

# Systematic CMA of the U-slot Patch with FEKO

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**Abstract**—Since its introduction 25 years ago, the probe-fed U-slot patch antenna has remained popular. Recently, Characteristic Mode Analysis (CMA) revealed these devices are governed by Coupled Mode Theory (CMT). Although this principle is conceptually simple, achieving this understanding is only possible through a systematic analysis using CMA. This paper uses the U-slot patch to illustrate a general process for analyzing electrically small antennas using CMA with the software package FEKO.

**Keywords**—characteristic mode analysis, coupled mode theory, FEKO, U-slot patch antenna.

## I. INTRODUCTION

Over the past 25 years, the probe-fed U-slot patch antenna [1] has remained popular in government and academia. Recently, CMA revealed these devices are governed by CMT; coupling between the conventional  $TM_{01}$  patch mode and the “uncoupled slot resonator” (a lumped LC resonator involving the slot and the probe) results in stagger-tuned in-phase and anti-phase modes that together produce broad impedance bandwidth and stable radiation pattern [2]. Although conceptually simple, achieving this understanding is only possible via a systematic analysis using CMA. This paper illustrates such a process for the U-slot patch using the software package FEKO [3].

CMA is a modal decomposition based on the method of moments (MoM) [4]. Within CMA, a set of real orthogonal basis currents  $J_n$  result from  $[X]J_n = \lambda_n [R]J_n$  where  $[Z] = [R] + j[X]$  is the MoM impedance matrix and  $\lambda_n$  is the eigenvalue. A mode is resonant when  $\lambda_n = 0$  [4]. Currents driven by a source  $E_{tan}^i$  may be represented as a sum of modes:  $J_{total} = \sum_n \alpha_n J_n$  where the modal weighting coefficient (MWC)  $\alpha_n = \langle J_n, E_{tan}^i \rangle / (1 + j\lambda_n)$  [4]. Thus, the driven admittance of a structure is the sum of all modal admittances.

## II. CMA PROCESS

The usual steps of entering the geometry, meshing, and solving the structure are performed. It is important to mesh the probe and provide an excitation in these geometries, not only because the probe can affect the characteristic modes (CMs) but also because having an excitation gives clues (e.g., admittance & MWC) as to what modes are important. Note that CMA often benefits from a finer mesh than is required for solving a driven problem (for this analysis, triangle edge lengths were between  $\lambda/25$  and  $\lambda/50$ ). In FEKO, a driven MoM solve (i.e., using a 1V excitation) can be performed alongside a CMA solve.

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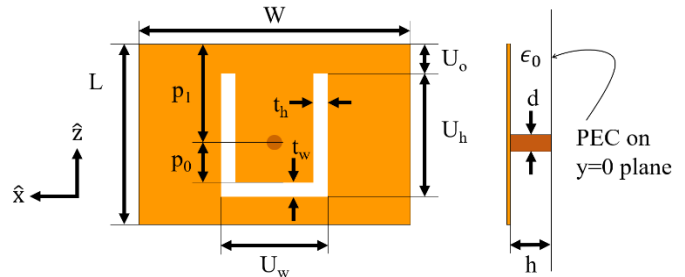


Fig. 1. U-slot patch geometry of [1];  $(W, L, h, U_w, U_h, U_o, t_h, t_w, d, p_0) = (220, 124, 26.9, 68.6, 82.2, 22.9, 10.2, 8.89, 3.05, 33.9)$  mm.

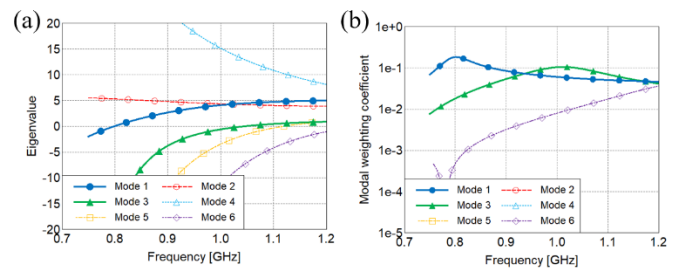


Fig. 2. Eigenvalues (a) and modal weighting coefficient  $\alpha_n$  (b) of the first 6 modes of the Fig. 1 geometry calculated by FEKO.

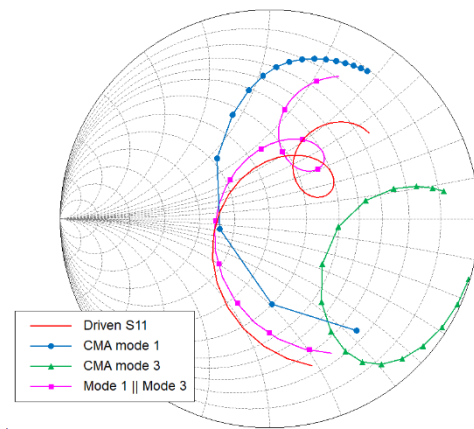


Fig. 3. The sum of mode 1 & 3 admittances replicates the driven admittance.

Data interpretation begins by viewing the eigenvalue spectrum shown in Fig. 2 (a). The eigenvalue traces are inspected for mode-tracking errors (which greatly complicate data interpretation) and other phenomena such as eigenvalue crossing avoidance. Modes that are not resonant within the band of interest are often of secondary (if any) interest. We see that of six modes requested of FEKO, only modes 1, 3, 5 and 6 are resonant. For a 1V gap source excitation at the base of the probe, FEKO calculates  $\alpha_n$  as seen in Fig. 2 (b). Only modes 1, 3 and 6 are excited significantly and warrant further study.

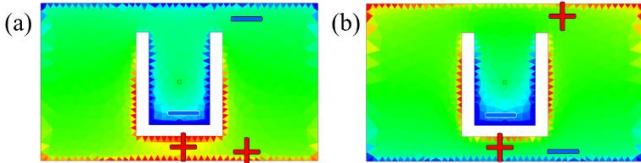


Fig. 4. Mode 1/in-phase (a) and Mode 3/anti-phase (b) charge distributions.

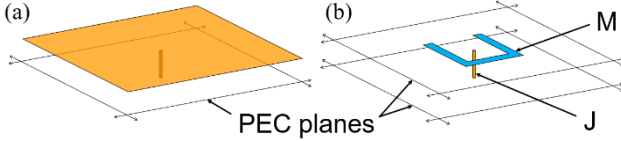


Fig. 5. Uncoupled patch (a) and uncoupled slot (b) resonator geometries.

We next seek to replicate the driven input admittance by a sum of modal admittances. We find the mode 6 admittance is much lower than  $1/50\Omega$ , but that the sum of mode 1 and 3 admittances (calculated using a POSTFEKO Lua script) replicates the driven admittance locus well, as seen in Fig. 3. If the driven admittance cannot be adequately represented by the sum of mode admittances, then more CMs should be requested of FEKO, and the above process repeated.

We turn to physical interpretation of modes 1 and 3. FEKO plots the modal charge distributions (at phase angle  $\phi = 90^\circ$ ) near resonance; these are much simpler than current distributions and greatly aid in physical interpretation. We note in-phase and anti-phase charge accumulations on the patch and slot edges for modes 1 and 3, respectively, as seen in Fig. 4 (a) and Fig. 4 (b). This suggests that CMT underlies the U-slot patch, which was demonstrated rigorously in [2]; coupling is due to the mutual inductance of the slot.

At this point, the modal far-fields are compared to those of the driven problem to ensure the selected modes appropriately capture the radiation behavior of the driven structure.

With the important modes identified, we enquire about the nature of the *uncoupled* resonators (i.e., the isolated patch and slot); this process often requires some ingenuity. For example, it is simple to remove the *slot from the patch* and identify the conventional  $TM_{01}$  patch mode resonant near the full U-slot patch structure center frequency, however it is less obvious how to remove the *patch from the slot*. Increasing  $W$  and  $L$  to infinity, as shown in Fig. 5 (b), gives results useful for design purposes [2].

FEKO can calculate CMs of the Fig. 5 (b) structure using the planar multilayer Green's function and the planar Green's function aperture features (to represent magnetic currents on the U-slot), resulting in a mode resonant near the full U-slot patch structure center frequency. Infinite conducting planes are accounted for analytically; this is of critical importance, as large meshed ground planes supporting current explicitly as MoM unknowns result in low-frequency CMs that are of no interest (i.e., modes of the ground plane plates, rather than of the slot-probe structure). To avoid a FEKO error, the dielectric between the infinite conducting planes is set to  $\epsilon_r = 1.01$  as seen in Fig. 6; in our experience, the numerical solution remains stable and the difference from the case where  $\epsilon_r = 1$  is negligible. Of course, U-slot patches on dielectric substrates with  $\epsilon_r > 1$  are

easily addressed by FEKO.

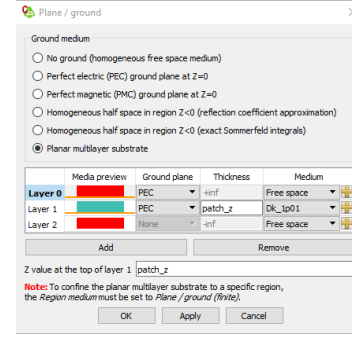


Fig. 6. FEKO planar multilayer Green's function configuration for Fig. 5(b).

To explain the uncoupled slot resonator behavior, CMA and CMT are utilized once again. The geometry of Fig. 5 (b) is mirrored through the  $y=0$  plane; one resulting CM satisfies a PEC boundary condition on  $y=0$  and another a PMC boundary condition. The CM resonant frequencies and interpretation of the modal nearfields lead to a simple, four element LC equivalent circuit for the uncoupled slot resonator [2].

A volumetric grid of resonant modal near-fields  $E_{1,2}$  and  $H_{1,2}$  encompassing the uncoupled slot and patch resonators may be used to calculate the coupling between them via [5]:

$$\kappa = \frac{\int \epsilon E_1 \circ E_2 dV}{\sqrt{W_{e1}W_{e2}}} + \frac{\int \mu H_1 \circ H_2 dV}{\sqrt{W_{m1}W_{m2}}}, \quad (1)$$

where  $W_{e1,2} = \int \epsilon |E_{1,2}|^2 dV$  and  $W_{m1,2} = \int \mu |H_{1,2}|^2 dV$ , giving  $|\kappa| = 0.19$ —similar to that calculated via the coupled resonances  $\omega_{\pm}$  as  $\kappa = (\omega_+^2 - \omega_-^2)/(\omega_+^2 + \omega_-^2) = 0.26$  in [2].

### III. CONCLUSION

A systematic approach to applying CMA to electrically small antenna problems was presented using FEKO and the U-slot patch: (1) include the excitation/mesh the probe and run driven MoM problem alongside CMA, (2) select resonant modes, (3) select modes with significant MWCs, (4) replicate driven admittance with sum of selected modal admittances, (5) check modal far-fields, (6) use CM current and charge distributions to obtain physical insight, and, for multi-modal antennas, (7) establish uncoupled resonator geometries and calculate coupling using near-fields.

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