Parametric Investigation and Analysis of an Electric-LC Resonator by Using LC Circuit Model

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Abstract **─** Electric-LC resonators (ELCs) metamaterials, as a kind of common structures, have been extensively investigated from microwave to terahertz frequencies. In this paper, we present a LC circuit model to analyze electric-LC resonator. With the reliable and closed formulas of the effective inductance and capacitance, the expressions of electric and magnetic resonance frequencies were obtained, which is suitable to discuss the resonance characteristic under the normal incidence case. Meanwhile, the mutual relationships among the permittivity, permeability, refractive index, and structure parameters can be explored by using the obtained expressions. Numerical simulations and theoretical calculations reveal that the width and length of the gaps are some of the critical parameters determining the resonator frequency of the example metamaterial. This study provides valuable information for designing the desired left-hand metamaterial at some specific frequency points.

Index Terms **─** Circuit model, metamaterial, resonator.

I. INTRODUCTION

In the past few years, research on metamaterials, especially on left-handed metamaterials (LHMs), has attracted considerable attention from microwave to optical frequencies due to their exotic intriguing physical properties that don't exist in natural materials. The electromagnetic response of a specific metamaterial can be predicted via retrieved the effective permittivity and permeability [1]. The intriguing features of the metamaterials by a combination of artificial "electric atoms" and "magnetic atoms" have conceptualized many novel devices such as perfect absorbers [2-6], invisible cloak [7, 8], superlens [9, 10], and so on.

As is generally known, the development of research on metamaterials is usually connected with design. Over the last decade, various kinds of artificial LHMs were proposed and investigated either for in-plane or normal incidence [11-14]. Meanwhile, some theoretical approaches, such as equivalent circuit theory [15-17] and effective medium theory [18, 19], have been exploited to explain the physical mechanism. These theoretical analysis methods have significant effects on the design and analysis of the LHMs. However, they still not manage to offer the specific principles for selecting structural parameters of the desired LHMs. As a matter of fact, it is more important for engineers to understand the relationship between the structure parameters and the resonant frequency.

In this communication, we utilized a LC circuit model to reveal the influences of the structure parameters on the resonance frequency. It must be pointed that the proposed LHM has a structural similarity to the ELC resonator introduced in Ref. [20], but given that the major aim of this paper is excavating the physical mechanism. We use the LC circuit to analyze the achieved numerical simulation response. By this method, a distinct and intuitive understanding of the parametric response of ELC was obtained, which can lead to guidelines for the optimization of the related metamaterial structures.

II. SIMULATION AND THEORY

The proposed ELC unit cell is illustrated in Fig. 1 (a). The metallic pattern is printed on one side of a FR4 substrate with the relative permittivity $\varepsilon_r = 4.4$ and $\tan\delta = 0.02$. The metallization is copper with a thickness of $t = 17\ \mu m$. The other geometrical dimensions are shown in Fig. 1 (a).

The simulations were performed using the full-wave

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finite element solver. As Refs. [21, 22], a theoretical model for normal-to-side incidence is developed based on an artificial waveguide with two ideal magnetic conductor vertical planes and two ideal electric conductor horizontal planes at boundaries [Fig. 1 (b)], which is equivalent to an infinite layer medium illuminated by a normal incident plane wave. The transmission and reflection characteristics of electromagnetic waves can be conveniently obtained by evaluating the *S* parameters. Then, by the standard algorithm [23], effective electromagnetic parameters of LHM were retrieved.

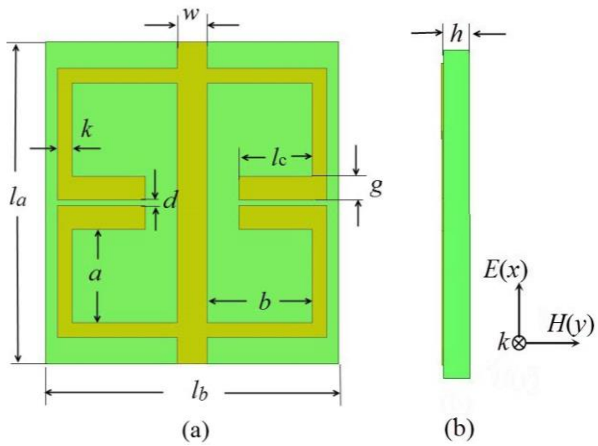


Fig. 1. (a) Schematic representation of unit cell of the ELC structure. (b) Side view. The geometry of ELC are as follows: l_a=11 mm, l_b=10 mm, l_c=2.5 mm, w=1 mm, k=0.5 mm, g=0.8 mm, a=3.2 mm, b=3.6 mm, d=0.2 mm, h=0.8 mm.

The retrieved effective parameters of the LHM structure are plotted in Fig. 2. There is one double-negative passband due to the magnetic resonance. Figures 2 (a) and 2 (b) show that the effective permittivity is negative in 3.4–5 GHz while the frequency range of negative effective permeability is 3.05–3.6 GHz, much narrower than the negative effective permittivity range. Fig. 2 (c) clearly shows that the negative refractive index band is between 3.3 GHz and 4 GHz, and negative refraction bandwidth is 600 MHz. But note that in the frequency range between 3.4 and 3.6 GHz, where both the effective permeability and permittivity are negative, a LH band is anticipated.

When a plane electromagnetic wave incident on the unit cell with its wave vector is parallel to the plane of the ELC and the magnetic field is perpendicular to the ELC, currents *I* flow around ELC will be induced. From the equivalent circuit [Fig. 3 (a)], the self-inductance (L_1) in the middle conducting strip has the form [24, 25]:

$$L_1 \approx \frac{\mu_0 l_a}{2\pi}\left[\ln(\frac{2l_a}{w})+\frac{1}{2}+\frac{w}{3l_a}-\frac{w^2}{24l_a^2}\right]. \qquad (1)$$

The inductance L_2 of the metal arms can be approximately calculated by:

$$L_2 \approx \frac{\mu_0(a+b+l_c)}{\pi}\ln\left[\frac{6.6(a+b+l_c)}{k}\right], \qquad (2)$$

and the calculation expression of effective capacitance between the two metal arms is as follows:

$$C_e \approx \varepsilon_0 \frac{0.017(l_c+k)}{d}. \qquad (3)$$

Then, the electric resonant frequency can be given by:

$$f_e = \frac{1}{2\pi\sqrt{L_e C_e}} \propto \frac{d}{(a+b+l_c)(l_c+k)}. \qquad (4)$$

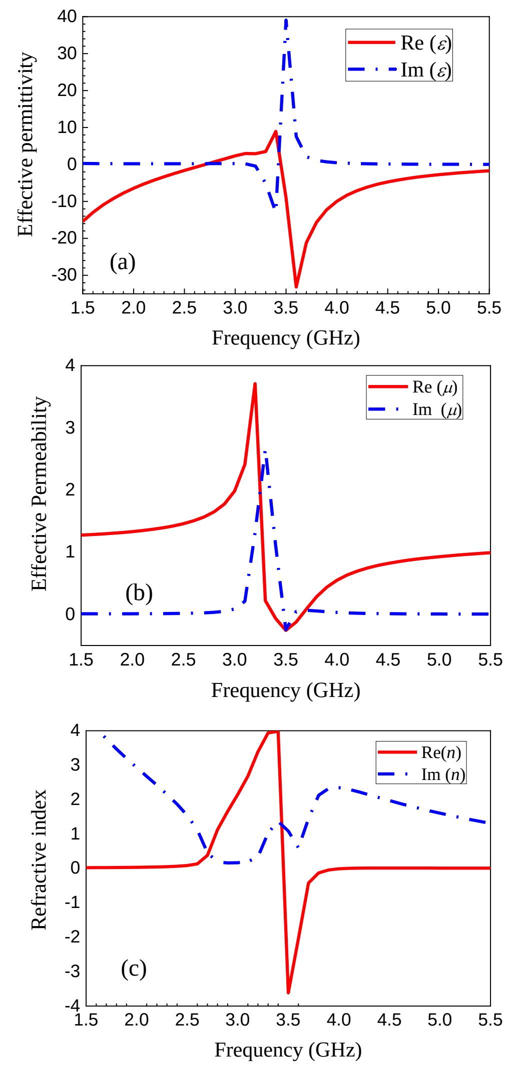


Fig. 2. Retrieved effective parameters of ELC: (a) complex permittivity, (b) complex permeability, and (c) complex refractive index.

The circuit model of magnetic resonance is shown in Fig. 3 (b), which is similar to the circuit model of electric resonance model. The inductance $L_2^{'}$ for an ELC unit is:

$$L_2^{'} = 4\mu_0 ab/l , \tag{5}$$

where l is the periodicity of this ELC array in the y direction. The magnetic resonance frequency can be described as $f_m = 1\big/2\pi\sqrt{L_m C_m}$, where capacitance $C_m \approx C_e$. According to Eqs. (1-3), we can deduce that:

$$f_m = \frac{1}{2\pi\sqrt{L_m C_m}} \propto \frac{1}{abl_a l_c}. \tag{6}$$

Equations (4) and (6) indicate that not only structural parameters a, b, l_a, l_c but also k and d have an important impact on the electric and magnetic resonant frequencies of the ELC resonator, and they can be employed to understand the working mechanism of the LHM metamaterials.

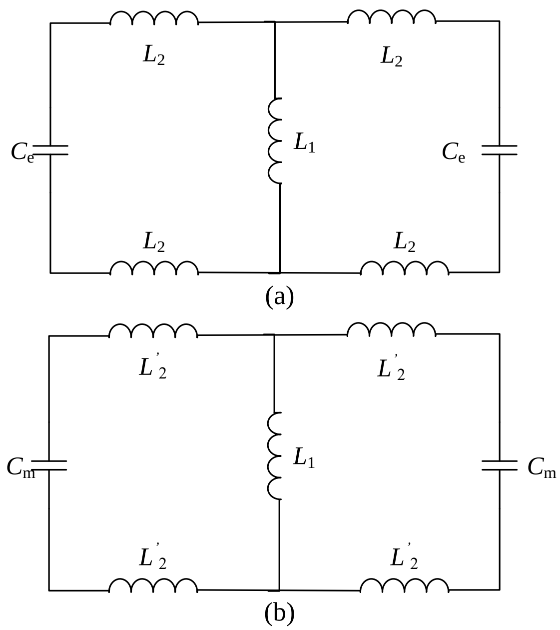
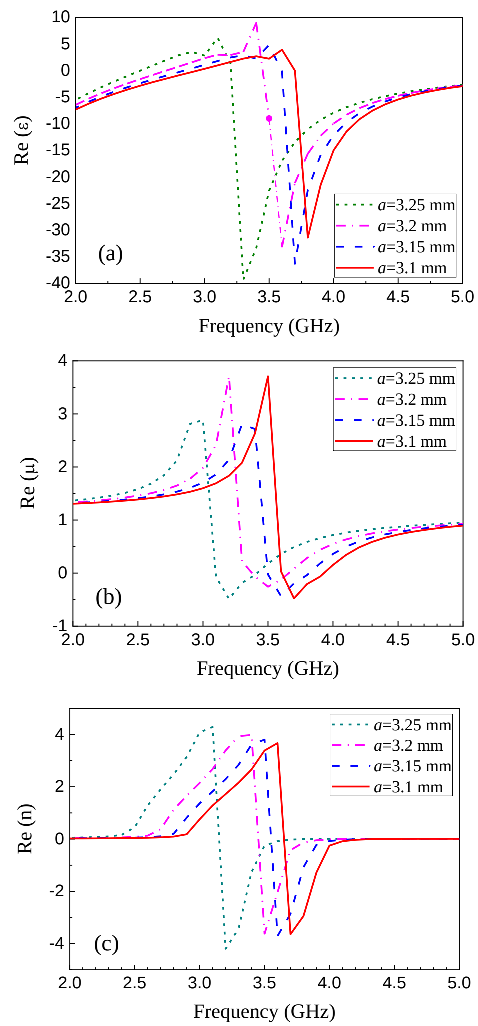


Fig. 3. Equivalent circuits for: (a) electric resonance, and (b) magnetic resonance.

III. RESULT AND DISCUSSION

To better understand the resonant mechanism and gain physical insight into this ELC resonator, now we use the derived equivalent circuit model to investigate the influence of the constitutive structural parameters on the resonant frequencies, and to reveal some strategies for designing metamaterials. According to the Eqs. (4) and (6), it can find that all the structural parameters have a significant impact on the resonant frequencies of the metamaterial. Due to there are six factors affecting the resonant frequencies, for simplicity but without losing generality, we select two parameters a and l_c to research this proposed ELC resonator. Other factors have similar results.

The retrieved equivalent parameters of the ELC resonator are shown in Fig. 4. In the simulation, the wire length a varies from 3.1 mm to 3.25 mm. Figure 4 (a) shows the influence of the length a on the real part of the permittivity. It can find that the electrical resonance f_e shifts to the lower frequency with increasing the wire length, displaying an exact linear relation as indicated by Eq. (4). The permeability μ and refractive index n dependence of the wire length a shown in Figs. 4 (b) and (c) have a similar behavior as observed in the Fig. 4 (a). In order to show clearly the effect of the value of length a, we plot the dependence of the resonance frequency on a in Fig. 4 (d). It is clear that the electric and magnetic resonance is inversely proportional to the length of a. From Eqs. (4) and (6), the electric and magnetic resonant frequencies are inversely proportional to the length a of the wires. Resonance frequencies are determined by numerical simulation with various parameter a show a good agreement with the formulas (4) and (6).



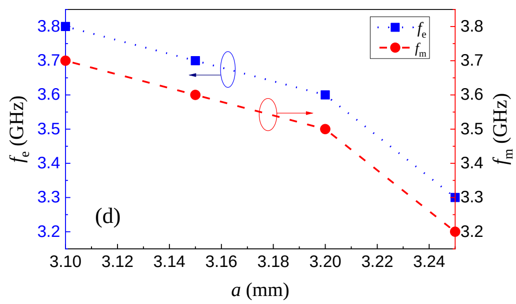
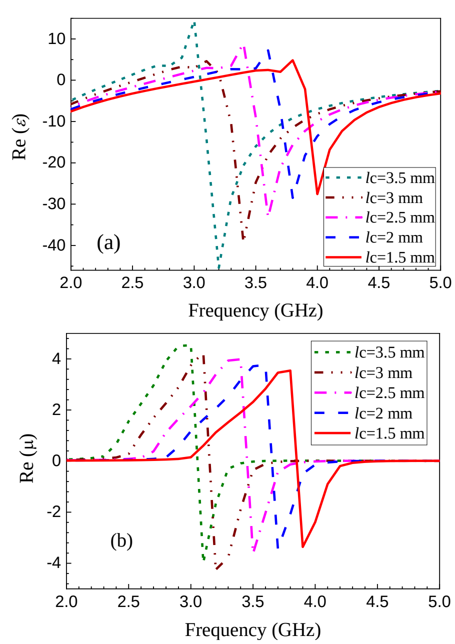


Fig. 4. The real part of the equivalent parameters for length variation of metallic wire *a*. (a) Permittivity, (b) permeability, (c) refractive index, and (d) electric resonant frequencies and magnetic resonant frequencies as a function of *a*. Dashed line denoted the linear dependence of the wire length *a* for the resonant frequency predicted by relations (4) and (6).

Figure 5 displays the effect of the length l_c on the electric and magnetic resonance frequencies. The length l_c is varied from1.5 mm to 3.5 mm. From Fig. 5 (a), we can see that the electric resonance peak values decrease as increasing of length l_c. According to the Eqs. (3) and (4), the increase of metallic wire length l_c leads to the inductance L_2 of the arms and the capacitance of the gap between the two arms decrease.

It is believed that the value of the magnetic resonance frequency f_m is inversely proportional to the length l_c, and the result is presented in Fig. 5 (b). As expected by Eq. (6), the length l_c of the metal wire increase causes a reduction of the resonance frequency. Figures 5 (c) and 5 (d) show a similar behavior as observed in Figs. 4 (c) and 4 (d).



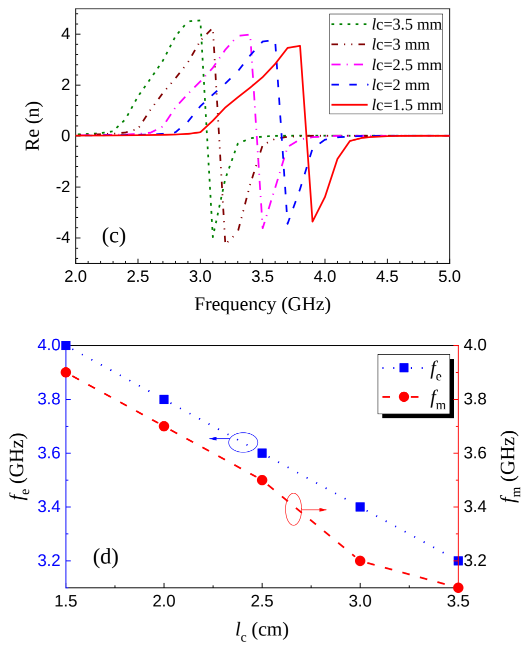


Fig. 5. The real part of the equivalent parameters for length variation of metallic wire l_c. (a) Permittivity, (b) permeability, (c) refractive index, and (d) electric resonant frequencies and magnetic resonant frequencies as a function of l_c. Dashed line denoted the linear dependence of the wire length *a* for the resonant frequency predicted by relations (4) and (6).

IV. CONCLUSION

In summary, we have utilized an equivalent circuit model to analyze the ELC resonator. From the derived approximate relationship for the electric and magnetic resonant frequencies as a function of the structural parameters, influence on metallic wire width *a* and l_c were analyzed. It is shown that the simulation results are satisfactory agreement theoretical analysis using LC circuit model for the proposed ELC resonator. With this circuit model and analytical method, it is easy to understand the physical mechanism and may provide helpful guidance in the future developments of more advanced metamaterial devices for microwave applications.

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