Field-to-Wire Coupling Model for Wire Bundles with Strongly Non-uniform Path

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Abstract ─ This paper presents a field-to-wire coupling model for wire bundles with strongly non-uniform path. Previous studies on multi-conductor transmission lines (MTL) are mainly confined to uniform or weakly non-uniform path, which is sometimes not the case in practice. In this paper, the external and internal characteristics of the wire bundle are decomposed by a mode transformation method, of which the advantage is the transformation matrices do not vary with the non-uniform path. The external characteristics correspond to common-mode (CM) components, modeled as an equivalent single wire running in the same path with the bundle above the reference ground. The internal characteristics correspond to differential-mode (DM) components, modeled as a uniform MTL system composed of the original wires in the bundle. In this way, the effects of the non-uniform path and the external field only exist in the CM model. Mode conversion caused by the dielectric coating and terminals is modeled with equivalent circuits. The proposed model is validated with a bundle of curved wires above a PCB board.

Index Terms ─ Field-to-wire coupling, mode conversion, mode transformation, strongly non-uniform path, wire bundle.

I. INTRODUCTION

The wire bundle is a widely used medium for data/power transmission between different devices, PCBs or equipment in information and communication systems. Field-to-wire coupling is a main path for electromagnetic interferences entering the systems. Many methods have been developed to deal with the field coupling problem for wire bundles. The most fundamental one is the multi-conductor transmission line theory (MTLT) [1]. Other methods made improvements based on this theory, such as the reduction technique at high frequency [2], the modeling method for random bundle of twisted-wire pairs [3], the modeling method for cable bundle with lacing cords [4] and the analysis of undesired asymmetries and non-uniformities [5]. In these studies, the wiring path of the bundle is assumed to be uniform or weakly non-uniform. It is sometimes not the case in practice. Wires inside a bundle are often parallel to each other, but the whole bundle may not always be parallel to the reference ground. It means that the per-unit-length parameters in MTLT vary greatly along the wire bundle. Moreover, the height of the bundle from the reference ground may not be electrically small. In this case, only full-wave methods or other methods derived from Maxwell's theory, such as transmission-line super theory (TLST) [6], can deal with this problem, but they are complex and not so efficient. To this end, a modeling method for field-to-wire coupling of wire bundles with strongly non-uniform path is proposed in this paper.

The proposed method decomposes the wire bundle and its reference ground into two parts. One is an equivalent single wire running in the same path with the bundle above the reference ground; the other is a uniform MTL system composed of the original wires in the bundle. In this way, the non-uniform path only exists in the first part and can be dealt with by method of moments (MoM) [7]. The above model decomposition is achieved by mode transformation. Different from the modal analysis in MTLT, the mode transformation proposed here is independent with the height of the bundle from the reference ground. Thus, it is suitable for the analysis of wire bundles with non-uniform path. The two parts of the model are called common-mode (CM) and differential-mode (DM) components in mode domain respectively, since they are corresponding to the external and internal characteristics of the wire bundle. The coupling between CM and DM components will occur if terminals are unbalanced or dielectric coating exists [8]. This is characterized by controlled sources in this paper.

The rest of this paper is organized as follows. In

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Section II, the fundamental mode transformation method for a wire bundle with strongly non-uniform path is introduced. Models for the resulted CM and DM components are built. Mode conversion caused by the dielectric coating and terminals is characterized by equivalent circuits. The proposed modeling method is experimentally validated in Section III. Conclusions are drawn in Section IV.

II. MODELING METHOD

A wire bundle with strongly non-uniform path above the ground is shown in Fig. 1. To model this system, mode transformation is applied to decompose the system into CM and DM components. This is illustrated in the case of bare wires firstly. Then, the dielectric coating and the terminal are considered and modeled with equivalent circuits.

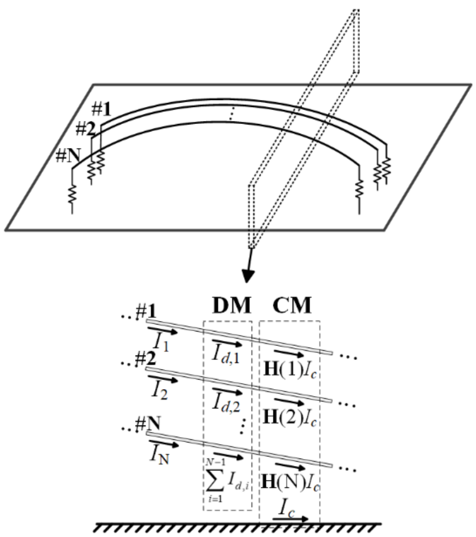

Fig. 1. A wire bundle with strongly non-uniform path above the ground.

A. Mode transformation

In this subsection, the wires are assumed to have no coating. An infinitely small area of the system under analysis is shown in Fig. 1. Currents of the wires are divided into CM current I_c and DM currents $I_{d,i}$ $(i=1,2,...,N-1)$ in the following way:

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_{N-1} \\ I_N \end{bmatrix} = \underbrace{\begin{bmatrix} & 1 & 0 & \cdots & 0 \\ & 0 & 1 & \cdots & 0 \\ \mathbf{H} & \vdots & \vdots & \ddots & \vdots \\ & 0 & 0 & \cdots & 1 \\ & -1 & -1 & \cdots & -1 \end{bmatrix}}_{\mathbf{T}_I} \begin{bmatrix} I_c \\ I_{d,1} \\ I_{d,2} \\ \vdots \\ I_{d,N-1} \end{bmatrix}, \quad (1)$$

where $\mathbf{H}$ is the current division vector. The ith $(i=1,2,...,N)$ element of $\mathbf{H}$, $\mathbf{H}(i)$, denotes the ratio of CM current in wire #i to total CM current. $\mathbf{H}$ is similar to the current division factor h (also known as imbalance factor [9]). Concerning to two-conductor transmission lines (TLs) with the reference ground, $\mathbf{H}=[h;1-h]$. Voltages of the wires are also divided into CM voltage V_c and DM voltages $V_{d,i}$ $(i=1,2,...,N-1)$. The corresponding relationship can be obtained by the equivalence of the transmitted power:

$$\begin{aligned} &\left[V_1,V_2,\ldots,V_N\right]\cdot\left[I_1^*,I_2^*,\ldots,I_N^*\right]^T \\ &=\left[V_c,V_{d,1},\ldots,V_{d,N-1}\right]\cdot\left[I_c^*,I_{d,1}^*,\ldots,I_{d,N-1}^*\right]^T \end{aligned}, \quad (2)$$

and is,

$$\begin{bmatrix} V_c \\ V_{d,1} \\ V_{d,2} \\ \vdots \\ V_{d,N-1} \end{bmatrix} = \underbrace{\begin{bmatrix} & & \mathbf{H}^T & & \\ 1 & 0 & \cdots & 0 & -1 \\ 0 & 1 & \cdots & 0 & -1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -1 \end{bmatrix}}_{\mathbf{T}_V^{-1}} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_{N-1} \\ V_N \end{bmatrix}. \quad (3)$$

In the above mode transformation, the external field and DM components are decoupled, and CM and DM components are decoupled, if the dielectric coating and the terminal are not considered. CM components of the system can be equivalent to a single wire running in the same path with the bundle above the reference ground, and DM components can be modeled as uniform N-conductor TLs with wire #N as the reference conductor, shown in Fig. 2. In this way, CM components can be modeled by MoM, and DM components can be modeled by MTLT.

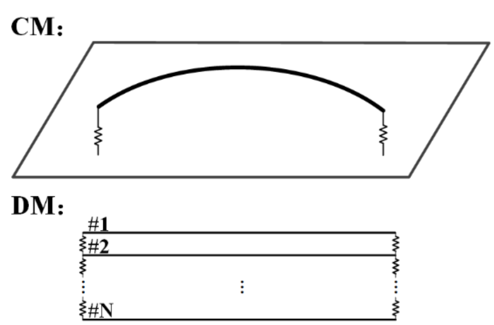

Fig. 2. Models for CM and DM components of the system in Fig. 1.

Then the left problem is how to obtain $\mathbf{H}$. Since CM voltages on all conductors are the same, the following relationship can be obtained:

$$\sum_{k=1}^{N} j\omega L_{ik}\mathbf{H}(k)I_c = \sum_{k=1}^{N} j\omega L_{mk}\mathbf{H}(k)I_c \quad (i,m=1,2,...,N), \quad (4)$$

where L_{ik} (L_{mk}) is the per-unit-length self-inductance of wire #i (#m) if $i = k$ ($m = k$), or the per-unit-length mutual-inductance between wire #i (#m) and #k if $i \neq k$ ($m \neq k$). For the infinitely small area in Fig. 1, L_{ik} (L_{mk}) can be expressed by the formulas in [1]:

$$\mathbf{L} = \frac{\mu_0}{2\pi}\begin{bmatrix} \ln\frac{2D}{r_1} & \ln\frac{2D}{d_{12}} & \cdots & \ln\frac{2D}{d_{1N}} \\ \ln\frac{2D}{d_{21}} & \ln\frac{2D}{r_2} & \cdots & \ln\frac{2D}{d_{2N}} \\ \vdots & \vdots & \ddots & \vdots \\ \ln\frac{2D}{d_{N1}} & \ln\frac{2D}{d_{N2}} & \cdots & \ln\frac{2D}{r_N} \end{bmatrix}, \tag{5}$$

where D is the distance between the wire bundle and the ground, d_{ij} $(i, j = 1, 2, ..., N)$ is the distance between wire #i and #j, and r_i is the radius of wire #i. (5) is based on the assumption that the wire bundle is not close to the ground, and the wires are not close to each other. Generally, the results will be accurate enough if $D \geq 2d$ and $d \geq 4r$. (4) can be simplified as:

$$\mathbf{L} \cdot \mathbf{H} = [c]_{N\times 1}, \tag{6}$$

where c is an unknown constant. Thus, combining (5), (6) with $\sum_{k=1}^{N}\mathbf{H}(k) = 1$, **H** can be calculated by:

$$\begin{bmatrix} \ln r_1 & \ln d_{12} & \cdots & \ln d_{1N} \\ \ln d_{21} & \ln r_2 & \cdots & \ln d_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \ln d_{N1} & \ln d_{N2} & \cdots & \ln r_N \end{bmatrix}\mathbf{H}_0 = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}, \tag{7}$$

$$\mathbf{H} = \mathbf{H}_0 \Big/ \sum_{i=1}^{N}\mathbf{H}_0(i). \tag{8}$$

It can be seen that **H** is independent with D, and the transformation matrices $\mathbf{T}_I$, $\mathbf{T}_V$ are invariant despite the non-uniform wiring path. The above derivation of **H** can also be expressed with the aid of the per-unit-length capacitance **C** in a similar way for $\mathbf{C} = \mu_0\varepsilon_0\mathbf{L}^{-1}$. The equivalent radius r_c of the single wire in the CM model can be obtained by combining the following relationship with (7):

$$j\omega\frac{\mu}{2\pi}\ln\frac{2D}{r_c}\cdot[1]_{N\times 1}\cdot I_c = j\omega\mathbf{L}\cdot\mathbf{H}\cdot I_c, \tag{9}$$

and is,

$$r_c = e^{1\Big/\sum_{i=1}^{N}\mathbf{H}_0(i)}. \tag{10}$$

B. Model for the effect of the dielectric coating

For wires with dielectric coating, $\mathbf{C} \neq \mu_0\varepsilon_0\mathbf{L}^{-1}$. Thus, different **H** will be obtained when we calculate it from **L** and **C**. In this case, mode conversion between CM and DM occurs, as described in [8].

Before the introduction of the model for dielectric coating, **H** derived from **C** (denoted as $\mathbf{H}_e$) is described. The coated wire is equivalent to a bare wire with the equivalent electric radius r_e [10]:

$$r_e = a^{\frac{1}{\varepsilon_r}} b^{1-\frac{1}{\varepsilon_r}}, \tag{11}$$

where a is the radius of the conductor, b is the radius of the whole wire, and ε_r is the relative permittivity of the coating. In this way, $\mathbf{H}_e$ can be calculated by (7) and (8) with r replaced by r_e .

Corresponding to $\mathbf{H}_e$, **H** derived from **L** is denoted as $\mathbf{H}_m$, and the magnetic radius is denoted as r_m (equal to the conductor radius a). If $\mathbf{H}_e$ is chosen as the **H** in transformation matrices $\mathbf{T}_I$, $\mathbf{T}_V$, the mode conversion caused by coating will be represented by series voltage sources. If $\mathbf{H}_m$ is chosen, the mode conversion will be represented by grounded parallel voltage sources. The two representation are both feasible, and the former is chosen in this paper. Thus, the transformation matrices are:

$$\mathbf{T}_I = \left(\mathbf{T}_V^{-1}\right)^T = \begin{bmatrix} & 1 & 0 & \cdots & 0 \\ & 0 & 1 & \cdots & 0 \\ \mathbf{H}_e & \vdots & \vdots & \ddots & \vdots \\ & 0 & 0 & \cdots & 1 \\ & -1 & -1 & \cdots & -1 \end{bmatrix}. \tag{12}$$

The series and parallel voltages of CM and DM components generated by the CM current are:

$$\left[V_{sc}, V_{sd,1}, V_{sd,2}, \ldots, V_{sd,N-1}\right]^T = \left[V_{sc}, \mathbf{V}_{sd}\right]^T = \mathbf{T}_V^{-1}\cdot\mathbf{L}\cdot j\omega\mathbf{H}_e I_c, \tag{13}$$

$$\begin{aligned}\left[V_{pc}, V_{pd,1}, V_{pd,2}, \ldots, V_{pd,N-1}\right]^T &= \left[V_{pc}, \mathbf{V}_{pd}\right]^T \\ &= \mathbf{T}_V^{-1}\cdot j\omega\mathbf{H}_e\Delta I_c \big/ \mathbf{C}(r_e).\end{aligned} \tag{14}$$

A transfer inductance vector $\mathbf{L}_t$ is defined to describe the relation between series voltage sources and the CM current, $\mathbf{V}_{sd} = j\omega\mathbf{L}_t I_c$. $\mathbf{L}_t$ can be derived from (13):

$$\mathbf{L}_t = \frac{\mu_0}{2\pi}\begin{bmatrix} \ln\frac{d_{N1}}{r_1} & \ln\frac{d_{N2}}{d_{12}} & \cdots & \ln\frac{d_{N,N-1}}{d_{1,N-1}} & \ln\frac{r_N}{d_{1N}} \\ \ln\frac{d_{N1}}{d_{21}} & \ln\frac{d_{N2}}{r_2} & \cdots & \ln\frac{d_{N,N-1}}{d_{2,N-1}} & \ln\frac{r_N}{d_{2N}} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \ln\frac{d_{N1}}{d_{N-1,1}} & \ln\frac{d_{N2}}{d_{N-1,2}} & \cdots & \ln\frac{d_{N,N-1}}{r_{N-1}} & \ln\frac{r_N}{d_{N-1,N}} \end{bmatrix}\mathbf{H}_e. \tag{15}$$

The equivalent circuit for this effect is shown in Fig. 3. Theoretically, there should also be CM voltage sources induced by DM currents, but this effect is weak and neglected for simplification.

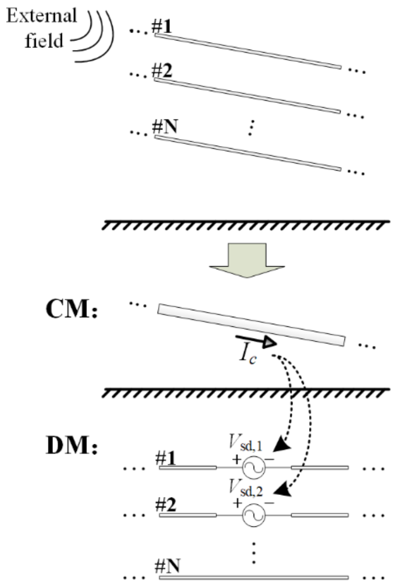

Fig. 3. Model for the effect of the dielectric coating.

The equivalent magnetic and electric radii of CM components, r_{cm} and r_{ce}, can be obtained from the V_{sc} and V_{pc} in (13) and (14):

$$r_{cm} = e^{\mathbf{H}_e^T \cdot \mathbf{A}_m \cdot \mathbf{H}_e}, \tag{16}$$

$$r_{ce} = e^{\mathbf{H}_e^T \cdot \mathbf{A}_e \cdot \mathbf{H}_e}, \tag{17}$$

where

$$\mathbf{A}_{m(e)} = \begin{bmatrix} \ln r_{m(e)1} & \ln d_{12} & \cdots & \ln d_{1N} \\ \ln d_{21} & \ln r_{m(e)2} & \cdots & \ln d_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \ln d_{N1} & \ln d_{N2} & \cdots & \ln r_{m(e)N} \end{bmatrix}. \tag{18}$$

C. Model for the terminal

Terminals of the CM and DM models in Fig. 2 are obtained by [11]:

$$\mathbf{Z}_{cd} = \mathbf{T}_v^{-1} \cdot \mathbf{Z} \cdot \mathbf{T}_I = \begin{bmatrix} z_c & [z_{ti}] \\ [z_{ti}]^T & [z_{dij}] \end{bmatrix} \quad (i, j = 1, 2, \ldots, N-1), \tag{19}$$

where $\mathbf{Z}$ is the original terminal impedance matrix and $\mathbf{Z}_m$ is the impedance matrix in mode domain. z_c is the CM load, z_{dij} ($i = j$) is the DM load between wire #i and #N, and z_{dij} ($i \neq j$) is the DM load between wire #i and #j. Unbalanced terminals will also cause mode conversion [12]. z_{ti} represents the coupling (or conversion) between CM and DM on terminals. The corresponding model is shown in Fig. 4.

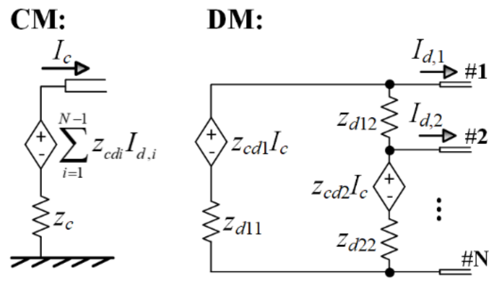

Fig. 4. Model for the terminal.

III. EXPERIMENTAL VALIDATION

The proposed modeling method in Section II is validated with a bundle of four curved wires shown in Fig. 5.

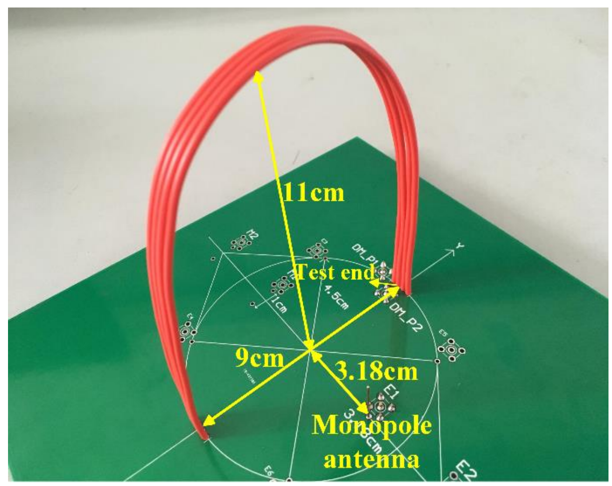

Fig. 5. A bundle of curved wires above a PCB board for experimental validation.

The wire bundle is connected to a double-face PCB. The four wires are the same. Their conductor radius is 0.465 mm, the outer radius is 1.05 mm, and the relative permittivity of the coating is 3.4. The distance between adjacent wires is about 3 mm. The length of the wires is 27 cm. Both ends of each wire are terminated with 51 Ω chip fixed resistors, except the test end. A spectrum analyzer (Keysight N9918A FieldFox Handheld Microwave Combination Analyzer) is connected to the test end through a coaxial connector on the back of the PCB board. External fields are generated by the monopole antenna on the PCB. The antenna is fed by a signal generator (R&S SMF 100A signal generator) providing 0-dBm power. The length of the antenna is 1.7 cm. The induced currents measured on the test end are compared with the proposed model. The results are shown in Fig. 6. It can be seen that the results obtained from measurements and the proposed model agree well.

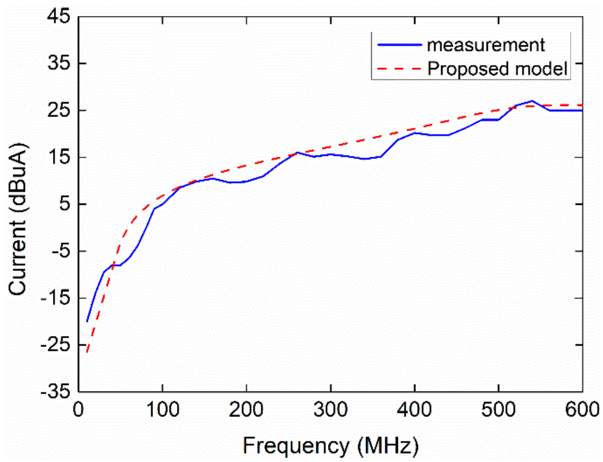

Fig. 6. The induced currents on the test end obtained from measurements and the proposed model.

IV. CONCLUSION

A field-to-wire coupling model for wire bundles with strongly non-uniform path is built in this paper. As the wiring path is one of the external characteristics, a mode transformation method is proposed to decompose the external and internal characteristics of the wire bundle. In this way, the corresponding CM and DM components are obtained. The former is modeled as an equivalent single wire running in the same path with the bundle above the reference ground. The latter is modeled as a uniform MTL system composed of the original wires in the bundle. External fields only excite the CM components. The non-uniform path only exists in the CM model and can be easily dealt with by MoM. Mode conversion caused by the dielectric coating and terminals are modeled as controlled voltage sources. Although the presented method focuses on the field-to-wire coupling problem, it can be readily extended to the counterpart radiation problem.

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