# Novel Reduced Matrix Equation Constructing Method Accelerates Iterative Solution of Characteristic Basis Function Method

## Zhonggen Wang [1*], Qiang Chen [2], WenYan Nie [3], and Han Lin [1]

[1] School of Electrical and Information Engineering
Anhui University of Science and Technology, Huainan, 232001, China
zgwang@ahu.edu.cn[*], hanlin@aust.edu.cn

[2] Department of Electrical and Communications Engineering
Tohoku University, Sendai, 980-8579, Japan
chenq@ecei.tohoku.ac.jp

[3] School of Mechanical and Electrical Engineering
Huainan Normal University, Huainan, 232001, China
wynie5240@163.com

*Abstract ─* In this paper, a new construction method of reduced matrix equation is proposed to improve the iterative solution efficiency of characteristic basis function method (CBFM). Firstly, the singular value decomposition (SVD) technique is applied to compress the incident excitations and these new excitations retained on each block after SVD are defined as the excitation basis functions (EBFs). Then, the characteristic basis functions (CBFs) of each block are solved from these EBFs. Lastly, these EBFs and CBFs are used as the testing functions and the basis functions to construct the reduction matrix equation, respectively. The diagonal sub-matrices of the reduced matrix constructed by the proposed method are all identity matrices. Thus, the condition of the reduced matrix is improved resulting in a smaller number of iterations required for the solution of the reduced matrix equation. The numerical results validate the accuracy of the proposed method. Compared with the traditional CBFM, the iterative solution efficiency of the reduced matrix equation constructed by the proposed method is significantly improved.

*Index Terms ─* Characteristic basis functions, characteristic basis function method, reduced matrix equation, singular value decomposition, testing functions.

## I. INTRODUCTION

The method of moments (MoM) is known as an effective method to solve the electromagnetic scattering problems. However, the computational time and memory requirement of MoM increase significantly while dealing with large problems. In order to mitigate these problems, a number of acceleration algorithms have been proposed, such as fast multipole method (FMM) [1], multilevel fast multipole method (MLFMM) [2-4], adaptive integral method (AIM) [5], adaptive cross approximation (ACA) algorithm [6,7], precorrected-fast Fourier transform (P-FFT) [8] method, and fast dipole method (FDM) [9]. These methods effectively utilize the matrix-vector products (MVPs) and can handle large number of unknowns. Another essentially different family of techniques reduces the number of degrees of freedom (DoFs) by employing macro basis functions and domain-decomposition schemes, instead of utilizing the rapid computation of the MVPs. This family of techniques includes domain decomposition method (DDM) [10-12], synthetic function expansion technique (SFX) [13], accurate sub-entire-domain (ASED) basis function method [14], characteristic mode (CM) [15,16], and characteristic basis function method (CBFM) [17-19]. Among these techniques, the CBFM has been successfully and widely applied to printed circuits and scattering problems. The main computational of the CBFM consists of three parts: characteristic basis functions (CBFs) generation; reduced matrix construction; and reduced matrix equation solution. In recent years, several techniques have been proposed to improve the performance of the CBFM. In [20], multistep angular-derived CBFs generation technique has been proposed to reduce the singular value decomposition (SVD) time of generating the CBFs. In [21], the ACA-SVD has been adapted to efficiently generate the CBFs, which reduces both the time of generating the initial CBFs and the SVD time of initial CBFs. In [22], an improved primary CBFM (IP-CBFM) has been proposed to reduce the amount of memory used for reduced matrix by combining the secondary CBFs with the primary CBFs. In [23], the high level CBFs have been proposed to

improve the iterative solution efficiency of CBFM. Furthermore, some hybrid methods have been presented, such as CBFM-FMM [24], CBFM-MLFMM [25], CBFM-ACA [26], and CBFM-FDM [27] to accelerate the vector-matrix-vector products (VMVPs) in the construction of the reduced matrix. To some extent, these methods can save time and reduce the storage requirement. However, the size of reduced matrix in CBFM increases when analyzing the electrically large problems. Therefore, the solution of the reduced matrix equation should be performed by an iterative method. In this paper, a new construction method of reduced matrix equation is proposed to improve the iterative solution efficiency of CBFM. The diagonal sub-matrices of the reduced matrix constructed by the proposed method are all identity matrices, which improve the condition of reduced matrix and reduce the number of iterations.

## II. CHARACTERISTIC BASIS FUNCTION METHOD

The CBFM divides the target into $M$ blocks, where each block is solved as an independent domain. For each block, the CBFs can be obtained as:

$$\mathbf{Z}_{ii}^e \cdot \mathbf{J}_i^{\text{CBFs}} = \mathbf{E}_i, \tag{1}$$

where $\mathbf{Z}_{ii}^e$ denotes the self-impedance of the extended block $i$, with dimensions $N_i^{eb} \times N_i^{eb}$, for $i = 1, 2, \cdots, M$. The $N_i^{eb}$ represents the number of Rao-Wilton-Glisson (RWG) basis functions belonging to the extended block $i$, $\mathbf{E}_i$ is the excitation matrix with dimensions $N_i^{eb} \times N_{\text{pws}}$, and $N_{\text{pws}}$ is the number of incident excitations. In order to eliminate the redundant information in $\mathbf{J}_i^{\text{CBFs}}$ caused by overestimation, the SVD is used to reduce the redundancy of the initial CBFs. This factorization yields the following result:

$$\mathbf{J}_i^{\text{CBFs}} = \mathbf{U}_i \mathbf{W}_i \mathbf{V}_i^{\text{T}}, \tag{2}$$

where $\mathbf{U}_i$ and $\mathbf{V}_i$ are orthogonal matrices with dimensions $N_i^{eb} \times N_i^{eb}$ and $N_{\text{pws}} \times N_{\text{pws}}$, respectively, and $\mathbf{W}_i$ is a diagonal matrix with dimensions $N_i^{eb} \times N_{\text{pws}}$. The superscript T denotes the transpose operation. Suppose, the same number $B$ of CBFs is retained on each block after SVD, where $B$ is smaller than $N_{\text{pws}}$, the surface current of the target can be expressed as a liner combination of these CBFs as:

$$\mathbf{J} = \begin{bmatrix} \mathbf{J}_1 \\ \vdots \\ \mathbf{J}_i \\ \vdots \\ \mathbf{J}_M \end{bmatrix} = \sum_{k=1}^{B} a_1^k \begin{bmatrix} \mathbf{J}_1^k \\ [0] \\ \vdots \\ [0] \end{bmatrix} + \cdots + \sum_{k=1}^{B} a_M^k \begin{bmatrix} [0] \\ \vdots \\ [0] \\ \vdots \\ [\mathbf{J}_M^k] \end{bmatrix}, \tag{3}$$

where $a_i^k$ are the unknown expansion coefficients and $\mathbf{J}_i^k$ is the $k$th CBF of block $i$. The Galerkin method [28] is used to determine the unknown expansion coefficients and a $BM \times BM$ reduced matrix for the $BM$ unknown expansion coefficients is obtained. Then the reduced matrix equation $\mathbf{Z}^{\text{R}} \cdot \boldsymbol{\alpha} = \mathbf{E}^{\text{R}}$ can be constructed as follow:

$$\begin{bmatrix} \mathbf{Z}_{11}^{\text{R}} & \mathbf{Z}_{12}^{\text{R}} & \cdots & \mathbf{Z}_{1M}^{\text{R}} \\ \mathbf{Z}_{21}^{\text{R}} & \mathbf{Z}_{22}^{\text{R}} & \cdots & \mathbf{Z}_{2M}^{\text{R}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{Z}_{M1}^{\text{R}} & \mathbf{Z}_{M2}^{\text{R}} & \cdots & \mathbf{Z}_{MM}^{\text{R}} \end{bmatrix} \begin{bmatrix} \boldsymbol{\alpha}_1 \\ \boldsymbol{\alpha}_2 \\ \vdots \\ \boldsymbol{\alpha}_M \end{bmatrix} = \begin{bmatrix} \mathbf{E}_1^{\text{R}} \\ \mathbf{E}_2^{\text{R}} \\ \vdots \\ \mathbf{E}_M^{\text{R}} \end{bmatrix}, \tag{4}$$

where $\boldsymbol{\alpha}_i = (\boldsymbol{\alpha}_i^1, \boldsymbol{\alpha}_i^2, \cdots, \boldsymbol{\alpha}_i^B)^{\text{T}}$ represent the unknown weights of the CBFs on block $i$, $\mathbf{E}_i^{\text{R}}$ is the excitation vector, and $\mathbf{Z}_{ij}^{\text{R}}$ is the sub-matrix containing the coupling terms between blocks $i$ and $j$. The each element of the sub-matrix can be written as:

$$\begin{aligned} \mathbf{Z}_{ij}^{\text{R}}(m,n) &= \left\langle \mathbf{F}_{i,m}, L(\mathbf{F}_{j,n}) \right\rangle \\ &= \left\langle \sum_{p=1}^{N_i} \mathbf{J}_i(p,m) \mathbf{f}_{i,p}(\mathbf{r}), L\left( \sum_{q=1}^{N_j} \mathbf{J}_j(q,n) \mathbf{f}_{j,q}(\mathbf{r}) \right) \right\rangle \\ &= \sum_{p=1}^{N_i} \sum_{q=1}^{N_j} \mathbf{J}_i(p,m)^* \left\langle \mathbf{f}_{i,p}(\mathbf{r}), L(\mathbf{f}_{j,q}(\mathbf{r})) \right\rangle \mathbf{J}_j(q,n), \\ &= \sum_{p=1}^{N_i} \sum_{q=1}^{N_j} \mathbf{J}_i(p,m)^* \mathbf{Z}_{ij}(p,q) \mathbf{J}_j(q,n) \end{aligned} \tag{5}$$

where $\left\langle \mathbf{F}_{i,m}, L(\mathbf{F}_{j,n}) \right\rangle$ denotes the coupling term between the $m$th CBF on block $i$ and $n$th CBF on block $j$. $\mathbf{F}_{i,m}$ and $\mathbf{F}_{j,n}$ are the $m$th and $n$th CBFs on blocks $i$ and $j$, respectively. $\mathbf{f}_{i,p}(\mathbf{r})$, and $\mathbf{f}_{j,q}(\mathbf{r})$ are the $p$th and $q$th RWG basis functions on blocks $i$ and $j$, respectively. $\mathbf{Z}_{ij}(p,q)$ stands for the coupling term between the $p$th RWG on block $i$ and $q$th RWG on block $j$. The coefficient $\mathbf{J}_i(p,m)$ denotes the value of the $m$th CBF, included on the block $i$, and sampled at the center of the $p$th RWG. $N_i$ and $N_j$ are the numbers of RWG basis functions on blocks $i$ and $j$, respectively. Equation (5) enables us to express the coupling terms between the CBFs of two blocks as a simple product between matrices:

$$\mathbf{Z}_{ij}^{\text{R}} = \mathbf{J}_i^{\text{H}} \mathbf{Z}_{ij} \mathbf{J}_j, \tag{6}$$

where H stands for conjugated transpose, $\mathbf{Z}_{ij}$ is the matrix containing the coupling terms between the RWGs on blocks $i$ and $j$. Likewise, the voltage vector is then computed as:

$$\mathbf{E}_i^{\text{R}} = \mathbf{J}_i^{\text{H}} \mathbf{E}_i. \tag{7}$$

It can be found from Eqs. (5), (6), and (7) that the CBFs ( $\mathbf{J}_i$ ) are used as higher level testing and basis functions in terms of RWG basis functions in the process of constructing the reduced matrix equation. For moderate size problems, the reduced matrix equation (Eq. (4)) can be solved via a single LU-decomposition and one matrix-vector product per excitation. However, for large size problems, the dimensions of the reduced matrix become so large that an iterative method should be used to solve the reduced matrix equation.

## III. NEW REDUCED MATRIX EQUATION CONSTRUCTION METHOD

Firstly, the SVD is applied to deal with the excitation matrix before generating the CBFs:

$$\mathbf{E}_i = \mathbf{UWV}^\mathrm{T}, \qquad (8)$$

where $\mathbf{U}$ and $\mathbf{V}$ are the orthogonal matrices of dimensions $N_i^{eb} \times N_i^{eb}$ and $N_\mathrm{pws} \times N_\mathrm{pws}$, respectively. $\mathbf{W}$ is an $N_i^{eb} \times N_\mathrm{pws}$ diagonal matrix whose elements are the singular values of $\mathbf{E}_i$. Setting an appropriate threshold (typically 0.001), a new set of incident excitations will be obtained retaining only those with relative singular values above the threshold. Hence, a new excitation matrix named $\mathbf{E}_i^\mathrm{new}$ is obtained and the number of excitations is decreased. These new excitations are defined as the new testing functions of the block and denoted as excitation basis functions (EBFs). For simplicity, it is assumed that all the blocks contain the same number $K$ of EBFs. The dimensions of $\mathbf{E}_i^\mathrm{new}$ are $N_i^{eb} \times K$, and $K$ is always smaller than $N_\mathrm{pws}$. Replacing $\mathbf{E}_i$ in Eq. (1) with $\mathbf{E}_i^\mathrm{new}$, a new equation can be constructed as follows:

$$\mathbf{Z}_{ii}^e \mathbf{J}_i^\mathrm{new} = \mathbf{E}_i^\mathrm{new}. \qquad (9)$$

By solving Eq. (9), $K$ CBFs ( $\mathbf{J}_i^\mathrm{new}$ ) can be obtained on each block and are defined as the new basis functions. The total number of matrix equation solutions is $M \cdot K$, which is smaller than $M \cdot N_\mathrm{pws}$ in the CBFM as $K \ll N_\mathrm{pws}$. The time required to generate the CBFs in the proposed method is reduced compared to the traditional CBFM. By using the EBFs and CBFs as the testing and the basis functions, the coupling terms between the CBFs of two blocks can be rewritten as:

$$\mathbf{Z}_{ij}^{\mathrm{R}^\mathrm{new}} = \left(\mathbf{E}_i^\mathrm{new}\right)^\mathrm{H} \mathbf{Z}_{ij} \mathbf{J}_j^\mathrm{new}, \qquad (10)$$

where $\mathbf{E}_i^\mathrm{new}$ is an unitary matrix. When $i = j$, $\mathbf{Z}_{ii}^{\mathrm{R}^\mathrm{new}} = \left(\mathbf{E}_i^\mathrm{new}\right)^\mathrm{H} \mathbf{E}_i^\mathrm{new} = \mathbf{I}$, where $\mathbf{I}$ represents the identity matrix. Eq. (4) can be rewritten as:

$$\begin{bmatrix} \mathbf{I} & \mathbf{Z}_{12}^{\mathrm{R}^\mathrm{new}} & \cdots & \mathbf{Z}_{1M}^{\mathrm{R}^\mathrm{new}} \\ \mathbf{Z}_{21}^{\mathrm{R}^\mathrm{new}} & \mathbf{I} & \cdots & \mathbf{Z}_{2M}^{\mathrm{R}^\mathrm{new}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{Z}_{M1}^{\mathrm{R}^\mathrm{new}} & \mathbf{Z}_{M2}^{\mathrm{R}^\mathrm{new}} & \cdots & \mathbf{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{\alpha}_1 \\ \boldsymbol{\alpha}_2 \\ \vdots \\ \boldsymbol{\alpha}_M \end{bmatrix} = \begin{bmatrix} \mathbf{E}_1^{\mathrm{R}^\mathrm{new}} \\ \mathbf{E}_2^{\mathrm{R}^\mathrm{new}} \\ \vdots \\ \mathbf{E}_M^{\mathrm{R}^\mathrm{new}} \end{bmatrix}, \quad (11)$$

where $\mathbf{E}_i^{\mathrm{R}^\mathrm{new}} = (\mathbf{E}_i^\mathrm{new})^\mathrm{H} \mathbf{E}_i^\mathrm{new}$, it can be seen from Eq. (11) that the sub-matrix containing the coupling terms between the CBFs of each block becomes the identity matrix because of the orthogonal properties of the EBFs. Compared with the traditional CBFM, the condition number of the reduced matrix is improved, and the number of iterations required in the solution process is reduced accordingly.

## IV. NUMERICAL RESULTS

In this section, three test samples are presented to demonstrate the accuracy and efficiency of the proposed method. All simulations are executed on a PC with an Intel(R) Core(TM) i5-6200 CPU with 2.3 GHz (only one core was used) and 48 GB RAM. The bi-conjugated stabilized gradient (BiCGStab) is selected as iterative solver with a residual error of 0.001. The relative error *Err* is introduced and defined as follows:

$$Err = \left(\left|\mathrm{RCS}_x - \mathrm{RCS}_\mathrm{FEKO}\right| / \left|\mathrm{RCS}_\mathrm{FEKO}\right|\right) \times 100\%, \quad (12)$$

where $\mathrm{RCS}_\mathrm{FEKO}$ are the simulation results from the software FEKO, and $\mathrm{RCS}_x$ are results computed by the traditional CBFM or the proposed method.

Firstly, the scattering problem of a PEC plate with a side length of 2 m is considered at a frequency of 500 MHz. The geometry is divided into 8246 triangular patches with an average length of $\lambda/10$ leading to 15286 unknowns. The geometry is divided into 9 blocks, with each block extended by $\Delta = 0.15\lambda$ in all directions. For both methods, 800 incident excitations are set. The total numbers of CBFs and the relative errors of two methods under different SVD thresholds are shown in Table 1. It can be seen from the table that the relative error of the proposed method converges faster with reducing SVD threshold than the traditional CBFM. In order to compare the iterative solution efficiency of the two methods, the reduced matrix dimensions of the proposed method and the traditional CBFM are 865×865 and 872×8721 when the SVD thresholds are selected as 0.002 and 0.001, respectively. The condition number of CBFM matrix is 11009, while the condition number of the proposed method matrix is only 2219. Figure 1 illustrates the iterative convergence of the two methods. It can be seen from the figure that the proposed method achieves a good convergence. In order to confirm the higher convergence of the proposed method, the observation of the eigenvalues distributions is presented in Fig. 2. The

figure shows that the convergence of the iterative solutions of the proposed method improves as the eigenvalues move away from the origin. The bistatic RCS in horizontal polarization calculated by the two methods are shown in Fig. 3. It is clear from the RCS curves that the results calculated by the proposed method agree well with the one by FEKO, and also the one by CBFM. The total number of iterations of CBFM is 63, while the one of proposed method is only 37. Thus, a 41% of iterations number reduction is obtained.
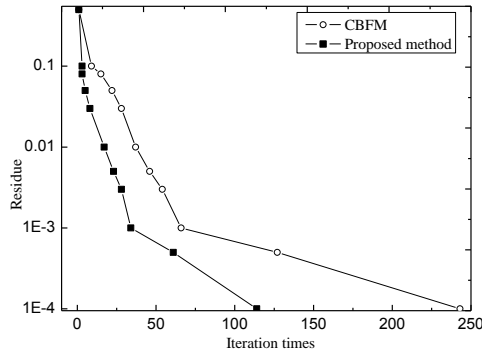


Fig. 1. Iteration times at different residues of two methods.
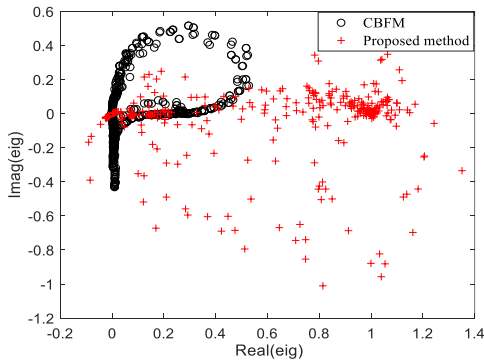


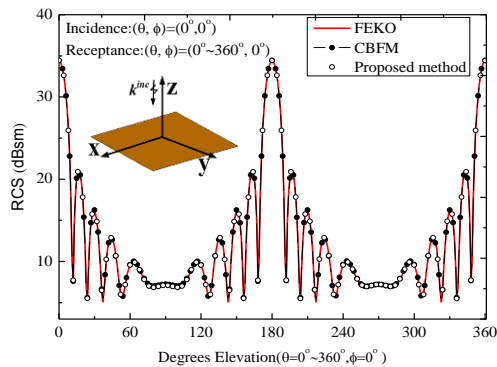Fig. 2. Eigenvalues distribution of two methods.



Fig. 3. Bistatic RCS of a PEC plate in horizontal polarization.

Next, the scattering problem of a PEC missile with a length of 1 m, a wingspan of 0.64 m, and a height of 0.22 m is considered at a frequency of 3 GHz. The total number of unknowns is 85217, and the geometry is divided into 52 blocks. Each block is excited by using multiple 800 incident excitations. The CBFs produced by the CBFM and the proposed method are 6025 and 6194, respectively. The condition number of CBFM matrix is 68341, and of proposed method matrix is only 7213. The average number of iterations of the conventional CBFM is 607 by using the iterative BiCGStab method without using the preconditioning techniques, while the proposed method requires only 299 iterations. The CPU time spent in the solution of the reduced matrix equation is reduced from 311.9 second with CBFM to 161.2 second in the proposed method. The bistatic RCS in horizontal polarization calculated by the two methods are depicted in Fig. 4. It can be seen from the figure that the RCS curve of proposed method is in a good agreement with that of traditional CBFM. A good agreement with the result of FEKO is also achieved except around $\theta = 115°$ and $\theta = 69°$.
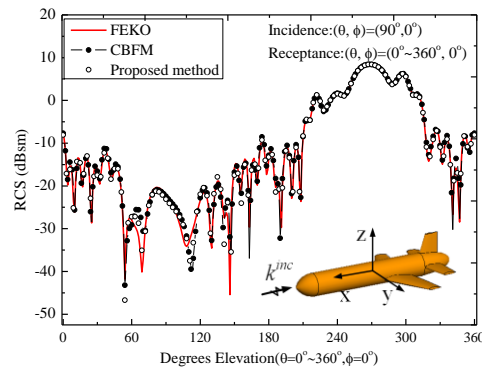


Fig. 4. Bistatic RCS of a PEC missile in horizontal polarization.

Finally, the monostatic RCS of a cone-sphere with gap (referenced from [29]) at a frequency of 10 GHz is calculated. Total 181 observation directions for $\theta = 90°$ and ranging from $\phi = 0°$ to $\phi = 180°$ are considered. The number of unknowns is 158881, while 11949 and 11832 CBFs are obtained for the CBFM and the proposed method, respectively. The sparse approximate inverse preconditioner is applied to accelerate the iterative solution of reduced matrix equation. The average number of iterations per direction is 59.7 for the traditional CBFM, while this number is reduced to 32.1 iterations when the proposed method is applied. A substantial reduction in the CPU-time is obtained because of the better conditioned reduced matrix. Figure 5 shows a comparison of the results obtained by applying FEKO, CBFM, and the proposed method for monostatic RCS in

horizontal polarization. It can be observed from the figure that the results obtained by the proposed method agree well with that obtained by the FEKO.
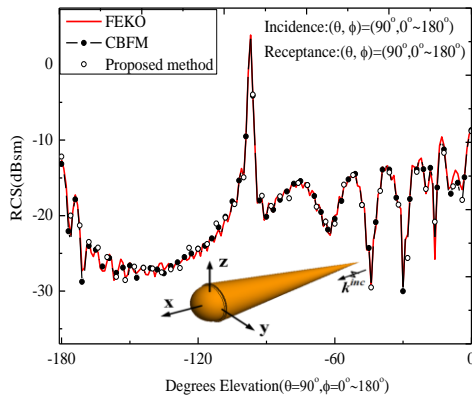


Fig. 5. Monostatic RCS of the cone-sphere with gap in horizontal polarization.

The CPU-times of the above three test examples using the CBFM and the proposed method are summarized in Table 2. Compared with the CBFM, the CPU-time of CBFs generation using the proposed method is reduced because the number of matrix equation solutions is significantly reduced. Moreover, the reduced matrix solution time is reduced because of the better conditioned system of equation using the proposed method. The CBFs generation time and the reduced matrix solving time are remarkably reduced and the gains are about 19.3% and 45.1%, respectively.

## V. CONCLUSION

A new construction method of reduced matrix equation is proposed in this paper to improve the iterative solution efficiency of characteristic basis function method (CBFM). In the proposed method, the excitation basis functions (EBFs) are first constructed by using the singular value decomposition (SVD) technique. Then, the characteristic basis functions (CBFs) are obtained by using the EBFs. These EBFs and CBFs are defied as the testing and the basis functions, respectively. The diagonal sub-matrices of the reduced matrix constructed by the new testing and basis functions are all identity matrices, which improves the condition of reduced matrix. Thus, the total number of iterations to achieve reasonable results is significantly reduced. Numerical simulations are conducted to validate the performance of the proposed method. The results demonstrate that the number of iterations required by the proposed method is noticeably less than that by the traditional CBFM due to the better conditioned system of equation. Furthermore, the proposed method can also be combined with MLFFM, AIM, P-FFT, FDM and other algorithms to further improve the efficiency of the characteristic basis function method for analyzing the electromagnetic scattering characteristics of electrically large targets.

Table 1: The total CBFs number and the relative error of two methods under different SVD threshold

| SVD Threshold | | 0.8 | 0.5 | 0.1 | 0.05 | 0.01 | 0.005 | 0.002 | 0.001 |
|---|---|---|---|---|---|---|---|---|---|
| CBFM | CBFs Number | 20 | 112 | 369 | 446 | 623 | 699 | 796 | 872 |
| | $Err$(%) | 68.85 | 34.82 | 16.01 | 7.62 | 1.58 | 0.96 | 0.82 | 0.51 |
| Proposed Method | CBFs Number | 28 | 171 | 421 | 492 | 679 | 756 | 865 | 912 |
| | $Err$(%) | 56.09 | 18.71 | 5.12 | 3.14 | 0.86 | 0.61 | 0.57 | 0.49 |

Table 2: CPU time of the CBFM and proposed method for different calculation steps

| Problems | Method | Impedance Matrix Calculation (s) | CBFs Generation (s) | Reduced Matrix Calculation (s) | Solving Matrix (s) | Total Time (s) |
|---|---|---|---|---|---|---|
| Plate | CBFM | 239.1 | 747.7 | 84.1 | 0.8 | 1071.7 |
| | Proposed Method | 242.9 | 479.7 | 83.3 | 0.5 | 806.4 |
| Missile | CBFM | 2205.6 | 6675.4 | 986.7 | 311.9 | 10179.6 |
| | Proposed Method | 2209 | 5749.8 | 994.9 | 137.2 | 9090.9 |
| Cone-Sphere with Gap | CBFM | 5040.3 | 21240.7 | 4516.3 | 277.2×181 | 80970.5 |
| | Proposed Method | 5048.2 | 19661.8 | 4373.6 | 160.5×181 | 58134.1 |

# REFERENCES

[1] R. Coifman, V. Rokhlin, and S. Wandzura, "The fast multipole method for the wave equation: A pedestrian prescription," *IEEE Antennas Propag Mag.*, vol. 53, no. 3, pp.7-12, 1993.

[2] M. Chen, R. S. Chen, and X. Q. Hu, "Augmented MLFMM for analysis of scattering from PEC object with fine structures," *Applied Computational Electromagnetics Society (ACES) Journal*, vol. 26, no. 5, pp. 418-428, 2011.

[3] X. M. Pan, S. L. Huang, and X. Q. Sheng, "Wide angular sweeping of dynamic electromagnetic responses from large Targets by MPI parallel skeletonization," *IEEE Trans. Antennas Propag.*, vol. 66, no. 3, pp. 1619-1623, 2018.

[4] Y. N. Liu, X. M. Pan, and X. Q. Sheng, "Skeletonization accelerated MLFMA solution of volume integral equation for plasmonic structures," *IEEE Trans. Antennas Propag.*, vol. 66, no. 3, pp. 1590-1594, 2018.

[5] E. Bleszynski, M. Bleszynski, and T. Jaroszewicz, "Adaptive integral method for solving large-scale electromagnetic scattering and radiation problems," *Radio Science*, vol. 31, no. 5, pp. 1225-1251, 1996.

[6] K. Zhao, M. N. Vouvakis, and J. F. Lee, "The adaptive cross approximation algorithm for accelerated method of moments computations of EMC," *IEEE Transactions on Electromagnetic Compatibility*, vol. 47, no. 4, pp. 763-773, 2005.

[7] Z. Liu, R. Chen, J. Chen, and Z. Fan, "Using adaptive cross approximation for efficient calculation of monostatic scattering with multiple incident angles," *Applied Computational Electromagnetics Society (ACES) Journal*, vol. 26, no. 4, pp. 325-333, 2011.

[8] J. R. Phillips and J. K. White, "A precorrected-FFT method for electrostatic analysis of complicated 3-D structures," *IEEE Trans. Comput. Aided Des. Integr. Circuits Syst.*, vol. 16, no. 10, pp. 1059-1072, 1997.

[9] X. Chen, C. Gu, Z. Niu, Y. Niu, and Z. Li, "Fast dipole method for electromagnetic scattering from perfect electric conducting targets," *IEEE Trans. Antennas Propag.*, vol. 60, no. 2, pp. 1186-1191, 2012.

[10] Z. Peng, X. Wang, and J. F. Lee, "Integral equation based domain decomposition method for solving electromagnetic wave scattering from non-penetrable objects," *IEEE Trans Antennas Propag.*, vol. 59, no. 9, pp. 3328-3338, 2011.

[11] J. Hu, R. Zhao, M. Tian, *et al.*, "Domain decomposition method based on integral equation for solution of scattering from very thin conducting cavity," *IEEE Trans. Antennas Propag.*, vol. 62, no.10, pp. 5344-5348, 2014.

[12] A. Freni, P. De Vita, P. Pirinoli, *et al.*, "Fast-factorization acceleration of MoM compressive domain-decomposition," *IEEE Trans. Antennas Propag.*, vol. 59, no. 1 2, pp. 4588-4599, 2011.

[13] L. Matekovits, V. A. Laza, and G. Vecchi, "Analysis of large complex structures with the synthetic-functions approach," *IEEE Trans Antennas Propag.*, vol. 55, no. 9, pp. 2509-2521, 2007.

[14] W. B. Lu, T. J. Cui, Z. G. Qian, *et al.*, "Accurate analysis of large-scale periodic structures using an efficient sub-entire-domain basis function method," *IEEE Trans. Antennas Propag.*, vol. 52, no. 11, pp. 3078-3085, 2004.

[15] F. G. Hu and C. F. Wang, "Integral equation formulations for characteristic modes of dielectric and magnetic bodies," *IEEE Trans. Antennas Propag.*, vol. 64, no. 11, pp. 4770-4776, 2016.

[16] L. Guan, Z. He, D. Z. Ding, and R. S. Chen, "Efficient characteristic mode analysis for radiation problems of antenna arrays," *IEEE Trans. Antennas Propag.*, vol. 67, no. 1, pp. 199-206, 2019.

[17] V. V. S. Prakash and R. Mittra, "Characteristic basis function method: A new technique for efficient solution of method of moments matrix equations," *Microw. Opt. Technol. Lett.*, vol. 36, no. 2, pp. 95-100, 2003.

[18] E. Lucente, A. Monorchio, and R. Mittra, "An iteration free MoM approach based on excitation independent characteristic basis functions for solving large multiscale electromagnetic scattering problems," *IEEE Trans. Antennas Propag.*, vol. 56, no. 4, pp. 999-1007, 2008.

[19] Y. F. Sun, K. Lu, and G. H. Wang, "Analysis of electromagnetic scattering from dielectric objects using multilevel characteristic basis function method," *Chinese Journal of Radio Science*, vol. 28, no. 1, pp. 92-96, 2013.

[20] C. Delgado, M. F. Catedra, and R. Mittra, "Efficient multilevel approach for the generation of characteristic basis functions for large scatters," *IEEE Trans. Antennas Propag.* vol. 56, no. 7, pp. 2134-2137, 2008.

[21] X. Chen, C. Gu, Z. Niu, Y. Niu, and Z. Li, "Efficient iterative solution of electromagnetic scattering using adaptive cross approximation enhanced characteristic basis function method," *IET Microwaves, Antennas & Propagation*, vol. 9, no. 3, pp. 217-223, 2015.

[22] T. Tanaka, Y, Inasawa, Y, Nishioka, and H. Miyashita, "Improved primary characteristic basic function method for monostatic radar cross section analysis of specific coordinate plane," *IEICE Transactions on Electronics*, vol. E99-C, no. 1, pp. 28-35, 2016.

[23] E. García, C. Delgado, and F. Cátedra, "A novel and efficient technique based on the characteristic basis functions method for solving scattering problems," *IEEE Trans. Antennas Propag.*, vol. 67, no.5, pp. 3241-3248, 2019.

[24] C. Craeye, "A fast impedance and pattern computation scheme for finite antenna arrays," *IEEE Trans. Antennas Propag.*, vol. 54, no. 10, pp. 3030-3034, 2006.

[25] E. García, C. Delgado, I. González, and M. F. Catedra, "An iterative solution for electrically large problems combining the characteristic basis function method and the multilevel fast multipole algorithm," *IEEE Trans. Antennas Propag.*, vol. 56, no. 8, pp. 2363-2371, 2008.

[26] R. Maaskant and R. Mittra, "Fast analysis of large antenna arrays using the characteristic basis function method and the adaptive cross approximation algorithm," *IEEE Trans. Antennas Propag.* vol. 56, no. 11, pp. 3440-3451, 2008.

[27] X. Chen, Z. Li, Z. Niu, and C. Gu, "A hybrid fast dipole method and adaptive modified characteristic basis function method for electromagnetic scattering from perfect electric conducting targets," *J Electromagn Waves Appl.*, vol. 25, no. 14, pp. 1940-1952, 2011.

[28] B. G. Galerkin, "On electrical circuits for the approximate solution of the Laplace equation," *Vestnik Inzh.*, vol. 19, pp. 897-908, 1915.

[29] A. C. Woo, H. T. G. Wang, M. J. Schuh, and M. L. Sanders, "Benchmark radar targets for the validation of computational electromagnetics programs," *IEEE Antennas Propag. Mag.*, vol. 35, no. 6, pp. 84-89, 1993.

**Qiang Chen** received the B.E. degree from Xidian University, Xi'an, China, in 1986, and the M.E. and D.E. degrees from Tohoku University, Sendai, Japan, in 1991 and 1994, respectively. He is currently a Chair Professor with the Electromagnetic Engineering Laboratory, Department of Communications Engineering, Tohoku University. His current research interests include antennas, microwave and millimeter wave, electromagnetic measurement, and computational electromagnetics. He is a member of the Institute of Electronics, Information and Communication Engineers (IEICE). He is currently the Chair of the IEICE Technical Committee on Wireless Power Transfer. He was the Secretary and the Treasurer of the IEEE Antennas and Propagation Society Tokyo Chapter in 1998, the Secretary of the Technical Committee on Electromagnetic Compatibility of the IEICE from 2004 to 2006, and the Secretary of the Technical Committee on Antennas and Propagation of the IEICE from 2008 to 2010. He was an Associate Editor of the IEICE Transactions on Communications from 2007 to 2012.



**Wenyan Nie** received the B.S. and M.S degrees from Anhui University of Science and Technology in 2007 and 2012, respectively. She is currently an Associate Professor at Huainan Normal University. Her research interests include computational electromagnetic methods, antenna theory and design.



**Han Lin** is a Lecturer at Anhui University of Science and Technology. She received the Ph.D. degree in Electromagnetic Field and Microwave Technique from the Nanjing University of Aeronautics and Astronautics, Nanjing, P. R. China, in 2015. Her research interests include computational electromagnetic methods, antenna theory and design.



**Zhonggen Wang** received the Ph.D. degree in Electromagnetic Field and Microwave Technique from the Anhui University of China (AHU), Hefei, P. R. China, in 2014. Since 2014, he has been with the School of Electrical and Information Engineering, Anhui University of Science and Technology, where he is currently an Associate Professor. Since 2018, he is currently with the Department of Communications Engineering, Tohoku University, Sendai, Japan, as a Visiting Scholar. His research interests include computational electromagnetics, array antennas, and reflectarrays.