

Unified 3D Analytical Models of Surface-Inserted Axial Flux Machines With Odd-/Even-Piece Halbach Arrangements

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Abstract – Considering the 3-dimensional (3D) air-gap fringing field, unified 3D analytical models equivalent to linear machines are proposed for surface-inserted axial flux machines (SIAFMs) with odd-/even-piece Halbach arrangements. The 3D scalar potential governing equations for several regions can be uniquely solved according to Fourier decomposition coefficient matrixes derived from field boundary conditions. Based on this, the electromagnetic parameters can be derived. Predictions indicate that, compared with the non-optimized SIAFMs with three-/four-piece Halbach arrangements, the optimized SIAFMs with three-/four-piece Halbach arrangements exhibit excellent electromagnetic performances. The 3D finite element (FE) and analytical predictions for the SIAFM models are compared.

Index Terms – Odd-/even-piece Halbach arrangements, surface-inserted axial flux machine, unified 3D analytical model.

I. INTRODUCTION

Different from the non-optimized radial flux permanent magnet (PM) machines, axial flux machines with PMs have not only smaller volume but also higher torque and power density although they may lead to larger torque ripple [1, 2]. With the progress and development of society, especially extensive use of rare earth materials and improvement of control techniques, more axial flux machines, which are used in fields like electric vehicles, aerospace, and wind power [2–4], are needed.

A great variety of academic research on surface-mounted axial flux machines (SMAFMs) with PMs is carried out with the aim of improving their performance. Halbach arrangements that not only weaken torque ripple but also gather the magnetic field [4], are used for SMAFMs in [4–9], and the combination of magnet circumferential displacement with various pole-arc

ratios and magnet axial shape design are utilized to reduce pulsating torque in [10]. Surface-inserted axial flux machines (SIAFMs) with different thickness Halbach magnets and soft magnetic materials are proposed in [11]. Axial flux machines combining surface-inserted and surface-mounted magnets are analyzed in [7].

As a numeric solution approach, the 3-dimensional (3D) finite element (FE) analysis takes a long computation time, especially for optimization. In order to reduce the computing time, a few approaches have been presented. An approach with 2-dimensional (2D) scalar potential equivalent to linear motor for SMAFM is proposed in [5, 12, 13]. A 2D analytical model with vector potential is presented for achieving the magnetic field in SMAFMs with Halbach arrangements in [14, 15], and a method with 2D vector potential equivalent to linear machine is proposed in [4, 6, 16–18]. However, these approaches, except [13, 17], ignore the fringing effect in radial direction, and the methods with correction coefficient proposed in [13, 17] still have a degree of error. Then, 3D scalar potential techniques based on Bessel function in [18–20] or virtual argument Bessel function in [8, 9, 21] are presented. These 3D techniques can only solve the fields in SMAFMs and have non-negligible errors with the equivalent method in [7]. In addition, an equivalent method with the 3D scalar potential equivalent to linear motors has been proposed in [22–25], but it cannot be used for SIAFMs. The approach proposed in [26] only applies to surface-inserted radial flux PM machines.

In this paper, unified 3D analytical models are proposed for SIAFMs with odd-/even-piece Halbach arrangements. By means of solving the Fourier decomposition of boundary condition matrix in equivalent linear machines, the field governing equations in different regions are solved. The stator slotting effect is taken into account by Carter's coefficient. Different from 2D radial flux machines, the air-gap fringing effect for SIAFMs

be expressed as

$$\begin{cases} M_x^j = \frac{B_{re}}{\mu_0} \sin(\theta_2) & -\frac{\alpha_0 l}{2\alpha} < x < -\frac{\alpha_1 l}{2\alpha} \\ M_y^j = 0 & -\frac{qn_t}{2} < y < \frac{qn_t}{2} \\ M_z^j = \frac{B_{re}}{\mu_0} \cos(\theta_2) \end{cases}, \quad (2)$$

$$\begin{cases} M_x^j = 0 & -\frac{\alpha_1 l}{2\alpha} < x < \frac{\alpha_1 l}{2\alpha} \\ M_y^j = 0 & -\frac{qn_t}{2} < y < \frac{qn_t}{2} \\ M_z^j = \frac{B_{re}}{\mu_0} \end{cases}, \quad (3)$$

$$\begin{cases} M_x^j = -\frac{B_{re}}{\mu_0} \sin(\theta_2) & \frac{\alpha_1 l}{2\alpha} < x < \frac{\alpha_0 l}{2\alpha} \\ M_y^j = 0 & -\frac{qn_t}{2} < y < \frac{qn_t}{2} \\ M_z^j = \frac{B_{re}}{\mu_0} \cos(\theta_2) \end{cases}, \quad (4)$$

$$\begin{cases} M_x^j = -\frac{B_{re}}{\mu_0} \sin(\theta_2) & -\frac{\alpha_0 l}{2\alpha} + l < x < -\frac{\alpha_1 l}{2\alpha} + l \\ M_y^j = 0 & -\frac{qn_t}{2} < y < \frac{qn_t}{2} \\ M_z^j = -\frac{B_{re}}{\mu_0} \cos(\theta_2) \end{cases}, \quad (5)$$

$$\begin{cases} M_x^j = 0 & -\frac{\alpha_1 l}{2\alpha} + l < x < \frac{\alpha_1 l}{2\alpha} + l \\ M_y^j = 0 & -\frac{qn_t}{2} < y < \frac{qn_t}{2} \\ M_z^j = -\frac{B_{re}}{\mu_0} \end{cases}, \quad (6)$$

$$\begin{cases} M_x^j = \frac{B_{re}}{\mu_0} \sin(\theta_2) & -\frac{\alpha_1 l}{2\alpha} + l < x < \frac{\alpha_0 l}{2\alpha} + l \\ M_y^j = 0 & -\frac{qn_t}{2} < y < \frac{qn_t}{2} \\ M_z^j = -\frac{B_{re}}{\mu_0} \cos(\theta_2) \end{cases}, \quad (7)$$

$$\begin{cases} M_x^j = 0 \\ M_y^j = 0 \quad \text{others,} \\ M_z^j = 0 \end{cases} \quad (8)$$

$$\begin{cases} l = \frac{\alpha \pi R_j}{p} \\ q = \frac{R_{out} - R_{in}}{n_t} \end{cases}, \quad (9)$$

where M_x^j , M_y^j , and M_z^j are x -, y -, and z -axis components of the j -th group of magnets, B_{re} is PM remanence, μ_0 is vacuum permeability, p is pole-pair number, and h is PM length in the axial direction.

Using double Fourier decomposition, the magnetizations in regions $Q_0(-1/2 \leq x \leq 3/2$ and $qn_t \leq z \leq qn_t)$ are expressed by

$$\begin{cases} M_x^j = \sum_{m=0}^{\infty} \sum_{i=1,3,5}^{\infty} M_{xmi}^j \sin(\omega_i^j x) \cos \left\{ \omega_m^j \left[\left(y + \left(j - \frac{n_t+1}{2} \right) q \right) \right] \right\} \\ M_y^j = 0 \\ M_z^j = \sum_{m=0}^{\infty} \sum_{i=1,3,5}^{\infty} M_{zmi}^j \cos(\omega_i^j x) \cos \left\{ \omega_m^j \left[\left(y + \left(j - \frac{n_t+1}{2} \right) q \right) \right] \right\} \end{cases}, \quad (10)$$

where i is an odd number, m is a nonnegative integer, and ω_m^j and ω_i^j can be given as

$$\begin{cases} \omega_m^j = \frac{m\pi}{R_{out} - R_{in}} \\ \omega_i^j = \frac{ip}{\alpha R_j} \end{cases}, \quad (11)$$

where M_{zmi}^j and M_{xmi}^j are the Fourier decomposition coefficients and can be written as

$$M_{xmi}^j = \begin{cases} \frac{1}{lqn_t} \iint_Q M_x^j \sin(\omega_i^j x) & m \neq 0 \\ \cos \left\{ \omega_m^j \left[\left(y + \left(j - \frac{n_t+1}{2} \right) q \right) \right] \right\} dx dy, \\ \frac{1}{2lqn_t} \iint_Q M_x^j \sin(\omega_i^j x) dx dy & m = 0 \end{cases}, \quad (12)$$

$$M_{zmi}^j = \begin{cases} \frac{1}{lqn_t} \iint_Q M_z^j \cos(\omega_i^j x) \\ \cos \left\{ \omega_m^j \left[\left(y + \left(j - \frac{n_t+1}{2} \right) q \right) \right] \right\} dx dy, & m \neq 0. \\ \frac{1}{2lqn_t} \iint_Q M_z^j \cos(\omega_i^j x) dx dy, & m = 0 \end{cases} \quad (13)$$

III. 3D ANALYTICAL MODEL FOR HALBACH SIAFMS

A. Slotless 3D field prediction

In order to ensure the feasibility of 3D field calculation, several assumptions are made: infinite permeability for the stators and rotor, linear demagnetization property for PM, and neglected eddy current.

The whole 3D solution domain consists of air region $Q_1(-q/2 < y < q/2$ and $h < z < h + g)$ and magnet region $Q_2(-l/2 + l/\alpha \leq x \leq 3l/2 + l/\alpha, -q/2 < y < q/2$ and $0 < z < h)$. The 3D governing equations in these

two regions are written as

$$\begin{cases} \frac{\partial^2 \varphi_1^j}{\partial x^2} + \frac{\partial^2 \varphi_1^j}{\partial y^2} + \frac{\partial^2 \varphi_1^j}{\partial z^2} = 0, & \text{in } Q_1 \\ \frac{\partial^2 \varphi_2^j}{\partial x^2} + \frac{\partial^2 \varphi_2^j}{\partial y^2} + \frac{\partial^2 \varphi_2^j}{\partial z^2} = \frac{1}{\mu_r} \frac{\partial M_x^j}{\partial x}, & \text{in } Q_2 \end{cases}, \quad (14)$$

where μ_r is relative permeability of PMs, φ is scalar potential, subscript numbers denote the region, superscript j means j -th group of magnets, and g is axial length of air gap. In the linear analytical model, permeability of the magnet is generally considered as a constant.

In a Cartesian coordinate system, according to the symmetry and period boundary conditions, with setting a zero scalar potential value on the interface, the 3D scalar potential equations are expressed as

$$\begin{cases} \varphi_1^j(x, y, z) = -\varphi_1^j(x + \pi R_j / p, y, z) \\ \varphi_2^j(x, y, z) = -\varphi_2^j(x + \alpha \pi R_j / p, y, z) \\ \varphi_{1,2}^j(x, y, z) = \varphi_{1,2}^j(x, y + 2R_{\text{out}} - 2R_{\text{in}}, z) \\ \varphi_1^j(x, y, h + g) = \varphi_2^j(x, y, 0) = 0 \end{cases}. \quad (15)$$

On the basis of (15) and the separation of variables, general solutions of 3D scalar potential equations for (14) are

$$\begin{cases} \varphi_1^j = \sum_{m=0}^{\infty} \sum_{n=1,3}^{\infty} A_{1mn}^j (e^{k_1^j z} - e^{2k_1^j h + 2k_1^j g - k_1^j z}) \cos(\omega_n^j x) \\ \quad \times \cos \left\{ \omega_m^j \left[\left(y + \left(j - \frac{n_t + 1}{2} \right) q \right) \right] \right\} \\ \varphi_2^j = \sum_{m=0}^{\infty} \sum_{n=1,3}^{\infty} \left[A_{2mi}^j (e^{k_2^j z} - e^{-k_2^j z}) - \frac{\omega_i^j M_{xmi}^j}{\mu_r k_2^j} (1 - e^{-k_2^j z}) \right] \\ \quad \times \cos(\omega_i^j x) \cos \left\{ \omega_m^j \left[\left(y + \left(j - \frac{n_t + 1}{2} \right) q \right) \right] \right\} \end{cases}, \quad (16)$$

$$\begin{cases} k_1^j = \sqrt{\omega_m^j{}^2 + \omega_n^j{}^2} \\ k_2^j = \sqrt{\omega_m^j{}^2 + \omega_i^j{}^2} \end{cases}, \quad (17)$$

$$\begin{cases} \omega_m^j = \frac{m\pi}{R_{\text{out}} - R_{\text{in}}} \\ \omega_n^j = \frac{n\pi}{R_j} \\ \omega_i^j = \frac{i\pi}{\alpha R_j} \end{cases}, \quad (18)$$

where A_{1mn}^j and A_{2mi}^j are unknown coefficients of the j th group of magnets depending on the boundary conditions and n is an odd number.

Moreover, in the 3D spatial Cartesian coordinates, the relationships between scalar potential, magnetic field intensity, and flux density are

$$\begin{cases} H_x = -\frac{\partial \varphi}{\partial x} \\ H_y = -\frac{\partial \varphi}{\partial y} \\ H_z = -\frac{\partial \varphi}{\partial z} \end{cases}, \quad (19)$$

$$\begin{cases} B_x = -\mu_0 \mu_r \frac{\partial \varphi}{\partial x} + \mu_0 M_x \\ B_y = -\mu_0 \mu_r \frac{\partial \varphi}{\partial y} + \mu_0 M_y \\ B_z = -\mu_0 \mu_r \frac{\partial \varphi}{\partial z} + \mu_0 M_z \end{cases}, \quad (20)$$

For the unique solution of undetermined coefficients in (16), the 3D boundary conditions between two different regions are applied as

$$\begin{cases} H_{1x,y}^j(x, y, h) = H_{2x,y}^j(x, y, h) \\ B_{1z}^j(x, y, h) = B_{2z}^j(x, y, h) \end{cases}, \quad (21)$$

where subscript numbers denote the region, and subscript letters denote the x -, y -, and z -axis components.

Because of the rotor teeth being in the magnet region, Fourier decompositions are made for the boundary conditions. Separate Fourier decomposition coefficients need to be in agreement. Thus, (21) can be expanded as

$$\begin{cases} \int_{-\frac{l}{2}}^{\frac{l}{2}} \int_{-\frac{q\pi}{2}}^{\frac{q\pi}{2}} H_{1x}^j(x, y, h) \sin(\omega_n^j x) \\ \quad \times \cos \left\{ \omega_m^j \left[\left(y + \left(j - \frac{n_t + 1}{2} \right) q \right) \right] \right\} dx dy \\ = \int_{-\frac{l}{2}}^{\frac{l}{2}} \int_{-\frac{q\pi}{2}}^{\frac{q\pi}{2}} H_{2x}^j(x, y, h) \sin(\omega_n^j x) \\ \quad \times \cos \left\{ \omega_m^j \left[\left(y + \left(j - \frac{n_t + 1}{2} \right) q \right) \right] \right\} dx dy \\ \int_{-\frac{l}{2}}^{\frac{l}{2}} \int_{-\frac{q\pi}{2}}^{\frac{q\pi}{2}} B_{1z}^j(x, y, h) \cos(\omega_i^j x) \\ \quad \times \cos \left\{ \omega_m^j \left[\left(y + \left(j - \frac{n_t + 1}{2} \right) q \right) \right] \right\} dx dy \\ = \int_{-\frac{l}{2}}^{\frac{l}{2}} \int_{-\frac{q\pi}{2}}^{\frac{q\pi}{2}} B_{2z}^j(x, y, h) \cos(\omega_i^j x) \\ \quad \times \cos \left\{ \omega_m^j \left[\left(y + \left(j - \frac{n_t + 1}{2} \right) q \right) \right] \right\} dx dy \end{cases}. \quad (22)$$

According to (16)–(20), both the magnetic field intensity components H_{1x}^j and H_{2x}^j and the flux density

components B_{1z}^j and B_{2z}^j in (22) are the superposition of multiple harmonics. Therefore, considering the orthogonality of trigonometric functions, (22) can be simplified to a matrix equation as

$$\begin{bmatrix} \mathbf{A}_{1m}^j \\ \mathbf{A}_{2m}^j \end{bmatrix} = \begin{bmatrix} \mathbf{C}_m^j & \mathbf{D}_m^j \\ \mathbf{E}_m^j & \mathbf{F}_m^j \end{bmatrix}^{-1} \cdot \begin{bmatrix} \mathbf{G}_m^j \\ \mathbf{H}_m^j \end{bmatrix}, \quad (23)$$

where $\mathbf{A}_{1m}^j$ is a matrix with $(n_{\max} + 1)/2 \times 1$, $\mathbf{A}_{2m}^j$ is a matrix with $(i_{\max} + 1)/2 \times 1$, $\mathbf{C}_m^j$ is a matrix with $(n_{\max} + 1)/2 \times (n_{\max} + 1)/2$ related to m and n , $\mathbf{D}_m^j$ is a matrix with $(n_{\max} + 1)/2 \times (i_{\max} + 1)/2$ related to m , i and n , $\mathbf{E}_m^j$ is a matrix with $(i_{\max} + 1)/2 \times (n_{\max} + 1)/2$ related to m , i and n , $\mathbf{F}_m^j$ is a matrix with $(i_{\max} + 1)/2 \times (i_{\max} + 1)/2$ related to m and i , $\mathbf{G}_m^j$ is a matrix with $(n_{\max} + 1)/2 \times 1$ related to m , i and n , and $\mathbf{H}_m^j$ is a matrix with $(i_{\max} + 1)/2 \times 1$ related to m and i . Note that the elements in $\mathbf{G}_m^j$ are the superposition of different i values. In fact, for different values of m , the left term with unknown coefficients in (23) is rewritten as

$$\begin{bmatrix} \mathbf{A}_1^j \\ \mathbf{A}_2^j \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{1m=0}^j & \mathbf{A}_{1m=1}^j & \cdots & \mathbf{A}_{1m=m_{\max}^j}^j \\ \mathbf{A}_{2m=0}^j & \mathbf{A}_{2m=1}^j & \cdots & \mathbf{A}_{2m=m_{\max}^j}^j \end{bmatrix}, \quad (24)$$

where $\mathbf{A}_{1m}^j$ is a matrix with $(n_{\max} + 1)/2 \times m$, $\mathbf{A}_{2m}^j$ is a matrix with $(i_{\max} + 1)/2 \times m$, and subscript max denotes the maximum value.

Once the field due to j -th group of magnets is calculated, the total flux densities are the superposition of flux density generated by multiple groups of magnets and are given as

$$\begin{cases} B_{1\text{sum}} = \sum_{j=1}^{n_t} B_1^j \\ B_{2\text{sum}} = \sum_{j=1}^{n_t} B_2^j \end{cases}. \quad (25)$$

B. Slotted 3D field prediction

Considering the slotting influence, an equivalent approach is brought in. Figure 4 shows five flux paths denoted (a)–(e), where γ is an angle and $x_1, x_2, x_3, x_4, x_{rb}, x_{rc}$, and x_{rd} are length parameters. Accounting for the shortest magnetic circuit principle, it is worth noting that there exist at most five paths. The expressions of these flux paths are given as

$$\begin{cases} \eta_a = g \\ \eta_b = g + \frac{\pi x_{rb}}{2} \\ \eta_c = g + x_3 + 2\pi x_{rc} \cdot \frac{\gamma + 90}{360} \\ \eta_d = g + x_4 \cdot \tan \gamma + \frac{\pi x_{rd}}{2} + x_3 \\ \eta_e = g + x_1 - x_2 \end{cases}. \quad (26)$$

The magnetic conductance expressions of five paths are

$$\begin{cases} \Lambda_a = \mu_0 \int_{S_a} \frac{1}{\eta_a} dS = \frac{\mu_0 S_a}{g} \\ \Lambda_b = \mu_0 \int_{S_b} \frac{1}{\eta_b} dS = \frac{2\mu_0 (R_{\text{out}} - R_{\text{in}})}{\pi} \ln \left(1 + \frac{\pi x_\alpha}{2g} \right) \\ \Lambda_c = \mu_0 \int_{S_c} \frac{1}{\eta_c} dS = \frac{\mu_0 (R_{\text{out}} - R_{\text{in}})}{k_p} \\ \quad \times \ln \left[1 + \frac{k_p (x_\beta - x_\alpha)}{g + x_3 + k_p x_\alpha} \right] \\ \Lambda_d = \mu_0 \int_{S_d} \frac{1}{\eta_d} dS = \frac{2\mu_0 (R_{\text{out}} - R_{\text{in}})}{\pi} \\ \quad \times \ln \left[1 + \frac{\pi (x_\delta - x_\beta)}{k_t + \pi x_{rd}} \right] \\ \Lambda_e = \mu_0 \int_{S_e} \frac{1}{\eta_e} dS = \frac{\mu_0 S_e}{g + x_1 - x_3} \end{cases}, \quad (27)$$

where S_a, S_b, S_c, S_d , and S_e are the areas of five paths, x_α, x_β , and x_δ are the specific values for the equal lengths for paths (b) and (c), those for paths (c) and (d) and those for paths (d) and (e), respectively. In addition, k_p and k_t are written as

$$\begin{cases} k_p = \frac{\pi(\gamma + 90)}{180} \\ k_t = 2g + 2x_4 \cdot \tan \gamma + 2x_3 \end{cases}. \quad (28)$$

Instead of the slotted stator, the equivalent area S_f , the air gap length η_f , and the magnetic conductance Λ_f for the slotless stator are given as

$$\begin{cases} S_f = \pi (R_{\text{out}}^2 - R_{\text{in}}^2) \frac{1}{2N_s} \\ \eta_f = g_f \\ \Lambda_f = \mu_0 \int_{S_f} \frac{1}{\eta_f} dS = \frac{\mu_0 S_f}{g_f} \end{cases}. \quad (29)$$

With the same total magnetic conductance, the equivalent air gap can be written as

$$g_f = \frac{\mu_0 S_f}{\Lambda_a + \Lambda_b + \Lambda_c + \Lambda_d + \Lambda_e} = C_s \cdot g, \quad (30)$$

where C_s is Carter's coefficient and is computed by

$$C_s = \frac{\mu_0 S_f}{g \cdot (\Lambda_a + \Lambda_b + \Lambda_c + \Lambda_d + \Lambda_e)}. \quad (31)$$

For a period of one slot with $2l_c = 2\pi(R_{\text{out}} + R_{\text{in}})/2N_s$, the coefficient C'_s related to C_s can be defined

as

$$C'_s = \begin{cases} C_s \frac{\pi}{N_s} \frac{R_{out} + R_{in}}{2} - \beta < x < \frac{\pi}{N_s} \frac{R_{out} + R_{in}}{2} + \beta, \\ 1 & \text{others} \end{cases} \quad (32)$$

where β is the circumferential length of one half of slot and N_s is the stator slot number.

In order to predict the slotted model, using Fourier decomposition, the coefficient is expressed as

$$C'_s = \sum_{n_{cs}=0}^{\infty} c_{sn} \cos\left(n_{cs} \frac{\pi x}{l_c}\right), \quad (33)$$

where n_{cs} is a nonnegative integer and c_{sn} can be defined by

$$C_{sn} = \begin{cases} \frac{1}{l_c} \int_0^{2l_c} C'_s \cos\left(n_{cs} \frac{\pi x}{l_c}\right) dx & n_{cs} \neq 0 \\ \frac{1}{2l_c} \int_0^{2l_c} C'_s dx & n_{cs} = 0 \end{cases}. \quad (34)$$

Thus, the slotted flux density in the air-gap is calculated as

$$B_{1\text{slotted}} = \frac{B_{1\text{sum}}}{C'_s}. \quad (35)$$

As shown in Fig. 2, considering the winding end being in the 3D solution region, the air-gap fringing effect for SIAFMs needs to be considered. As shown in Fig. 5, there exists the air-gap fringing magnetic field. The air-gap fringing field coefficient is defined as

$$C_{fr} = 1 + \frac{\pi(2|y| - qn_t)}{4g}. \quad (36)$$

In order to calculate the back-EMF, according to (16), (19), (20), and (25), the slotless flux density in the axial direction at $z = h + g$ can be written as

$$B_{1z}|_{h+g} = -2\mu_0 \sum_{j=1}^{n_t} \sum_{m=0}^{\infty} \sum_{n=1,3}^{\infty} A_{1mn}^j k_1^j e^{k_1^j(h+g)} \cos(\omega_n^j x) \times \cos\left\{\omega_m^j \left[y + \left(j - \frac{n_t+1}{2}\right)q\right]\right\}, \quad (37)$$

where A_{2mi}^j and B_{2mi}^j are the undetermined coefficients in the region of the layer magnets near the air-gap.

And the ranges of region Q_3 influencing the back EMF of the s -th slot are $W - R_j\alpha_{cp}/2 < x < W + R_j\alpha_{cp}/2$ and $q < y < q$

$$W = W(t) = R_j[-\omega_r t + \alpha_{cp}(s-1)], \quad (38)$$

$$\alpha_{cp} = \frac{2\pi}{N_s}, \quad (39)$$

$$\omega_r = \frac{2\pi n_e}{60}, \quad (40)$$

where n_e is the rated rotational speed.

Therefore, the slotted flux density in the axial direction at $z = h + g$ can be expressed as

$$B'_{1z}|_{h+g} = \begin{cases} \frac{B_{1z}|_{h+g}}{C_s C_{fr}} \begin{cases} W - \frac{R_j\alpha_{cp}}{2} \leq x \leq W - \frac{R_j\alpha_{cp}}{2} + \beta \\ -qn_t \leq y \leq -\frac{qn_t}{2} \text{ or } \frac{qn_t}{2} \leq y \leq qn_t \end{cases} \\ \frac{B_{1z}|_{h+g}}{C_{fr}} \begin{cases} W - \frac{R_j\alpha_{cp}}{2} + \beta \leq x \leq W + \frac{R_j\alpha_{cp}}{2} - \beta \\ -qn_t \leq y \leq -\frac{qn_t}{2} \text{ or } \frac{qn_t}{2} \leq y \leq qn_t \end{cases} \\ \frac{B_{1z}|_{h+g}}{C_s C_{fr}} \begin{cases} W + \frac{R_j\alpha_{cp}}{2} - \beta \leq x \leq W + \frac{R_j\alpha_{cp}}{2} \\ -qn_t \leq y \leq -\frac{qn_t}{2} \text{ or } \frac{qn_t}{2} \leq y \leq qn_t \end{cases} \\ \frac{B_{1z}|_{h+g}}{C_s} \begin{cases} W - \frac{R_j\alpha_{cp}}{2} \leq x \leq W - \frac{R_j\alpha_{cp}}{2} + \beta \\ -\frac{qn_t}{2} \leq y \leq \frac{qn_t}{2} \end{cases} \\ B_{1z}|_{h+g} \begin{cases} W - \frac{R_j\alpha_{cp}}{2} + \beta \leq x \leq W + \frac{R_j\alpha_{cp}}{2} - \beta \\ -\frac{qn_t}{2} \leq y \leq \frac{qn_t}{2} \end{cases} \\ \frac{B_{1z}|_{h+g}}{C_s} \begin{cases} W + \frac{R_j\alpha_{cp}}{2} - \beta \leq x \leq W + \frac{R_j\alpha_{cp}}{2} \\ -\frac{qn_t}{2} \leq y \leq \frac{qn_t}{2} \end{cases} \end{cases}. \quad (41)$$

Then, using the slotted flux density, the flux linkage for the s -th coil with N -turn is written as

$$\Psi = N\Phi = N \iint_{Q_3} B'_{1z}|_{h+g} dS. \quad (42)$$

In addition, the back EMF of each coil can be obtained as

$$e_s = -\partial\Psi/\partial t = 4Nu_0 \sum_{j=1}^{n_t} \sum_{m=0}^{+\infty} \sum_{n=1,3}^{+\infty} \times A_{1mn}^j k_1^j e^{k_1^j(h+g)} \omega_r R_j \cdot \text{Int}(t) \cdot \text{Int}, \quad (43)$$

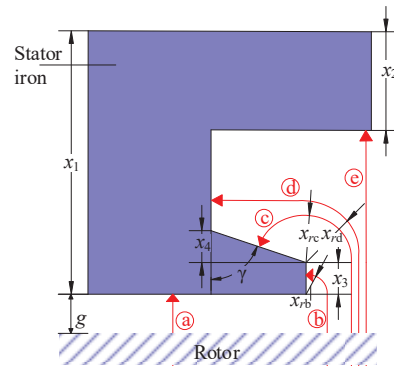


Fig. 4. Ideal paths in one half of slot and tooth.

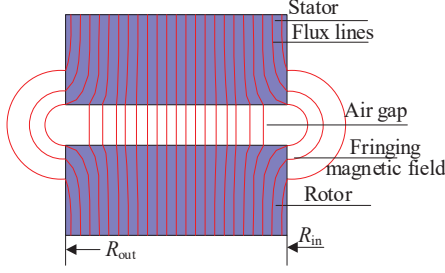


Fig. 5. Air-gap fringing magnetic field for SIAFMs.

where

$$Int(t) = \sin\{\omega_n R_j [-\omega_r t + \alpha_{cp}(s-1)]\} \times \left\{ \frac{2}{C_s} \sin\left(\frac{\omega_n \beta}{2}\right) \cos\left[\frac{\omega_n(R_j \alpha_{cp} - \beta)}{2}\right] + \sin\left[\frac{\omega_n(R_j \alpha_{cp} - 2\beta)}{2}\right] \right\}, \quad (44)$$

$$Int = \begin{cases} qn_t + \frac{4g}{\pi} \ln\left(1 + \frac{\pi qn_t}{4g}\right), & m = 0 \\ 2 \frac{\sin\left(\frac{\omega_m qn_t}{2}\right)}{\omega_m} & m \neq 0 \\ \times \cos\left[\omega_m \left(j - \frac{n_t + 1}{2}\right) q\right] \\ + Int_1 + Int_2, \end{cases}, \quad (45)$$

$$\begin{cases} Int_1 = \int_{-qn_t}^{-qn_t/2} \frac{1}{C_{fr}} \cdot \cos\left\{\omega_m \left[y + \left(j - \frac{n_t + 1}{2}\right) q\right]\right\} dy \\ Int_2 = \int_{qn_t/2}^{qn_t} \frac{1}{C_{fr}} \cdot \cos\left\{\omega_m \left[y + \left(j - \frac{n_t + 1}{2}\right) q\right]\right\} dy \end{cases}. \quad (46)$$

Figure 6 exhibits the stator coil distribution and the current directions of a 20-pole/ 24-slot SIAFM. The instantaneous balanced three phase back-EMFs are written as

$$\begin{cases} e_A = 2(-e_1 + e_2 + e_7 - e_8 - e_{13} + e_{14} + e_{19} - e_{20}) \\ e_B = 2(e_3 - e_4 - e_9 + e_{10} + e_{15} - e_{16} - e_{21} + e_{22}) \\ e_C = 2(-e_5 + e_6 + e_{11} - e_{12} - e_{17} + e_{18} + e_{23} - e_{24}) \end{cases}, \quad (47)$$

where e_A, e_B , and e_C are the three-phase back-EMFs, and the subscripts 1, 2, ..., 24 are the coil numbers.

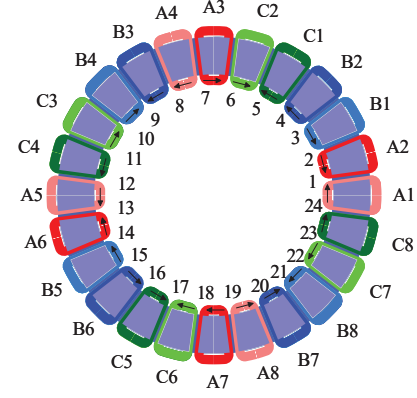


Fig. 6. Coil distribution and current directions of 20-pole/24-slot SIAFMs.

Therefore, the electromagnetic torque for SIAFMs is

$$T_e = \frac{e_A i_A + e_B i_B + e_C i_C}{\omega_r}, \quad (48)$$

where i_A, i_B , and i_C are the instantaneous three-phase currents.

Table 1: Design parameters of SIAFMs

Symbol	Parameter	Value
R_1/R_2	Inner/outer radii of rotor iron	52/68
r_1/r_2	Inner/outer radii of magnets	52/68
p/N_s	Numbers of pole-pairs and slot	20/24
I	Rated phase current	4 A
N	Number of coil turns	16
α_0	Arc ratio of whole pole magnets	0.7
α	Arc ratio of distance between	0.7
g	Air gap length	1 mm
h	Magnet thickness	5 mm
B_{re}/μ_r	Magnet property	1.1
n_e	Rated rotational speed	3000
P_N	Rated power	270 W

IV. OPTIMIZATION RESULTS AND FE VALIDATION

Table 1 lists the main design parameters of 20-pole/ 24-slot SIAFMs. In order to obtain the relatively better performance of the SIAFMs, optimization is made. The optimization object with the maximum average electromagnetic torque and the optimization constraint conditions are written as

$$\begin{cases} \text{Max}\{\text{Average}[T_e(\alpha_1, \dots, \alpha_\lambda, \theta_1, \dots, \theta_{\lambda+1})]\} \\ \text{Subject to. } \begin{cases} 0 \leq \alpha_1, \dots, \alpha_\lambda \leq \alpha_0 \leq 1 \\ 0^\circ \leq \theta_1, \dots, \theta_{\lambda+1} \leq 90^\circ \end{cases} \end{cases}. \quad (49)$$

Based on the parameter scanning technique, Fig. 7 shows the analytically obtained average electromagnetic torque against two optimization variables (magnetization angle and pole-arc ratio) for the three-piece Halbach arrangements, and Fig. 8 shows that against three optimization variables (two magnetization angles and pole-arc ratio) for the four-piece Halbach arrangements, respectively.

Table 2: Optimized parameters for three-/four-piece Halbach

Optimized Parameters	Three-Piece	Four-Piece
α_1	0.29	0.39
θ_1	/	17.1°
θ_2	53.0°	60.0°
$T_{\max}$ (Nm)	0.847	0.862

According to Figs. 7 and 8, for the three-/four-piece Halbach SIAFM, the optimization variables and the largest average value of electromagnetic torque are given in Table 2.

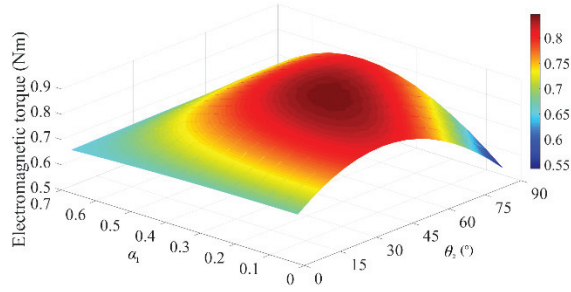


Fig. 7. Analytically obtained average electromagnetic torque against pole-arc ratio and magnetization angle for three-piece Halbach.

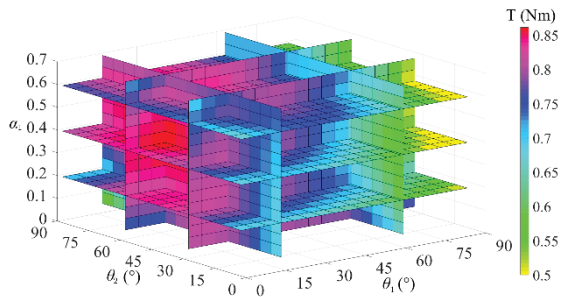


Fig. 8. Analytically obtained average electromagnetic torque against pole-arc ratio and two magnetization angles for four-piece Halbach.

Figure 9 shows a comparison of axial flux density waveforms of SIAFMs with optimized three-/four-piece Halbach arrangements at different radii by 3D FE and

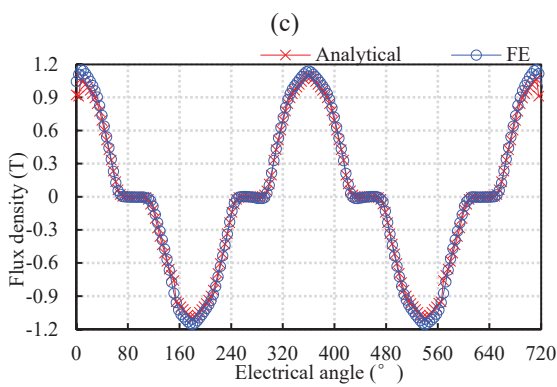
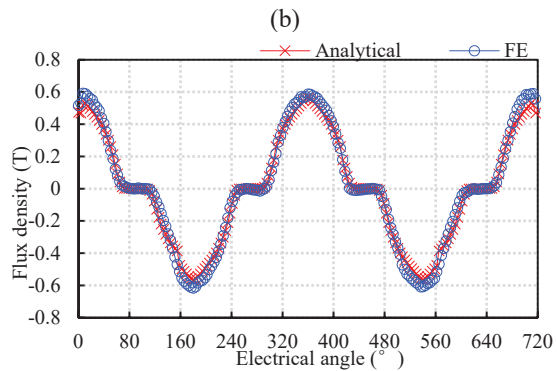
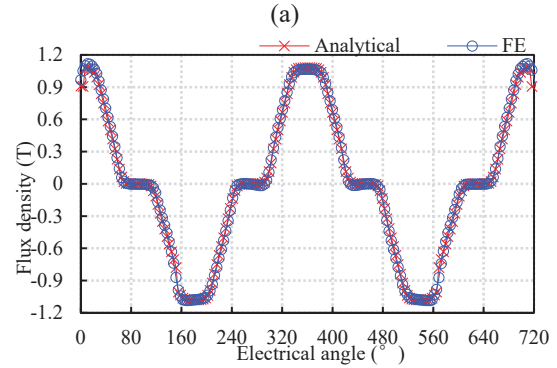
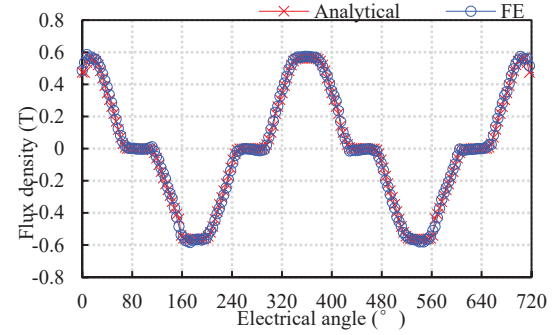


Fig. 9. Flux density waveform comparison of SIAFMs with optimized three-/four-piece Halbach arrangements at different radii in the radial direction and at 5.5 mm (axial air gap center) in axial direction by 3D analytical and FE models: (a) three-piece at 26 mm, (b) three-piece at 30 mm, (c) four-piece at 26 mm, and (d) four-piece at 30 mm.

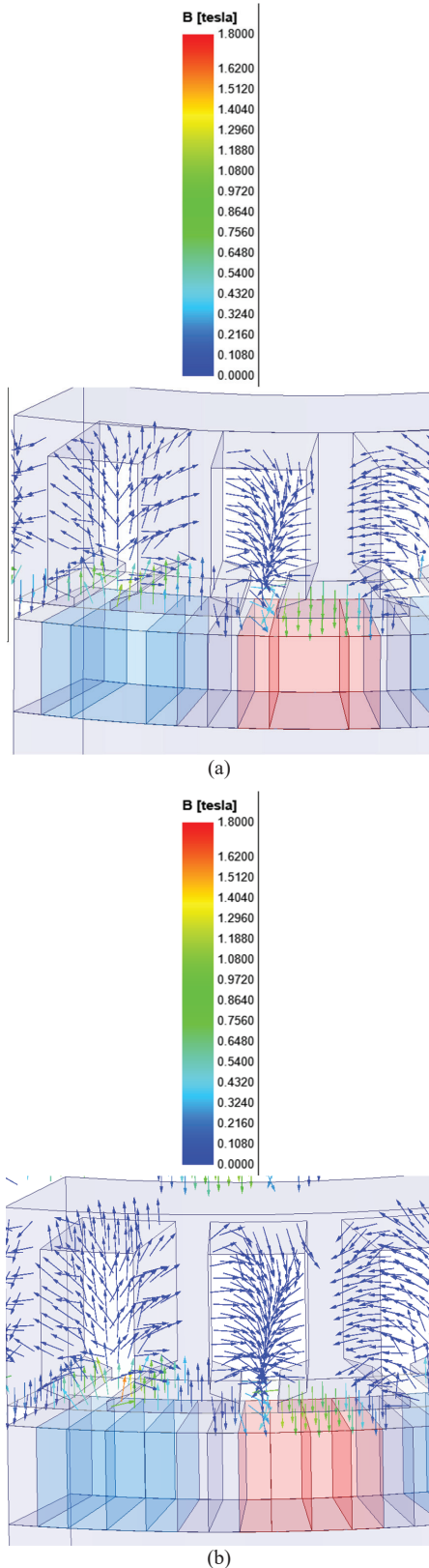


Fig. 10. Magnetic flux density vector of SIAFMs with optimized three-/four-piece Halbach arrangements in 3D FE models: (a) three-piece and (b) four-piece.

analytical models to verify the feasibility of the method by equivalent reluctance coefficients. It can be seen that, independent of the radius, each two waveforms calculated by two models are in great agreement.

Figure 10 shows the magnetic flux density vector of air gap and slots in SIAFMs with optimized three-/four-piece Halbach arrangements in 3D FE models. Figure 11 presents the comparison of no-load back-EMF waveforms for SIAFMs with optimized three-/four-piece Halbach arrangements from 3D FE and analytical models. It is clearly seen that great accuracy can be achieved with the use of Carter's coefficient and the air-gap fringing field coefficient for SIAFMs.

Because the amplitude of reluctance torque for SIAFMs is lower than 1 mNm, its waveform is not given. Compared with the electromagnetic torque, its value can be ignored. Figure 12 shows that the instantaneous electromagnetic torque waveforms for 3D FE and analytical models are in good agreement for SIAFMs with both three- and four-piece Halbach arrangements. Thus, the proposed 3D analytical models are validated.

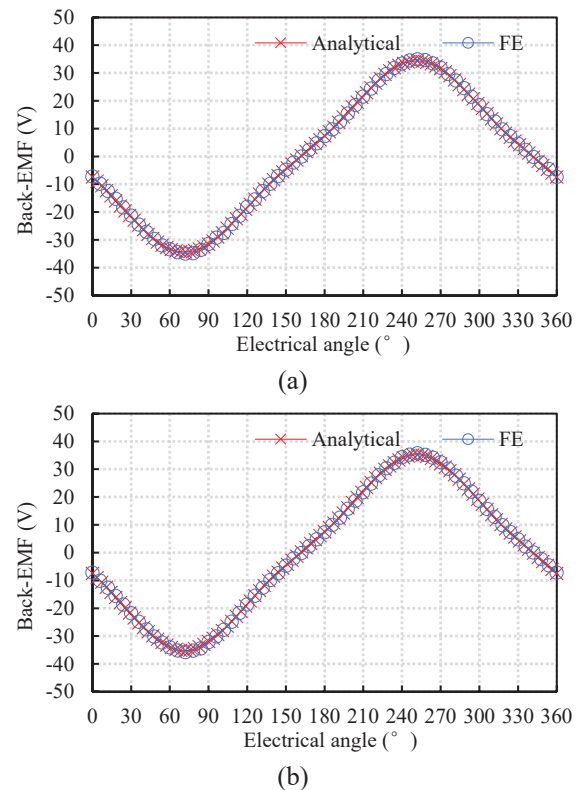


Fig. 11. Comparison of back-EMF waveforms of SIAFMs with optimized three-/four-piece Halbach arrangements by 3D FE and analytical models: (a) three-piece Halbach and (b) four-piece Halbach.

Due to the presence of the radial air gap edge magnetic field, accuracy values of analytical models

with/without air gap edge magnetic field are different. For SIAFMs with optimized three-/four-piece Halbach arrangements, the errors in back-EMF amplitude reduce from 9.22%/8.97% to 1.61%/1.34%, respectively. In terms of the average electromagnetic torque, the errors decrease from 7.73%/8.03% to 0.06%/0.35%, respectively.

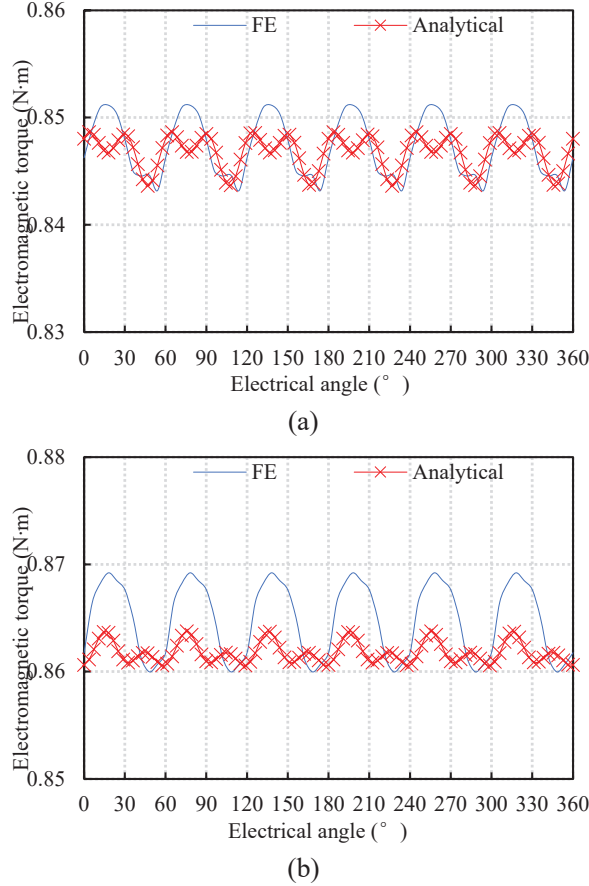


Fig. 12. Comparison of electromagnetic torque waveforms of SIAFMs with optimized three-/four-piece Halbach arrangements for 3D FE and analytical models: (a) three-piece Halbach and (b) four-piece Halbach.

V. OPTIMIZATION RESULTS AND FE VALIDATION FOR SMAFMS

Similarly, Figs. 13 (a) and (b) show the PM design parameters of 2D cross section for the proposed surface-mounted odd-/even-piece Halbach arrangements, and the PM design parameters are same as those for surface-inserted odd-/even-piece Halbach arrangements. In particular, if $\alpha = 1$, the SIAFMs are equivalent to the SMAFMs.

Figure 14 shows a comparison of axial flux density of slotted SMAFMs with optimized three-/four-piece Halbach arrangements by FE and 3D analytical models.

It can be seen that the waveforms calculated by the two models referred to above are in great agreement.

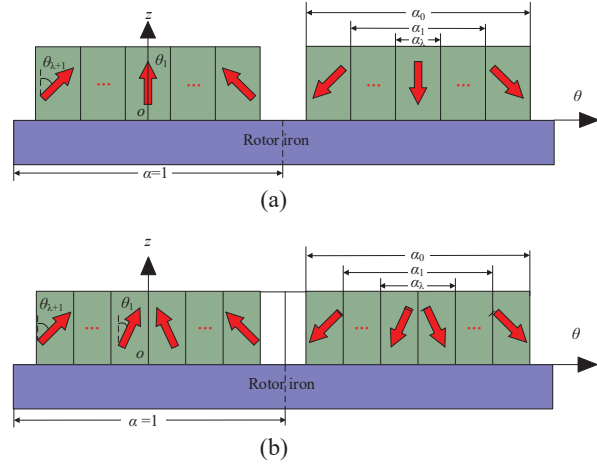


Fig. 13. Design parameters of surface-mounted odd-/even-piece Halbach arrangements: (a) odd-piece Halbach and (b) even-piece Halbach.

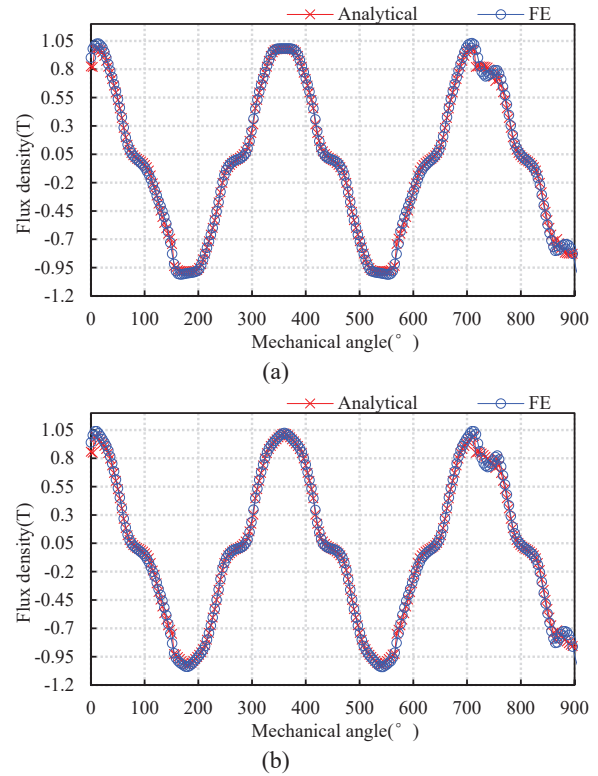


Fig. 14. Flux density waveform comparison of SMAFMs with optimized three-/four-piece Halbach arrangements by FE and 3D analytical models: (a) three-piece and (b) four-piece.

Figure 15 presents a comparison of no-load back-EMF waveforms of slotted SMAFMs with optimized

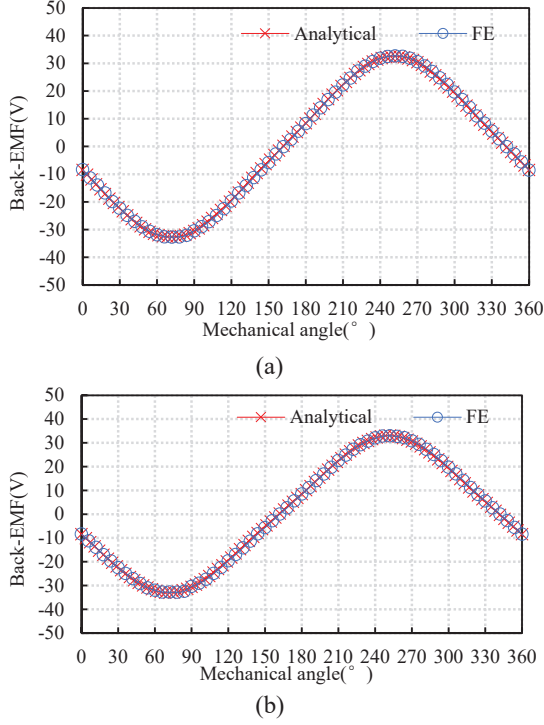


Fig. 15. Comparison of back-EMF waveforms of SMAFMs with optimized three-/four-piece Halbach arrangements by FE and 3D analytical models: (a) three-piece Halbach and (b) four-piece Halbach.

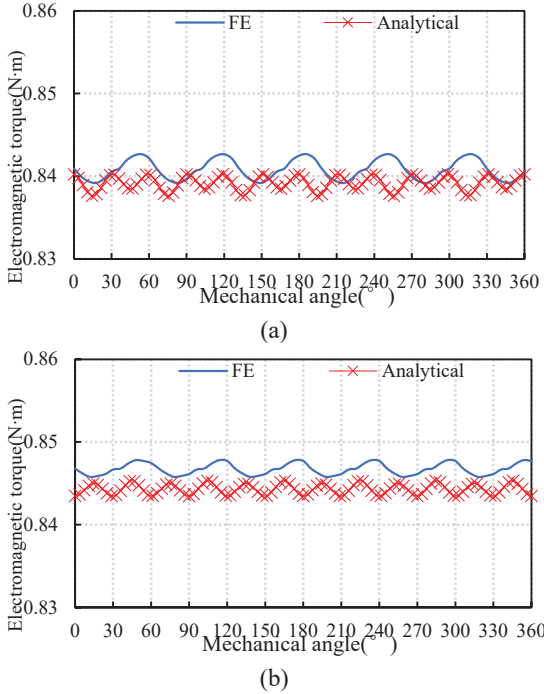


Fig. 16. Comparison of electromagnetic torque waveforms of SMAFMs with optimized three-/four-piece Halbach arrangements by FE and 3D analytical models: (a) three-piece Halbach and (b) Four-piece Halbach.

three-/four-piece Halbach arrangements from FE and 3D analytical models. It can be clearly seen that the use of Carter's coefficient and the air-gap outside leakage magnetic field coefficient for slotted SMAFMs exhibit great computation accuracy.

Figure 16 shows that instantaneous electromagnetic torque waveforms from FE and 3D analytical models are in excellent agreement for SMAFMs with both three- and four-piece Halbach arrangements. Thus, the 3D analytical models proposed above are validated.

VI. ANALYTICAL COMPARISON WITH NON-OPTIMIZED HALBACH SIAFMS

To analyze the effects of the optimized variables and methods mentioned in section IV, this section compares the traditional Halbach array with the optimized Halbach array in terms of back EMF, harmonics, and electromagnetic torque. Figures 17 (a) and (b) show the PM design parameters of the non-optimized three-/four-piece Halbach arrangements for SIAFMs, and the detailed parameter values are listed in Table 3.

Table 3: PM parameters for non-optimized SIAFMs with three-/four-piece Halbach arrangements

Symbol	Three-Piece	Four-Piece	
		Combination 1	Combination 2
α	0.7	0.7	0.7
α_0	0.7	0.7	0.7
α_1	7/30	0.35	0.35
$\theta_1(^{\circ})$	0	30	45
$\theta_2(^{\circ})$	45	45	60

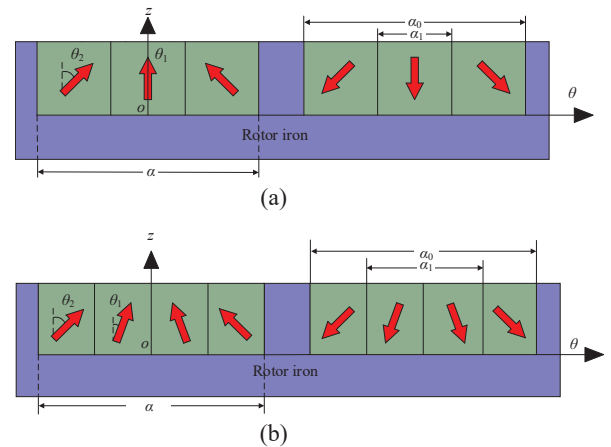


Fig. 17. Design parameters of non-optimized three-/four-piece Halbach arrangements for SIAFMs: (a) three-piece Halbach and (b) four-piece Halbach.

The prediction shows that for four-piece Halbach, the performance of combination 1 is better than that

of combination 2. Therefore, combination 1 is chosen for the comparison with the optimized results. Figure 18 exhibits and compares the back-EMF for the non-optimized/optimized 3D SIAFM models with three-/four-piece Halbach arrangements.

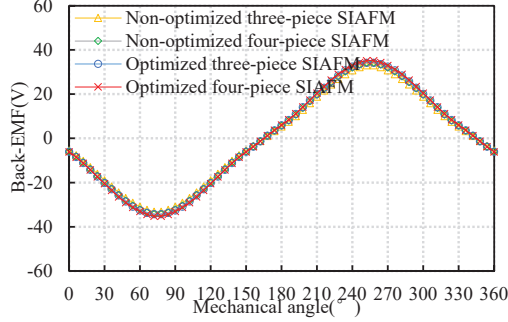


Fig. 18. Comparison of back-EMF waveforms for SIAFMs with non-optimized/proposed three-/four-piece Halbach.

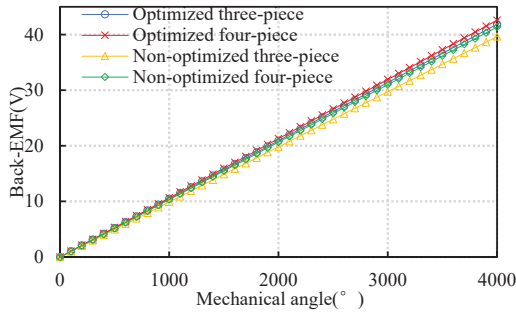


Fig. 19. Comparison of fundamental component of back-EMF with respect to speed for SIAFMs with non-optimized/proposed three-/four-piece Halbach.

Figure 19 exhibits and compares the differences of fundamental component of back-EMF with respect to speed for SIAFMs with non-optimized/optimized three-/four-piece Halbach. Tables 4 and 5 list the back-EMF harmonics, total harmonic distortion (THD), and fundamental amplitude (FA) for the non-optimized/optimized SIAFMs with Halbach arrangements. It can be clearly observed that the back-EMFs of the optimized 3D SIAFM models have higher fundamental and less harmonic components than those of the non-optimized ones. Compared with the values of the electromagnetic torque, it can be seen that the amplitudes of cogging torque waveforms for both Halbach arrays are small and can be ignored.

Figure 20 compares the instantaneous electromagnetic torque for the 3D SIAFM models with non-optimized/optimized Halbach arrangements. As far as the electromagnetic torque values are concerned, the optimized model parameters are much better.

Table 4: Back-EMF harmonics for non-optimized/optimized three-/four-piece Halbach

Harmonic Order	Three-Piece		Four-Piece	
	Non-Optimized	Optimized	Non-Optimized	Optimized
1	100.00%	100.00%	100.00%	100.00%
3	10.71%	9.49%	9.96%	9.73%
5	0.05%	0.13%	0.09%	0.12%
7	0.26%	0.04%	0.16%	0.03%
9	0.43%	0.04%	0.31%	0.31%
11	0.17%	0.35%	0.20%	0.08%
13	0.22%	0.18%	0.21%	0.17%
15	0.04%	0.04%	0.03%	0.01%
17	0.01%	0.01%	0.01%	0.01%

Table 5: Back-EMF parameters for non-optimized/optimized three-/four-piece Halbach

	Three-Piece		Four-Piece	
	Non-Optimized	Optimized	Non-Optimized	Optimized
FA (V)	29.73	31.39	31.01	31.94
THD	10.58%	9.50%	9.97%	9.74%

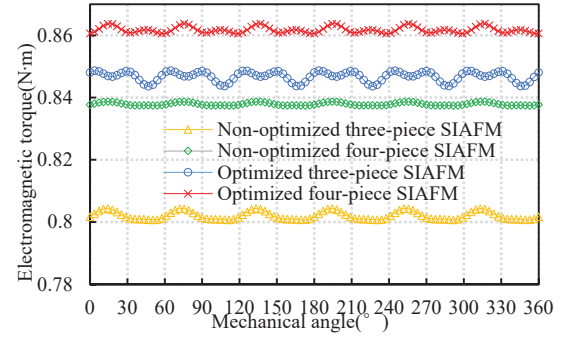


Fig. 20. Comparison of torque waveforms for SIAFMs with non-optimized/optimized three-/four-piece Halbach.

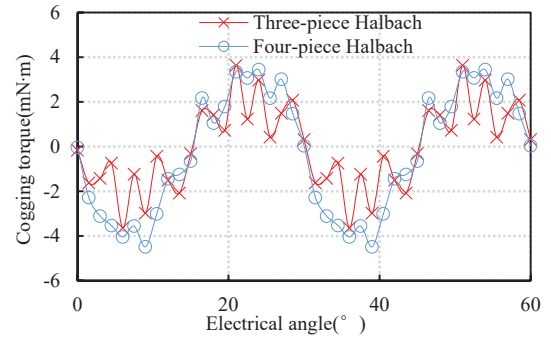


Fig. 21. Cogging torque for SIAFMs with three-/four-piece Halbach.

Figure 21 shows the cogging torque for the 3D SIAFM models with proposed three-/four-piece Halbach. Figure 22 exhibits the efficiency of the machine over whole speed and torque range for the 3D SIAFM models. When the 3D SIAFM models are in constant

power operation, the current and speed are inversely proportional. Therefore, as can be seen from the Fig. 22, the efficiency increases with the increase of speed.

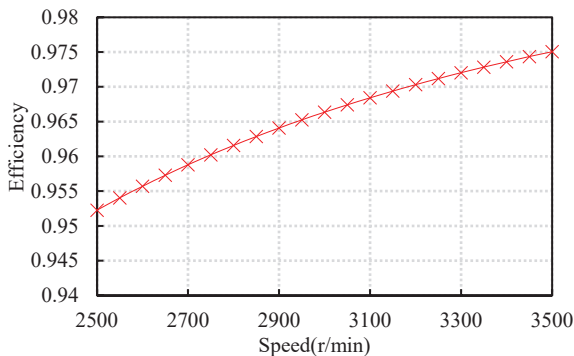


Fig. 22. Efficiency of SIAFMs over whole speed and torque range.

VII. DISCUSSION

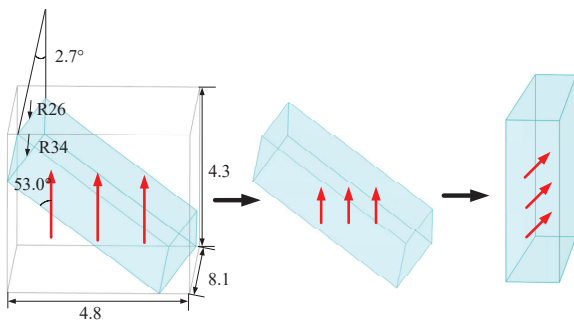


Fig. 23. Manufacturing process of one Halbach magnet.

The manufacture of a Halbach magnet with specific magnetization angle is difficult and time-consuming and expensive. Taking one Halbach magnet segment with a specific magnetization angle as an example, the manufacturing process is shown in Fig. 23. First, a relatively large rectangular prism with parallel magnetization is manufactured. Second, the required portion of magnet is exactly cut out. Third, this magnet segment with a specific magnetization direction is placed at the desired position. Finally, all the magnet segments of Halbach array are assembled using a kind of superglue. For magnet bonding, it is essential to choose a suitable adhesive, depending on the magnet material, service environment, and strength requirements. Epoxy resin adhesive is one of the commonly used options, offering high strength, heat resistance, and chemical resistance. However, it requires mixing of two components (A and B) and has a relatively long curing time (typically 24 hours). It is often used for permanent bonding of neodymium (rare-earth) magnets and ferrite magnets to metals or plastics.

A considerable part of each magnet segment is redundant and wasteful. It can be also seen that the manufacturing process is complex and expensive. Although Halbach arrays possess excellent electromagnetic performance, their practical implementation is restricted by complicated and costly manufacturing techniques. This work focuses on applying the 3D analytical model to axial flux PM machines, using a rectangular coordinate system for derivation to avoid the complicated calculations with Bessel functions in cylindrical coordinates. It also unifies surface-mounted and surface-inserted structures into one model to evaluate the application value of Halbach arrays in axial flux PM machines. Since our research focuses on analytical modeling and design analysis rather than experimental prototyping, no physical prototype is developed in this work.

VIII. CONCLUSION

Different from 2D radial flux machines, the air-gap fringing effect for SIAFMs needs to be considered. Accounting for the air-gap fringing field, unified 3D analytical models are proposed for SIAFMs with odd-/even-piece Halbach arrangements. In order to obtain the correct analytical solution with high accuracy, the slotting effect is taken into account by Carter's coefficient. Based on the proposed 3D analytical models, the constructed equations according to the interface conditions between different regions are solved and the optimized electromagnetic performances of SIAFMs with Halbach arrangements are obtained. The comparison of 3D analytical and FE predictions shows good agreement. The proposed 3D analytical models can be applied for design and optimization of SIAFMs with any-piece Halbach arrangements.

REFERENCES

- [1] A. Echle, A. Neubauer, and N. Parspour, "Design and comparison of radial flux and axial flux brushless DC motors for power tool applications," in *2018 XIII International Conference on Electrical Machines (ICEM)*, Alexandroupoli, Greece, pp. 125–130, 2018.
- [2] A. S. Patil and R. T. Ugale, "Design and comparative analysis of axial flux and radial flux permanent magnet brushless DC motor for a 2-wheeler electric vehicle application," in *2022 IEEE Conference on Interdisciplinary Approaches in Technology and Management for Social Innovation (IATMSI)*, Gwalior, India, pp. 1–4, 2022.
- [3] S. Kumar, W. Zhao, Z. S. Du, T. A. Lipo, and B.-I. Kwon, "Design of ultrahigh speed axial-flux permanent magnet machine with sinusoidal back EMF for energy storage application," *IEEE Trans. Magn.*, vol. 51, no. 11, pp. 1–4, Nov. 2015.
- [4] J. Zhu, G. Li, D. Cao, Z. Zhang, and S. Li, "Comparative analysis of coreless axial flux permanent

- magnet synchronous generator for wind power generation,” *J. Electr. Eng.*, vol. 15, no. 2, pp. 727–735, 2020.
- [5] B. Guo, Z. Djelloul-Khedda, and F. Dubas, “Non-linear analytical solution in axial flux permanent magnet machines using scalar potential,” *IEEE Trans. Industr. Electron.*, vol. 71, no. 4, pp. 3383–3393, 2024.
 - [6] B. Guo, Y. Huang, F. Peng, and J. Dong, “An analytical model for axial flux permanent magnet machines with Halbach array,” *Chin. Soc. Electr. Eng.*, vol. 39, no. 1, pp. 289–295+343, 2019.
 - [7] Y. Cao, L. Feng, R. Mao, and K. Li, “Analysis of analytical magnetic field and flux regulation characteristics of axial-flux permanent magnet memory machine,” *IEEE Trans. Magn.*, vol. 58, no. 9, pp. 1–9, 2022.
 - [8] P. Jin, Y. Yuan, Q. Xu, S. Fang, H. Lin, and S. L. Ho, “Analysis of axial-flux Halbach permanent-magnet machine,” *IEEE Trans. Magn.*, vol. 51, no. 11, pp. 1–4, 2015.
 - [9] T. Okita and H. Harada, “3D analytical model of axial-flux permanent magnet machine with segmented multipole-Halbach array,” *IEEE Access*, vol. 11, pp. 2078–2091, 2023.
 - [10] S. Wu, S. Zuo, X. Wu, F. Lin, and J. Shen, “Magnet modification to reduce pulsating torque for axial flux permanent magnet synchronous machines,” *Applied Computational Electromagnetics Society (ACES) Journal*, vol. 31, no. 3, pp. 294–303, 2016.
 - [11] L. Yang, K. Yang, S. Sun, Y. Luo, F. Xiong, and C. Luo, “Study on the influence of a combined-Halbach array for the axial flux permanent magnet electrical machine with yokeless and segmented armature,” *IEEE Trans. Magn.*, vol. 60, no. 3, pp. 1–5, 2024.
 - [12] Y. Huang, B. Guo, Y. Guo, J. Zhu, A. Hemeida, and P. Sergeant, “Analytical modeling of axial flux PM machines with eccentricities,” *Int. J. Appl. Electromagn. Mech.*, vol. 53, no. 4, pp. 757–777, 2017.
 - [13] F. Yi, C. Zhang, S. Qiu, L. Wu, and G. Yang, “Quasi-3D analytical method of open-circuit magnetic field for yokeless and segmented armature machine,” *IEEE Trans. Magn.*, vol. 59, no. 12, pp. 1–10, 2023.
 - [14] H. Tiegna, A. Bellara, Y. Amara, and G. Barakat, “Analytical modeling of the open-circuit magnetic field in axial flux permanent-magnet machines with semi-closed slots,” *IEEE Trans. Magn.*, vol. 48, no. 3, pp. 1212–1226, 2012.
 - [15] A. Bellara, Y. Amara, G. Barakat, and P. Reghem, “Analytical modelling of the magnetic field in axial flux permanent magnet machines with semi-closed slots at no load,” in *XIX International Conference on Electrical Machines (ICEM)*, Rome, Italy, pp. 1–6, 2010.
 - [16] W. Xu, Y. Hu, Y. Zhang, and J. Wang, “Design of an integrated magnetorheological fluid brake axial flux permanent magnet machine,” *IEEE Trans. Transp. Electrification*, vol. 10, no. 1, pp. 1876–1886, 2024.
 - [17] M. Sun, R. Tan, X. Han, and W. Tong, “Analysis and modeling for open circuit air gap magnetic field prediction in axial flux permanent magnet machines,” *Chin. Soc. Electr. Eng.*, vol. 38, no. 5, pp. 1525–1533, 2018.
 - [18] F. Zhao, Z. Shao, and H. Fan, “Analytical calculation and low-order component optimization of axial electromagnetic force for an axial flux motor,” *IEEE Trans. Magn.*, vol. 59, no. 11, pp. 1–5, 2023.
 - [19] J. H. Kim, Y. S. Kwon, S. Lee, B. M. Kang, J. M. Park, J. M. Sung, and J. Y. Yoon, “3D hybrid segmented layer modeling of on-load magnetic fields and torques for coreless axial-flux permanent magnet synchronous motor,” *IEEE Trans. Energy Convers.*, vol. 38, no. 3, pp. 1815–1826, 2023.
 - [20] Y. Huang, B. Ge, J. Dong, H. Lin, J. Zhu, and Y. Guo, “3D analytical modeling of no-load magnetic field of ironless axial flux permanent magnet machine,” *IEEE Trans. Magn.*, vol. 48, no. 11, pp. 2929–2932, 2012.
 - [21] P. Jin, Y. Yuan, M. Jin, S. Fang, H. Lin, H. Yang, and S. L. Ho, “3D analytical magnetic field analysis of axial flux permanent-magnet machine,” *IEEE Trans. Magn.*, vol. 50, no. 11, pp. 1–4, 2014.
 - [22] A. Zeroul, L. Hadjout, Y. Ouazir, H. Bensaidane, A. Benbekai, and O. Chaouch, “3D analytical model to compute the electromagnetic torque of axial flux magnetic coupler with a rectangular-shaped magnet,” in *2018 International Conference on Electrical Sciences and Technologies in Maghreb (CISTEM)*, Algiers, Algeria, pp. 1–4, 2018.
 - [23] B. Dolisy, S. Mezani, T. Lubin, and J. L  v  que, “A new analytical torque formula for axial field permanent magnets coupling,” *IEEE Trans. Energy Convers.*, vol. 30, no. 3, pp. 892–899, 2015.
 - [24] B. Dolisy, T. Lubin, S. Mezani, and J. L  v  que, “Three-dimensional analytical model for an axial-field magnetic coupling,” *Prog. Electromagn. Res.*, vol. 35, pp. 173–182, 2014.
 - [25] A. Hemeida and P. Sergeant, “Analytical modeling of surface PMSM using a combined solution of Maxwell’s equations and magnetic equivalent circuit,” *IEEE Trans. Magn.*, vol. 50, no. 12, pp. 1–13, 2014.
 - [26] Z. Q. Zhu, D. Howe, and Z. P. Xia, “Prediction of open-circuit airgap field distribution in brushless machines having an inset permanent magnet rotor topology,” *IEEE Trans. Magn.*, vol. 30, no. 1, pp. 98–107, 1994.



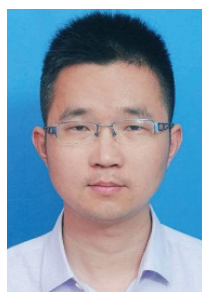
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