An Efficient Numerical Model for the Radiation Analysis of Microstrip Patch Antennas

Lu Liu and Zaiping Nie

School of Electrical Science and Engineering
University of Electronic Science and Technology of China, Chengdu, Sichuan, 610054, China
liulu1412@163.com, zpnie@uestc.edu.cn

Abstract **─** An accurate integral equation method based on quasi-static relationship (QSR), thin dielectric sheet (TDS) method and high-order hierarchical Legendre (HOHL) basis function is proposed in this paper for fast analysis of microstrip antennas radiation problems. This technique employs the QSR of the current on the parallel plate capacitor to describe the field continuity boundary condition and to embody the tight coupling between the radiation patch, the ground plane and the substrate of microstrip antenna. The frequency offset problem of conventional VSIE can be effectively eliminated. Moreover, combined with the TDS and the HOHL basis function, the proposed model can provide high accuracy in input impedance and far-field performance with faster convergence speed and lower computational cost. Numerical results are presented to show the accuracy and efficiency of the proposed method.

Index Terms **─** Microstrip antenna and array, numerical modelling, radiation analysis, quasi-static relationship, thin dielectric sheet method.

I. INTRODUCTION

Integral equation methods (IEM) and their numerical solutions have been widely used for simulating electromagnetic scattering and radiation problems [1-3]. However, when IEMs are used for modelling the radiation performance of metal-dielectric composite structures (microstrip patch antennas, for example), the number of the unknowns to be solved, iteration convergence and the accuracy of the solution are still challenging due to complex excitations, multi-scale structures and strong mutual coupling. Uniform plane wave is commonly employed as the excitation condition for the scattering target. Conversely, a forced voltage or current is typically used as the excitation at the feed port of the radiation problem, resulting in a strong and complex field distribution near the feed port. Due to the complex field distribution and strong mutual coupling, the modelling of radiation problems is more difficult than scattering problems. Therefore, constraint conditions describing the physical mechanisms of radiation structures may be very helpful for efficient modelling of radiation problems.

The volume-surface integral equation (VSIE) [1,4] is particularly convenient for modelling microstrip antennas because it is more accurate for modelling the thin dielectric, corners and edges than the commonly used surface integral equation (SIE) [5]. However, the traditional VSIE numerical solution has two major drawbacks that prevent its application to the radiation analysis of microstrip antennas. First, the convergence is very slow, and a significant shifting of the resonant frequency can be observed in the numerical solution. In addition, most existing VSIE models apply low-order and sub-domain basis functions to expand the unknown current [6-9], and geometrically model the detailed structure of the antenna by means of many electrically small elements to match the structure precisely, all leading to a large number of unknowns and huge computational costs, especially for antenna arrays.

Makarov et al. [6] first proposed that the failure description of the boundary condition on the interface between metal and dielectric is the main reason for the resonant frequency shifting problem of the VSIE model. They enforced an explicit boundary condition for the volume bases in contact with the metal surface to address this issue successfully. According to this idea, Zhang et al. [8] proposed a new hybrid basis function to explicitly enforce the boundary condition at the interface. A coupled surface-volume integral equation approach proposed by Lu and Chew [3] used the current continuity equation $\hat{n}\cdot\vec{D}=\nabla\cdot\vec{J}_s/i\omega$ as a boundary condition to formulate the relationship between the current $\vec{J}_s$ on the metal side of the interface and the electric flux density $\vec{D}$ on the dielectric side. This formulation is valid for composite objects with three-dimensional metal body. However, it may not be accurate for the metal sheet with “zero-thickness”, such as the antenna patch and its ground plane.

To reduce the number of unknowns and memory requirements, Cai [10] used curved tetrahedral/triangular elements for geometric modelling and the higher order

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hierarchical vector basis functions for the volume/surface current modelling. The entire domain basis function with high-precision edge condition has been used in [11] to represent the surface current distribution on the patch to reduce computational requirements and achieve the fast analysis of microstrip antennas. However, it is not convenient for the analysis of strongly coupled environments, such as arrays. Thin dielectric sheet (TDS) approximation is also an effective and reliable method for reducing unknowns when VSIE is used to deal with thin dielectric problems [12-14]. The coupled PEC-TDS approach proposed in [13] has been used to analyze the radiation of the patch antenna, and high efficiency has been observed. However, the total number of unknowns is still large because this approach is based on low-order basis functions, and the unknowns for the volume are required.

In order to eliminate the frequency offset problem, improve the convergence performance, and reduce the unknowns simultaneously, an efficient numerical technique based on the quasi-static relationship (QSR) of the current on the parallel plate capacitor in conjunction with TDS approach is proposed for the fast analysis of radiation problems of microstrip antennas in this work. QSR is employed to embody the tight coupling properties and describe the current continuity condition between two sides of the metal-dielectric interface, thereby improving the accuracy of the numerical solution. Besides, by combining the TDS approach with the QSR formulation, the need to solve for $\vec{D}$ in the substrate is removed. Therefore, the unknowns can be greatly reduced and only exist on the metal surface. Moreover, the unknowns can be further reduced by using higher order hierarchical legendre (HOHL) basis function to expand the current. Furthermore, since the direction of HOHL basis function is consistent with the edge of the patch, the edge singular current on the patch can be well described, and the convergence property can be significantly improved compared to the commonly used RWG basis functions. In general, when dealing with the radiation problems of microstrip patch antennas, arrays and electrically large antennas, the new model proposed in this work provides less error in resonant frequency with fewer unknowns and faster convergence speed compared with the traditional VSIE model.

II. FORMULATION

The following VSIE model is commonly used to describe the electromagnetic properties of microstrip patch antenna with a thin finite dielectric substrate as shown in Fig. 1:

$$\begin{aligned} &\vec{E}^i(\vec{r}) + \vec{E}^s(\vec{r})\,|_{\tan} = 0 \qquad (\vec{r} \in S) \\ &\vec{E}^i(\vec{r}) + \vec{E}^s(\vec{r}) = \vec{E}(\vec{r}) \qquad (\vec{r} \in V), \end{aligned} \tag{1}$$

where V is the dielectric volume, S denotes the metal surface, $\vec{E}^i$ represents the incident electric field due to the applied source and $\vec{E}^s$ is the total scattered field.

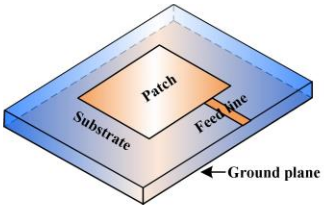


Fig. 1. Microstrip patch antenna model.

The scattered field produced by the surface current on the conducting patch $\vec{J}_s$ and the equivalent volume current in V are given by:

$$\vec{E}^s_{pec}(\vec{r}) = ik_0\eta_0 \int_S [\vec{J}_s(\vec{r}') + \frac{1}{k_0^2}\nabla\nabla' \cdot \vec{J}_s(\vec{r}')]G(\vec{r},\vec{r}')d_{s'}\,, \tag{2}$$

and

$$\vec{E}^s_{die}(\vec{r}) = ik_0\eta_0 \int_V [i\omega\chi(\vec{r}')\vec{D}(\vec{r}') + \frac{i\omega}{k_0^2}\nabla\nabla' \cdot \chi(\vec{r}')\vec{D}(\vec{r}')]G(\vec{r},\vec{r}')d_{v'}\,, \tag{3}$$

respectively. Where $\vec{D}$ is the electric flux density in V and η_0 is the wave impedance in free space. The contrast ratio χ is defined as $\chi(\vec{r}) = [\varepsilon(\vec{r}) - \varepsilon_0]/\varepsilon(\vec{r})$. The harmonic time convention e^{-iwt} is adopted in this work.

In the above VSIE model, the metals and dielectrics are considered to be independent of each other. Field continuity boundary condition on the metal-dielectric interface may not be satisfied if the commonly used SWG basis functions are employed to expand the electric flux density [6]. It may cause problems of resonant frequency offset and poor convergence when it is used in radiation analysis.

Based on the cavity model proposed by Lo [15], the electric field within each volume element can be assumed to have only one significant vertical component for a microstrip patch antenna with a thin dielectric substrate. In this work, the vertical component of the electric flux density in substrate is described by the surface currents at the top patch and the ground plane by using the parallel plate capacitor QSR shown in Fig. 2. The QSR formula [16] can be written as:

$$\vec{D}_n = \begin{cases} \hat{z}\dfrac{1}{2i\omega}(\nabla \cdot \vec{J}_s^- - \nabla \cdot \vec{J}_s^+) \text{ between ground plane and patch} \\ \hat{z}\dfrac{1}{2i\omega}(\nabla \cdot \vec{J}_s^- - 0) \text{ between ground plane and vacuum} \end{cases}, \tag{4}$$

where $\vec{J}_s^+$ and $\vec{J}_s^-$ are the surface current density on the metal radiation patch and the metal ground plane respectively. In more detail, the electric flux density $\vec{D}$ in the dielectric is modelled by the currents on the radiation patch and the ground plane. Therefore, the field continuity and the tight coupling between the radiation patch, the ground plane and the substrate can be described.

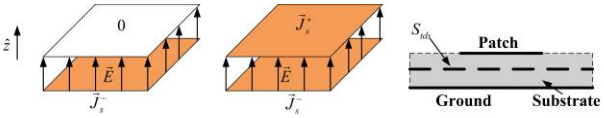


Fig. 2. Quasi-static electric field distribution in dielectric substrate.

When the thickness of the substrate τ is very small compared to the wavelength, the volume integral in (3) can be approximated by surface integral $d_{V'} \approx \tau d_{S'}$ to reduce the requirement of the computational resources, according to the idea of TDS approach [17]. The scattering field (3) introduced by the volume equivalent current can be calculated by surface integral and can be written as:

$$\begin{aligned}\vec{E}^s_{die}(\vec{r}) = ik_0\eta_0 \Bigg\{ \int_{S_{tds}} i\omega\chi\tau(\vec{r}')\vec{D}_n(\vec{r}')G(\vec{r},\vec{r}')d_{s'} \\ -\frac{i\omega}{k_0^2}\int_S \chi(\vec{r}')\vec{D}_n(\vec{r}')\nabla[G_\tau(\vec{r},\vec{r}') - G(\vec{r},\vec{r}')]d_{s'}\Bigg\},\end{aligned} \tag{5}$$

where S_{tds} is the middle surface between the top patch and the ground plane as shown in Fig. 2, and $G_\tau(\vec{r},\vec{r}') = G(\vec{r},\vec{r}'+\tau\hat{n})$. Combined with (4), equation (5) can be further expressed as:

$$\begin{aligned}\vec{E}^s_{die}(\vec{r}) = ik_0\eta_0 \Bigg\{ \int_{S_{tds}} \frac{\tau\chi(\vec{r}')}{2}\cdot\hat{n}\cdot[\nabla\cdot\vec{J}_s^- - \nabla\cdot\vec{J}_s^+]G(\vec{r},\vec{r}')d_{s'} \\ -\frac{1}{k_0^2}\int_S \frac{\chi(\vec{r}')}{2}[\nabla\cdot\vec{J}_s^- - \nabla\cdot\vec{J}_s^+]\cdot\nabla[G_\tau(\vec{r},\vec{r}') - G(\vec{r},\vec{r}')]d_{s'}\Bigg\}.\end{aligned} \tag{6}$$

The above equation indicates that the field continuity condition and tight coupling can be rigorously embedded by expressing $\vec{D}$ in the substrate using the current on the radiation patch and ground plane. Moreover, as the unknowns only exist on the metal surface and no volume unknown is required in this model, the number of total unknowns can be significantly reduced. Since QSR and TDS are used in this model, it is suitable for planar microstrip antennas with thin dielectric substrates, and may lose accuracy when processing conformal microstrip antennas with high curvature.

RWG basis function is typically used in VSIE method to expand the current on the metal surface. RWG is a zeroth-order basis function, and the direction of the basis function is perpendicular to the common side of the adjacent elements. When the microstrip antenna operates at the resonant frequency, there are two singular edge currents parallel to the two edges of the radiating patch. Therefore, the mesh size of the RWG basis function needs to be small enough to accurately describe the edge current on the radiating patch, which results in a large number of unknowns. In this work, HOHL basis function [18] is used to describe the surface current because the direction of the HOHL basis function is consistent with the edge of the radiating patch, which can well describe the singular edge currents on the radiating patch and improve the convergence characteristics of the system. In addition, the mesh size of the HOHL basis function is always much larger than the RWG basis function, therefore the unknowns and memory costs can be further reduced. The HOHL basis function is defined as:

$$\vec{J}_s = \vec{J}_s^u\vec{a}_u + \vec{J}_s^v\vec{a}_v, \tag{7}$$

where $\vec{a}_u$ and $\vec{a}_v$ are the co-variant unitary vectors. The definition of u-directed current $\vec{J}_s^u$ or v-directed current $\vec{J}_s^v$ can be found in [18].

III. NUMERICAL RESULTS

A. Microstrip patch antenna radiation analysis

Consider a microstrip patch antenna excited by a transmission line as shown in Fig. 3. A rectangular patch is placed over a dielectric substrate with $\varepsilon_r = 2.2$. The bottom plane of the substrate is covered by a finite ground plane. Finite-width feeding model [19] is employed in this section to improve convergence and accuracy. The input impedance performances obtained by the proposed approach and the VSIE model described in equation (1) (discretized by RWG and SWG) are shown in Fig. 4 (a). The Ansoft HFSS finite element (FEM) simulation results are also given as a reference. A significant positive frequency offset (about 3%) of the traditional VSIE solution can be observed, which is consistent with the result described in [6]. However, both the real and imaginary part of the input impedance calculated by the proposed approach over 7.0GHz to 9.0GHz are in good agreement with the FEM solutions. Only a frequency shift of 0.087% is observed, which indicates that the frequency offset can be effectively eliminated by the proposed approach. The comparison of the radiation gain pattern at the phi=0º plane computed by the proposed model and the FEM is shown in Fig. 4 (b). Although a difference of 2dB exists in the back lobe of the pattern, excellent agreement can be observed.

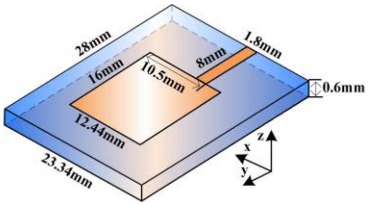
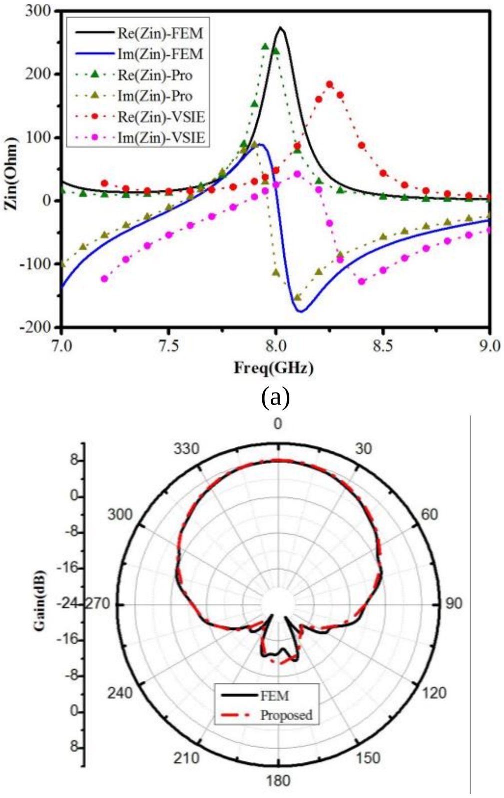


Fig. 3. The geometry of a microstrip patch antenna working at X band.

The computational details of both the proposed approach and traditional VSIE model under the same computing platform are listed in Table 1. For the VSIE model, the patch antenna requires 499 triangles for conductive plates and 1391 tetrahedrons for dielectric substrate modelling, resulting in 3990 unknowns in total. However, the proposed model consists of 34 quadrilateral elements and results in 238 unknowns in total when 2 order HOHL basis functions are used. The memory of the impedance matrix is reduced from 12.1MB to 0.43MB and the total CUP time is reduced from 1185s to 27s. In addition, the convergence properties of the proposed model and VSIE model are represented by the red and black line, respectively, in Fig. 5. The proposed approach converges much faster than the traditional VSIE. Both equations are solved by the GMRES iterative method and no preconditioned method is employed. As a consequence, the proposed approach provides higher accuracy and requires fewer computational resources than traditional VSIE model, when analyzing the radiation problem of the microstrip antenna.



(a)

(b)

Fig. 4. Input impedance and far-field radiation performance. (a) Comparison of the input impedance calculated by FEM, VSIE and the proposed model. (b) Comparison of the radiation patterns obtained by FEM and proposed model at 7.75GHz, phi=0°.

Table 1: Comparison of CPU time and memory requirement of the proposed model and the traditional model

	Number of Unknowns	Memory of Impedance Matrix (MB)	Total CPU Time (s)
Traditional VSIE	3990	121.46	1184
Proposed model	238	0.43	27

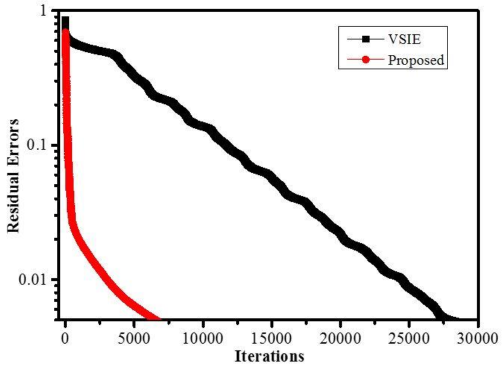
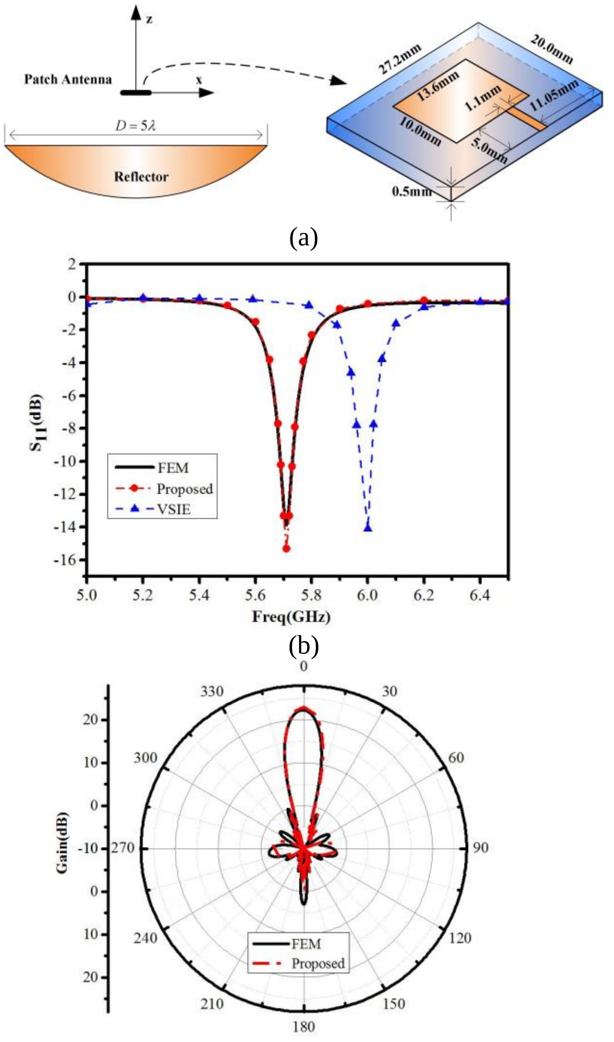


Fig. 5. The convergence properties of the proposed model and traditional VSIE.

B. Reflector antenna with a patch antenna as feed

The proposed approach is also a good choice for the EM modelling of electrically large antennas, such as the reflector antenna with a patch antenna as the feed. The geometries and dimensions of the reflector and patch antenna are shown in Fig. 6 (a). The dielectric constant of the patch antenna substrate is 3.5. FEM is very efficient for analyzing electrically small antenna. However, it is not suitable for analyzing electrically large antennas due to the large number of unknowns generated by the bounding box. VSIE model with low-order basis functions is also not a good choice because of the frequency shift problem and the large number of unknowns. VSIE needs more than 20196 unknowns to describe this example and takes more than two hours of solving time when MLFMA and Block-Diagonal precondition are used for acceleration. However, only 2658 unknowns is required for the proposed method when 2 order HOHL basis function is selected, and the solving time can be reduced to 508s. The return loss (with respect to 50Ω) calculated by VSIE, the proposed method and FEM are compared in Fig. 6 (b). Similarly, a significant frequency shift of about 4.9% is observed for VSIE result, however, the S_{11} obtained from the proposed approach consists well with FEM solution. The comparison of radiation gain patterns is shown in Fig. 6 (c). Good agreement in the main lobe of the pattern is observed, although the side and back lobes are slightly different.



(a)

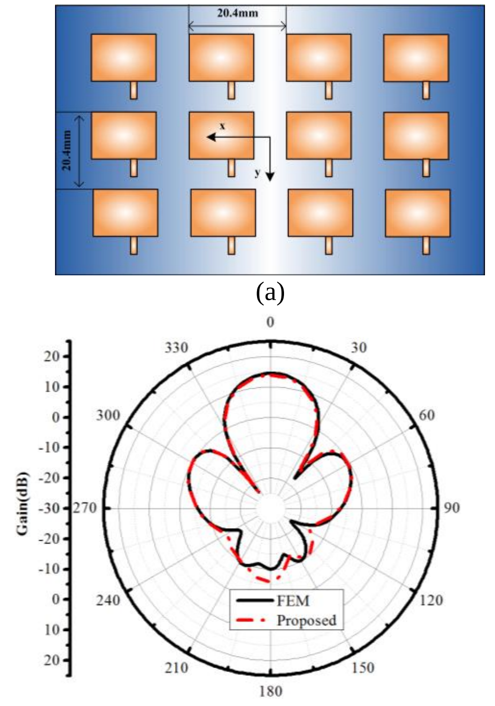
(b)

(c)

Fig. 6. Reflector antenna with a microstrip patch antenna as feed. (a) Geometry of the reflector antenna with ${}^{f}\!/_{D} = 0.38$ and the patch antenna at the focus plane. (b) Comparison of the return loss obtained by FEM, VSIE and the proposed mode, S_{11}. (c) Comparison of the radiation patterns obtained by FEM and the proposed model at 5.71GHz, phi=0°.

C. Microstrip patch antenna array

As the final example, a 3×4 two-dimensional patch array shown in Fig. 7 (a) is considered. The size of each antenna element and the dielectric constant of the substrate are the same as the patch antenna described in the second example. The distance between two adjacent elements is 20.4mm. The excitation voltages of each element is set as 1.0V. The proposed model contains a total of 3108 unknowns and takes 1851 seconds to solve this problem when MLFMA is employed. However, VSIE model requires 35678 unknowns and takes more than 10 hours of solving time due to its poor convergence. The radiation performance of this antenna array in phi=0° plane computed by the proposed model is compared with the result of FEM in Fig.7 (b). The main and side lobes are in good agreement, and a slight difference exists in the back lobe of the pattern. The results show that the proposed model has high accuracy and efficiency even when analyzing the strongly coupled antenna array radiation problems.



(a)

(b)

Fig. 7. 3×4 microstrip patch antenna array. (a) Geometry of the array. (b) Comparison of the radiation patterns obtained by FEM and the proposed model at 5.71GHz, phi=0°.

IV. CONCLUSION

Based on the physical properties of microstrip antennas, an effective numerical model is proposed for fast analyzing the radiation of planar microstrip patch antennas with thin dielectric substrates. In this model, the current on metal surface is described by the HOHL basis function. QSR formulation is applied to embody the physical properties of the microwave resonant structure. TDS method combined with QSR is employed to reduce computing resources and improve efficiency. Numerical examples have demonstrated that this model is able to remove the frequency shift problem, reduce the number of unknowns and speed up the iteration convergence significantly compared to traditional VSIE model. Since this model has fewer unknowns and can be easily combined with the existing fast algorithms, it is promising for simulation of electrically large antennas and antenna arrays with big number of elements.

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