Necessity of Charge Measurement for Radiation Evaluation of Transmission Lines

Bing Li [1], **Xinwei Song** [2], **and Donglin Su** [1*]

[1] School of Electronic and Information Engineering
Beihang University, Beijing, 100191, China
bingli@buaa.edu.cn, *sdl@buaa.edu.cn

[2] School of Electrical and Information Engineering
Beijing University of Civil Engineering and Architecture, Beijing, 102616, China
songxinwei@bucea.edu.cn

Abstract **─** This paper attempts to arouse people's attention to charge measurement in electromagnetic compatibility, especially when evaluating the radiation of transmission lines (TLs). Usually the total current (or common-mode current) is supposed to represent the potential radiation of a TL system. However, it is proved that the measurement of charge, which is the dual source quantity of current, is also necessary to evaluate radiation in this paper. Only when the current and charge are both obtained, the radiation field could be determined accurately. First of all, it is pointed out that charge information could not be properly obtained by current measurement. Although charge could be derived from current theoretically, the error transferred from current to charge could be great for measurement. Then, the error transferred from current to near field (which reflects the charge distribution) is studied by simulation of a typical TL case. And it is proved that such error could be reduced effectively if current is modified by charge. In addition, another important reason for charge measurement is given as limited measurement points, since the standing waveform on TLs cannot be determined by current only. Finally, a possible method for charge measurement is proposed.

Index Terms **─** Charge measurement, radiation, transmission lines.

I. INTRODUCTION

The radiation of transmission lines is an important problem in electromagnetic compatibility (EMC). In many studies, the radiation of wires, cables or other transmission line systems is considered to be determined by current [1]–[6]. For example, a cable is modeled according to the terminal current in [1]. In MIL-STD-461 [7] and CISPR 25 [8], the high frequency conducted emission (CE) is based on current measured with current probe. However, compared with current, the other field source quantity—charge receives very little attention. Current (without charge) is considered sufficient to describe everything in transmission line radiation. Usually, there are two main reasons for this idea:

1) The charge conservation law indicates that charge and current could be derived from each other, which means the charge information may be obtained by current.

2) The electric field generated by the given current distribution [9] is:

$$\boldsymbol{E} = -j\omega\mu \oiint_S \boldsymbol{J}\frac{e^{-jkR}}{4\pi R}\mathrm{d}s - \nabla\left(\frac{1}{\varepsilon}\oiint_S \sigma\frac{e^{-jkR}}{4\pi R}\mathrm{d}s\right), \tag{1}$$

which indicates that the contribution of charge to electric field gets lower at higher frequencies.

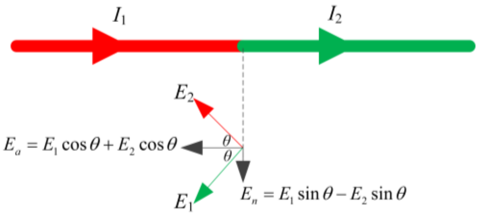


Fig. 1. Axial and normal components of the electric field generated by two current segments on the symmetric plane.

However, the two reasons are not always correct. Here we take the electric field as the example to explain. As a simple example, the electric field generated by two connected current segments on the symmetric plane are shown in Fig. 1. In the near-field region, the axial components of the electric fields generated by the two segments have the same direction, hence $E_a \propto I_1 + I_2$. For E_a, the relative error (RE) is limited:

Submitted On: October 14, 2018
Accepted On: December 25, 2018

$$\frac{\Delta E_a}{E_a} = \frac{\Delta I_1 + \Delta I_2}{I_1 + I_2} \le \max\left\{\frac{\Delta I_1}{I_1}, \frac{\Delta I_2}{I_2}\right\}. \tag{2}$$

However, for E_n, the RE is:

$$\frac{\Delta E_n}{E_n} = \frac{\Delta I_1 - \Delta I_2}{I_1 - I_2}. \tag{3}$$

Obviously, the error is out of control and even approaches infinity since $I_1 - I_2 \approx 0$ if $I_1 \approx I_2$. That means the electric field generated by charge does not always reduce as the frequency gets higher. The same problem exists for Reason 1, since the charges at the connection node is also determined by $I_1 - I_2$. (Actually the normal field E_n just right represents the node charge.)

The error transfer from current to electric field is analyzed and discussed in Section II. After that, current is modified with charge, which proves to effectively reduce the errors in Section III. An additional reason for charge measurement is given in Section IV. Finally, a possible method for charge measurement is proposed in Section V.

II. ERRORS TRANSFER FROM CURRENT TO ELECTRIC FIELD

For different parameter settings, the field errors caused by current errors are shown and explained. This discussion helps to understand how the errors happen, and when the errors have to be corrected.

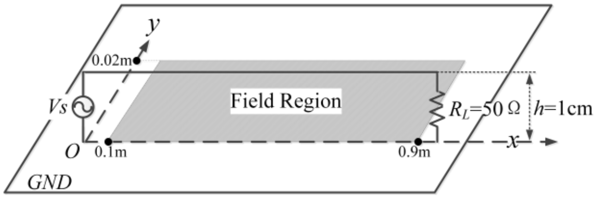


Fig. 2. The transmission line system and the field region under test.

The transmission line system in Fig. 2 is taken as the example. By default, the length of the wire is L = 1m, the height from the ground plane is h = 1cm and the radius is r = 1mm. A voltage source V_s = 1V is connected at the left terminal. The load is connected at the right terminal, which is set to be $R_L = Z_c$ by default. $Z_c = 60\ln(2h/r)$ is the characteristic impedance of the transmission line. Electric fields in the rectangle range $0.1\text{m} \le x \le 0.9\text{m}$ and $0 \le y \le 0.2\text{m}$ on the ground (GND) plane are calculated based on integral equations [9]. Since the system is symmetric by the *xOz* plane, only one side on the *y* axis is considered. Fields near the two terminals are not considered, because the currents are calculated by transmission line theory (TLT) [10], which are violated on the terminals and may lead to incorrect results.

The default frequency is set to be f = 100MHz. With the default settings, the current distribution along the wire could be easily obtained with TLT:

$$I(x) = \frac{1}{Z_c}\left(Ae^{-j\beta x} - Be^{j\beta x}\right), \tag{4a}$$

where

$$A = \frac{1}{2}\frac{(R_L + Z_c)e^{j\beta L}}{R_L\cos\beta L + Z_c \cdot j\sin\beta L}V_s, \tag{4b}$$

$$B = \frac{1}{2}\frac{(R_L - Z_c)e^{-j\beta L}}{R_L\cos\beta L + Z_c \cdot j\sin\beta L}V_s, \tag{4c}$$

$$\beta = \frac{2\pi f}{v}. \tag{4d}$$

Here v is the phase velocity of the transmitted wave, which is usually equal to velocity of light c.

Then the wire is divided into 80 segments with known currents. With (1), the electric field generated by each segment and its mirror segment current could be calculated , then sums up to be the total electric field on the field point, as shown in Fig. 3.

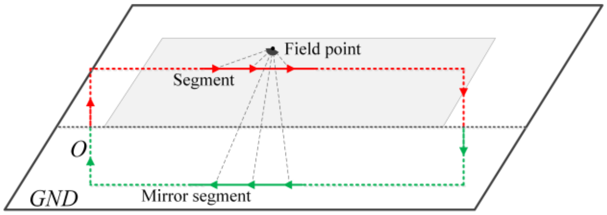


Fig. 3. Field calculation of the transmission line system.

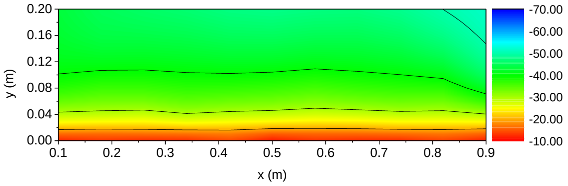
To observe the error transferring from current to field, a random 1% error is added to each segment current and the simulation is redone. To avoid the fortuity, the same process is done by 100 times and average values of the REs are taken as the results.

Firstly, it is obvious that 1% (40dB) errors for currents could lead to much larger errors for fields. This is because fields are determined by $I_1 - I_2$ (or the charge) to a certain extent as introduced in Section I. Another phenomenon is that the REs get smaller as field points get further from the wire. The reason is that the field close to the wire tends to be determined by the local charge density, while the field far from the wire tends to be determined by the general charge density. For a certain segment current, the effect caused by the error is carrying the charge from one end to the other. As further from the wire, the field becomes less sensitive to such charge movement, which means the errors would be smaller.

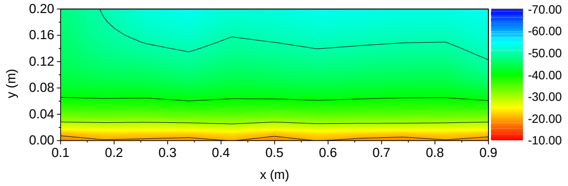
There-in-after, RE of the electric field would be shown for each group of parameter settings.

A. Frequency *f*

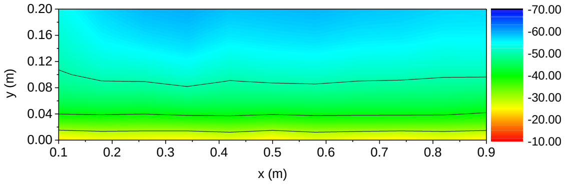
Besides the default 100MHz, three other frequencies, 25MHz, 50MHz and 200MHz, are also selected to demonstrate the influence of frequency on the REs of the electric field. The electric fields for the four cases are all shown in Fig. 4. Obviously, the errors become smaller as frequency gets higher. When frequency gets higher, differences between adjacent segment currents ($I_1 - I_2$) become larger, which means real value of charges and electric fields would be larger. Simultaneously, the absolute errors keep invariant, which leads to the reduction of the REs.



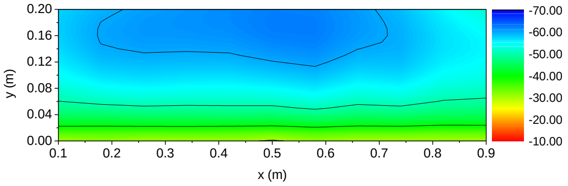
(a) *f* = 25MHz



(b) *f* = 50MHz



(c) *f* = 100MHz

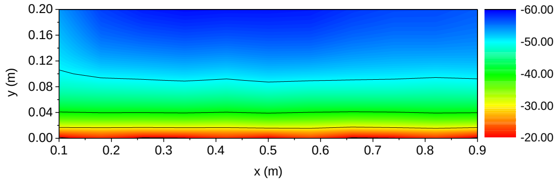


(d) *f* = 200MHz

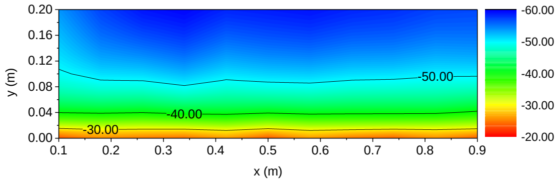
Fig. 4. Relative errors (dB) of the field at different frequencies.

B. Height *h*

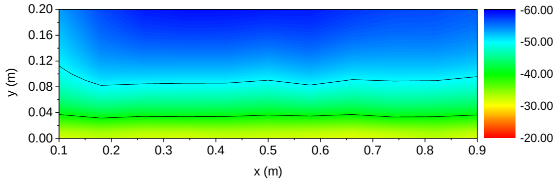
The REs of the electric field for three heights 0.5cm, 1cm (default) and 2cm are shown in Fig. 5. At $y = 0$, the REs of electric field tightly rely on h. However, at $y = 0.04$m, the REs of the three cases are all about –40dB. In other words, the influence of h declines very fast as the field point gets further from the wire. The explanation is that the REs depend on the strict point-wire distance $d' = \sqrt{d^2 + h^2}$. When d is small (close to the wire), $d' \approx h$; when d is large (far from the wire), $d' \approx d$.



(a) *h* =0.5cm



(b) *h* =1cm



(c) *h* =2cm

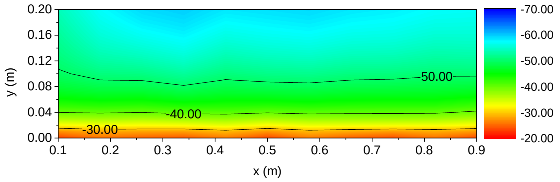
Fig. 5. Relative errors (dB) of the field with different heights.

C. Radius *r*

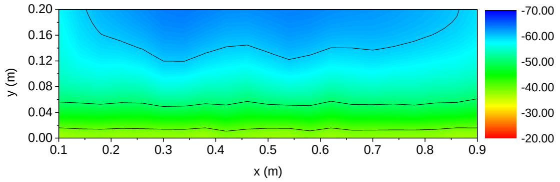
The radius actually has no influence on the REs. In TLT, r is only related to Z_c, where $Z_c = 60 ln\,(2h/r)$. As long as Z_L keeps equal to Z_c, the current distribution has no relation with the specific value of r.

D. Coating

Dielectric coating is quite common for transmission lines. In the aspect of current/charge distribution, the effect of coating is that it decreases the phase velocity v of the transverse electromagnetic (TEM) wave traveling along the wires. Here, the phase velocity is set as v=0.5c (c is the light speed in vacuum). The REs of electric fields is given in Fig. 6 (calculated with the modified Green function in [11]), compared with the default case.



(a) v=c



(b) v=0.5c

Fig. 6. Relative errors (dB) of the field with different phase velocities.

Obviously, the existence of coating reduces the REs. Similar to *f*, the coating could also change the differences of adjacent segment currents $I_1 - I_2$. $I_1 - I_2$ depends on β, where $\beta = 2\pi f / v$. In other words, both *f* and the coating influence the REs by changing β. The REs in Fig. 4 (d) (*f*=200MHz) and Fig. 6 (b) (*v*=0.5*c*) are quite similar, which also proves this standpoint.

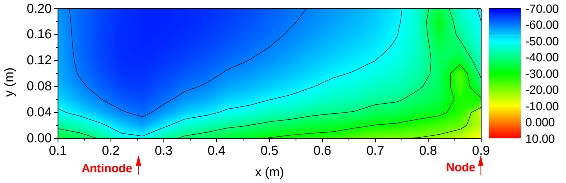
The permittivity of the coating may be frequency dependent, which means *v* varies at different frequencies. However, this effect is not evident at the low frequencies the transmission line analysis concerns. Another possible effect of the coating is dielectric loss, which may lead to the attenuation of the wave in transmission. In this condition, the true value and error change in equal proportion and RE remains unchanged.

E. Terminal Load R_L

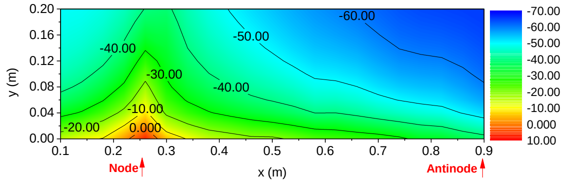
In the default settings, R_L is set to be equal to Z_c, which makes the magnitude of current along the wire invariant. This setting helps to demonstrate the effects of other parameters. But in practice, the terminal load is more possible to be open or short. To avoid the extreme conditions, a 10Ω load and a 10kΩ load are used to represent the short and open states, respectively. The REs of electric fields is given in Fig. 7, compared with the default cases.

Since R_L is set as nearly short (10Ω) or open (10kΩ), standing wave forms along the wire. The nodes and antinodes of the voltage have been marked in Fig. 7. The REs near the voltage nodes are quite large while those near the antinodes are much smaller. At the voltage nodes (which are also the current antinodes), the current is largest while the voltage and charge density are the smallest. Therefore, the RE becomes evidently large. Yet at the voltage antinodes, the condition is the exact opposite. That is why such a big difference of REs forms along the wire.

Here it seems that the error is not severe at voltage antinodes. However, that is because till now we have been discussing relative errors, where the small current here makes the added error value also small there. Considering absolute errors, the problem may be as serious at voltage antinodes. Actually, in many cases the errors in the measurement system are absolute, such as the receiver thermal noise. For a low-current system, like open-ended wire or dipole antenna, the current may be too small and submerged by the noise, hence charge becomes the only measurable quantity.



(a) R_L=10Ω



(b) R_L =10kΩ

Fig. 7. Relative errors (dB) of the field along the wire with approximately short and open terminal loads. (Nodes and antinodes of the voltage are marked).

III. CURRENT MODIFICATION WITH CHARGE DENSITY

Since small errors of current may lead to large errors of electric field, an applicable way to eliminate this effect is to directly measure the current difference ($I_1 - I_2$), which actually represents the charge deposited on the connection node of the two current segments $Q = 1/j\omega(I_1 - I_2)$. That is why charge measurement is necessary for such cases.

Another problem left is that how to include charge density information in the modeling. Currents have already been measured and charge densities are another group of constraints for the currents. This makes the problem over determined. Here, we manage to modify the currents with the charge density with minimum changes on the currents.

Still taking the case in Fig. 2 as the example, the current vector $\boldsymbol{I} = [I_1; I_2; \ldots I_N]$ (the number of segments *N*=80 here) represents segment currents from left to right on the wire. The 1% error has been added to each element of $\boldsymbol{I}$. Then the charge density vector $\boldsymbol{Q} = [Q_1; Q_2; \ldots Q_{N-1}]$, which represents charges on connection nodes, could be obtained:

$$\mathbf{Q} = \frac{1}{j\omega}\mathbf{TI}, \tag{5}$$

where

$$\mathbf{T} = \begin{bmatrix} 1 & -1 & & & \\ & 1 & -1 & & \\ & & \ddots & \ddots & \\ & & & 1 & -1 \end{bmatrix}_{(N-1)\times N}. \tag{6}$$

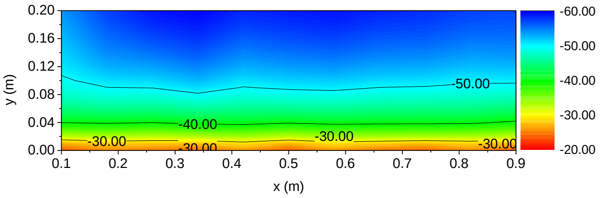
Assuming the measured charge density is $\boldsymbol{Q}'$, the modification current vector $\Delta\boldsymbol{I}$ could be obtained with

$$\boldsymbol{T}\Delta\boldsymbol{I} = \boldsymbol{Q}' - \boldsymbol{Q}. \tag{7}$$

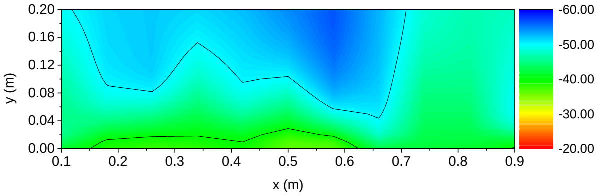
Notably, (7) is an underdetermined equation. With the command $\Delta\boldsymbol{I} = \boldsymbol{T} \backslash (\boldsymbol{Q}' - \boldsymbol{Q})$ in Matlab, the minimum norm solution of $\Delta\boldsymbol{I}$ could be obtained, which means changes on the currents is minimized. After that, the modified current vector could be derived:

$$\boldsymbol{I}' = \boldsymbol{I} + \Delta\boldsymbol{I}. \tag{8}$$

The REs of electric field before and after the current modification are shown in Fig. 8. It could be seen that the errors are effectively suppressed (about 20dB) at the field points close to the wire.



(a) Original



(b) Modified

Fig. 8. Relative errors (dB) of the field calculated with the original and modified currents at f =100MHz.

IV. ANOTHER REASON FOR CHARGE MEASUREMENT

The discussion above is all about the error caused by the lack of charge measurement, where we assume that the current along the whole transmission line could be measured. However, the measuring positions are usually limited in practice because of site conditions or time-consuming. In many cases, the measurement could only be done close to the source port. This is another reason for charge measurement, because we will see only current cannot determine the standing waveform of a transmission line. The charge information is also needed.

First of all, we must point out that for the common-mode of a transmission line system, the terminals are usually short or open. Therefore, standing-wave is the normal state. As shown in Fig. 9, if only the current value is known, how could we know the total current distribution is A, B or C?

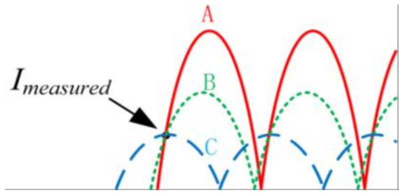


Fig. 9. Possible current distributions on a transmission line if current is measured at only one point.

However, if the charge is also known on the test point, the distribution could be determined. The spatial differentiation of current distribution $\boldsymbol{I}$ is related to charge distribution $\boldsymbol{\rho}$ with:

$$\frac{\partial}{\partial x}\boldsymbol{I} = j\omega\boldsymbol{\rho}, \qquad (9)$$

which helps to determine the waveform. As shown in Fig. 10, the waveform could be determined as B (the green curve) this time. Therefore, charge measurement is also necessary in the conditions that measuring points are limited.

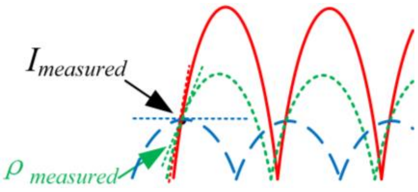
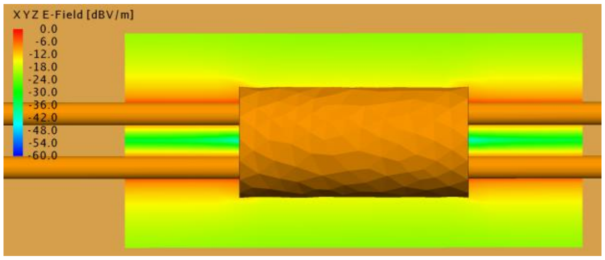


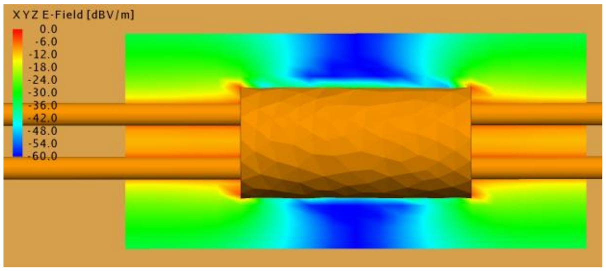
Fig. 10. The waveform could be determined with both $\boldsymbol{I}$ and $\boldsymbol{\rho}$ measured.

V. A PROPOSED METHOD FOR CM CHARGE MEASUREMENT

The current probe measures the total current (or the so-called common-mode current) by sensing the magnetic field surrounding the transmission lines. Similarly, the charge could be measured by testing electric field, which could be done by a near-field electric probe. However, there is another problem here. What we want is only the CM component, while measurement with the electric probe inevitably contains the DM component. To eliminate the DM component, a simple method is introduced here: adding a thin metal ring around the wires. Similar to a 'Faraday cage', the metal ring could sum up the charges on all wires, which could simultaneously eliminate the DM component and homogenize the CM component around the lines.



(a) With only CM excitation



(b) With only DM excitation

Fig. 11. Normalized field generated by only CM or DM component near the metal ring.

To indicate the effect of the metal ring, the near field of the wire pair with only CM or DM component is shown in Fig. 11 (simulated by FEKO). For the DM component, the ring suppress the field effectively, since we can see the field with the ring exceeds the field without the ring for about 10dB. While for the CM component, the ring has even no influence on the field value along the wire. This result proves that a circular metal ring with an electric field probe could be a possible device for charge measurement.

VI. CONCLUSION

This paper aims to discuss the necessity of charge measurement for the radiation evaluation of a transmission line. The key point here is whether current information is enough to accurately predict the near field. Usually we believe current is enough because charge information is implicitly included in current except DC. This assertion is right theoretically. However, the error transferred from current to charge could be very large, since the charge is determined by the difference of the current. The error transferred from the current to the field (or the charge) in a transmission line system is discussed in detail. And it is also proved that the error may be suppressed effectively if the charge information is added. Another reason for charge measurement is that only current cannot determine the whole standing wave along a transmission line and charge information is also needed. A possible method for charge measurement is also proposed in this paper.

In computational electromagnetics (CEM), current is also the data that usually used to characterize the potential radiation of transmission lines, such as in CST Cable Studio. However, it could be better to store or exchange both current and charge information for lower frequencies. Using only current means much higher accuracy of current is needed to represent the charge correctly. Moreover, note that current accuracy may not completely reflect the performance of a CEM method or software.

ACKNOWLEDGMENT

This work is sponsored by National Natural Science Foundation of China (No. 61771032) and Civil Aircraft Projects of China (No. MJ-2017-F-11).

REFERENCES

[1] K. W. Kam, A. Radchenko, and D. Pommerenke, “On different approaches to combine cable information into near-field data for radiated-field estimation,” *IEEE Trans. Electromagn. Compat.*, vol. 56, no. 2, pp. 276-285, Apr. 2014.

[2] C. Su and T. H. Hubing, “Imbalance difference model for common-mode radiation from printed circuit boards,” *IEEE Trans. Electromagn. Compat.*, vol. 53, no. 1, pp. 150-156, Feb 2011.

[3] T. Matsushima, T. Watanabe, Y. Toyota, R. Koga, and O. Wada, “Prediction of EMI from two-channel differential signaling system based on imbalance difference model,” *2010 IEEE International Symposium on Electromagnetic Compatibility*, pp. 413-418, July 2010.

[4] H. Ke, K. Morishita, T. Hubing, N. Kobayashi, and T. Harada, “Modeling radiated emissions due to power bus noise from circuit boards with attached cables,” *IEEE Trans. Electromagn. Compat.*, vol. 51, no. 2, pp. 412-416, May 2009.

[5] C. A. Nucci, F. Rachidi, and M. Rubinstein, “An overview of field-to-transmission line interaction,” *ACES Newsletter*, vol. 22, no. 1, pp. 9-27, 2007.

[6] J. Jia, Z. Lai, J. Lei, and Q. Wang, “Stochastic Radiation Model for Cable Bundle with Random Wires,” *ACES Journal*, vol. 31, no. 3, pp. 279-287, Mar. 2016.

[7] *MIL-STD-461F: Requirements for the Control of Electromagnetic Interference Characteristics of Subsystems and Equipment*, U.S. Dept. of Defense Std., 2007.

[8] *CISPR 25: Limits and Methods of Measurement for the Protection of Onboard Receivers*, 3rd, International Electrotechnical Commission (IEC) Std., 2008.

[9] R. F. Harrington, *Field Computation by Moment Methods*. IEEE Press Series on Electromagnetic Waves, New York: IEEE Press, 1993.

[10] C. R. Paul, *Analysis of Multiconductor Transmission Lines*. 2nd ed., New York: Wiley-IEEE Press, Oct. 2008.

[11] B. D. Popović, M. B. Dragović, and A. R. Djordjević, *Analysis and Synthesis of Wire Antennas*. New York: Research Studies Press, 1982.

Bing Li received the B.S. and Ph.D. degrees in Electronic and Information Engineering from Beihang University, Beijing, China, in 2011 and 2017, respectively. He currently works as a Post-doctoral Scholar with the School of Electronic and Information Engineering, Beihang University, Beijing, China.

His research interests include numerical modeling of transmission lines, field coupling analysis, and computational electromagnetics.

Xinwei Song received the B.S. and Ph.D. degrees in Electronic and Information Engineering from Beihang University, Beijing, China, in 2012 and 2018, respectively. She is currently a Lecturer with the School of Electrical and Information Engineering, University of Civil Engineering and Architecture, Beijing, China.

Her research interests include numerical modeling and measurements of transmission lines, and electromagnetic compatibility.

Donglin Su received the B.S., M.S., and Ph.D. degrees in Electronic Engineering from Beihang University (BUAA), Beijing, China, in 1983, 1986, and 1999, respectively. In 1986, she joined the Faculty of School of Electronics and Information Engineering, BUAA, where she is currently a Full Professor.

Her research interests include the numerical methods for microwave and millimeter-wave integrated circuits and systematic electromagnetic compatibility design of various aircrafts.

She is a senior member of the Chinese Institute of Electronics. She is the General Chair of 2018 International ACES Symposium. She is the Chair of Beijing Chapter of the IEEE Antennas and Propagation Society and the Deputy Chair of the Antennas Society, Chinese Institute of Electronics.