A *p*-norm-like Constraint LMS Algorithm for Sparse Adaptive Beamforming

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Abstract **─** In this paper, a *p*-norm-like constraint normalized least mean square (PNL-CNLMS) algorithm is proposed for sparse adaptive beamforming. The proposed PNL-CNLMS algorithm inherits the good capacity of the conventional constrained least mean square (CLMS) algorithm in adaptive beamforming, i.e., forming ideal beam patterns. Also, the proposed PNL-CNLMS algorithm utilizes a *p*-norm-like constraint to exploit sparse property of the corresponding antenna array. In the derivation procedure, the Lagrange multiplier approach and the gradient descent method are utilized to obtain the devised updating equation. Numerical simulations reveal the superiority of the proposed PNL-CNLMS algorithm.

Index Terms **─** Array beamforming, constrained LMS algorithm, *p*-norm-like constraint, sparse adaptive beamforming.

I. INTRODUCTION

With the ability of forming the desired beampattern in the sector of interest while suppressing the influences from the unexpected interferences, adaptive beamforming has been an important application for array processing in the last decades. Because of the good capacity, adaptive beamforming is widely applied to radar, sonar, mobile communications, seismic sensing and other fields [1-2]. The strategy for an adaptive beamformer to acquire a better signal-to-interference-plus-noise ratio (SINR) is to form a main lobe in the interested direction to get a high gain, meanwhile, to form nulls to attenuate the interferences [3].

The wide spread linearly constrained minimum variance (LCMV) algorithm developed by Frost provides an excellent beamforming performance, which can provide the mentioned properties, i.e., dynamically adjusting the array weight vectors to adaptively capture the signals of interest (SOI) and suppress the interferences [3]. Then, the normalized adaptive version of LCMV, namely the constraint normalized least-mean-square (CLMS) algorithm is developed in [4], through which the output power is minimized, and the unintended interferences are reduced. Meanwhile, the CLMS algorithm remains a maximum gain in the desired direction.

Always, however, in real-life applications, especially in radar system, enormous arrays are essentially needed for realizing the desired performance. Where the fact is, enormous arrays face the problem of limited power supply and insufficient computation ability. As conventional adaptive beamforming algorithms fail to meet the requirements of enormous arrays computations, sparse adaptive beamforming algorithms have been proposed [5-9] which aim to find sparse solution for adaptive beamforming with little effect on the beampattern capacity. The first proposed sparse adaptive beamforming algorithm is inspired by the Compressive Sensing [10] and the Least Absolutely Shrinkage and Selection Operator [11]. Then, with the development of sparse signal pressing [12-21], scholars use the zero attracting technique to exploit the sparse characteristics of the antenna array and force the minor entries of the weight vector towards zero [12-15].

Sparse signal processing algorithms exploit the sparse characteristics existing in many scenarios, which attributes to the fact that they have particular advantages on both convergence rate and performance. Sparse signal processing technique is a hot research point and has been widely investigated in recent years. From the representative zero-attracting LMS (ZA-LMS) algorithm, which introduces a zero-attractor into the traditional iteration equation of the LMS algorithm, an enormous number of algorithms have been studied for sparse system applications [13-22]. The zero-attractor forces all zero-filter taps to zero, so that the convergence rate is accelerated. However, the zero-attractor in the ZA-LMS, which is generate by the

Submitted On: May 2, 2019
Accepted On: October 23, 2019

*l*1-norm penalty, unable to distinguish dominant coefficients and attenuate all the coefficients. In this regard, the reweighted ZA-LMS (RZA-LMS) is proposed to introduce different zero attractors for different taps, i.e., the trivial coefficients are force to zero more quickly.

Inspired by the zero-attractor techniques, an *l*1-norm CNLMS (L1-CNLMS) algorithm and a weighted *l*1- norm CNLMS (L1-WCNLMS) have been proposed for sparse adaptive beamforming [5]. Recently, many reweighted *l*1-norm penalties are proposed and considered in [12-15, 19-20], and a new reweighted *l*1-norm CNLMS (RL1-CNLMS) algorithm is proposed [8]. In [23-24], a *p*-norm-like diversity measure is proposed for sparse system identification, which holds a better performance than that of the *l*1-norm based algorithms, resulting in that it possible to improve the *l*1-norm based sparse adaptive beamforming algorithms.

In this paper, we develop a *p*-norm-like constraint normalized least mean square (PNL-CNLMS) algorithm for sparse adaptive beamforming. Simulation results demonstrate that the proposed algorithms can get a better beamforming performance and use less antenna array elements.

II. ARRAY PROCESSING FUNDAMENTALS

In this paper, a planar antenna array is considered, in which the antenna elements are half wavelength spaced. Figure 1 is the model of an adaptive beamforming system, while Fig. 2 provides the array elements coordinate diagram. P*m* (*m*=1, 2,…, *M*) is the positions of the sensors, and *d* is the interval between antenna elements, which is equals to half wavelength. The received signals have the directions of θs and θi (i=1, 2,…, *N*) which corresponds to the SOI and interferences, respectively. It is obvious that the objective of the adaptive beamforming algorithms is to generate main beam in θs and nulls in θi (i=1, 2,…, *N*). One of the basic assumptions for the system is that the receiving signals, including SOI and interferences are far-field narrow-band signals. In this way, the receiving signals can be regarded as plane waves. The sensor array is composed of *M* omnidirectional antennas, and each antenna corresponds to a so-called weight coefficient. Then, the designed sparse adaptive beamforming algorithm is used to find out the final sparse solution, i.e., to acquire the sparse weight vector. The optimal weight coefficients will be introduced in the next section.

Under the paradigm mentioned above, the receiving signals at time index *k* can be written as:

$$\mathbf{x}(k)=\mathbf{a}_s\mathbf{s}(k)+\mathbf{a}_i\mathbf{i}(k)+\mathbf{n}(k). \quad (1)$$

Where a_s and a_i are the SOI and interferences steering matrix, s(*k*) and i(*k*) are the complex signal envelope vectors n(*k*) is the zero-mean white Gaussian noise vector. It should be pointed out that the SOI, interferences and the noise are assumed to be statistically independent.

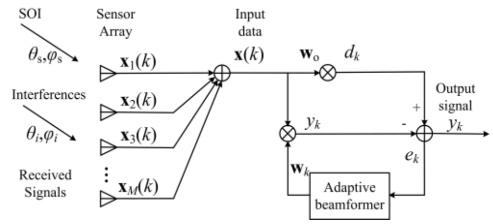

Fig. 1. Adaptive beamforming system.

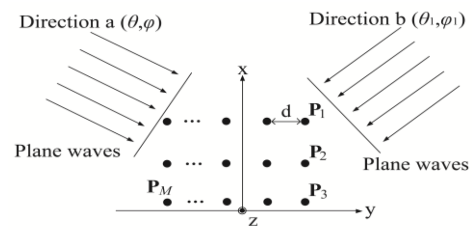

Fig. 2. Sensor array coordinate graph.

The output signal of the adaptive beamforming system then becomes:

$$\mathrm{y}(k)=\mathbf{w}^{\mathrm{H}}\mathbf{x}(k), \quad (2)$$

where w represents the weight vector of the adaptive beamforming system.

For a given direction (θ, φ), the beampattern is given by:

$$B(\theta,\varphi)=\mathbf{w}^{\mathrm{H}}\exp\left\{-j\frac{2\pi\mathbf{c}^T\mathbf{P}_m}{\lambda}\right\}, \quad (3)$$

where c=[-sinθcosφ,-sinθsinφ]$^{\mathrm{T}}$ is an unit vector and λ is the wavelength.

The output SINR of the adaptive beamformer is calculated by using:

$$\mathrm{SINR}=\frac{\sigma_s^2|\mathbf{w}^{\mathrm{H}}\mathrm{a}_s|^2}{\mathbf{w}^{\mathrm{H}}\mathbf{R}_{\mathrm{n+i}}\mathbf{w}}. \quad (4)$$

In our notation, $\sigma_s{}^2$ is the power of SOI and $\mathrm{R}_{\mathrm{n+i}}$ denotes the interference-plus-noise covariance matrix which is given by:

$$\mathbf{R}_{\mathrm{n+i}}=E\left\{(\mathbf{i}(k)+\mathbf{n}(k))(\mathbf{i}(k)+\mathbf{n}(k))^{\mathrm{H}}\right\}, \quad (5)$$

where $E\{\cdot\}$ is the expectation operator and $(\cdot)^{\mathrm{H}}$ represents the Hermitian operator.

III. THE CNLMS ALGORITHM

A. The CLMS algorithm

The well-known classical beamforming algorithm LCMV present a solution when the direction of SOI and interferences are given [1]. The weight vector in LCMV algorithm is expressed as:

$$\mathbf{w}_o = \mathbf{R}^{-1}\mathbf{C}(\mathbf{C}^H\mathbf{R}^{-1}\mathbf{C})^{-1}\mathbf{f}, \tag{6}$$

where R, C and f are the covariance matrix of input signal, the constraint matrix, and the constraint vector, respectively. It should be pointed out that the constraint matrix C contains the information of direction. In the constraint vector f, the elements associated to the SOI are set to 1 and entries corresponding to interferences are selected as 0. Then the adaptive version of LCMV algorithm is proposed, namely the CLMS algorithm which can adaptively give the desired beam pattern according to the direction of SOI and interferences [4]. The objective of CLMS algorithm is to solve:

$$\min_{\mathbf{w}} E\left[|e_k|^2\right] \quad \text{subject to} \quad \mathbf{C}^\mathbf{H}\mathbf{w} = \mathbf{f}, \tag{7}$$

with $d_k = \mathbf{w}_o^H\mathbf{x}_k$ and $e_k = d_k - \mathbf{w}^H\mathbf{x}_k$ denote the desired output signal and the estimation error, respectively.

To find out the solution of (7), the Lagrange multiplier method is utilized, and then (7) is transformed into the following cost function:

$$\xi_{CLMS}(k) = E\left[|e_k|^2\right] + \boldsymbol{\lambda}^H(\mathbf{C}^H\mathbf{w}_k - \mathbf{f}), \tag{8}$$

where λ is the Lagrange multiplier.

For the obtained cost function, namely (8), a close-form solution is unavailable. In this case, the gradient descent principle is utilized to iteratively seek for the solution. Then, the updating equation can be constructed as:

$$\mathbf{w}_{k+1} = \mathbf{w}_k - \mu\nabla_{\mathbf{w}}\xi_{CLMS}(k), \tag{9}$$

where μ represents the step size in each iteration and ∇_{w} is the gradient operation in terms of the weight vector.

To simplify the updating equation, the instantaneous estimation of the gradient vector is utilized. In this way, the gradient vector can be expressed as:

$$\hat{\nabla}_{\mathrm{w}}\xi_{CLMS}(k) = -2e_k^*\mathbf{x}_k + \mathbf{C}\boldsymbol{\lambda}_1. \tag{10}$$

From (7), one can get the constraint condition, i.e., $\mathbf{C}^\mathbf{H}\mathbf{w} = \mathbf{f}$. Use this constraint condition, one can derive the updating function after several straight-forward calculations, which is given by:

$$\mathbf{w}_{k+1} = \mathbf{P}\left[\mathbf{w}_k + \mu e_k^*\mathbf{x}_k\right] + \mathbf{f}_c, \tag{11}$$

where

$$\mathbf{P} = \mathbf{I}_{N\times N} - \mathbf{C}(\mathbf{C}^\mathbf{H}\mathbf{C})^{-1}\mathbf{C}^\mathbf{H}, \tag{12}$$

which is the projection matrix with $\mathrm{I}_{N\times N}$ is the identity matrix, and f_c is the constraint hyperplane which is given by:

$$\mathbf{f}_c = \mathbf{C}(\mathbf{C}^\mathbf{H}\mathbf{C})^{-1}\mathbf{f}. \tag{13}$$

B. The CNLMS algorithm

In CLMS algorithm, it can be seen that the step size, which is also referred as convergence factor, is a constant. As a consequence, the convergence rate of CLMS algorithm can be accelerated. Minimize the instantaneous posteriori squared error in terms of the step size [21]:

$$\frac{\partial[|e_{ip}(k)|^2]}{\partial\mu_k^*} = 0, \tag{14}$$

where

$$e_{ip}(k) = e_k\left(1 - \mu_k\mathbf{x}_k^H\mathbf{P}\mathbf{x}_k\right). \tag{15}$$

Solving (15), yields,

$$\mu_k = \frac{\mu_0}{\mathbf{x}_k^H\mathbf{P}\mathbf{x}_k + \varepsilon}. \tag{16}$$

In (16), ε is a small positive constant which can prevent overflowing when $\mathbf{x}_k^H\mathbf{P}\mathbf{x}_k$ is too small, and μ_0 is the initialized convergence factor.

Finally, the update function is obtained:

$$\mathbf{w}_{k+1} = \mathbf{P}[\mathbf{w}_k + \mu_k\frac{e_k\mathbf{x}_k}{\mathbf{x}_k^H\mathbf{P}\mathbf{x}_k + \varepsilon}] + \mathbf{f}_c. \tag{17}$$

C. The *p*-norm-like diversity measure

Different from the conventional Euclidean norm noted as $\|.\|_p$ or L_p, the *p*-norm-like diversity measure is a general effective criterion developed in [24], which is expressed as:

$$\|\mathbf{x}\|_{p-like} = \sum_{j=1}^{n}|x(j)|^p, \quad 0 \le p \le 1. \tag{18}$$

As (18) shows, it is clearly to see that the so-called *p*-norm-like diversity measure is not a classical norm, but they have close connection to provide sparse solution and can be used for sparse array beamforming. In [24-25], numerical simulation results have shown that the *p*-norm-like diversity measure outperforms the conventional l_1-norm optimal method for sparse system identification. Hence, in this paper the *p*-norm-like is utilized to exploit the sparsity characteristic of weight vector in adaptive beamforming algorithm.

D. Derivation of the PNL-CNLMS algorithm

The proposed PNL-CNLMS algorithm employs the *p*-norm-like diversity measure to develop the sparse adaptive beamforming algorithm, which is to solve:

$$\min_{\mathbf{w}} E\left[|e_k|^2\right] \quad \text{s.t.} \quad \begin{cases}\mathbf{C}^\mathbf{H}\mathbf{w}_k = \mathbf{f}; \\ \|\mathbf{w}_k\|_{p-like} = z,\end{cases} \tag{19}$$

where z acts as the constraint factor which lies in the range (0, 1), while e_k, C, w_k and f have the same meaning which are mentioned earlier in this paper.

Then, to solve (19), the Lagrange multiplier method is employed to acquire the objective function corresponding to (19):

$$\begin{aligned}\xi_{p\text{-}like}(k) = {} & E\left[|e_k|^2\right] + \boldsymbol{\lambda}_1^H(\mathbf{C}^H\mathbf{w}_k - \mathbf{f}) \\ & + \lambda_{p-like}[\|\mathbf{w}_k\|_{p-like} - z],\end{aligned} \tag{20}$$

where λ_1 and $\lambda_{p\text{-}like}$ are vector and scalar, respectively, which are the Lagrange multipliers.

Again, it is hard to obtain a close-form solution for (20). Similar to (8), (9), and (10), instantaneous estimation is used to implement the gradient of (20), which yields:

$$\hat{\nabla}_{\mathbf{w}}\xi_{p\text{-}like}(k) = -2e_k^*\mathbf{x}_k + \mathbf{C}\boldsymbol{\lambda}_1 + \lambda_{p-like}\mathbf{Q}_{p-like}, \tag{21}$$

with

$$\mathbf{Q}_{p-like} = \frac{\partial \| \mathbf{w}_k \|_{p-like}}{\partial \mathbf{w}_k} = \frac{\text{sgn}(\mathbf{w}_k)}{|\mathbf{w}_k|^{1-p}}, \tag{22}$$

where sgn(·) is a sign function whose definition is:

$$\text{sgn}(x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0, \\ 0, & x = 0 \end{cases} \tag{23}$$

Based on the principle of gradient descent concepts shown in (9), we can get the final updating equation given by:

$$\mathbf{w}_{k+1} = \mathbf{w}_k - \mu\hat{\nabla}_{\mathbf{w}}\xi_{p\text{-}like}(k), \tag{24}$$

where $\hat{\nabla}_{\mathbf{w}}\xi_{p\text{-}like}(k)$ is given in (21).

An upper bound is imposed on (22) to avert divergence when the entries of w_k become zero. This is an essential step especially when the algorithm itself is aimed to exploit sparse characteristic of the weight vector. As a consequence, $Q_{p\text{-}like}$ is expressed as:

$$\mathbf{Q}_{p-like} = \frac{\text{sgn}(\mathbf{w}_k)}{\varepsilon_{p-like} + |\mathbf{w}_k|^{1-p}}, \tag{25}$$

where $\varepsilon_{p\text{-}like}$ is a small positive constant.

The next task is to acquire the Lagrange multipliers. When the algorithm has converged, i.e., $w_{k+1}=w_k$, then we can rewrite the constraints in (19) to be:

$$\begin{cases} \mathbf{C}^{\mathrm{H}}\mathbf{w}_{k+1} = \mathbf{C}^{\mathrm{H}}\mathbf{w}_k = \mathbf{f}, \\ \mathbf{Q}_{p-like}\mathbf{w}_{k+1} = \mathbf{Q}_{p-like}\mathbf{w}_k = \| \mathbf{w}_k \|_{p-like} = z. \end{cases} \tag{26}$$

Take (21) into (24), and premultiplying (24) by C^H and $Q_{p\text{-}like}$ respectively, the Lagrange multipliers λ_1 and $\lambda_{p\text{-}like}$ are available:

$$\begin{cases} \boldsymbol{\lambda}_1 = \mathbf{G}(2e_k^*\mathbf{x}_k - \lambda_{p-like}\mathbf{Q}_{p-like}), \\ \lambda_{p-like} = (\frac{-2}{n\mu})z_e + \frac{2e_k^*\mathbf{Q}^{\mathrm{H}}_{p-like}\mathbf{P}\mathbf{x}_k}{n}, \end{cases} \tag{27}$$

with

$$\begin{cases} z_e = (z - \mathbf{Q}^{H}_{p-like}\mathbf{w}_k), \\ \mathbf{G} = (\mathbf{C}^{\mathrm{H}}\mathbf{C})^{-1}\mathbf{C}^{\mathrm{H}}, \\ n = \|\mathbf{P}\mathbf{Q}_{p-like}\|_2^2. \end{cases} \tag{28}$$

Then consider the normalizing approach in [26], the final updating formulation for the proposed PNL-CNLMS algorithm can be written as:

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \mu_k e_k^*\mathbf{W} + z_e(\frac{\mathbf{P}\mathbf{Q}_{p-like}}{v}), \tag{29}$$

where

$$\begin{cases} q = \mathbf{Q}^{\mathrm{H}}_{p-like}\mathbf{P}\mathbf{x}_k, \\ v = \mathbf{Q}^{\mathrm{H}}_{p-like}\mathbf{P}\mathbf{Q}_{p-like}, \\ \mu_k = \dfrac{\mu_0\left[e_k - z_e(\frac{\mathbf{P}\mathbf{Q}_{p-like}}{v})\mathbf{x}_k\right]}{e_k\mathbf{W}^{\mathrm{H}}\mathbf{x}_k + \varepsilon_{p-like}}, \\ \mathbf{P} = \mathbf{I}_{N\times N} - \mathbf{C}(\mathbf{C}^{\mathbf{H}}\mathbf{C})^{-1}\mathbf{C}^{\mathbf{H}}, \\ \mathbf{W} = \mathbf{P}(\mathbf{x}_k - \frac{q\mathbf{Q}_{p-like}}{v}). \end{cases} \tag{30}$$

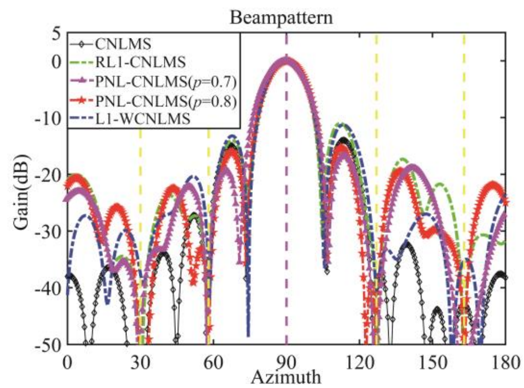

Fig. 3. Beampatterns of the proposed algorithms versus the CNLMS algorithm and the existing algorithms in [5, 8]. Purple line is the SOI, yellow lines are interferences.

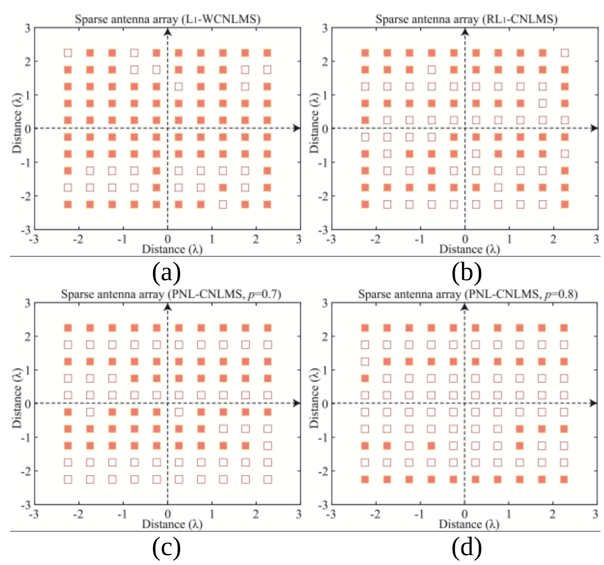

Fig. 4. Sparse arrays thinned by the proposed algorithms and the algorithm developed in [5]: (a) L_1-WCNLMS in [5], (b) RL_1-CNLMS algorithm in [8], (c) PNL-CNLMS algorithm with p=0.7, and (d) PNL-CNLMS algorithm with p=0.8.

IV. SIMULATION RESULTS

The proposed algorithm is expected to provide a better performance than the existing sparse adaptive beamforming algorithms [5, 8]. To evaluate its performance, several numerical simulations are carried out. The SOI as well as the interferences are QPSK signals from the azimuth of 90°, 30°, 58°, 127° and 163°, respectively, with an identical elevation angle of 45°. The signals are received by a rectangular array (RA) which contains 100 antenna elements with 10 rows and 10 columns. The signal-to-noise ratio (SNR) is set to 30 dB and the initialized convergence factor for L_1-WCNLMS, RL_1-CNLMS, CNLMS and PNL-CNLMS are 5×10^{-3}, 2×10^{-2}, 5×10^{-3} and 7×10^{-3}, respectively. The constraint factor z is selected as 0.8 uniformly. The iteration index is 6×10^3, while $\varepsilon_{p\text{-}like}$ is equal to 5×10^{-3}.

Figure 3 depicts the comparison of beam patterns. All the algorithms can form a mean lobe in the direction of SOI and generate nulls to attenuate interferences which are similar with that of the non-sparse classical CNLMS beamforming algorithm. Nevertheless, the side lobe level (SLL) for the proposed PNL-CNLMS algorithm as well as the L_1-WCNLMS and RL_1-CNLMS algorithms are a little higher than the CNLMS algorithm. However, the proposed new algorithm shows lower SLL against the existing sparse adaptive beamforming algorithms. It is found that for p=0.7 and p=0.8, the proposed algorithm shows a better balance between array sparsity and beampattern performance.

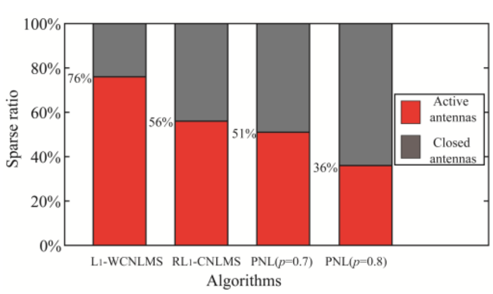

Fig. 5. Final sparse array ratio for the algorithms presented in Fig. 3 and Fig. 4.

Figure 4 illustrates the final thinned sensor array obtained by the proposed PNL-CNLMS algorithm and the existing adaptive sparse beamforming algorithms [5, 8]. In this paper, sparse ratio is defined as the percentage of active antenna elements taking account of the total antenna elements. The final sparse ratio is provided in Fig. 5. The figures indicate that all the algorithms hold the ability for realizing sparse adaptive beamforming. Nevertheless, the proposed algorithm can exploit a higher sparse level, it has a better performance in terms of beampattern in comparison with the existing algorithms, though the proposed algorithm has a better performance. This is because that the p-norm-like diversity measure can effectively exploit the sparse characteristic than the $L1$-norm and the reweighted $L1$-norm. In addition, simulation results reveal that there is no particular correlation between parameter p and the sparsity of the antenna array. In a word, our proposed adaptive beamformer can turn off the trivial antenna elements in order to reduce the power supply and keep a similar performance in the RA beamforming.

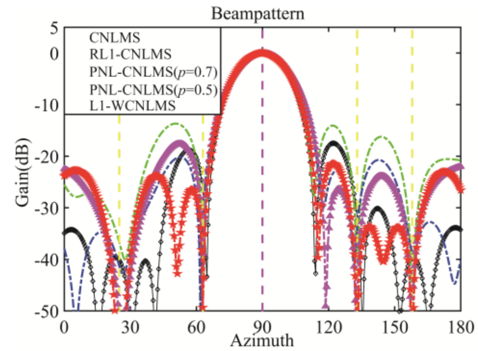

Fig. 6. Beampatterns of the proposed algorithms versus the CNLMS algorithm and the existing algorithms in [5, 8]. Purple line is the SOI, yellow lines are interferences.

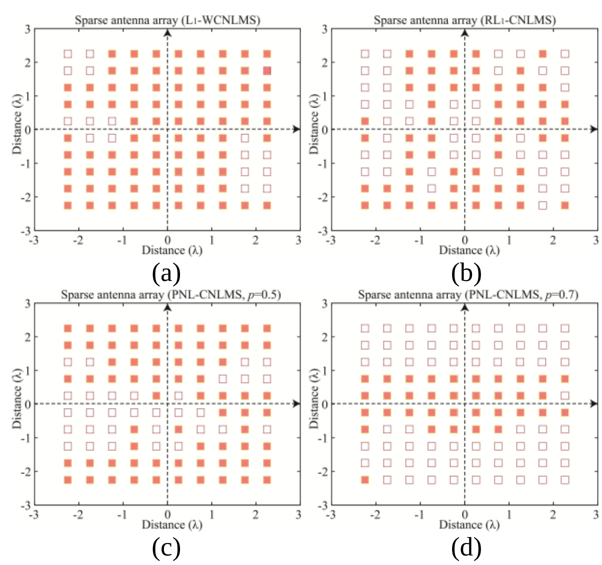

Fig. 7. Sparse arrays thinned by the proposed algorithms and the algorithm developed in [5]: (a) L_1-WCNLMS in [5], (b) RL_1-CNLMS algorithm in [8], (c) PNL-CNLMS algorithm with p=0.5, and (d) PNL-CNLMS algorithm with p=0.7.

Herein, to verify the effectiveness of the proposed algorithm, another example is presented. In this case, the directions of signals are 25°, 63°, 90°, 133°, and 158°, respectively. The elevation is set as 30°. p=0.5 and p=0.7 are selected, while other parameters are consistent with the first example. Figure 6 depicts the beampatterns of the proposed algorithm and other related sparse beamforming algorithms. The finalized sparse array is illustrated in Fig. 7. In this example, the proposed algorithm can get the ideal beampattern based a sparse array. What's more, it is clearly seen that the proposed PNL-CNLMS algorithm can provide a compromise between beampattern and array sparsity.

V. CONCLUSION

In this paper, a *p*-norm-like constraint normalized least mean square (PNL-CNLMS) algorithm is proposed for sparse adaptive beamforming. Two experiments are provided in the simulation to discuss the performance of the proposed algorithm. The proposed PNL-CNLMS algorithm can provide a similar beampattern with that of the conventional non-sparse adaptive beamforming algorithm using less antenna elements. For the sake of comparison with the existing sparse adaptive beamforming algorithms, the proposed PNL-CNLMS algorithm has a better beamforming performance and provides higher sparse level, which verifies the superiority of the proposed algorithm. Besides, by adjusting the parameter *p*, a trade-off between beampattern and sparse array is achieved. Still, the proposed algorithm shows potential to be further improved, e.g., reduce the SLL. What's more, the convergence rate for the proposed algorithm can be improved if a variable parameter *p* is employed, and the task is how to exploit the sparsity. In the future, the proposed algorithm can be further developed under impulsive noise and it can be used for MIMO antenna arrays [27-34].

ACKNOWLEDGMENT

This work was partially supported by the National Key Research and Development Program of China (2016YFE111100), Key Research and Development Program of Heilongjiang (GX17A016), the Science and Technology innovative Talents Foundation of Harbin (2016RAXXJ044), the Natural Science Foundation of Beijing (4182077), China Postdoctoral Science Foundation (2017M620918 and 2019T120134), the Ph.D. Student Research and Innovation Fund of the Fundamental Research Funds for the Central Universities (3072019GIP0808), the Fundamental Research Funds for the Central University (HEUCFG201829 and 2072019CFG0801), and Natural Science Foundation of Heilongjiang Province, China (F2017004).

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