An Effective Iterative Algorithm to Correct the Probe Positioning Errors in a Non-Redundant Plane-Rectangular Near-Field to Far-Field Transformation

F. D'Agostino, F. Ferrara, C. Gennarelli, R. Guerriero, and M. Migliozzi

Department of Industrial Engineering
University of Salerno, via Giovanni Paolo II, 132-84084, Fisciano, Italy
cgennarelli@unisa.it

Abstract ─ An algorithm for the effective compensation of known positioning errors, affecting the samples acquired by the probe in a non-redundant plane-rectangular (PR) near-field to far-field (NFFF) transformation, is presented and fully assessed by experimental tests. This transformation adopts a non-conventional PR scan, named planar wide-mesh scan (PWMS), characterized by meshes widening more and more as their distance from the measurement plane center increases, and uses a non-redundant sampling representation of the probe voltage. Such a representation is obtained by considering the antenna as contained in an oblate spheroid, to precisely determine the input NF data for the traditional PR NFFF transformation from the PWMS samples. These samples are unavailable in presence of positioning errors, but, as it will be shown, can be effectively retrieved from the errors affected ones by applying an iterative procedure.

Index Terms ─ Antenna measurements, non-conventional plane-rectangular scanning, non-redundant sampling representations, plane-rectangular near-field to far-field transformation, positioning errors compensation.

I. INTRODUCTION

In recent years, the near-field to far-field (NFFF) transformation techniques [1-5] have become extensively employed and well-assessed tools to precisely predict the far field radiated by an electrically large antenna from measurements performed in its NF region. As a matter of fact, the accurate measurement of the antenna radiated electromagnetic (EM) fields can be done only in a controlled environment, such as an anechoic chamber, where the propagation condition in free-space is very well approximated, by significantly reducing the field reflected from its walls, as well as the EM interferences from external sources. However, for an antenna under test (AUT) having large sizes as compared to the wavelength, it is practically impossible to fulfill the FF distance requirements in an anechoic chamber, so that only NF measurements can be carried out. Accordingly, the use of NFFF transformation techniques becomes mandatory to evaluate the radiated far field. Among these techniques, that adopting the plane-rectangular (PR) scan [6, 7] is undoubtedly the most simple one from the analytical and computational viewpoint. In fact, from the fast Fourier transform (FFT) of the complex voltages measured by the probe in two its orientations in a suitable lattice of the PR scanning surface, it is possible to determine the plane waves spectrum of the AUT field and then the radiated far field [6, 7]. This scan, as all the planar ones [5], is suitable to very directive AUTs, radiating pencil beam patterns, as e.g., those recently proposed in [8, 9], since, due to the truncation of the scanning area, a good FF reconstruction is obtained only in the angular region specified by the limits of this last and the AUT edges. As well-known, in the classical PR scanning, the spacings between two adjacent points of the lattice are constant and bounded by $\lambda/2$, λ being the wavelength. Accordingly, these spacings are determined only by the operating frequency of the AUT and are independent of its dimension and geometry. On the contrary, the knowledge of the geometric characteristics and sizes of the antenna is suitably taken into account in the non-redundant (NR) PR transformations [10, 11] employing a novel planar scan, named planar wide-mesh scan (PWMS), characterized by rectangular meshes which widen more and more as their distance from the measurement plane center grows. These NFFFs transformations have been developed by applying the NR sampling representations of the EM fields [12, 13] to the voltage revealed by the probe and modeling a volumetric AUT with a sphere and a quasi-planar one with a surface formed by joining together two circular bowls with the same aperture (double bowl) or with an oblate spheroid. The related two-dimensional optimal sampling interpolation (OSI) expansions allow an efficient and very precise reconstruction of the massive input NF data for the traditional PR NFFF [7] from the NR ones gathered using the PWMS. A drastic savings in the measurement time, as compared to that needed by the classical PR scan, is so achieved and this result is very important, since the acquisition time is today by far greater than the computing one to execute the NFFF transformation.

Submitted On: May 8, 2019
Accepted On: September 7, 2019

1054-4887 © ACES

It must be pointed out that, as a result of a not accurate control of the probe and AUT positioners, as well as of their finite resolution, a precise acquisition of the NF data at the points set by the NR sampling representation it is not always possible. In any case, the use of laser interferometric techniques makes possible to exactly determine the real locations of the acquired NF data. Hence, it becomes of key importance to have at one's disposal an efficient and stable method for the precise recovery of the NF data required to execute the classical PR NFFF transformation from the probe positioning errors affected ones acquired by using the PWMS. In this framework, an algorithm, adopting the conjugate gradient iteration technique and taking advantage from the fast Fourier transform for not equispaced data [14], has been utilized in [15] and [16] to compensate known positioning errors affecting the NF data necessary for the classical NFFF transformations with PR and spherical scans, respectively. By following the Yen's approach [17], an interpolation technique, that allows the correction of small position errors corrupting the NF data in the traditional PR NFFF transformation, has been proposed in [18]. In any case, these techniques are not suited to the NFFF transformations with PWMS [10, 11], where the massive PR NF data are recovered by interpolating the acquired NR ones, so that a different approach has to be applied. As underlined in [19], the direct reconstruction of the PR NF data from the positioning errors affected and, as consequence, irregularly spaced (non-uniform) samples is inopportune. A more convenient and viable approach is the retrieving of the regularly spaced (uniform) PWMS samples from the collected non-uniform ones and after that the accurate reconstruction of the PR NF data via an effective OSI algorithm [19]. To reach this objective, an iterative algorithm has been exploited for recovering the uniform samples from the not evenly distributed ones in PR [19], cylindrical or spherical lattices [20]. However, the existence of a one-to-one correspondence between every uniform sampling point and the closest non-uniform one is needed for its convergence. To overcome this shortcoming, a different approach, using the singular value decomposition (SVD) technique, has been afterward proposed and properly employed in the NR NFFF transformations with cylindrical [21], plane-polar [22], and bi-polar [23] scans from positioning errors affected NF data. Such an approach allows one to take advantage from the redundancy of data to improve the algorithm stability with respect to random errors affecting them, but requires that the retrieving of the uniform samples can be split in two separate one-dimensional problems. If this is not the case, a very large computational effort is needed due to the remarkable increase in the involved matrices dimensions. Both approaches have been compared through numerical simulations and experimentally assessed in [24] with reference to a NR spherical NFFF transformation, while their effectiveness in the NR NFFF transformations with cylindrical and plane-polar scannings has been experimentally assessed in [25] and [26, 27], respectively. Finally, the efficacy of the SVD-based approach, to retrieve the uniform (positioning errors free) samples from the positioning errors affected samples, in the NR NFFF transformation with PWMS employing an oblate spheroid as antenna modeling, has been proven through numerical simulations in [28] and via experimental tests in [29].

The aim of this paper is to suitably exploit the iterative algorithm to correct known positioning errors affecting the collected NF samples in the NFFF transformation with PWMS based on the oblate spheroidal model of the AUT (see Fig. 1) and to provide the experimental validation of the developed procedure.

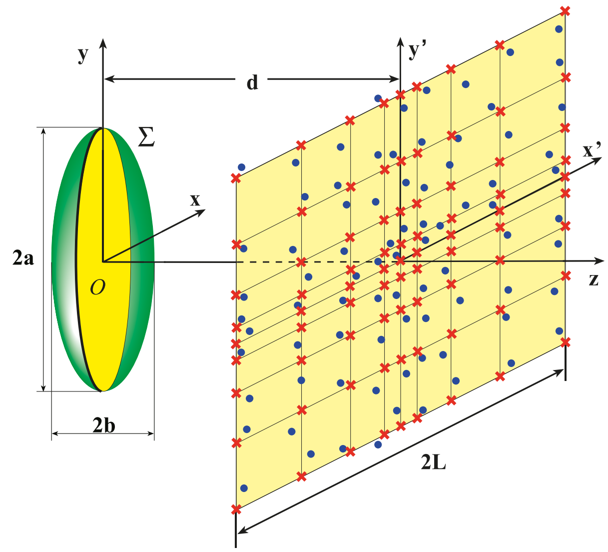

Fig. 1. PWMS scanning. Red crosses: uniform samples. Blue dots: non-uniform samples.

II. NON-REDUNDANT REPRESENTATION ON A PLANE FROM PWMS SAMPLES

An efficient NR representation of the voltage, detected by an electrically small probe on a plane d away from a quasi-planar AUT through a PWMS NF measurement system, and the corresponding OSI expansion are briefly recalled in this section for reader's convenience. In the following, two Cartesian coordinate systems are introduced to tackle this issue. The former (x, y, z) is used to denote a generic observation point and has its origin O at the center of the AUT aperture, while the latter (x', y', z'), having its z' axis coincident with the z one and the origin O' in correspondence of the scanning plane center at distance d from O (Fig. 1), is adopted to specify a point P belonging to the plane. A spherical reference system (r, ϑ, φ) with the origin again at O is also introduced. Because the voltage measured by a probe, whose dimensions are small with respect to the wavelength, has about the same spatial bandwidth of the EM field radiated by the AUT [30], the NR sampling representations [12] can be exploited to get the requested representation using a minimum number of samples. According to these representations, the AUT must be first of all modeled by an appropriate convex rotational surface Σ, enclosing it and fitting well its shape. In fact, the minimum number of samples needed to represent the voltage on any rotational surface containing the AUT is proportional to the area of Σ. Afterward, a suitable parameterization $\underline{r} = \underline{r}(\tau)$ must be adopted to describe any curve Γ lying on the plane and an appropriate phase factor $e^{-j\psi(\tau)}$ has to be singled out from the voltage V detected by the probe (V_y) and by the rotated probe (V_x), thus obtaining the so called "reduced voltage":

$$\tilde{V}(\tau) = V(\tau)\, e^{j\psi(\tau)}, \qquad (1)$$

which is a function spatially almost bandlimited to W_τ [12]. This means that the error made when approximating it by a function bandlimited to $\chi' W_\tau$ can be made negligible [12] by properly choosing the enlargement bandwidth factor $\chi' > 1$. Since the PWMS, as all the planar scans, is well suited for AUTs characterized by a quasi-planar geometry, a double bowl or an oblate spheroid can be chosen as modeling surface Σ. The choice between which of the two models is more convenient to adopt falls on that minimizing the area of the surface Σ. Anyhow, the oblate spheroidal model gives rise to a simpler NR representation, conversely, the double bowl one is more flexible, allowing a better fitting of antennas characterized by a non-symmetrical profile with respect to the plane identified by their maximum transverse dimension. In the following, an oblate spheroid, with major and minor semi-axes a and b, is chosen as surface Σ. In such a case, the expressions of the bandwidth W_τ, parameter τ, and function ψ for a meridian curve Γ, as the x' (or y') axis, are [10-12]:

$$W_\tau = \frac{4a}{\lambda} E\left(\frac{\pi}{2} \middle| \varepsilon^2\right), \qquad \tau = \frac{\pi E\left(\sin^{-1} u \,|\, \varepsilon^2\right)}{2E\left(\pi/2 \,|\, \varepsilon^2\right)}, \qquad (2)$$

$$\psi = \frac{2\pi a}{\lambda}\left[v\sqrt{\frac{v^2-1}{v^2-\varepsilon^2}} - E\left(\cos^{-1}\sqrt{\frac{1-\varepsilon^2}{v^2-\varepsilon^2}} \,\middle|\, \varepsilon^2\right)\right], \qquad (3)$$

where $\varepsilon = f/a$ is the eccentricity of the spheroid, $2f$ is its focal distance, $E(\cdot \,|\, \cdot)$ is the second kind elliptic integral, and $u = (r_1 - r_2)/2f$, $v = (r_1 + r_2)/2a$ are the elliptic coordinates, $r_{1,2}$ being the distances from the observation point P to the foci.

It must be underlined that, to allow the factorization of the two-dimensional OSI reconstruction algorithm into one-dimensional interpolations along lines, it is indispensable that the same parameterization $x' = x'(\xi)$ or $y' = y'(\eta)$, with the optimal parameters ξ and η defined by (2), has to be employed for describing all lines parallel to the x' or y' axis, respectively [10, 11]. As a consequence, the samples spacing for all lines parallel to the x' (or y') axis coincides with that corresponding to the x' (or y') axis. Accordingly, the PWMS lattice has rectangular meshes, which widen more and more as their distance from the center of the measurement plane increases (see Fig. 1). Regarding the phase function ψ, it depends only on the distances $r_{1,2}$, i.e., it is the same which would be used when interpolating along the radial line passing through P and, accordingly, it can be evaluated [10, 11] by applying (3).

In light of these results, the voltage V can be accurately determined at any point $P(x', y')$ via the following two-dimensional OSI expansion [8, 9]:

$$V(\xi(x'),\eta(y')) = e^{-j\psi(x',y')} \sum_{m=m_0-p+1}^{m_0+p} \Bigg\{ G(\eta,\eta_m,\bar{\eta},N,N'') \cdot \sum_{n=n_0-q+1}^{n_0+q} \tilde{V}(\xi_n,\eta_m)\, G(\xi,\xi_n,\bar{\xi},N,N'') \Bigg\}, \qquad (4)$$

wherein $m_0 = \text{Int}(\eta/\Delta\eta)$, $n_0 = \text{Int}(\xi/\Delta\xi)$, $2q \times 2p$ is the number of the considered reduced voltage samples $\tilde{V}(\xi_n,\eta_m) = V(\xi_n,\eta_m)\, e^{j\psi(x'_n,\, y'_m)}$ closest to P:

$$\eta_m = m\Delta\eta = 2\pi m/(2N''+1), \quad \xi_n = n\Delta\xi = n\Delta\eta, \qquad (5)$$

$$N'' = \text{Int}(\chi N') + 1, \quad N' = \text{Int}(\chi' W_\tau) + 1, \qquad (6)$$

$$N = N'' - N', \quad \bar{\eta} = p\Delta\eta, \quad \bar{\xi} = q\Delta\xi. \qquad (7)$$

$\text{Int}(\alpha)$ staying for the greatest integer less than or equal to α and $\chi > 1$ being an oversampling factor needed for the control of the truncation error [12, 13]. Moreover:

$$G(\alpha,\alpha_i,\bar{\alpha},L,L'') = \Omega_L(\alpha-\alpha_i,\bar{\alpha})\, D_{L''}(\alpha-\alpha_i), \qquad (8)$$

is the interpolation function of the OSI expansion, where $\Omega_L(\bullet,\bullet)$ and $D_{L''}(\bullet)$ are the Tschebyscheff and Dirichlet sampling functions [12].

The two-dimensional OSI expansion (4) can be suitably exploited to accurately recover V_x and V_y at the points needed by the standard PR NFFF transformation [7]. The corresponding probe compensated formulas in the adopted reference frame when the used probe is an open-ended rectangular waveguide, fed by the TE_{10} mode and characterized as described in [31], are shown in [32].

III. RETRIEVING OF THE UNIFORM SAMPLES

In the following, the collected PWMS samples are assumed as affected by known positioning errors, so that they are irregularly spaced on the plane. It is also supposed that there exists a bijection associating each of the $M = M_x \times M_y$ uniform sampling points to the closest non-uniform one. As explicitly emphasized in the Introduction, the iterative procedure can be suitably applied in these hypotheses. According to such an approach, by exploiting the OSI formula (4), the reduced voltage value in correspondence of each non-uniform sampling point (δ_k, σ_j) is expressed as function of the unknown ones at the closest uniform sampling points (ξ_n, η_m), thus obtaining the linear system:

$$\tilde{V}(\delta_k, \sigma_j) = \sum_{m=m_0-p+1}^{m_0+p} \Big\{ G(\sigma_j, \eta_m, \bar{\eta}, N, N") \cdot \sum_{n=n_0-q+1}^{n_0+q} \tilde{V}(\xi_n, \eta_m)\; G(\delta_k, \xi_n, \bar{\xi}, N, N") \Big\} \quad ,$$

$$k = 1, \ldots, M_x\, ; \quad j = 1, \ldots, M_y \ , \tag{9}$$

that, in matrix form, becomes $\underline{\underline{C}}\;\underline{X} = \underline{B}$, wherein $\underline{\underline{C}}$ is a $M \times M$ sized sparse matrix, whose elements are linked to the interpolating functions of the OSI expansion (4), $\underline{X}$ the unknown uniform samples vector, and $\underline{B}$ the known non-uniform samples one. As emphasized in the Introduction, it is not convenient to solve this system via the SVD method, due to the large dimensions of the matrix $\underline{\underline{C}}$. Conversely, its solution can be efficiently achieved by applying an iterative procedure, whose derivation is shown below. As first step, the matrix $\underline{\underline{C}}$ is subdivided into its diagonal and non-diagonal parts $\underline{\underline{C}}_D$ and $\underline{\underline{\Delta}}$, respectively. After that, both sides of the system $\underline{\underline{C}}\;\underline{X} = \underline{B}$ are multiplied by $\underline{\underline{C}}_D^{-1}$, thus obtaining:

$$\underline{X} + \underline{\underline{C}}_D^{-1} \underline{\underline{\Delta}}\, \underline{X} = \underline{\underline{C}}_D^{-1} \underline{B}\,. \tag{10}$$

By rearranging the terms of this last relation, the following iterative scheme is finally attained:

$$\underline{X}^{(\mu)} = \underline{\underline{C}}_D^{-1} \underline{B} - \underline{\underline{C}}_D^{-1} \underline{\underline{\Delta}}\, \underline{X}^{(\mu-1)} = \underline{X}^{(0)} - \underline{\underline{C}}_D^{-1} \underline{\underline{\Delta}}\, \underline{X}^{(\mu-1)}\, , \tag{11}$$

with $\underline{X}^{(\mu)}$ being the uniform samples vector attained at the μth iteration. To assure the convergence of the iterative scheme (11), it is necessary but not sufficient, as underlined in [19], that the magnitude of each element of the main diagonal of the matrix $\underline{\underline{C}}$ be non-zero and greater than the magnitudes of those belonging to the same column or row. It is easy to recognize that these conditions are certainly fulfilled, in the assumed hypothesis of existence of a bijection associating the uniform sampling points to the closest non-uniform ones. In explicit form, relation (11) becomes:

$$\tilde{V}^{(\mu)}(\xi_n, \eta_m) = \frac{1}{G(\sigma_m, \eta_m, \bar{\eta}, N, N")\, G(\delta_n, \xi_n, \bar{\xi}, N, N")} \cdot \Big\{ \tilde{V}(\delta_n, \sigma_m) - \sum_{\substack{i=i_0-p+1 \\ (i \neq m)}}^{i_0+p} \; \sum_{\substack{s=s_0-q+1 \\ \wedge\, (s \neq n)}}^{s_0+q} G(\sigma_m, \eta_i, \bar{\eta}, N, N") \cdot G(\delta_n, \xi_s, \bar{\xi}, N, N")\, \tilde{V}^{(\mu-1)}(\xi_s, \eta_i) \Big\}, \tag{12}$$

wherein:

$$i_0 = \begin{cases} m & \text{if } \sigma_m \geq \eta_m \\ m-1 & \text{if } \sigma_m < \eta_m \end{cases}, \quad s_0 = \begin{cases} n & \text{if } \delta_n \geq \xi_n \\ n-1 & \text{if } \delta_n < \xi_n \end{cases}. \tag{13}$$

IV. EXPERIMENTAL TESTING

Some results of experimental proofs assessing the validity of the developed iterative algorithm to effectively compensate even severe positioning errors, affecting the collected NF samples in the NR PR NFFF transformation with PWMS based on the oblate spheroidal AUT model, are shown below. The proofs have been performed by means of the flexible NF measurement system available in the anechoic chamber of the Antenna Characterization Laboratory of the University of Salerno. This NF acquisition system is equipped with a vertical linear positioner and several rotators, so that, by properly arranging these elements, it is possible to collect the NF data as they would be acquired in a cylindrical, spherical, plane-polar, bi-polar, and PR scanning NF facility. Moreover, by means of continuous and synchronized movements of the probe and AUT positioning systems, it is also possible to perform the helicoidal, as well as the planar and spherical spiral scannings. The acquisition of the classical PR NF data and of the PWMS ones is carried out by using the same arrangement allowing the plane-polar scanning. In such an arrangement, the probe is mounted on the linear vertical positioner and the AUT on a rotator with its rotation axis normal to the vertical positioner. This rotator is anchored to an L-shaped bracket, placed on a horizontal slide, so that the scanning plane distance can be suitably modified. Another rotator is placed between the positioner and the probe, to allow that the probe axes remain parallel to the AUT ones, as requested in the PWMS and in the classical PR scanning. The walls of the anechoic chamber, $8\text{m} \times 5\text{m} \times 4\text{m}$ sized, are coated with pyramidal absorbers, guaranteeing a reflectivity smaller than – 40 dB in the X band. An Anritsu vector network analyzer is used to perform the magnitude and phase measurements of the voltage acquired by an open-ended WR90 rectangular waveguide, employed as probe. The AUT considered in the reported experimental results is a dual pyramidal horn antenna with vertical polarization, operating at 10 GHz. The horn apertures ($8.9\text{cm} \times 6.8\text{cm}$ sized) are located on the plane z = 0 and the distance between their centers is 26.5 cm. An oblate spheroid with a = 18.3 cm and b = 6.3 cm has been chosen to model this antenna. To assess the efficacy of the proposed iterative approach, the PWMS samples have been gathered in a circle of radius 110 cm, lying on the plane at z = 16.5 cm, in such a way that their positions are intentionally affected by severe errors. In particular, the shifts in the x' and y' directions between the positions of the error affected PWMS samples and those of the corresponding error free ones are random variables with uniform distribution in $(-\Delta\eta/3, \Delta\eta/3)$.

In Figs. 2 and 3, the magnitude and phase of the voltage V_x, directly measured along the scanning plane line y' = 0 cm, are compared with those recovered via the iterative scheme from the PWMS positioning errors affected samples, while Figs. 4 and 5 show the analogous comparisons relevant to the line $y' = 3.6$ cm. For completeness, in Fig. 6, it is shown the reconstruction of the magnitudes of V_x and V_y along the line $y' = 10.0$ cm. As can be clearly observed, despite the considerable and pessimistic enforced positioning errors, very good reconstructions result, save for small discrepancies arising in the zones characterized by a very small voltage level, wherein the recovered voltages patterns appear smoother and more regular. This behaviour is imputable to the peculiar feature of the OSI function of cutting away the noise sources harmonics which exceed the spatial bandwidth of the AUT. It must be stressed that these reconstructions have been obtained by using 10 iterations, enough to ensure the convergence of the algorithm with very low errors [24].

The efficacy of the presented approach for the correction of the positioning errors is confirmed by the comparison (see Figs. 7 and 8) of the E-plane and H-plane FF patterns recovered from the 2601 positioning errors affected PWMS data through the iterative scheme with those, assumed as references, got from the 11025 PR NF data directly collected at 0.45λ spacing on the square of side 140cm, inscribed in the scanning circle. For sake of comparison, the related FF patterns got without using the iterative scheme are shown in Figs. 9 and 10. As can be observed, these last recoveries appear severely deteriorated, thus further confirming the efficacy of the proposed procedure. This efficacy is even more evident by comparing the very low errors in the reconstructed magnitudes shown in Figs. 7 and 8 with the significantly larger ones clearly visible in Figs. 9 and 10.

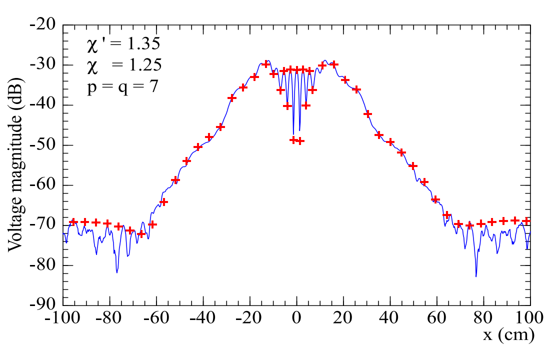

Fig. 2. Magnitude of V_x along the line $y' = 0$ cm. Solid line: measured. Crosses: recovered from the positioning errors affected PWMS measurements.

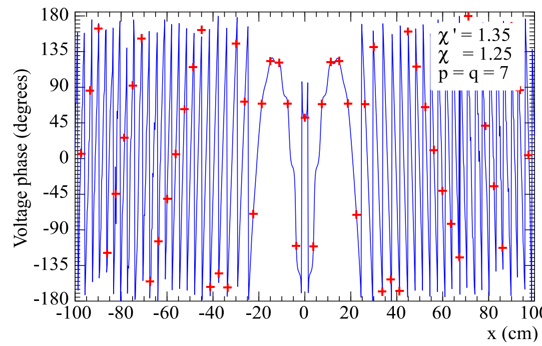

Fig. 3. Phase of V_x along the line $y' = 0$ cm. Solid line: measured. Crosses: recovered from the positioning errors affected PWMS measurements.

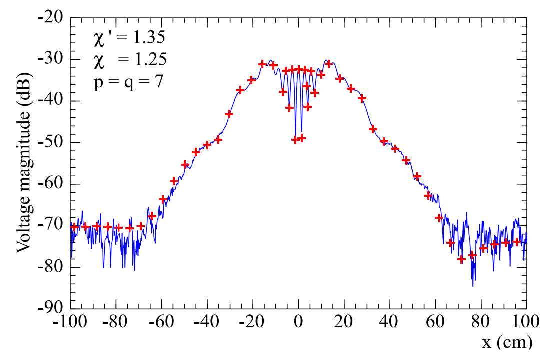

Fig. 4. Magnitude of V_x along the line $y' = 3.6$ cm. Solid line: measured. Crosses: recovered from the positioning errors affected PWMS measurements.

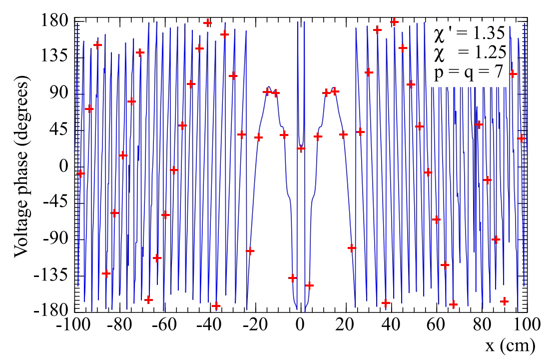

Fig. 5. Phase of V_x along the line $y' = 3.6$ cm. Solid line: measured. Crosses: recovered from the positioning errors affected PWMS measurements.

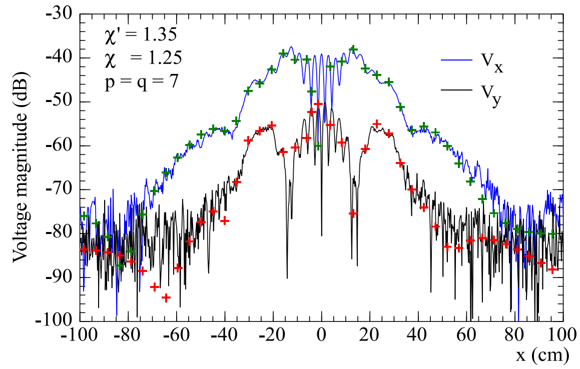

Fig. 6. Magnitudes of V_x and V_y along the line $y' = 10.0$ cm. Solid lines: measured. Crosses: recovered from the positioning errors affected PWMS measurements.

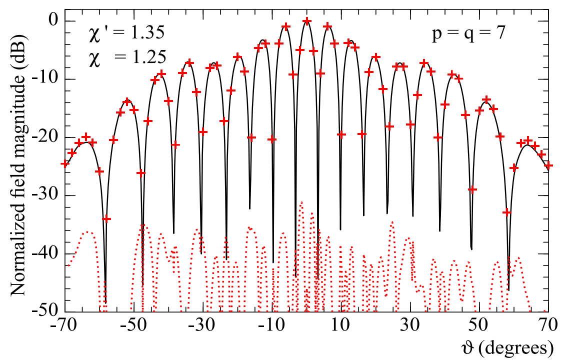

Fig. 7. E-plane pattern. Solid line: reference. Crosses: attained from the positioning errors affected PWMS samples applying the iterative scheme. Dashes: reconstruction error.

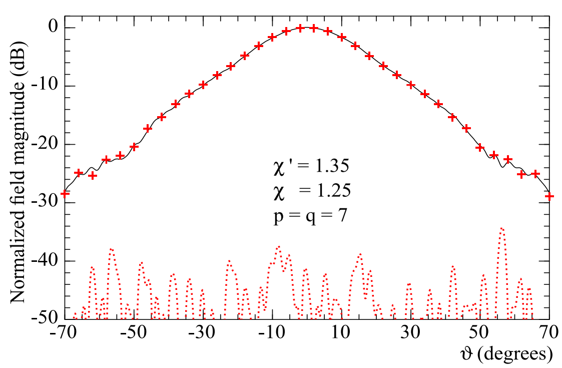

Fig. 8. H-plane pattern. Solid line: reference. Crosses: attained from the positioning errors affected PWMS samples applying the iterative scheme. Dashes: reconstruction error.

The interested reader can find in [33] a further set of laboratory results, which assess the efficacy of the proposed procedure for correcting known positioning errors and relevant to a different AUT.

V. CONCLUSION

An effective algorithm that allows the accurate compensation of known positioning errors affecting the acquired NF samples in the NFFF transformation with PWMS, using the oblate spheroidal modeling of the AUT, has been presented and experimentally assessed. Such an algorithm relies on an iterative procedure, which allows the accurate retrieving of the voltage samples at the points prescribed by the NR sampling representation from the gathered, positioning errors affected, PWMS samples. Once the positioning errors free PWMS samples have been determined, the massive NF data necessary for the classical PR NFFF transformation are accurately evaluated via an efficient OSI formula. The very accurate NF and FF reconstructions attained when such an algorithm is applied even to correct large positioning errors, as compared to the remarkably worsened FF reconstructions achieved when it is not used, assess its efficacy.

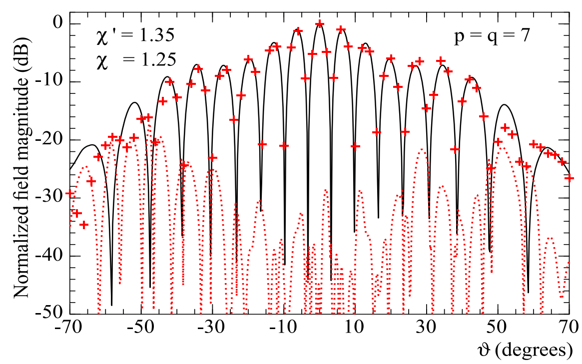

Fig. 9. E-plane pattern. Solid line: reference. Crosses: attained from the positioning errors affected PWMS samples without applying the iterative scheme. Dashes: reconstruction error.

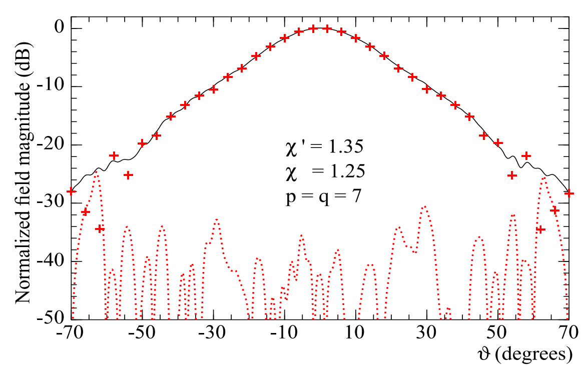

Fig. 10. H-plane pattern. Solid line: reference. Crosses: attained from the positioning errors affected PWMS samples without applying the iterative scheme. Dashes: reconstruction error.

REFERENCES

[1] A. D. Yaghjian, "An overview of near-field antenna measurements," *IEEE Trans. Antennas Prop*., vol. AP-34, pp. 30-45, Jan. 1986.

[2] E. S. Gillespie, "Special issue on near-field scanning techniques," *IEEE Trans. Antennas Prop.*, vol. 36, pp. 727-901, June 1988.

[3] M. H. Francis and R. W. Wittmann, *Near-Field Scanning Measurements: Theory and Practice*, in *Modern Antenna Handbook*, C.A. Balanis, Ed., John Wiley & Sons, Hoboken, NJ, USA, pp. 929-976, 2008.

[4] M. H. Francis, *IEEE Recommended Practice for Near-Field Antenna Measurements*, IEEE Standard 1720-2012, 2012.

[5] F. Ferrara, C. Gennarelli, and R. Guerriero, *Near-Field Antenna Measurement Techniques*, in *Handbook of Antenna Technologies*, Z. N. Chen, D. Liu, H. Nakano, X. Qing, and T. Zwick, Eds., Springer, Singapore, pp. 2107-2163, 2016.

[6] D. T. Paris, W. M. Leach Jr., and E. B. Joy, "Basic theory of probe-compensated near-field measurements," *IEEE Trans. Antennas Prop*., vol. AP-26, pp. 373-379, May 1978.

[7] E. B. Joy, W. M. Leach, Jr., G. P. RodrigueZ, and D. T. Paris, "Application of probe-compensated near-field measurements," *IEEE Trans. Antennas Prop*., vol. AP-26, pp. 379-389, May 1978.

[8] A. Lalbakhsh, M. U. Afzal, and K. P. Esselle, "Multi-objective particle swarm optimization to design a time-delay equalizer metasurface for an electromagnetic band-gap resonator antenna," *IEEE Antennas Wireless Prop. Lett.*, vol. 16, pp. 912-915, 2017.

[9] A. Lalbakhsh, M. U. Afzal, K. P. Esselle, and S. L. Smith, "Wideband near-field correction of a Fabry-Perot resonator antenna," *IEEE Trans. Antennas Prop.*, vol. 67, pp. 1975-1980, Mar. 2019.

[10] F. Ferrara, C. Gennarelli, R. Guerriero, G. Riccio, and C. Savarese, "An efficient near-field to far-field transformation using the planar wide-mesh scanning," *Jour. Electromagn. Waves Appl.*, vol. 21, no. 3, pp. 341-357, 2007.

[11] F. D'Agostino, I. De Colibus, F. Ferrara, C. Gennarelli, R. Guerriero, and M. Migliozzi, "Far-field pattern reconstruction from near-field data collected via a nonconventional plane-rectangular scanning: Experimental testing," *Int. Jour. Antennas Prop.*, vol. 2014, ID 763687, 9 pages, 2014.

[12] O. M. Bucci, C. Gennarelli, and C. Savarese, "Representation of electromagnetic fields over arbitrary surfaces by a finite and nonredundant number of samples," *IEEE Trans. Antennas Prop*., vol. 46, pp. 351-359, Mar. 1998.

[13] O. M. Bucci and C. Gennarelli, "Application of nonredundant sampling representations of electromagnetic fields to NF-FF transformation techniques," *Int. Jour. Antennas Prop*., vol. 2012, ID 319856, 14 pages, 2012.

[14] A. Dutt and V. Rohklin, "Fast Fourier transforms for nonequispaced data," *Proc. SIAM Jour. Scie. Comput.*, vol. 14, pp. 1369-1393, Nov. 1993.

[15] R. C. Wittmann, B .K. Alpert, and M. H. Francis, "Near-field antenna measurements using nonideal measurement locations," *IEEE Trans. Antennas Prop*., vol. 46, pp. 716-722, May 1998.

[16] R. C. Wittmann, B. K. Alpert, and M. H. Francis, "Near-field, spherical-scanning antenna measurements with nonideal probe locations," *IEEE Trans. Antennas Prop*., vol. 52, pp. 2184-2186, Aug. 2004.

[17] J. L. Yen, "On nonuniform sampling of bandwidth-limited signals," *IRE Trans. Circuit Theory*, vol. CT-3, pp. 251-257, Dec. 1956.

[18] V. Dehghanian, M. Okhovvat, and M. Hakkak, "A new interpolation technique for the reconstruction of uniformly spaced samples from non-uniformly spaced ones in plane-rectangular near-field antenna measurements," *Prog. Electromagn. Res.*, vol. 72, pp. 47-59, 2007.

[19] O. M. Bucci, C. Gennarelli, and C. Savarese, "Interpolation of electromagnetic radiated fields over a plane from nonuniform samples," *IEEE Trans. Antennas Prop*., vol. 41, pp. 1501-1508, Nov. 1993.

[20] O. M. Bucci, C. Gennarelli, G. Riccio, and C. Savarese, "Electromagnetic fields interpolation from nonuniform samples over spherical and cylindrical surfaces," *IEEE Proc. Microw. Antennas Prop.*, vol. 141, pp. 77-84, Apr. 1994.

[21] F. Ferrara, C. Gennarelli, G. Riccio, and C. Savarese, "NF-FF transformation with cylindrical scanning from nonuniformly distributed data," *Microw. Optic. Technol. Lett.*, vol. 39, pp. 4-8, Oct. 2003.

[22] F. Ferrara, C. Gennarelli, G. Riccio, and C. Savarese, "Far field reconstruction from nonuniform plane-polar data: A SVD based approach," *Electromagnetics*, vol. 23, pp. 417-429, 2003.

[23] F. D'Agostino, F. Ferrara, C. Gennarelli, R. Guerriero, and M. Migliozzi, "On the correction of the probe positioning errors in a non-redundant bi-polar near to far-field transformation," *Appl. Comp. Electromagn. Soc. Jour.*, vol. 33, pp. 1374-1382, Dec. 2018.

[24] F. D'Agostino, F. Ferrara, C. Gennarelli, R. Guerriero, and M. Migliozzi, "Probe position errors corrected near-field-far-field transformation with spherical scanning," *Appl. Comp. Electromagn. Soc. Jour.*, vol. 31, pp. 106-117, Feb. 2016.

[25] F. D'Agostino, F. Ferrara, C. Gennarelli, R. Guerriero, and M. Migliozzi, "On the compensation of probe positioning errors when using a nonredundant cylindrical NF-FF transformation," *Prog. Electromagn. Res. B*, vol. 20, pp. 321-335, 2010.

[26] F. D'Agostino, F. Ferrara, C. Gennarelli, R.

Guerriero, and M. Migliozzi, "Two efficient procedures to correct the positioning errors in the plane-polar scanning," *IET Microw., Antennas Prop.*, vol. 10, no. 13, pp. 1453-1458, 2016.

[27] F. D'Agostino, F. Ferrara, C. Gennarelli, R. Guerriero, and M. Migliozzi, "Near to far-field plane-polar transformation from probe positioning error affected data," *Appl. Comp. Electromagn. Soc. Jour.*, vol. 33, pp. 419-429, Apr. 2018.

[28] F. D'Agostino, F. Ferrara, C. Gennarelli, R. Guerriero, M. Migliozzi, and G. Riccio, "A singular value decomposition based approach for far-field reconstruction from irregularly spaced planar wide-mesh scanning data," *Microw. Optic. Technol. Lett.*, vol. 49, pp. 1768-1772, July 2007.

[29] F. D'Agostino, F. Ferrara, C. Gennarelli, R. Guerriero, and M. Migliozzi, "An SVD-based approach to reconstruct the planar wide-mesh scanning NF data from inaccurately probe-positioned samples," *IEEE Antennas Wireless Prop. Lett.*, vol. 17, pp. 641-644, Apr. 2018.

[30] O. M. Bucci, G. D'Elia, and M. D. Migliore, "Advanced field interpolation from plane-polar samples: Experimental verification," *IEEE Trans. Antennas Prop.*, vol. 46, pp. 204-210, Feb. 1998.

[31] A. D. Yaghjian, "Approximate formulas for the far field and gain of open-ended rectangular waveguide," *IEEE Trans. Antennas Prop.*, vol. AP-32, pp. 378-384, Apr. 1984.

[32] F. D'Agostino, F. Ferrara, C. Gennarelli, R. Guerriero, S. McBride, and M. Migliozzi, "Fast and accurate antenna pattern evaluation from near-field data acquired via planar spiral scanning," *IEEE Trans. Antennas Prop.*, vol. 64, pp. 3450-3458, Aug. 2016.

[33] F. D'Agostino, F. Ferrara, C. Gennarelli, R. Guerriero, and M. Migliozzi, "An iterative technique to retrieve the planar wide-mesh scanning near-field data from those affected by an inaccurate positioning," *Proc. of MIKON*, Poznan, Poland, pp. 145-148, May 2018.

Francesco D'Agostino received the Laurea degree in Electronic Engineering from the University of Salerno in 1994, where in 2001 he received the Ph.D. degree in Information Engineering. From 2002 to 2005, he was Assistant Professor at the Engineering Faculty of the University of Salerno where, in October 2005, he was appointed Associate Professor of Electromagnetics and joined the Department of Industrial Engineering, where he is currently working. His research activity includes application of sampling techniques to electromagnetics and to innovative NF-FF transformations, diffraction problems, radar cross section evaluations, Electromagnetic Compatibility. In this area, he has co-authored 4 books and over 230 scientific papers, published in peer-reviewed international journals and conference proceedings. He is a regular reviewer for several journals and conferences and has chaired some international events and conferences. D'Agostino is a member of AMTA, EurAAP, and IEEE.

Flaminio Ferrara was born near Salerno, Italy, in 1972. He received the Laurea degree in Electronic Engineering from the University of Salerno in 1999. Since the same year, he has been with the Research Group in Applied Electromagnetics at the University of Salerno. He received the Ph.D. degree in Information Engineering at the same University, where he is presently an Assistant Professor of Electromagnetic Fields. His interests include: application of sampling techniques to the efficient reconstruction of electromagnetic fields and to NF-FF transformation techniques; monostatic radar cross section evaluations of corner reflectors. Ferrara is co-author of more than 230 scientific papers, mainly in international journals and conference proceedings. In particular, he is co-author of 4 books on NF-FF transformation techniques and co-author of the chapter "Near-field antenna measurement techniques" of the Handbook of Antenna Technologies. He is reviewer for several international journals and member of the Editorial board of the International Journal of Antennas and Propagation. He is member of the IEEE society.

Claudio Gennarelli was born in Avellino, Italy, in 1953. He reeived the Laurea degree (*summa cum laude*) in Electronic Engineering from the University of Naples, Italy, in 1978. From 1978 to 1983, he worked with the Research Group in Electromagnetics at the Electronic Engineering Department of the University "Federico II" of Naples. In 1983, he became Assistant Professor at the Istituto Universitario Navale (IUN), Naples. In 1987, he was appointed Associate Professor of Antennas, formerly at the Engineering Faculty of Ancona University and subsequently at the Engineering Faculty of Salerno University. In 1999, he has been appointed Full Professor at the same University. The main topics of his scientific activity are: reflector antennas analysis, antenna measurements, diffraction problems, radar cross section evaluations, scattering from surface impedances, application of sampling techniques to electromagnetics and to NF–FF transformations. Gennarelli is co-author of more than 400 scientific papers, mainly in international journals and conference proceedings. In particular, he is co-author of 4 books on NF-FF transformation techniques and co-author of the chapter "Near-field antenna measurement techniques" of the Handbook of Antenna Technologies. He is a Senior Member of the IEEE since 2002 and member of the Editorial board of the Open Electrical and Electronic Engineering Journal and of the International Journal of Antennas and Propagation.

Rocco Guerriero received the Laurea degree in Electronic Engineering and the Ph.D. degree in Information Engineering from the University of Salerno in 2003 and 2007, respectively. Since 2003, he has been with the Research Group in Applied Electromagnetics of University of Salerno, where he is currently an Assistant Professor of Electromagnetic Fields. His interests include: application of sampling techniques to the efficient reconstruction of electromagnetic fields and to near-field-far-field transformation techniques; antenna measurements; inversion of ill-posed electromagnetic problems; analysis of microstrip reflectarrays; diffraction problems. Guerriero is co-author of about 190 scientific papers, mainly in international journals and conference proceedings. In particular, he has co-authored 3 books on NF-FF transformation techniques and is co-author of the chapter "Near-field antenna measurement techniques" of the Handbook of Antenna Technologies. He is reviewer for several international journals and member of the Editorial board of the International Journal of Antennas and Propagation. Since 2015, he is member of IEEE.

Massimo Migliozzi received the Laurea degree in Electronic Engineering from the University of Salerno in 1999. He received the Ph.D. degree in Information Engineering at the same University, where at the present time he is a Research fellow in Electromagnetic Fields. His scientific interests include: application of sampling techniques to the efficient reconstruction of electromagnetic fields and to NF-FF transformation techniques; antenna measurements; electromagnetic compatibility; antenna design; diffraction problems. Migliozzi is co-author of about 150 scientific papers, mainly in international journals and conference proceedings and reviewer for several international journals.