. 1. Firstly, let us consider two different spherical coordinate systems:

- the onset coordinate system $S(r,\vartheta,\varphi)$ having the origin $O$ at the center of the scanning spherical surface $M$;
- the offset coordinate system $S'(r',\vartheta',\varphi')$ obtained by translating the onset one $S$ by a distance $d_s$ along the $z$-axis, so that its origin $O'$ coincides with the center of the offset mounted antenna.

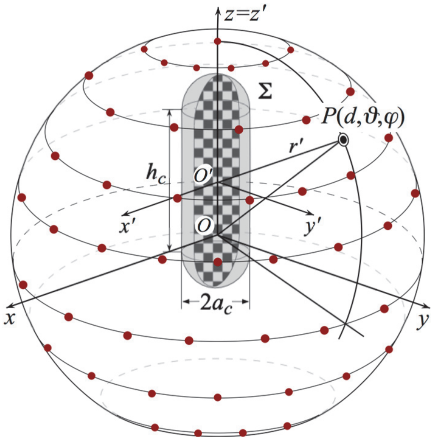

Fig. 1. Problem geometry.

While the onset coordinate system $S(r,\vartheta,\varphi)$ identifies the generic observation point $P(d,\vartheta,\varphi)$, the introduction of the offset one $S'$ allows us to develop an appropriate NR representation of the voltage on the sphere. Therefore, the NR sampling points are computed in the offset coordinate system $S'$ and subsequently mapped into the onset one $S$ through the following coordinate transformation relations linking the two reference systems:

$$\varphi=\varphi';\vartheta=\tan^{-1}[r'\sin\vartheta'/(r'\cos\vartheta'+d_s)], \quad (1)$$

$$r'=\sqrt{d^2-(d_s\sin\vartheta')^2}-d_s\cos\vartheta', \quad (2)$$

$$\vartheta'=\tan^{-1}[d\sin(\vartheta)/(d\cos(\vartheta)-d_s)]. \quad (3)$$

In accordance with the theory developed in [27], it is possible to define an NR sampling representation of the voltage $V$ (i.e. a representation requiring a minimum amount of samples) by adopting a suitable parameterization $\underline{r}=\underline{r}(\xi)$ to describe the observation curve and extracting an optimal phase factor $e^{j\psi(\xi)}$ from the expression of $V$, thus getting the so called "reduced voltage":

$$\tilde{V}(\xi)=e^{j\psi(\xi)}V(\xi), \quad (4)$$

which is a spatially quasi-bandlimited function. It is important to highlight that $V(\xi)$ is the voltage $V_p$ revealed by the probe or $V_r$ by the rotated probe. It follows that, when approximating $\tilde{V}(\xi)$ by a bandlimited function, a band-limitation error arises. However, this error has step-like behavior, being very small when the bandwidth is greater than a critical value $W_\xi$. In particular, it is possible to ensure the validity of this condition by considering an enlargement bandwidth factor $\chi'$, so that the bandwidth of the chosen bandlimited function is $\chi' W_\xi$. It is noteworthy that $\chi'$ values slightly greater than one can be adopted when AUT sizes are large in terms of wavelengths.

As shown in [27], the number of NR samples, at Nyquist rate, necessary to accurately represent $V(\xi)$ on a closed surface (even unbounded) encircling the surface $\Sigma$ modeling the antenna is proportional to the area of $\Sigma$. Therefore, to minimize the number of required samples, an appropriate choice of the most suitable rotational surface $\Sigma$ is in order. In particular, $\Sigma$ must be selected in such a way as to minimize any volumetric redundancy by best fitting the shape of the AUT. Since we are dealing with an elongated AUT, a suitable choice is represented by a rounded cylinder centered in $O'$ (see Figs. 1 and 2), namely a cylinder, of radius $a_c$ and height $h_c$, terminated with two hemispheres. As previously stated, the NR representation of the voltage $V$ on the scan sphere $M$, that is conveniently described by parallels and meridians, must be determined in $S'$. The spatial bandwidth relevant to a meridian, that is still a meridian curve in $S'$, i.e. a curve got as the intersection between $M$ and the meridian plane through the observation point $P$, is [27]:

$$W_\xi = \ell(C')/\lambda, \tag{5}$$

where $\ell(C')$ is the length of the curve $C'$, intersection between $\Sigma$ and the meridian plane, and $\lambda$ is the wavelength. Moreover, the optimal parameter and phase function are [27]:

$$\xi = (\pi/\ell(C'))[R_1 + s'(P_1) - R_2 + s'(P_2)], \tag{6}$$

$$\psi = (\pi/\lambda)[R_1 + s'(P_1) + R_2 - s'(P_2)], \tag{7}$$

where $R_1$ and $R_2$ are the distances from the point $P$ to the tangency points $P_1$ and $P_2$ on $C'$ and $s'(P_1)$ and $s'(P_2)$ are the related arclength abscissas (see Fig. 2). These parameters depend, of course, on the specific choice of the surface $\Sigma$. Having selected a rounded cylinder as a suitable surface modeling the AUT, we know that $\ell(C') = 2(h_c + \pi a_c)$ and hence:

$$W_\xi = 2(h_c + \pi a_c)/\lambda. \tag{8}$$

The related expressions for $\xi$ and $\psi$ are got by substituting in (6) and (7) the distances $R_{1,2}$ and the curvilinear abscissas $s'(P_{1,2})$, whose expressions depend on the positions of the points $P_1$ and $P_2$, which in turn change as the angle $\vartheta'$ varies. As shown in [34], three cases occur.

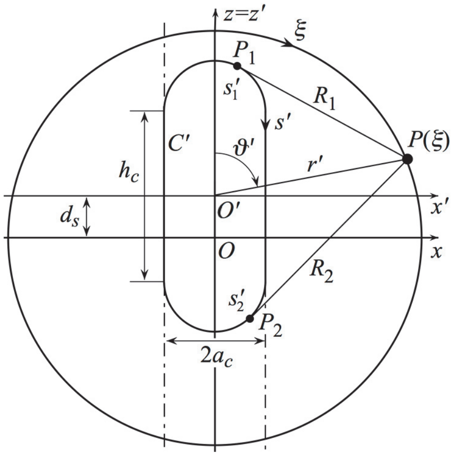

Fig. 2. Relevant to a meridian.

For $0 \le \vartheta' \le \sin^{-1}(a_c/r')$:

$$R_1 = \sqrt{(r' \sin\vartheta')^2 + (r' \cos\vartheta' - h_c/2)^2 - a_c^2}, \tag{9}$$

$$s'(P_1) = a_c \sin^{-1}\left(\frac{a_c r' \sin\vartheta' + R_1((h_c/2) - r' \cos\vartheta')}{R_1^2 + a_c^2}\right), \tag{10}$$

$$R_2 = R_1, \tag{11}$$

$$s'(P_2) = a_c \sin^{-1}\left(\frac{a_c r' \sin\vartheta' - R_2((h_c/2) - r' \cos\vartheta')}{R_2^2 + a_c^2}\right), \tag{12}$$

for $\sin^{-1}(a_c/r') \le \vartheta' \le \pi - \sin^{-1}(a_c/r')$, $R_1$ and $s'(P_1)$ are given by (9) and (10), while:

$$R_2 = \sqrt{(r' \sin\vartheta')^2 + (r' \cos\vartheta' + h_c/2)^2 - a_c^2}, \tag{13}$$

$$s'(P_2) = h_c + \\ + a_c\left[\pi - \sin^{-1}\left(\frac{a_c r' \sin\vartheta' + R_2((h_c/2) + r' \cos\vartheta')}{R_2^2 + a_c^2}\right)\right], \tag{14}$$

finally, for $\pi - \sin^{-1}(a_c/r') \le \vartheta' \le \pi$, $R_2$ and $s'(P_2)$ are given by (13) and (14), and:

$$R_1 = \sqrt{(r' \sin\vartheta')^2 + (r' \cos\vartheta' + h_c/2)^2 - a_c^2}, \tag{15}$$

$$s'(P_1) = h_c + \\ + a_c\left[\frac{\pi}{2} - \sin^{-1}\left(\frac{R_1 r' \sin\vartheta' + a_c((h_c/2) + r' \cos\vartheta')}{R_1^2 + a_c^2}\right)\right]. \tag{16}$$

For a parallel, the phase function is constant and $\varphi'$ can be effectively employed as an optimal parameter. The corresponding bandwidth is:

$$W_{\varphi'} = \frac{\pi}{\lambda}\max_{\bar{z}'}\left(\sqrt{(\rho+\rho'(\bar{z}'))^2+(z'-\bar{z}')^2} - \sqrt{(\rho-\rho'(\bar{z}'))^2+(z'-\bar{z}')^2}\right), \tag{17}$$

where $\rho = r'\sin\vartheta'$ and $\rho'(\bar{z}')$ defines the rounded cylinder equation.

The maximum is achieved at:

$$\bar{z}' = \begin{cases} z' & |z'| \le \dfrac{h_c}{2} \\ \left[\dfrac{h_c}{2} + \dfrac{(|z'|-h_c/2)a_c^2}{(r'\sin\vartheta')^2 + (|z'|-h_c/2)^2}\right]\operatorname{sgn}(z') & |z'| > \dfrac{h_c}{2} \end{cases}, \tag{18}$$

where $\operatorname{sgn}(\cdot)$ is the sign function.

The obtained NR samples are uniformly spaced with respect to the optimal parameter $\xi$ but not with respect to $\vartheta'$. This is why a suitable interpolation of the samples must be introduced to carry out the standard NF-FFT, which requires a set of uniformly distributed input data. It must be emphasized that interpolation of the NR voltage samples in the prescribed (uniformly spaced) positions on the sphere using the classical cardinal series would entail a massive computational effort. Indeed, this well-known technique is suboptimal when dealing with a great number of data, due to the use of the Dirichlet sampling functions, that, to prevent the truncation error, implicates the involvement of all input samples to determine any output one. Furthermore, this would also cause propagation of errors, which affect the samples, from high to low voltage values. Hence, a custom-made optimal interpolation algorithm, the 2-D OSI expansion [40], is used. Such an interpolation scheme allows us to reconstruct the voltage values at any point on the scan sphere by using a traveling sampling and adopting optimal self-truncating functions, which require us to retain only a small number of samples in the proximity of the considered output one to minimize truncation error. It is thus possible to lessen the computational load and to prevent error propagation from high to low voltage levels. Hence, probe voltage at the point $P(\vartheta', \varphi')$ can be reconstructed [36]:

$$V(\xi(\vartheta'),\varphi') = \mathrm{e}^{-\mathrm{j}\psi(\xi)} \sum_{n=n_0-q+1}^{n_0+q} \left\{ K(\xi,\xi_n,\bar{\xi},N,N'') \cdot \sum_{m=m_0-p+1}^{m_0+p} \tilde{V}(\xi_n,\varphi'_{m,n}) K(\varphi',\varphi'_{m,n},\bar{\varphi}',M_n,M''_n) \right\}, \tag{19}$$

where $n_0 = \mathrm{Int}(\xi/\Delta\xi)$, $m_0 = \mathrm{Int}(\varphi'/\Delta\varphi'_n)$, $2q \times 2p$ is the retained samples number, and:

$$\xi_n = n\Delta\xi = 2\pi n/(2N''+1); N'' = \mathrm{Int}(\chi N')+1, \tag{20}$$

$$N = N''-N'; \quad N' = \mathrm{Int}(\chi' W_\xi)+1; \quad \bar{\xi} = q\Delta\xi, \tag{21}$$

$$\varphi'_{m,n} = m\Delta\varphi'_n = 2\pi m/(2M''_n+1); \quad M''_n = \mathrm{Int}(\chi M'_n)+1, \tag{22}$$

$$M_n = M''_n - M'_n; \quad M'_n = \mathrm{Int}[\chi^* W_{\varphi'}(\xi_n)]+1, \tag{23}$$

$$\chi^* = 1+(\chi'-1)[\sin\vartheta'(\xi_n)]^{-2/3}; \quad \bar{\varphi}' = p\Delta\varphi'_n. \tag{24}$$

In particular, Int(x) stays for the integer part function, while $\chi > 1$ is the oversampling factor, introduced to control the truncation error. Furthermore:

$$K(\alpha,\alpha_t,\bar{\alpha},H,H'') = D_{H''}(\alpha-\alpha_t)\Omega_H[(\alpha-\alpha_t),\bar{\alpha}], \tag{25}$$

where

$$\Omega_H(\alpha,\bar{\alpha}) = \frac{T_H[2\cos^2(\alpha/2)/\cos^2(\bar{\alpha}/2)-1]}{T_H[2/\cos^2(\bar{\alpha}/2)-1]}, \tag{26}$$

$$D_{H''}(\alpha) = \frac{\sin[(2H''+1)\alpha/2]}{(2H''+1)\sin(\alpha/2)}, \tag{27}$$

are, respectively, the Tschebyscheff and the Dirichlet sampling functions [27, 36].

The interested reader can find, in [40], a detailed study on the error introduced by the OSI (plots of the normalized maximum and normalized mean-square reconstruction errors for various values of the oversampling factor and versus the retained samples number). Such an expansion is computationally simple and very fast. Its validity is widely demonstrated in many papers by the authors.

## III. EXPERIMENTAL VALIDATION

The showcased NR NF-FFT has been validated through laboratory proofs at the UNISA Antenna Characterization Laboratory. This facility has an anechoic chamber (8 × 5 × 4 m) sized for both NF and FF measurements, depending on the FF distance requirements. The chamber is paneled by pyramidal absorbers, guaranteeing a background noise level lower than −40 dB in the X band. Three rotary tables and a vertical linear scanner equip a versatile system (see Fig. 6 in [39]), allowing us to perform NF measurements not only using classical scans, like cylindrical, spherical, and plane-polar, but also the innovative helicoidal, spherical-spiral, and planar-spiral scans. A vector network analyzer (VNA) is utilized to perform accurate amplitude and phase measurements. Such a measurement facility provides an additional rotary table too, allowing us to execute direct FF or radar cross section measurements of electrically small AUTs/targets. A photo of the NF facility of the UNISA Antenna Characterization Laboratory is shown in Fig. 3. The interested reader can find a more detailed description of this NF facility and additional photos in Chapter 1 of [11].

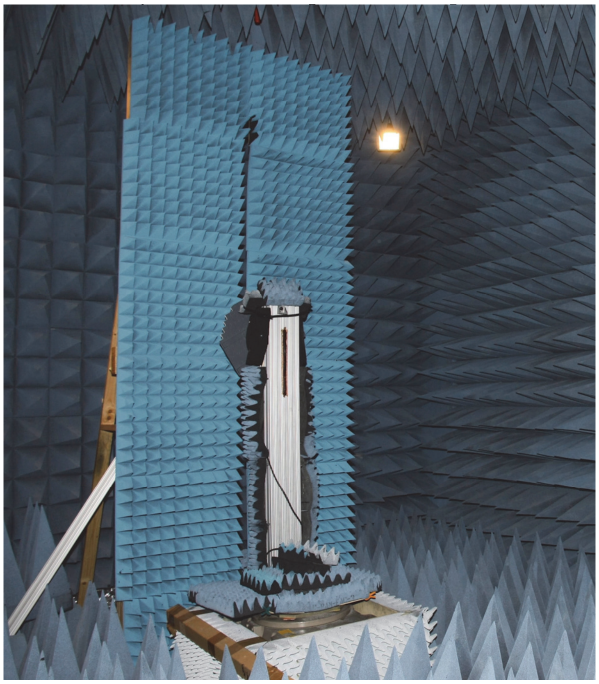

Fig. 3. Photo of the NF measurement system available at the UNISA Antenna Characterization Laboratory.

In the reported experimental results, the employed antenna is a slotted waveguide array operating at 10 GHz. It was obtained by realizing eight rounded-ended slots in the large walls of a tapered rectangular waveguide and feeding it by means of a coaxial to rectangular wave-guide transition. It has been mounted with its broad walls parallel to the $x = 0$ plane and its axis coinciding with the $z$ one. An open-ended WR90 rectangular waveguide is utilized as the scanning probe. The standard spherical NF-FFT [15] is carried out by the MI-3000 package and is employed to compute both the reference FF patterns, directly got by processing the considerably great amount of (offset) NF data collected at the classical lattice [15], and those attained from the NF data recovered by interpolating the drastically lower amount of gathered NR NF samples. In the considered experimental tests, the AUT has been shifted along the $z$-axis by a distance $d_s = 5.37\ \lambda$.

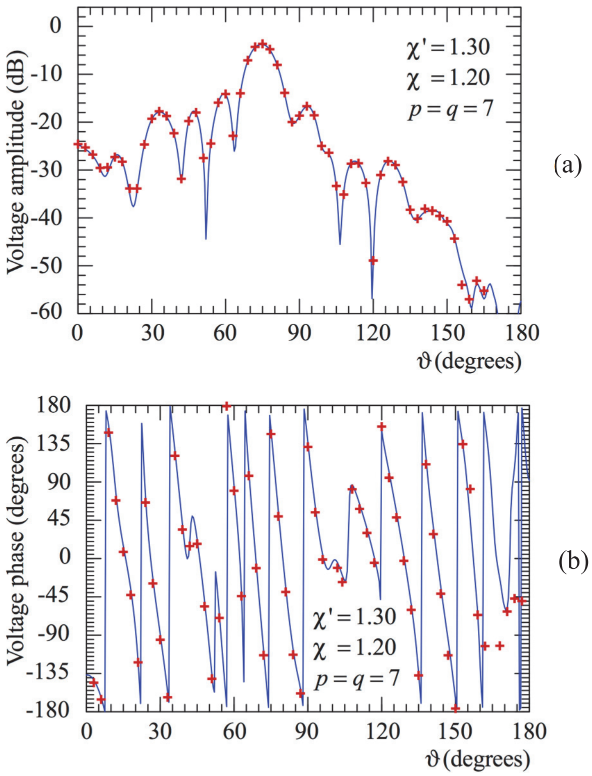

Fig. 4. Voltage amplitude (a) and phase (b) on the meridian at $\varphi = 0°$ — measured. + + + NR data.

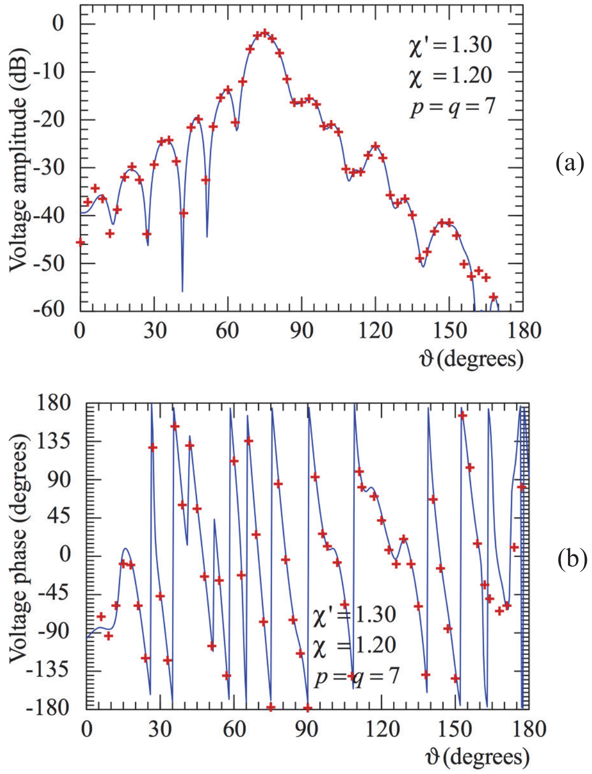

Fig. 5. Voltage amplitude (a) and phase (b) on the meridian at $\varphi = 90°$. — measured. + + + NR data.

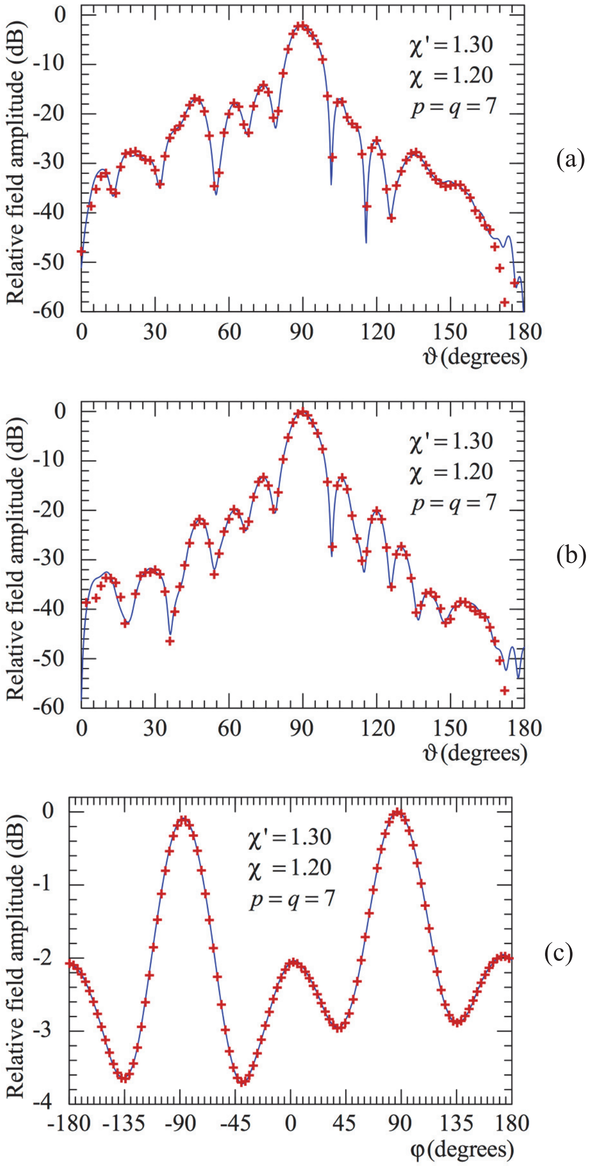

Fig. 6. FF patterns at $\varphi = 0^\circ$ (a), $\varphi = 90^\circ$ (b), and $\vartheta = 90^\circ$ (c). — reference. + + + NR data.

The parameters of the rounded cylinder employed for modeling the AUT are $h_c = 10\ \lambda$ and $a_c = 0.85\ \lambda$. The voltages have been measured on a sphere with radius $d = 20\ \lambda$ and the shown recoveries have been got by adopting $p = q = 7$, $\chi' = 1.30$, and $\chi = 1.20$. In Figs. 4 and 5, the amplitudes and phases of the recovered voltage $V_r$ on the meridians at $\varphi = 0^\circ$ and $\varphi = 90^\circ$ are compared with the directly acquired ones on the same meridians. The FF patterns at $\varphi = 0^\circ$, $\varphi = 90^\circ$, and $\vartheta = 90^\circ$, obtained from the NR NF samples, are compared in Figs. 6 (a–c) with those (references) taken from the massive number of offset data directly acquired on the NF grid for the standard NF-FFT. As can be clearly observed, both the NF and the FF recoveries results are very accurate, save for the regions where the voltage/field is very low, thus fully demonstrating the practical effectiveness of the approach.

It must be highlighted that the proposed technique allows a drastic decrease of the number of samples to be acquired and, consequently, a huge saving of acquisition time. Indeed, the AUT testing required only 705 NR NF samples, a remarkably lower amount of data than the 7320 required by the traditional spherical NF-FFT. It is interesting to compare the corresponding times employed to acquire the needed NF samples, namely 4.65 and 24.38 hours, respectively.

Further experimental results, which validate the effectiveness of the developed NR NF-FFT with spherical scanning for offset mounted elongated antennas and relevant to a different AUT and working frequency, are reported in [41].

## IV. CONCLUSION

In this paper, experimental validation of an NR NF-FFT technique with spherical scan for offset mounted long antennas is presented. The proposed approach, with respect to other existing techniques, requires the gathering of the same minimum number of NF samples fixed by the NR sampling for the onset mounting scenario, entailing not only a massive reduction in terms of needed NF data, but also of acquisition time. The experimental results are relevant to a rounded cylinder modeling of an elongated AUT, which represents the optimal choice since it suitably fits its shape without introducing any volumetric redundancy. Then, a custom-made 2-D OSI expansion allows us to accurately and efficiently reconstruct the NF data necessary to execute the standard spherical NF-FFT from the gathered NR ones. The accuracy and effectiveness of this innovative approach are experimentally confirmed by the very satisfactory agreement obtained in both the NF and FF reconstructions.

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