

Low-Cost Surrogate Modeling of Miniaturized Microwave Components Using Nested Kriging

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Abstract—In the paper, a recently reported nested kriging methodology is employed for modeling of miniaturized microwave components. The approach is based on identifying the parameter space region that contains high-quality designs, and, subsequently, rendering the surrogate in this subset. The results obtained for a miniaturized unequal-power-split rat-race coupler and a compact three-section impedance transformer demonstrate reliability of the method even for highly-dimensional parameter spaces, as well as its superiority over conventional modeling methods.

Keywords—design optimization, microwave design, miniaturized structures, nested kriging, surrogate modeling.

I. INTRODUCTION

Full-wave electromagnetic (EM) analysis plays an important role in design closure of microwave structures, especially compact devices [1]. Yet, solving EM-driven design tasks is CPU intensive due to massive simulations involved. Computational savings can be obtained by using fast surrogate models of the structure under design. Conventional modeling methods, e.g., neural networks [2], or kriging [3], are limited by the curse of dimensionality to handling circuits described by small numbers of parameters.

In [4] and [5], constrained modeling of microwave structures was presented with the model domain confined to a region containing a set of pre-existing reference designs, optimized for problem-specific figures of interest. The advantage is a remarkable reduction of the domain volume, which mitigates the problem of excessive number of training data samples required by the traditional setup. A recent nested kriging approach [6] brings further improvements over [4], [5] by enabling uniform sampling and model optimization in a straightforward manner. Here, we demonstrate the feasibility of this method for modeling of compact microwave components.

II. NESTED KRIGING MODELING FORMULATION

In this paper, we apply the nested kriging approach of [6] to cost-efficient modeling of compact microwave components. A brief formulation of the framework is provided in this section, followed by demonstration case studies discussed in Section III.

A. First-Level Model

We denote by f_k , $k = 1, \dots, N$, the figures of interest relevant to the design process (e.g., coupler operating frequency and/or power split ratio). We assume the existence of the reference designs $\mathbf{x}^{(j)} = [x_1^{(j)} \dots x_n^{(j)}]^T$, $j = 1, \dots, p$, optimized w.r.t. the performance vectors $\mathbf{f}^{(j)} = [f_1^{(j)} \dots f_N^{(j)}]$. The objective space F is defined by the ranges $f_{k,\min} \leq f_k^{(j)} \leq f_{k,\max}$, $k = 1, \dots, N$, to be covered by the surrogate. The first-level model $s_1(\mathbf{f})$ maps F into the design space X . It is implemented using kriging [3],

where $\{\mathbf{f}^{(j)}, \mathbf{x}^{(j)}\}$ are the training points. Figure provides a graphical illustration of these concepts.

B. Domain Definition

The model domain X_S is constructed by “fattening” the set $s_1(F) \subset X$ which approximates the region containing the designs that are optimum w.r.t. all $\mathbf{f} \in F$. This is realized by an orthogonal extension of $s_1(F)$ towards its normal vectors $\{\mathbf{v}_n^{(k)}(\mathbf{f})\}$, $k = 1, \dots, n - N$. Let $\mathbf{x}_{\max} = \max\{\mathbf{x}^{(k)}, k = 1, \dots, p\}$, $\mathbf{x}_{\min} = \min\{\mathbf{x}^{(k)}, k = 1, \dots, p\}$, and $\mathbf{x}_d = \mathbf{x}_{\max} - \mathbf{x}_{\min}$ (parameter variations within $s_1(F)$). We also define:

$$\alpha(\mathbf{f}) = [\alpha_1(\mathbf{f}) \dots \alpha_{n-N}(\mathbf{f})]^T = 0.5T \left[|\mathbf{x}_d \mathbf{v}_n^{(1)}(\mathbf{f})| \dots |\mathbf{x}_d \mathbf{v}_n^{(n-N)}(\mathbf{f})| \right]^T. \quad (1)$$

Here, T is a thickness parameter; α_k determine the boundaries of the domain X_S , located between the manifolds M_+ and M_- :

$$M_{\pm} = \left\{ \mathbf{x} \in X : \mathbf{x} = s_1(\mathbf{f}) \pm \sum_{k=1}^{n-N} \alpha_k(\mathbf{f}) \mathbf{v}_n^{(k)}(\mathbf{f}) \right\}. \quad (2)$$

Formally, we have:

$$X_S = \left\{ \begin{aligned} &\mathbf{x} = s_1(\mathbf{f}) + \sum_{k=1}^{n-N} \lambda_k \alpha_k(\mathbf{f}) \mathbf{v}_n^{(k)}(\mathbf{f}) : \mathbf{f} \in F, \\ &-1 \leq \lambda_k \leq 1, k = 1, \dots, n - N \end{aligned} \right\}. \quad (3)$$

The actual (second-level) surrogate is a kriging interpolation model set up in X_S , using a set of training data samples $\{\mathbf{x}_B^{(k)}, \mathbf{R}(\mathbf{x}_B^{(k)})\}_{k=1, \dots, N_B}$, where \mathbf{R} is the EM-simulation model of the compact structure of interest.

C. Design of Experiments

The data sampling can be readily implemented by exploiting (3) and an appropriate two-stage mapping H from the unit interval $[0,1]^n$ onto X_S . Let $\{\mathbf{z}^{(k)}\}$, $k = 1, \dots, N_B$, be the set of uniformly distributed data points obtained using LHS [6], with $\mathbf{z}^{(k)} = [z_1^{(k)} \dots z_n^{(k)}]^T$. The mapping h_1 :

$$\mathbf{y} = h_1(\mathbf{z}) = h_1([z_1 \dots z_n]^T) = [f_{1,\min} + z_1(f_{1,\max} - f_{1,\min}) \dots \dots f_{N,\min} + z_N(f_{N,\max} - f_{N,\min})] \times [-1 + 2z_{N+1} \dots -1 + 2z_n] \quad (4)$$

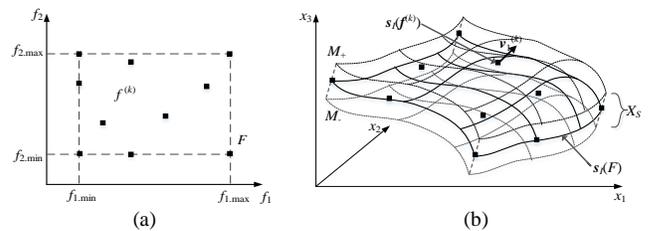


Fig. 1. The concept of nested kriging modeling: (a) reference designs and objective space F ; (b) the image $s_1(F)$ of the first-level surrogate model and the normal vector $\mathbf{v}_1^{(k)}$ at $\mathbf{f}^{(k)}$; the manifolds M_- and M_+ as well as the surrogate model domain X_S defined as the orthogonal extension of $s_1(F)$.

transforms the hypercube onto $F \times [-1,1]^{n-N}$ (here, \times denotes a Cartesian product). Subsequently, the function h_2 defined as:

$$\mathbf{x} = h_2(\mathbf{y}) = h_2([y_1 \dots y_n]^T) = s_l(\mathbf{y}) + \sum_{k=1}^{n-N} y_{N+k} \alpha_k(\mathbf{y}) \mathbf{v}_n^{(k)}(\mathbf{y}), \quad (5)$$

maps $F \times [-1,1]^{n-N}$ onto X_S . Uniformly distributed samples $\mathbf{x}_B^{(k)} \in X_S$ are then obtained as:

$$\mathbf{x}_B^{(k)} = H(\mathbf{z}^{(k)}) = h_2(h_1(\mathbf{z}^{(k)})). \quad (6)$$

The mapping H can also be used for optimization within X_S : regardless of the geometry of X_S , it is sufficient to operate within $F \times [-1,1]^{n-N}$ and apply (6) for EM evaluation purposes.

III. MICROWAVE MODELING USING NESTED KRIGING

We consider two compact microstrip structures, a rat-race coupler (RRC) shown in Fig. 2 (a) and the 50-to-100 Ohm impedance matching transformer shown in Fig. 2 (d), composed of the CMRCs of Fig. 2 (c). Both structures are implemented on RF-35 substrate ($\epsilon_r = 3.5$, $h = 0.762$ mm, $\tan \delta = 0.018$). The RRC parameters are: $\mathbf{x} = [l_1 \ l_2 \ l_3 \ d \ w \ w_1]^T$, with relative variable $d_1 = d + |w - w_1|$ and dimensions $d = 1.0$, $w_0 = 1.7$, $l_0 = 15$ fixed (all in mm). The parameters of the transformer are $\mathbf{x} = [l_{1,1} \ l_{1,2} \ w_{1,1} \ w_{1,2} \ w_{1,0} \ l_{2,1} \ l_{2,2} \ w_{2,1} \ w_{2,2} \ w_{2,0} \ l_{3,1} \ l_{3,2} \ w_{3,1} \ w_{3,2} \ w_{3,0}]^T$.

The goal is to model the RRC within the region covering optimum designs corresponding to the operating frequencies f_0 from 1 GHz to 2 GHz and the power split ratios K from -6 dB to 0 dB (equal power split). The transformer model is supposed to cover the operating bands $[f_1, f_2]$ for $1.5 \text{ GHz} \leq f_1 \leq 3.5 \text{ GHz}$, and $4.5 \text{ GHz} \leq f_2 \leq 6.5 \text{ GHz}$. The allocation of the reference designs for both structures are shown in Figs. 2 (b) and 2 (e), respectively. The lower and upper bounds for design variables are based on the reference designs. These are $\mathbf{l} = [2.0 \ 7.0 \ 12.5 \ 0.2 \ 0.7 \ 0.2]^T$, and $\mathbf{u} = [4.5 \ 12.5 \ 22.0 \ 0.65 \ 1.5 \ 0.9]^T$ for the RRC and $\mathbf{l} = [2.0 \ 0.15 \ 0.65 \ 0.35 \ 0.30 \ 2.70 \ 0.15 \ 0.44 \ 0.15 \ 0.30 \ 3.2 \ 0.15 \ 0.30 \ 0.15 \ 0.30]^T$, and $\mathbf{u} = [3.4 \ 0.50 \ 0.80 \ 0.55 \ 1.90 \ 4.00 \ 0.50 \ 0.67 \ 0.50 \ 1.55 \ 4.5 \ 0.26 \ 0.46 \ 0.27 \ 1.75]^T$ for the transformer.

The nested kriging surrogate has been constructed for various training data sets listed in Table I, using the thickness parameter $T = 0.05$. The model error was estimated with 100 independent test points. The results for the conventional kriging model set up in the interval $[\mathbf{l}, \mathbf{u}]$ are reported as well (see also Figs. 3 and 4). A considerable improvement of the modeling accuracy offered by the nested kriging surrogate over the conventional one can be observed. The comparable predictive power is obtained for much smaller training data sets, by a factor of four and higher. Note that in the case of the transformer, the accuracy of the conventional model is poor even for the largest data set consisting of 800 samples. This example is challenging due to a large number of parameters (fifteen) as well as wide parameter ranges.

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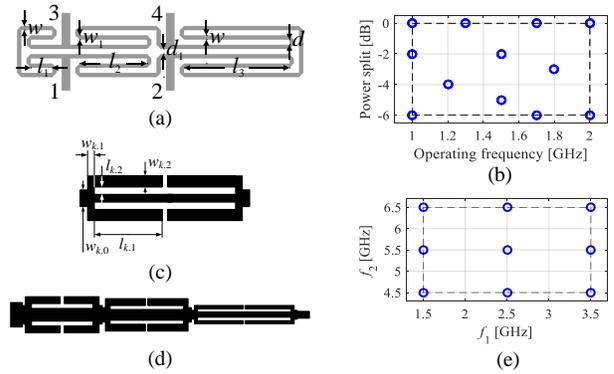


Fig. 2. Verification test cases: (a) microstrip rat-race coupler (RRC) [20], (b) allocation of the reference designs for the RRC, (c) compact cell (CMRC), (d) CMRC-based miniaturized three-section impedance transformer, and (e) allocation of the reference designs for the transformer.

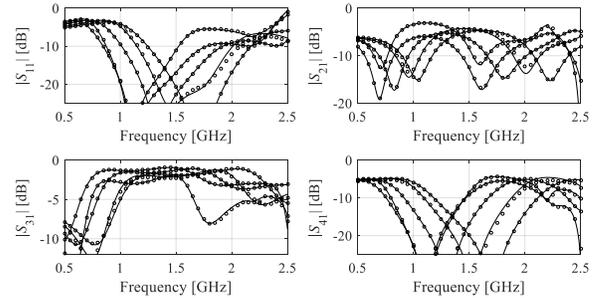


Fig. 3. Responses of the compact RRC of Fig. 2 (a) at the selected test designs for $N = 800$: EM model (—), nested kriging surrogate (o).

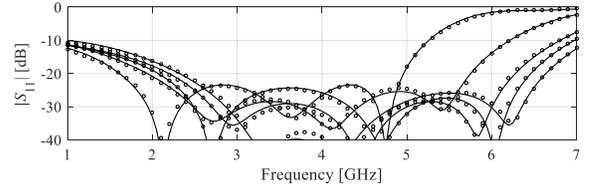


Fig. 4. Responses of the impedance transformer of Fig. 2 (d) at the selected test designs for $N = 800$: EM model (—), nested kriging surrogate (o).

TABLE I. MODELING RESULTS FOR RRC AND TRANSFORMER

Number of Training Samples	Relative RMS Error for Compact RRC		Relative RMS Error for Impedance Transformer	
	Conventional Kriging Model	Nested Kriging Model [This Work]	Conventional Kriging Model	Nested Kriging Model [This Work]
50	25.7%	6.9%	49.1%	17.3%
100	17.9%	5.7%	31.1%	13.9%
200	13.5%	3.8%	25.9%	10.3%
400	9.9%	3.5%	20.4%	7.4%
800	8.0%	3.1%	15.7%	6.1%

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