Plane Wave Scattering by a Dielectric Circular Cylinder in the Vicinity of a Conducting Strip (TM Case)

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Abstract **─** Scattering of plane electromagnetic waves by a dielectric circular cylinder in the vicinity of a conducting strip is presented. Two methods of solution are introduced. The first is an exact solution in which the scattered field from conducting strip is expressed in terms of Fourier series of radial and angular Mathieu function of unknown coefficient. Meanwhile the scattered field from the circular cylinder is expressed in terms of Fourier series of Bessel functions of unknown coefficient. The unknown coefficient can be obtained by enforcing the boundary conditions. The application of the boundary condition requires the use of the addition theorem of Mathieu to Bessel functions and vice versa. The second method is based in an asymptotic technique introduced by Karp and Russek for solving scattering by wide slit. The technique assumes the total scattered field from the strip and the dielectric cylinder as the sum of the scattered fields from the individual element due to a plane incident wave plus scattered fields from factious line sources of unknown intensity located at the center of every element. The line sources account for the multiple scattering effect. By enforcing the boundary conditions, the intensity of the line sources can be calculated. Numerical examples are calculated using both methods showing excellent agreement in all cases.

Index Terms **─** Dielectric cylinder and conducting strip, multiple-scattering, scattering cross section.

I. INTRODUCTION

The scattering of a plane electromagnetic waves by a perfectly conducting strip grating was the subject of many investigations [1] and [2]. Different methods have been used for solving such a problem, among them is the self-consistent method [3]. This method is based on the previous knowledge of the responses of the isolated objects in the multi-object scattering problem. In an approximate treatment the self-consistent method was used by Karp and Russek [4]. The solution is restricted to the case where the spacing between the objects is much greater than the maximum dimension of any one object. This technique has been extended to the case of scattering of plane waves by wide double wedges (Elsherbeni and Hamid [5]). Hansen [6] used the integral equation approach in order to calculate the diffracted field of a plane acoustic wave through two or more parallel slits in a plane screen.

Scattering by dielectric cylinders has also been widely investigated. Yousif and Kohler [7], presented Mueller scattering matrix elements and the cross sections for the scattering of an electromagnetic plane wave from two infinitely long, parallel, circular dielectric cylinders at oblique incidence. Dunster [8], studied the case of two parallel infinite dielectric cylinders, of equal diameter and dielectric constant, illuminated by a plane electromagnetic wave. Henin et al. [9], presented a rigorous semi-analytical solution for scattering from an array of circular dielectric cylinders due to an obliquely incident plane wave.

Recently, the scattering by two dielectric coated elliptic cylinders [10] and by metamaterial coated elliptic cylinders [11] has been addressed. In addition, pattern equation method has been developed for solving problems of scattered electromagnetic waves by dielectric coated conducting bodies [12] and [13]. Also scattering of electromagnetic waves by inhomogeneous dielectric gratings loaded with conducting strips were investigated by Yamasaki [14].

The TM and TE scattering by a conducting strip loaded by a circular dielectric cylinder was presented by Karunaratne et al. [15] and [16]. They restrict their solution to the case of radial orientation of the strip with respect to the dielectric cylinder center. They employed the integral equation formulation along with the moment method in solving this problem.

Two major objectives are addressed in the present paper. The first is to show the application of the addition theorem of Mathieu to Bessel function and vice versa. The second is to solve a mixed objects problem such as a dielectric cylinder and perfectly conducting strip as multiple scattering problem. The restriction in [15] has been released in our work and the conducting strip can take any orientation. An exact analytical solution based on boundary value method along with an approximate

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solution based on the technique in [4] of a plane electromagnetic wave incident a dielectric circular cylinder near a conducting strip are considered.

II. FORMULATION OF THE PROBLEM

Figure 1 shows the cross sections of an infinitely long dielectric circular cylinder near a conducting strip of infinite length with their axes parallel to the z-axis. The conducting strip has a width $2d$ and the dielectric cylinder has radius a and permittivity ε. The center of the of dielectric circular cylinder is located at the origin of the global coordinates $(\rho,\ \phi, z)$. In addition to the global coordinate system, a local coordinates (u, v, z) is defined at the center of the conducting strip which is located at $(\rho = b,\ \phi = 0)$.

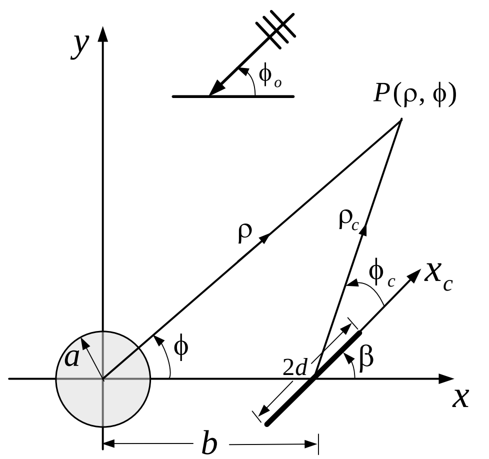

Fig. 1. Geometry of the problem.

A plane wave, with $e^{j\omega t}$ time dependence, is incident with an angle ϕ_o with respect to the x-axis of the global coordinate system and polarized in z-direction:

$$E_z^i = e^{-jk_o\rho\cos(\phi - \phi_o)}, \tag{1}$$

where k_o is the wave number of free space. The incident wave can be expressed in terms elliptic wave function of the local coordinates at the center of the conducting strip:

$$E_z^{inc} = \sqrt{8\pi}\, e^{-jk_o b\cos\phi_o} \sum_{n=0}^{\infty} j^{-n} \left[\frac{Je_n(h,\zeta_c)\quad Se_n(h,\eta_c)\, Se_n(h,\cos\phi_{01})}{N_n^{(e)}(h)} + \frac{Jo_n(h,\zeta_c)\, So_n(h,\eta_c) So_n(h,\cos\phi_{01})}{N_n^{(o)}(h)}\right], \tag{2}$$

where

$$\phi_{01} = \phi_o - \beta.$$

$Je_n(h,\zeta_c)$ and $Jo_n(h,\zeta_c)$ are respectively the even and the odd modified Mathieu functions of the first kind and order n. Also, $Se_n(h,\eta_c)$ and $So_n(h,\eta_c)$ are respectively the even and the odd angular Mathieu functions of order n. $N_n^{(e)}(h)$ and $N_n^{(o)}(h)$ are normalized functions. The arguments of the Mathieu functions are $h = k_o d$, and $\eta = \cos v$, where u and v are elliptical cylindrical coordinates defined by:

$$x = d\cosh u \cos v, \quad y = d\cosh u \sin v. \tag{3}$$

The incident wave is also expanded in terms of Bessel functions of the circular cylindrical coordinates:

$$E_z^{inc} = \sum_{n=0}^{\infty} 2\kappa_n j^{-n} J_n(k_o\rho)\cos n(\phi - \phi_o)$$

$$\kappa_n = \begin{cases} 0.5 & n = 0 \\ 1 & n > 0 \end{cases}. \tag{4}$$

The scattered field from the conducting strips can be expressed in terms of local coordinates as:

$$E_z^{ss} = \sqrt{8\pi} \sum_{n=0}^{\infty} A_n\, He_n^{(1)}(h,\zeta_c)\, Se_n(h,\eta_c). \tag{5}$$

While, the scattered field inside the dielectric circular cylinder is given by:

$$E_z^{sci} = \sum_{n=0}^{\infty} B_n\ J_n(k\rho)\ \cos n(\phi - \phi_o), \tag{6}$$

and the scattered field outside the dielectric cylinder is given by:

$$E_z^{sco} = \sum_{n=0}^{\infty} C_n\ H_n^{(1)}(k_o\rho)\ \cos n(\phi - \phi_o). \tag{7}$$

The total field outside the dielectric cylinder is:

$$E_z^{tot} = E_z^{inc} + E_z^{ss} + E_z^{sco}, \tag{8}$$

This total field must satisfy the boundary conditions:

$E_z^{tot} = 0$, on the surface of the conducting strip, (9)

$E_z^{tot} = E_z^{sci}$, on the surface of dielectric cylinder, (10)

$H_\phi^{tot} = H_\phi^{sci}$, on the surface of dielectric cylinder. (11)

To apply these boundary condition E_z and E_ϕ must be expressed in terms of circular cylindrical coordinate system $(\rho,\ \phi, z)$ when applying the boundary conditions on the cylinder surface, while they must be expressed in terms of elliptical coordinates when applying boundary conditions on the strip surface. This is done using the addition theorem of Mathieu function, from elliptical to circular cylindrical coordinates is:

$$He_p^{(1)}(h,\zeta)\, Se_p(h,\eta) = \sqrt{\frac{\pi}{2}} \sum_{m=0}^{\infty} \sum_{s=0}^{\infty} \kappa_s\ (j)^{m-p}\, De_m^p(a)\, J_s(k_o\rho)\ \ R_{s,m}, \tag{12}$$

$$R_{s,m} = \left[H_{s-m}^{(1)}(k_o b)\cos(s\phi - m\beta) + (-1)^m H_{s+m}^{(1)}(k_o b)\cos(s\phi + m\beta)\right],$$

and from circular cylindrical to elliptical coordinates is:

$$H_n^{(1)}(k_o\rho)\ \cos n(\phi - \phi_0) = \sum_{\ell=0}^{\infty} W_{\ell,n}\, Je_\ell(h,\zeta_c)\, Se_\ell(h,\eta_c) + \sum_{\ell=1}^{\infty} X_{\ell,n}\ Jo_\ell(h,\zeta_c)\, So_\ell(h,\eta_c) \tag{13}$$

$$W_{\ell,n} = \frac{\sqrt{2\pi}}{N_\ell^{(e)}(a_1)} \sum_{s=0}^{\infty} (j)^{(\ell-s)}\, De_s^\ell(a_1)\, N_{n,s},$$

$$N_{n,s} = H_{n+s}^{(i)}(kb)\cos(s\beta + n\phi_0) + (-1)^s H_{n-s}^{(i)}(kb)\cos(s\beta - n\phi_0),$$

$$X_{\ell,n} = \frac{\sqrt{2\pi}}{N_\ell^{(o)}(a_1)} \sum_{s=1}^{\infty} (j)^{(\ell-s)}\, Do_s^\ell(a_1) \{H_{n-s}^{(i)}(kb)\sin(s\beta - n\phi_0) + (-1)^s H_{n+s}^{(i)}(kb)\sin(s\beta + n\phi_0)\}.$$

Employing equation (13) into (7) and applying boundary condition (9), at $\zeta_c = 1$, then multiplying both sides of the resulting equation by $Se_m(h,\eta_c)$ and integrate over v from 0 to 2π, we obtain:

$$A_n = -\frac{Je_n(h,1)}{He_n^{(1)}(h,1)}\left(\frac{j^{-n}\ e^{-jkb\cos\phi_o}}{N_n^{(e)}(h)} Se_n(h,\cos\phi_{01}) + \frac{1}{\sqrt{8\pi}}\sum_{p=0}^{\infty} C_p W_{n,p}\right). \quad (14)$$

Employing (12) into (5), and applying Boundary condition (10), then multiplying both sides of the resulting equation by $\cos m\phi$ and integrating over ϕ from 0 to 2π, we obtain:

$$2j^{-m} J_m(k_o a)\cos(m\phi_o) + 2\pi J_m(k_o a) \sum_{n=0}^{\infty} A_n \sum_{p=0}^{\infty} (j)^{p-n} De_p^n(h)\ M_{m,p} + C_m\,\chi_m\, H_m^{(1)}(k_o a) \cos(m\phi_o) = \chi_m\, B_m\ \cos(m\phi_o) J_m(ka),$$

$$\chi_m = \begin{cases} 2 & m = 0 \\ 1 & m > 0 \end{cases}, \quad (15)$$

$$M_{m,p} = \left[H_{m-p}^{(1)}(k_o b) + (-1)^p\, H_{m+p}^{(1)}(k_o b)\right] \cos(p\beta).$$

To apply the boundary condition $H_\phi^{tot} = H_\phi^{sci}$ on the surface of the dielectric cylinder, one must obtain H_ϕ^{inc}, H_ϕ^{ss}, and H_ϕ^{sci}, using, $H_\phi = \frac{-j\omega\varepsilon}{k^2}\frac{\partial E_z}{\partial\rho}$.

Applying boundary condition (11), we obtain:

$$2j^{-m} J_m'(k_o a)\ \cos(m\phi_o) + 2\pi\, J_m'(k_o a) \sum_{n=0}^{\infty} A_n \sum_{p=0}^{\infty} (j)^{(p-n)} De_p^n + C_m\,\chi_m\ H_n^{(1)'}(k_o a)\cos(m\phi_o) = \sqrt{\varepsilon_r}\ B_m\ \cos(m\phi_o)\,\chi_m\ J_m'(ka). \quad (16)$$

From equations (14), (15) and (16), one can get:

$$[Z][C] = [G], \quad (17)$$

$$z_{i,j} = \sqrt{\frac{\pi}{2}}\frac{JJ_i(\varepsilon_r,a)}{JH_i(\varepsilon_r,a)} \sum_{n=0}^{\infty} \frac{Je_n(h,1)}{He_n^{(1)}(h,1)} W_{n,j} \sum_{s=0}^{\infty} (j)^{(s-n)}\ \ De_s^n(h) M_{i,s}, \quad (18)$$

$$z_{i,i} = \chi_i\ \cos(i\phi_o) + \sqrt{\frac{\pi}{2}}\frac{JJ_i(\varepsilon_r,a)}{JH_i(\varepsilon_r,a)} \sum_{n=0}^{\infty} \frac{Je_n(h,1)}{He_n^{(1)}(h,1)} W_{n,i} \sum_{\ell s=0}^{\infty} (j)^{(s-n)}\ \ De_s^n(h) M_{i,s}, \quad (19)$$

$$g_m = 2j^{-m}\ \cos(m\phi_o)\frac{JJ_m(\varepsilon_r,a)}{JH_m(\varepsilon_r,a)} - 2\pi\frac{JJ_m(\varepsilon_r,a)}{JH_m(\varepsilon_r,a)} \sum_{n=0}^{\infty} \frac{Je_n(h,1)}{He_n^{(1)}(h,1)} \frac{e^{-jkb\cos\phi_o}\ j^{-n}}{N_n^{(e)}(h)} Se_n(h,\cos\phi_{01}) \sum_{s=0}^{\infty} (j)^{(s-n)}\ \ De_s^n(h) M_{m,s}, \quad (20)$$

$$JJ_m(\varepsilon_r,a) = J_m(ka)\, J_m'(k_o a) - \sqrt{\varepsilon_r}\, J_m(k_o a)\, J_m'(ka), \quad (21)$$

$$JH_m(\varepsilon_r,a) = \sqrt{\varepsilon_r}\, H_m^{(1)}(k_o a)\, J_m'(ka) - H_m^{(1)'}(k_o a)\, J_m(ka), \quad (22)$$

The scattered electric field in the outer region can be obtain from (7), by employing asymptotic Hankel function expression $H_m^{(1)}(k_o\rho) = \frac{1}{\sqrt{k_o\rho}}\ e^{j(k_o\rho - (\frac{(2m+1)}{4})\pi)}$, as:

$$E_z^{totsc} = \sqrt{\frac{2}{\pi}}\frac{e^{jk_o\rho}}{\sqrt{k_o\rho}}\ e^{-j\pi/4)}\, P(\phi) = c(k_o\rho)P(\phi), \quad (23)$$

$$P(\phi) = 2\pi e^{-jk_o b\cos\phi} \sum_{n=0}^{\infty} A_n (-j)^n\, Se_n(h,\cos(\phi-\beta)) + \sqrt{\frac{\pi}{2}} \sum_{n=0}^{\infty} C_n(-j)^n \cos n(\phi-\phi_o), \quad (24)$$

III. APROXIMATE SOLUTION

The approximate solution follows the technique that was established by Karp and Russek [4]. To apply this technique one needs to obtain the far scattered field from conducting strip and the dielectric cylinder due to both plane wave and line source excitations. Although this solution is approximate, it produces accurate results and it is much easier to compute compared to the exact solution. This is because it doesn't have matrix equation to form and solve like the exact solution.

A. Plane wave excitation of the conducting strip

Consider the plane wave of equation (2), is incident on the conducting strip. The scattered field from the conducting strip is expressed in equation (5).

Applying the boundary conditions on the conducting strip surface and multiplying both sides of the resulting equation by $Se_n(h,\eta_c)$ and integrating over v_i from 0 to 2π, one obtains:

$$A_m = -j^{-m}\ e^{-jk_o b\cos\phi_o} \frac{Je_m(h,1)\, Se_m(h,\cos\phi_{01})}{He_m^{(1)}(h,1) N_m^{(e)}(h)}. \quad (25)$$

Once the coefficients are calculated the scattered electric field in the outer region is given by Equation (7). Since $He_m^{(1)}(h,\zeta) = \frac{1}{\sqrt{h\zeta}}\ e^{j(h\zeta - (\frac{2m+1}{4})\pi)}$ and for large $h\zeta$ it can be represented in terms of circular cylindrical coordinates ($h\zeta = k_o\rho_c$). In this case the total scattered field is given by:

$$E_z^{(s)} = c(k_o\rho_c)\, f_s(h,b,\phi_c,\phi_o), \quad (26)$$

$$f_s(h,b,\phi_c,\phi_o) = 2\pi\sum_{n=0}^{\infty}(-j)^n\ A_n Se_n(h,\cos\phi_c). \quad (27)$$

B. Line source excitation of the conducting strip

Consider a line source of unit intensity placed at (x_0, y_0) with respect to the coordinates at the center of the conducting strip, then the z-component of the electric field due to such a line source can be expressed as:

$$E_z^{inc} = 4\left[\sum_{m=0}^{\infty} \frac{Se_m(h,\eta_0)}{N_m^{(e)}(h)} Se_m(h,\eta) \begin{cases} Je_m(h,\zeta_0)\, He_m^{(1)}(h,\zeta) \\ Je_m(h,\zeta) He_m^{(1)}(h,\zeta_0) \end{cases} + \frac{So_m(h,\eta_0)}{N_m^{(o)}(h)} So_n(h,\eta) \begin{cases} Jo_m(h,\zeta_0)\, Ho_m^{(1)}(h,\zeta) & u > u_0 \\ Jo_m(h,\zeta)\, Ho_m^{(1)}(h,\zeta_0) & u < u_0 \end{cases}\right], \quad (28)$$

$$\zeta_0 = \left[\frac{1}{2}\left(\frac{s_0^2}{d^2}+1\right) + \left(\frac{1}{4}\left(\frac{s_0^2}{d^2}+1\right)^2 - \frac{x_0^2}{d^2}\right)^{1/2}\right]^{1/2}, \quad (29)$$

$$\eta_0 = \frac{x_0}{\zeta_0 d}, \quad \psi_0 = \tan^{-1}\left[\frac{y_0}{x_0}\right], \quad (30)$$

$$s_0 = ((x_0)^2 + (x_0)^2)^{1/2}. \quad (31)$$

The scattered field from the strip can be written as:

$$E_z^{(s)} = 4\sum_{n=0}^{\infty} D_n\, He_n^{(1)}(h,\zeta)\, Se_n(h,\eta). \quad (32)$$

Matching the boundary condition corresponding to

E_z and multiply both sides of the resulting equation by $Se_m(h,\eta)$ and integrating over v_i from 0 to 2π, we get:

$$D_n = -\frac{He_n^{(1)}(h,\zeta_0)\, Je_n(h,1)\, Se_n(h,\eta_0)}{He_n^{(1)}(h,1)\ N_n^{(e)}(h)}. \qquad (33)$$

Once the coefficients are calculated the scattered electric field in the outer region is:

$$E_z^{(z)} = c(k_o\rho_c)\, g_s(h,\phi_c,\zeta_0,\eta_0), \qquad (34)$$

where

$$g_s(h,\phi_c,\zeta_0,\eta_0) = \sqrt{8\pi}\sum_{n=0}^{\infty} j^{-n} D_n\, Se_n(h,cos\phi_c). \quad (35)$$

C. Plane wave excitation of the dielectric cylinder

The plane wave of equation (1) is considered incident on the dielectric cylinder, which can be expanded as in equation (4). The scattered field inside the dielectric circular cylinder is given by equation (6) while the scattered field outside the dielectric cylinder is given by equation (7). The total field outside the dielectric cylinder is:

$$E_z^{tot} = E_z^{inc} + E_z^{sco}. \qquad (36)$$

This total field must satisfy the boundary conditions: $E_z^{tot} = E_z^{sci}$ and $H_\phi^{tot} = H_\phi^{sci}$, on the surface of the dielectric cylinder. Applying these boundary condition:

$$j^{-n}J_n(k_oa) + \chi_m C_n H_n^{(1)}(k_oa) = \chi_m B_n J_n(ka)\,, \quad (37)$$

$$j^{-n}J'_n(k_oa) + \chi_m C_n H_n^{(1)\prime}(k_oa) = \chi_m B_n \sqrt{\varepsilon_r}\, J'_n(ka). \quad (38)$$

Solving (37) and (38), we obtain:

$$C_n = \frac{j^{-n}\left(\sqrt{\varepsilon_r}\, J_n(k_oa)\, J'_n(ka) - J'_n(k_oa) J_n(ka)\right)}{\chi_m\left(J_n(ka)\, H_n^{(1)\prime}(k_oa) - \sqrt{\varepsilon_r}\, J'_n(ka)\, H_n^{(1)}(k_oa)\right)}. \qquad (39)$$

Once the coefficients are calculated the scattered electric field in the outer region is:

$$E_z^{(s)} = c(k_o\rho)\, f_c(a,\varepsilon_r,\phi,\phi_o), \qquad (40)$$

$$f_c(a,\varepsilon_r,\phi,\phi_o) = \sqrt{\frac{\pi}{2}}\sum_{n=0}^{\infty}(-j)^n C_n\, cosn(\phi-\phi_o). \quad (41)$$

D. Line source excitation of the dielectric cylinder

Consider a line source of unit intensity placed at (ρ_0,ϕ_0) with respect to the coordinates at the center of the conducting strip, the *z*-component of the electric field due to such a line source can be expressed as:

$$E_z^{inc} = \begin{cases} \sum_{n=0}^{\infty} 2\kappa_n H_n^{(1)}(k_o\rho_o)\, J_n(k_o\rho) cosn(\phi-\phi_o) & ,\rho<\rho_o \\ \sum_{n=0}^{\infty} 2\kappa_n H_n^{(1)}(k_o\rho)\, J_n(k_o\rho_o) cosn(\phi-\phi_o) & ,\rho>\rho_o \end{cases}. \qquad (42)$$

The scattered field inside the dielectric circular cylinder is given equation (6), while the scattered field outside the dielectric cylinder is given by:

$$E_z^{sco} = \sum_{n=0}^{\infty} F_n\ H_n^{(1)}(k_o\rho)\, cosn(\phi-\phi_o). \qquad (43)$$

Applying boundary conditions (37), and (38), gives:

$$F_n = \frac{\left(\sqrt{\varepsilon_r}\, J_n(k_oa)\, J'_n(ka) - J'_n(k_oa) J_n(ka)\right)}{\chi_m\left(J_n(ka)\, H_n^{(1)\prime}(k_oa) - \sqrt{\varepsilon_r}\, J'_n(ka)\, H_n^{(1)}(k_oa)\right)} H_n^{(1)}(k_o\rho_o). \qquad (44)$$

Once the coefficients are calculated the scattered electric field in the outer region is:

$$E_z^{(s)} = c(k_o\rho)\, g_c(a,\varepsilon_r,\phi,\rho_o,\phi_o), \qquad (45)$$

$$g_c(a,\varepsilon_r,\phi,\rho_o,\phi_o) = \sqrt{\frac{\pi}{2}}\sum_{n=0}^{\infty}(-j)^n\ F_n\, cosn(\phi-\phi_o). \qquad (46)$$

Now consider the problem of the dielectric cylinder and the conducting strip shown in Fig. 1. Assuming a fictitious line source C_2 at the center of the conducting strip, the far scattered field from the dielectric cylinder is:

$$E^{sc} = c(k_o\rho)\,[f_c(a,\varepsilon_r,\phi,\phi_o) + C_2 g_c(a,\varepsilon_r,\phi,b,0)]. \quad (47)$$

The partial scattered field from the conducting strip can be determined in two ways. The first at $\phi = 0$, the value of $E^{sc}[c(k_o\rho)]^{-1}$ can be considered as the intensity of a line source times the response (43), i.e.,

$$E^{sc} = c(k_o\rho)\,[f_c(a,\varepsilon_r,\phi,\phi_o) + C_2 g_c(a,\varepsilon_r,\phi,b,0)]\ \ g_s(h,\phi_c,\zeta_0,\eta_0). \quad (47)$$

Second, this partial scattered field is given by:

$$E^{pss} = c(k_o\rho_c)\ C_1\, g_s(h,\phi_c,\zeta_0,\eta_0). \qquad (48)$$

From (43) and (44):

$$C_1 = f_c(a,\varepsilon_r,0,\phi_o) + C_2 g_c(a,\varepsilon_r,0,b,0). \qquad (49)$$

Similarly, the far scattered field from the conducting strip due to the fictitious line source C_1 and incident plane wave is given by:

$$E^{ss} = c(k_o\rho_c)\,[f_s(a,b,\phi_c,\phi_{01}) + C_1 g_s(h,\phi_c,\zeta_{0c},\eta_{0c}). \quad (50)$$

The partial scattered field from the dielectric cylinder can be determined in two ways. The first at $\phi_c = \pi-\beta$, the value of $E^{ss}[c(k_o\rho_c)]^{-1}$ can be considered as the intensity of a line source times the response (46), i.e.,

$$E^{psc} = c(k_o\rho)\,[f_s(h,b,(\pi-\beta),\phi_{01}) + C_1 g_s(h,(\pi-\beta),\zeta_{0c},\eta_{0c})]\ \ g_c(a,\varepsilon_r,\phi,\rho_0,\phi_0). \quad (51)$$

Second, this partial scattered field is given by:

$$E^{psc} = c(k_o\rho)\, C_2\ \ g_c(a,\varepsilon_r,\phi,\rho_0,\phi_0). \qquad (52)$$

From (70) and (71), one can obtain:

$$C_2 = f_s\big((h,b,(\pi-\beta),\phi_{01})\big) + C_1\, g_c(h,(\pi-\beta),\zeta_{0c},\eta_{0c}). \quad (53)$$

From (49) into (53):

$$C_1 = \frac{f_c(a,\varepsilon_r,0,\phi_0) + f_s(h,b,(\pi-\beta),\phi_{01}) g_c(a,\varepsilon_r,0,b,0)}{1 - g_s(h,(\pi-\beta),\zeta_{0c},\eta_{0c}) g_c(a,\varepsilon_r,0,b,0)}.$$

Once C_1 and C_2 are obtained, one can determine the z-component of the total scattered field from the present system as:

$$E_z^s = c(k_o\rho) P(\phi), \qquad (54)$$

$$P(\phi) = f_c(a,\varepsilon_r,\phi,\phi_0) + C_2 g_c(a,\varepsilon_r,\phi,b,0) + e^{-jk_ob\,cos\phi}\,[\,f_s(h,b,(\phi-\beta),\phi_{01}) + C_1 g_s(h,(\phi-\beta),\zeta_{0c},\eta_{0c})]. \qquad (55)$$

The plane wave scattering properties of a two-dimensional body of infinite length are conveniently described in terms of the echo width, i.e.,

$$W(\phi) = \frac{4}{k}|P(\phi)|^2. \tag{56}$$

IV. RESULTS AND DISCUSSION

In order to check our computational accuracy our results for both exact and approximate solution are compared with the published data b Karunaratne et al. [15]. In this example, $a = 0.5\,\lambda$, $d = 0.25\,\lambda$, $b = 0.75\,\lambda$, $\beta = 0^o$, $\varepsilon_r = 4.0$, and $\phi_o = 90^o$. Results shown in Fig. 2 corresponding to this case are in excellent agreement compared with those of the same case published in [15].

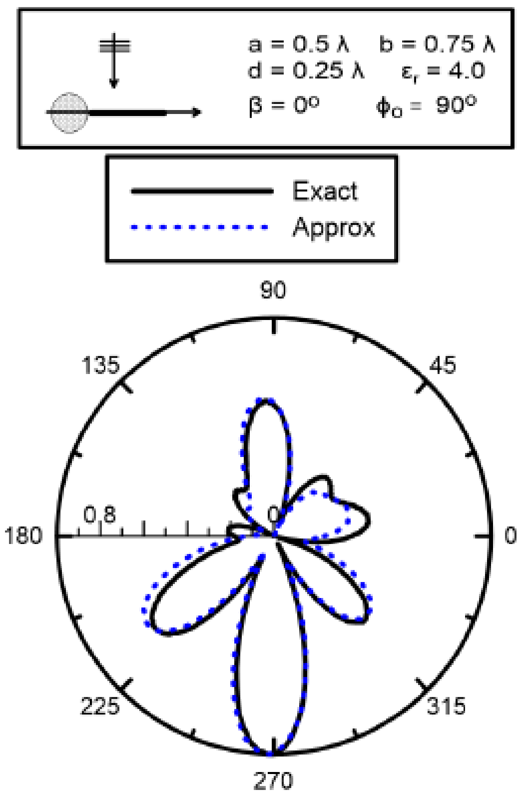

Fig. 2. Normalized far field pattern of a plane wave scattered by dielectric cylinder touching a conducting strip.

The results presented next are for some cases of different directions of the incident wave and different orientation of the conducting strip. Both methods presented earlier are used in our calculations of the normalized far field pattern for each case. In general excellent agreement for both methods is noticed. The first case is of a plane wave normally incident on a dielectric cylinder and conducting strip with $\beta = 0^o$. As can be seen from Fig. 3 the forward and backscattering echo widths are similar. Results of both methods agrees very well here at all angles. As the conducting strip orientation changes to $\beta = 20^o$ keeping the same direction of the incident wave at $\phi_o = 90^o$, the scattering field in the forward direction is changed as shown in Fig. 4. All other parameters in first and second case are kept the same as illustrated in Figs. 3 and 4.

The scattering field was calculated again for different conducting strip orientation in order to show the effect of the orientation on the forward and backscattering fields. Figure 5 illustrates three cases of the echo width scattering pattern at $\beta = 0^o$, $\beta = 45^o$, and $\beta = 90^o$, respectively. As one can see from Fig. 5, the forward scattered field is forming more lobes as β increases while the backscattered field is decreasing. That shows there is a substantial change in the forward and backscattering fields with the change of the conducting strip orientation.

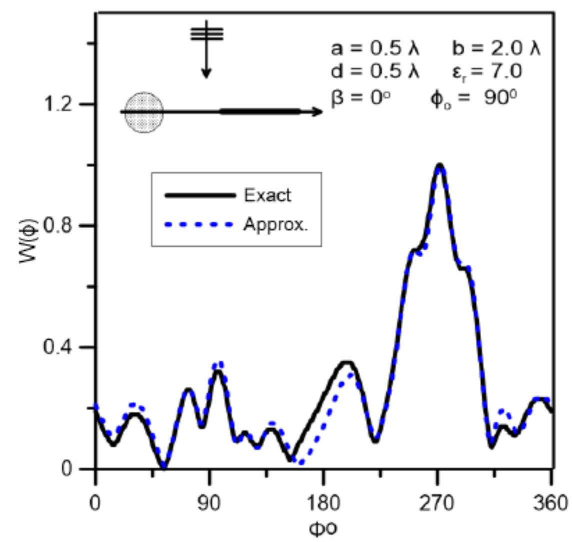

Fig. 3. Normalized far field pattern of a plane wave scattered by dielectric cylinder near a horizontal conducting strip.

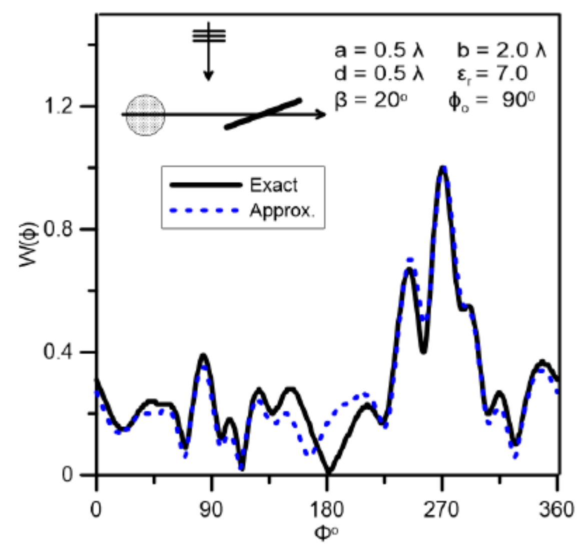

Fig. 4. Normalized far field pattern of a plane wave scattered by dielectric cylinder near a tilted conducting strip.

The direction of the incident wave is changes in the following two cases. Firstly, we are taking $\phi_o = 0^o$, and considering $\beta = 90^o$ in which the conducting strip is facing the incident plane wave followed by the dielectric cylinder. As shown in Fig. 6, the pattern shows a maximum forward scattered field while the backscattered field is minimal.

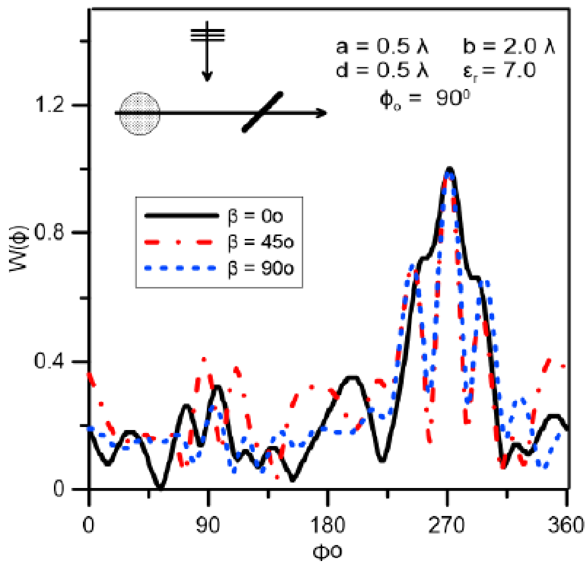

Fig. 5. Normalized far field pattern of a plane wave scattered by dielectric cylinder near a tilted conducting strip at different tilt angles.

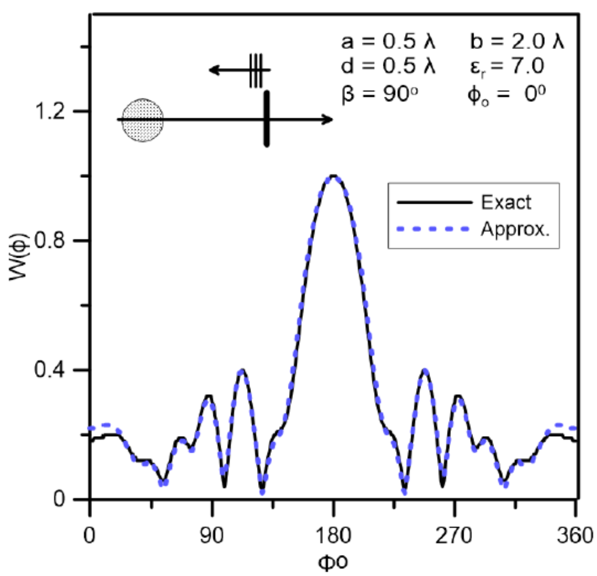

Fig. 6. Normalized far field pattern of a plane wave scattered by dielectric cylinder near a vertical conducting strip.

Secondly we are taking $\phi_o = 180^o$, and considering $\beta = 90^o$ in which the dielectric cylinder is facing the incident plane wave followed by conducting strip. As shown in Fig. 7, the pattern shows a maximum forward scattering echo width while the backscattering echo width is relatively higher than the previous case.

V. CONCLUSION

Scattering of an electromagnetic wave by a dielectric cylinder in the vicinity of a conducing strips is achieved. The far field pattern of this system is illustrated for different cases of strip orientation and different directions of the incident plane wave. It is found that back scattering echo width can be minimized by placing the conducting strip facing the incident plane wave followed by the dielectric cylinder.

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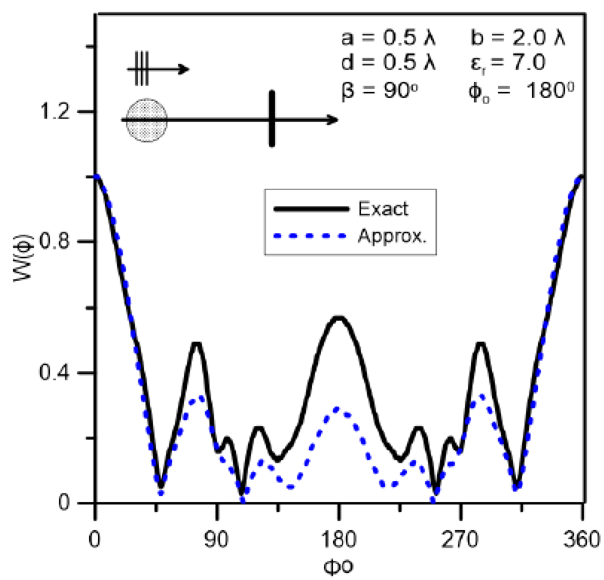

Fig. 7. Normalized far field pattern of a plane wave scattered by dielectric cylinder near a vertical conducting strip.

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