

# Debye Coefficients for Biological Tissues From 100 MHz to 100 GHz

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**Abstract**—Debye modeling of dispersive, biological tissues allows for the numerical analysis of electromagnetic waves in the vicinity of a human body using the FDTD method. Three-term Debye expansion coefficients for 55 human biological tissues are determined via a two-step numerical solver. The Debye coefficients obtained predict precisely the complex permittivity of the experimentally available tissue data.

**Index Terms**—Debye, dispersive media, FDTD method, frequency domain.

## I. INTRODUCTION

Several methods have been used to evaluate the EM energy absorbed/penetration in biological tissues. Among these methods is the finite-difference time-domain (FDTD) which is considered the most useful for calculating specific absorption rate (SAR) and temperature rise in human tissues due to electromagnetic radiation at multiple frequencies [1]. The traditional FDTD formulation can easily be modified to include the dispersive type properties of the human tissues, provided their parameters are presented in a Debye form [2].

This paper focuses on providing Debye coefficient for 55 different human tissues for electromagnetic applications supporting frequencies up to 100 GHz. This is an extension of similarly presented work which was only applicable to 20 GHz [3]. Similar procedure to what was developed in [4] and [3] is applied here on tissues data obtained from [5] for a frequency range from 100 MHz to 100 GHz. The accuracy of the generated three-term Debye coefficients is verified by comparison with the original data at different frequency ranges.

## II. EVALUATION OF DEBYE COEFFICIENTS

The Debye Model is best shown in the following compact form [6]:

$$\epsilon_r(\omega) = \epsilon_\infty + \sum_{k=1}^N \frac{\Delta\epsilon_k}{1+j\omega\tau_k} ; \Delta\epsilon_k = A_k(\epsilon_s - \epsilon_\infty), \quad (1)$$

where:

- N: Number of Debye terms
- $\epsilon_\infty$ : Permittivity at high frequency
- $\Delta\epsilon_k$ : Pole weight
- $\tau_k$ : Relaxation time.

The measured data of tissue parameters obtained from [5] are then used with equation (1) to calculate the Debye coefficients. Table I shows the relative permittivity and conductivity for a sample biological tissue within the range of 100 MHz to 100 GHz. The conductivity is converted to the corresponding imaginary part of the complex permittivity:

$$\epsilon_r(\omega) = \epsilon' - j\epsilon'' . \quad (2)$$

Table I. Aorta parameters from [5]

Frequency (GHz)	Relative Permittivity $\epsilon'$	Conductivity [S/m]
0.100	59.780	0.462
1	44.561	0.729
10	32.673	9.127
20	22.445	19.640
50	10.934	34.895
100	6.864	43.022

Since the number of data points obtained from [5] were limited for further numerical processing, an interpolation process to generate a larger number of points for each tissue is accomplished via the MATLAB function “pchip.” Next, a two-step process is performed. First, the MATLAB function “invfreqs” is used to calculate the frequency transfer function from the data points,

$$\epsilon_r(\omega) = \frac{\sum_{k=1}^N b_{k-1} \omega^{k-1}}{\sum_{k=1}^{N+1} a_{k-1} \omega^{k-1}}, \quad (3)$$

where N is the number of poles that is desired to have in the numerical fit and the  $a_k$  and  $b_k$  coefficients are computed using the “invfreqs” function. Next, the transfer function can be used to calculate the poles and weights for the Debye coefficients using the “residue” function as was previously done in [4] and [3].

## III. RESULTS AND DISCUSSION

The large frequency range from 100 MHz to 100 GHz was divided into three sub-ranges, 100 MHz to 2 GHz, 2 GHz to 20 GHz, and 20 GHz to 100 GHz and Debye coefficients were computed for each sub-range. The computed three-term Debye coefficients for some sampled tissues are listed in Table II for frequency range from 20 GHz to 100 GHz.

The maximum percentage errors of these sampled tissues did not exceed 0.1% and 0.27% for  $\epsilon'$  and  $\epsilon''$ , respectively. The complete set of permittivity coefficients for the three frequency sub-ranges have been computed and will be presented. Fig. 1 shows excellent coincidence between the generated and original measured permittivity for the aorta.

## IV. CONCLUSION

Three-term Debye coefficients for 55 biological tissues are determined from a small set of measured data for frequency range from 100 MHz to 100 GHz. With a three-term fit the maximum normalized errors related to the 55 tissue types was found to be 3.047% and 3.958% for  $\epsilon'$  and  $\epsilon''$ , respectively.

Table II. Three-term Debye parameters for frequency range of 20GHz-100GHz

Tissue	$\epsilon_{\infty}$	$\Delta\epsilon_1$	$\Delta\epsilon_2$	$\Delta\epsilon_3$	$\tau_1 \times 10^{-9}$	$\tau_2 \times 10^{-9}$	$\tau_3 \times 10^{-9}$
Aorta	4.325	2.311	20.134	19.363	0.002	0.006	0.017
Bladder	2.635	0.969	8.610	7.917	0.002	0.006	0.019
Blood	4.498	3.684	32.111	26.674	0.002	0.006	0.021
Bone	2.565	0.335	1.158	1.407	0.001	0.005	0.016
Brain	4.428	3.123	25.887	20.355	0.002	0.006	0.019
Breast	2.513	0.074	0.555	2.391	0.002	0.007	0.021
Cartilage	4.371	2.030	11.251	25.934	0.001	0.006	0.020
Cervix	4.410	3.058	25.632	19.530	0.002	0.006	0.018
Colon	4.477	3.485	28.886	22.688	0.002	0.006	0.019
Cornea	4.465	3.452	28.828	22.807	0.002	0.006	0.022
Dura	4.602	3.754	19.037	19.513	0.002	0.006	0.019
Eye	4.460	3.447	28.933	22.031	0.002	0.006	0.019
Fat	2.566	0.339	1.182	1.424	0.001	0.005	0.016
Bladder	4.246	2.373	42.606	16.075	0.002	0.007	0.024
Heart	4.499	3.591	29.594	23.813	0.002	0.006	0.020
Kidney	4.491	3.497	28.628	24.139	0.002	0.006	0.022
Lens	4.376	2.803	23.427	18.053	0.002	0.006	0.017
Liver	4.345	2.401	20.938	20.001	0.002	0.006	0.019
Lung	4.418	3.061	25.394	19.870	0.002	0.006	0.018
Lymph	4.494	3.673	30.674	23.645	0.002	0.006	0.017
Muscle	4.490	3.793	29.742	19.355	0.002	0.006	0.017
Nail	2.647	0.695	2.683	6.163	0.001	0.006	0.019
Nerve	4.251	1.827	15.117	11.972	0.002	0.006	0.019
Ovary	4.618	3.608	19.036	24.768	0.002	0.006	0.022
Pancreas	4.494	3.673	30.674	23.645	0.002	0.006	0.017
Prostate	4.504	3.766	31.554	24.101	0.002	0.006	0.019
Retina	4.460	3.447	28.933	22.031	0.002	0.006	0.019
Skin	4.030	0.125	32.419	22.833	0.001	0.007	0.161
Intestine	4.506	3.796	31.520	28.688	0.002	0.006	0.027
Stomach	4.542	4.036	33.756	25.919	0.002	0.006	0.018
Tendon	4.251	1.611	14.119	27.539	0.002	0.007	0.018
Testis	4.504	3.766	31.554	24.101	0.002	0.006	0.019
Thymus	4.494	3.673	30.674	23.645	0.002	0.006	0.017
Thyroid	4.494	3.673	30.674	23.645	0.002	0.006	0.017
Tongue	4.451	3.348	27.925	21.646	0.002	0.006	0.017
Tooth	2.647	0.695	2.683	6.163	0.001	0.006	0.019
Uterus	4.511	3.804	31.844	24.425	0.002	0.006	0.019

With three-term Debye coefficients along with the FDTD formulations for dispersive type material, it is now possible to address many applications at high frequencies such as biomedical imaging and effect of radiation from 5G devices on human body.

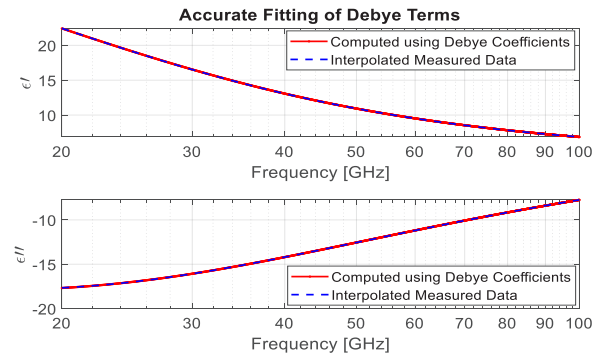


Fig. 1. Permittivity vs. frequency using Debye three-term fit and interpolated raw data for the human aorta tissue from 20 to 100 GHz.

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