# Design and Application of Coupled Line Cross-Shaped Resonator in Band-pass Filter

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**Abstract** — A coupled line cross-shaped resonator (CLCSR) is proposed, which consists of four parallel coupled lines. By using even- and odd-mode approach, this resonator is characterized and designed to build up a wide band-pass filter. There are three transmission poles in the pass-band and two transmission zeros out of the pass-band. The positions of the transmission zeros are fixed. The transmission poles are determined and adjusted by the parameters:  $z_1$ ,  $z_2$ ,  $k_1$ , and  $k_2$ . Then two coupled line cross-shaped resonators are cascaded to realize a wide band-pass filter. The sharper selectivity and better performance can be obtained by cascading two CLCSRs. Good S-parameters are achieved as demonstrated in both simulated and measured results.

*Index Terms* — Band-pass filter, coupled line cross-shaped resonator, transmission pole, transmission zero.

#### I. INTRODUCTION

The multi-mode resonator wide band-pass filter (BPF) design has become a research hotspot due to its simple filter topology, compact physical size and simple design process. In Ref. [1], an ultra-wideband BPF is designed with a cross-shaped resonator. An ultra-wideband BPF with the capacitively coupled stub-loaded resonator is introduced in Ref. [2]. Reference [3] proposes a differential wideband BPF which consists of slot-line multimode resonators. Cheng proposes a wideband BPF with reconfigurable bandwidth function [4]. The wideband BPF is composed of a parallel-coupled line structure and a cross-shaped resonator. In Refs. [5-11], some structures are added to the cross-shaped resonator, such as coupled lines, short stubs. Those band-pass filters provide better ideas for this

study. There are many studies based on cross-shaped resonators. However, the performance of the ordinary cross-shaped resonators is poor, and the frequency selectivity of the filters needs to be improved.

In this paper, a new coupled line cross-shaped resonator filter is proposed. The input impedance of its equivalent circuit can be calculated through the odd-even mode method. The proposed filter is miniaturized by bending parallel coupled lines. In order to increase the bandwidth and improve the out-of-band performance, two CLCSRs are cascaded in the wideband BPF design. The simulated results are basically consistent with the measured ones. Compared with single cross-shaped resonator, the frequency selectivity of the cascaded CLCSR BPF is improved.

## **II. BAND-PASS FILTER DESIGN**

Figure 1 (a) shows the proposed BPF which is composed of single CLCSR. The CLCSR consists of four parallel coupled lines. They are connected at the middle node position. The proposed BPF circuit is a symmetric structure. The odd mode characteristic impedance and even mode characteristic impedance of the parallel coupled lines are denoted as  $z_{ce1} = z_1 \sqrt{1 + k_1} / \sqrt{1 - k_1}$ ,  $z_{co1} = z_1 \sqrt{1 - k_1} / \sqrt{1 + k_1}$ . The odd mode characteristic impedance and even mode characteristic impedance and even mode characteristic impedance and even mode characteristic impedance of the other two coupled lines are denoted as  $z_{ce2} = z_2 \sqrt{1 - k_2} / \sqrt{1 - k_2}$ ,  $z_{co2} = z_2 \sqrt{1 - k_2} / \sqrt{1 + k_2}$ . The electrical length of all coupled lines is  $\theta$ .

The odd mode circuit is shown in Fig. 1 (b) and the even mode circuit is shown in Fig. (c). The normalized impedance parameters  $z_1 = Z_1 / Z_0$  and  $z_2 = Z_2 / Z_0$  are

used in the even-odd mode circuit. The input impedances  $z_{ine(o)}$  can be derived from formula (1) and (2) in Ref

[12]. According to the odd-even mode circuit, the even mode load impedance is in formula (2) and the odd mode load impedance is in formula (3):

$$z_{ine(o)} = \frac{z_{Le(o)} z_1 (1 - k_1^2) - (1 - k_1^2) \tan \theta z_{Le(o)} z_1 + j z_1^2 \tan \theta \sqrt{1 - k_1^2}}{(2 j \tan \theta z_{Le(o)} \sqrt{1 - k_1^2} + z_1 (1 + k_1))(1 - k_1)}, (1)$$
$$z_{Le} = \frac{z_2 \sqrt{1 - k_2^2}}{2 j \tan(\theta)(1 - k_2)}, (2)$$

$$z_{Lo} = 0. (3)$$



Fig. 1. (a) Ideal circuit of the CLCSR BPF, (b) odd-mode equivalent circuit, and (c) even-mode equivalent circuit.

In a symmetric two-port network, the normalized frequency response is

$$s_{11} = s_{22} = \frac{z_{ine} z_{ino} - 1}{(z_{ine} - 1)(z_{ino} - 1)},$$
(4)

$$s_{12} = s_{21} = \frac{z_{ine} - z_{ino}}{(z_{ine} - 1)(z_{ino} - 1)} \,.$$
(5)

When  $\theta$  is equal to  $\pi$ , the corresponding frequency is  $f_0$ , as shown in formula (6). When  $|s_{21}|$  is equal to zero, the transmission zeros of the CLCSR BPF can be obtained. The transmission zeros  $f_{z1}$  and  $f_{z2}$  are shown in formula (7a) and (7b). The positions of the CLCSR BPF's transmission zeros can be adjusted through the  $f_0$ :

$$\theta = \pi = \frac{2\pi f_0}{v} l, \qquad (6)$$

$$f_{z1} = \frac{1}{2} f_0 , \qquad (7a)$$

$$f_{z2} = \frac{3}{2}f_0$$
. (7b)

When  $|s_{11}|$  is equal to zero, the transmission poles of the CLCSR BPF can be obtained. The calculated results of the transmission poles are as shown in formula (8) and (9):

$$f_{p1} = \frac{2f_0 \arctan(\sqrt{u_p})}{\pi},\tag{8}$$

$$f_{p2} = f_0,$$
 (9)

$$f_{p3} = \frac{2f_0(\pi - \arctan(\sqrt{u_p}))}{\pi},$$
 (10)

$$u_{p} = \frac{(-\sqrt{1-k_{1}^{2}}z_{2}(-\frac{1}{2}z_{1}^{2}+k_{1}-1)\sqrt{1-k_{2}^{2}}+z_{1}(k_{1}^{2}-1)(k_{2}-1))(1+k_{1})}{z_{1}^{2}(-\frac{1}{2}z_{2}(-1+k_{1})\sqrt{1-k_{2}^{2}}\sqrt{1-k_{1}^{2}}-z_{1}(1+k_{1})(k_{2}-1))}.$$
 (11)

The transmission poles  $f_{p1}$  and  $f_{p3}$  are symmetric about the  $f_0$ . There are two transmission zeros out of the pass-band and three transmission poles in the pass-band. The zero-pole distribution of the ideal CLCSR BPF is shown in Fig. 2.



Fig. 2. Zero-pole distribution of the CLCSR BPF's circuit.

In Fig. 2, the relationship between the transmission poles and the transmission zeros in the frequency range is:

$$f_{z1} < f_{p1} < f_{p2} < f_{p3} < f_{z2}.$$
(12)

The influence of the design parameters on the return loss of the CLCSR BPF is given in Fig. 3. When one parameter changes, the other parameters remain unchanging. The transmission poles  $f_{p_1}$  and  $f_{p_3}$  are far away from each other when one parameter increases. In Fig. 3 (a), with the increases of  $k_1$ , the return loss decreases and the frequency selectivity is improved. The bandwidth increases and the return loss decreases with the increases of  $k_2$  in Fig. 3 (b). In Fig. 3 (c), the frequency selectivity is improved and the return loss decreases with the increase of  $z_1$ . In Fig. 3 (d), the bandwidth increases and the return loss decreases with the increase of  $z_2$ .

The physical structure parameters of the coupled lines

can be calculated based on Z and k. The characteristic impedance Z is smaller when the width of the coupled line is wider. When the gap between the coupled lines is wider, the value of k is usually smaller. Thus, the design parameters are selected as  $k_1 = k_2 = 0.6$ ,  $z_1 = z_2 = 1.5$ .





Fig. 3. The influence of  $z_1$ ,  $z_2$ ,  $k_1$ , and  $k_2$  on the return loss of the CLCSR BPF: (a)  $z_1$ ; (b)  $z_2$ ; (c)  $k_1$ ; (d)  $k_2$ .

## **III. RESULT AND DISCUSSION**

The final size of the CLCSR BPF is shown in Fig. 4 (a) and the photograph of the CLCSR BPF is shown in Fig. 4 (b).The coupled lines are bended for filter miniaturization. The resonant frequencies of the CLCSR BPF are simulated by using ANSYS HFSS. The EM method in simulation is finite element method. The CLCSR BPF is designed on Rogers RT5880 microwave dielectric board (h = 0.508 mm,  $\varepsilon_{re} = 2.2$ , tan  $\delta = 0.0009$ ). The line width g=1.54mm is chosen for the characteristic impedance of  $50\Omega$  for the input/output microstrip line. The characteristic impedance of SMA connector is  $50\Omega$ which matches the microstrip line.



Fig. 4. (a) The CLCSR BPF structure parameters (a=14.1, b=7.19, c=5, d=0.255, e=0.15, g=1.54, h=6.58, l=0.2, Unit: mm), (b) photograph of the CLCSR BPF.

Figure 5 shows the simulated and measured results of the CLCSR BPF. The measured results are obtained with Agilent N5230C vector network analyzer. As shown in Fig. 5, the measured results almost agree well with the simulation ones. The absolute bandwidth of the CLCSR BPF is 1.6 GHz, which is from 3.2 to 4.8 GHz. The return loss is bigger than 12dB in the whole pass-band. In addition, the stop-band rejection is better than 14dB from 2 to 3.2 GHz. Moreover the stop-band rejection is better than 12 dB from 4.8 to 7 GHz.

In order to improve the out-of-band performance and expand the pass-band frequency range, two CLCSRs are cascaded in wide-band BPF design. Figure 6 (a) shows the equivalent circuit of the cascaded CLCSR BPF. The cascaded CLCSR BPF structure parameters are slightly different from the CLCSR BPF in Fig. 1. The cascaded CLCSR BPF layout is shown in Fig. 6 (b) and the photograph of the cascaded CLCSR BPF is shown in Fig. 6 (c).



Fig. 5. Simulated and measured results of the CLCSR BPF.



Fig. 6. (a) Ideal circuit diagram for the cascaded CLCSR BPF, (b) actual size of the cascaded CLCSR BPF circuit (a=1.54, b=0.15, c=1.05, d=43.12, e=10.78, f=5.92, g=16.66, h=4.9, i=0.7, j=0.2, k=4.5, Unit: mm), (c) photograph of the cascaded CLCSR BPF.

The simulated and measured results of the cascaded CLCSR BPF are shown in Fig. 7. The absolute bandwidth of the cascaded CLCSR BPF is 2 GHz from 4.2 to 6.2 GHz. The return loss is bigger than 12dB in the passband. The out-of-band rejection levels are greater than 20 dB. The simulated results are basically consistent with the measured ones. The frequency discrepancy between the simulated and measured results is due to the machining error and the material parameters difference. The material parameters include the dielectric constant and the thickness of dielectric plate. The discrepancy of the return loss and the insertion loss is due to the conductor loss, the dielectric loss and the radiation loss. There are a few errors in the simulated and measured results, which also makes the simulated results different from the measured ones.



Fig. 7. Simulated and measured results of the cascaded CLCSR BPF.

Table 1 compares the proposed works with some previous works. Obviously, the proposed filters have some advantages, such as wide bandwidth, compact size and high performance.

## **IV. CONCLUSION**

In this paper, a CLCSR is proposed, which consists of four parallel coupled lines. By using the odd-even mode method, the CLCSR is analyzed and designed to construct a wide-band BPF. The cascaded CLCSR BPF is designed by cascading CLCSRs. Finally, the CLCSR BPF and the cascaded CLCSR BPF are simulated, fabricated and measured. The simulated and measured results for two proposed wide-band BPFs are basically the same.

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Reference	Center Frequency	TZs/TPs	Insertion Loss	Return Loss	Size $(\lambda_{g} * \lambda_{g})$
Ref. [1]	6.65	2/4	0.35	>20	0.5*0.79
Ref. [2]	6.85	4/5	0.6	>14	0.38*1.14
Ref. [3]	2.6	Not give	0.5	>10	1.4*1.4
Ref. [4]	5.7	2/3	1.4	>10	0.18*0.18
Ref. [5]	1.75	Not give	1.5	Not give	0.38*0.17
CLCSR BPF	4	2/3	0.4	>12	0.15*0.15
Cascaded CLCSR BPF	5.2	2/5	1.1	>12	0.19*0.15

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