Domain Decomposition Method for Scattering from an Aircraft with Jet Engine Inlet Cavity

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Abstract ─ Monostatic radar cross section (RCS) of an combat aircraft is analyzed using iterative Least Square Method weighted domain decomposition method (LSM weighted DDM). The model of the 200 wavelengths long aircraft is made of perfect electric conductor (PEC). Inlet cavity of the jet engine is included in the model. The inlet has realistic shape, whereas cavity is closed with a PEC on the outlet side. Corresponding model without the inlet is also analyzed—the inlet contour is smoothed, and then it is closed with a PEC. Method of moments (MoM) solution is used as a reference. It is shown that the LSM weighted DDM solution can provide very good solution in just a few iterations if problematic parts of the structure are extracted in separate subdomains.

Index Terms ─ Domain decomposition method, jet engine inlet, method of moments, monostatic radar cross section.

I. INTRODUCTION

It is shown that jet engines have significant influence on the monostatic radar cross section (RCS) of aircrafts. A jet engine alone is difficult for electromagnetic (EM) analysis, because its inlet cavity acts like resonant structure, and it features geometrically complex, fine details (e.g., blades). Thus, asymptotic techniques (like ray tracing) cannot provide sufficient accuracy, whereas numerically exact methods (like the Method of Moments—MoM and the Finite Element Method—FEM) are limited by their computational cost. In the last decade of 20th century different strategies were used for jet engine analysis [1]. Recently, domain decomposition methods (DDMs) coupled with accelerators such as the multilevel fast multipole algorithm (MLFMA) show as good candidates for such analysis [2].

A novel iterative DDM is introduced in [3]. The entire model is decomposed into subdomains. Solution is obtained as a linear combination of the subdomain solutions from the current iteration and from all previous iterations. Original excitation is used for the subdomain solutions in the 1st iteration, whereas the subdomains are excited with the residual error in the subsequent iterations. The weighting coefficients of the linear combination are determined in a way to minimize residual error of the MoM system of equations. It is not necessary to store or invert the entire MoM matrix, so the method is more efficient than the MoM. General theory of the Method of Moments weighted Domain Decomposition Method (MoM weighted DDM) is presented in [4]. The method in [3] is a special case of the MoM weighted DDM, and can be referred to as Least Square Method weighted Domain Decomposition Method (LSM weighted DDM).

In this paper the LSM weighted DDM method is used to obtain the monostatic RCS of the geometrically realistic aircraft model, made of perfect electric conductor (PEC). The model includes the inlet cavity, but without fan blades / non-trivial outlet terminations. The cavity is ended with a simple PEC wall. To illustrate the influence of the inlet on the convergence, we also analyzed the model without the inlet.

II. LSM WEIGHTED DDM FOR MULTIPLE EXCITATIONS

The MoM weighted DDM in [4] is presented for the single excitation case (bistatic RCS). Though multiple excitation case (monostatic RCS) can be processed as a sequence of single excitation analysis, the process can be speed up by dealing with all excitations (or a group of excitations) simultaneously. In that sense, the theory of LSM weighted DDM for multiple excitations is just a slightly adapted theory from [4], but will be presented here briefly, for completeness. More detailed explanation of the method can be found in [4].

The solution for the *j*th excitation (incident EM

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plane wave), $j = 1,..,N_{\mathrm{e}}$, is the solution of the MoM matrix equation:

$$\mathbf{Z}\mathbf{A}^{(j)} = \mathbf{V}^{(j)}, \quad (1)$$

such that the $N \times N$ matrix $\mathbf{Z}$ is a linear operator, the $N \times 1$ matrix $\mathbf{V}^{(j)}$ is the known excitation column vector, and the $N \times 1$ matrix $\mathbf{A}^{(j)}$ is the unknown response column vector to be determined. We solve (1) iteratively, using domain decomposition strategy.

The solution space of **A** is split into *M* overlapping subdomains, formally using $N_l \times N$ restriction matrices E_l, $l = 1,..,M$. A restriction matrix E_l has a single non-zero element in each row, such that,

$$\sum_{l=1}^{M} \mathbf{E}_l^{\mathrm{T}} \mathbf{E}_l = \mathbf{I}. \quad (2)$$

Restriction of **A** to the *l*th subdomain results in a column vector ($N_l \times 1$ matrix) $\mathbf{A}_l = \mathbf{E}_l \mathbf{A}$, whereas extension from the *l*th subdomain back to the entire domain is performed as $\mathbf{B} = \mathbf{E}_l^{\mathrm{T}} \mathbf{A}_l$. Restriction of **Z** to the *l*th subdomain results in a $N_l \times N_l$ matrix $\mathbf{Z}_{ll} = \mathbf{E}_l \mathbf{Z} \mathbf{E}_l^{\mathrm{T}}$.

If we know the solution for the *j*th excitation in the $(n-1)$th iteration, $\mathbf{A}^{(n-1,j)}$, residual vector is defined as:

$$\mathbf{V}^{(n-1,j)} = \mathbf{V} - \mathbf{Z}\mathbf{A}^{(n-1,j)}, \; n > 0. \quad (3)$$

To start the process, we adopt $\mathbf{A}^{(0,j)} = 0$. Solution for the *l*th subdomain (extended to the entire domain) in the *n*th iteration is:

$$\mathbf{B}_l^{(n,j)} = \mathbf{E}_l^{\mathrm{T}} \left(\mathbf{E}_l \mathbf{Z} \mathbf{E}_l^{\mathrm{T}} \right)^{-1} \mathbf{E}_l \mathbf{V}^{(n-1,j)}. \quad (4)$$

We calculate vector of weighting coefficients for the *j*th excitation in the *n*th iteration, $\mathbf{C}^{(n,j)}$, as:

$$\mathbf{C}^{(n,j)} = \left(\left(\mathbf{Z}\mathbf{B}^{(n,j)} \right)^{\mathrm{H}} \left(\mathbf{Z}\mathbf{B}^{(n,j)} \right) \right)^{-1} \left(\mathbf{Z}\mathbf{B}^{(n,j)} \right)^{\mathrm{H}} \mathrm{V}^{(n,j)}, \quad (5)$$

where $\mathbf{B}^{(n,j)}$ is a matrix which columns are $\mathbf{B}_l^{(n,j)}$ from (4). Approximate solution of (1) in the *n*th iteration can now be calculated as:

$$\mathbf{A}^{(n,j)} = \mathbf{B}^{(n,j)} \mathbf{C}^{(n,j)}, \quad (6)$$

and the residual vector in the *n*th iteration is:

$$\mathbf{V}^{(n,j)} = \mathbf{V} - \mathbf{Z}\mathbf{B}^{(n,j)} \mathbf{C}^{(n,j)}. \quad (7)$$

Normalized residuum is used as a measure of the accuracy of the solution. For the *j*th excitation, after the *n*th iteration, the normalized residuum is calculated as

$$R_{\mathrm{norm}}^{(n,j)} = \frac{\left\| \mathbf{V}^{(n,j)} \right\|^2}{\left\| \mathbf{V} \right\|^2}. \quad (8)$$

Essentially, solution $\mathbf{A}^{(n,j)}$ given by (6) is a linear combination of all subdomain solutions in all previous and the current iteration, for the *j*th excitation. Weighting coefficients $\mathbf{C}^{(n,j)}$ minimizes residuum (8) in the least square sense.

Note that a matrix $\mathbf{E}_l \mathbf{Z} \mathbf{E}_l^{\mathrm{T}}$ is not calculated using **Z**, but is obtained using a partial model for the *l*th subdomain. Also, $\mathbf{Z}\mathbf{B}^{(n,j)}$ in (5) is calculated using a single row (or a few rows) of **Z** at a time, and entire $\mathbf{Z}\mathbf{B}^{(n,j)}$ is then stored in order to calculate residual vector using (7), instead by using (3). In that way the entire **Z** matrix is never stored, relaxing memory demands in the case of electrically large problems.

The LSM weighted DDM is an iterative procedure, such that the *n*th iteration consists of three steps:

1) the subdomain solutions in the *n*th iteration are found using (4), subdomain by subdomain—inverted matrix for the *l*th subdomain is calculated once, then residual vectors for all excitations are replaced in (4),

2) The vectors of weighting coefficients are calculated using (5), excitation by excitation, and

3) the approximate solution in the *n*th iteration is calculated using (6), and the residual vector of the matrix equation (1) in the *n*th iteration is calculated using (7), both excitation by excitation.

The LSM weighted DDM iterative procedure is finished when a specified normalized residuum is reached, or a specified number of iterations is reached.

III. NUMERICAL RESULTS

The model of an aircraft is shown in Fig. 1. The model is symmetric, so only the half of the model is shown. Enlarged details of the model with and without the inlet are shown in the inset figures up. The model without the inlet has smooth inlet contour, in order to simplify meshing of the PEC cover of the cavity. The uniform automatic domain decomposition [4] results in 72 subdomains, as illustrated in the inset figure right. Since automatic domain decomposition is applied, the cavity does not belong to a single subdomain. The monostatic RCS, σ, is calculated in the symmetry plane (205 incident waves are used). The normalized monostatic RCS (σ/λ^2) is shown in the graphs. Analysis is performed at the single frequency of 4 GHz, at which the model is about 200 wavelengths long. The EM simulator WIPL-D Pro [5] is used for the MoM analysis. The MoM discretization of the model with the inlet, using higher order bases, results in 499618 unknowns. The number of unknowns for the model without the inlet is 484316.

The normalized monostatic RCS in the symmetry plane, obtained by using the MoM, for the models with and without the inlet is shown in Fig. 2. The inlet significantly changes RCS in the zone where incident waves enter the inlet cavity, whereas in the rest of the symmetry plane the RCS results for the models with and without the inlet are very similar.

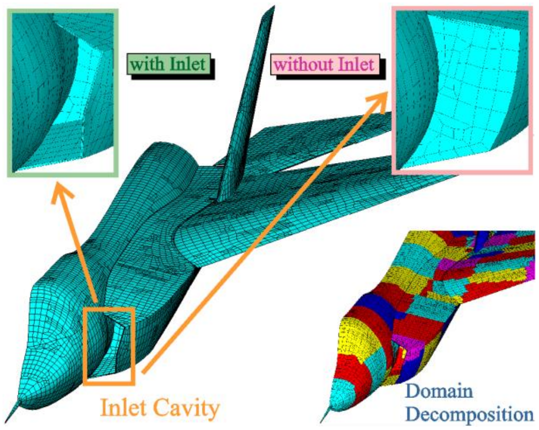

Fig. 1. A half of the symmetric aircraft model with/without inlet (inset up), and automatic domain decomposition of the model (inset right).

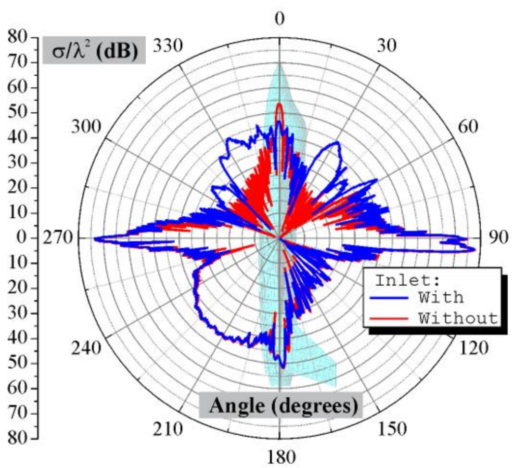

Fig. 2. Monostatic RCS, obtained by MoM, for the models with and without the inlet.

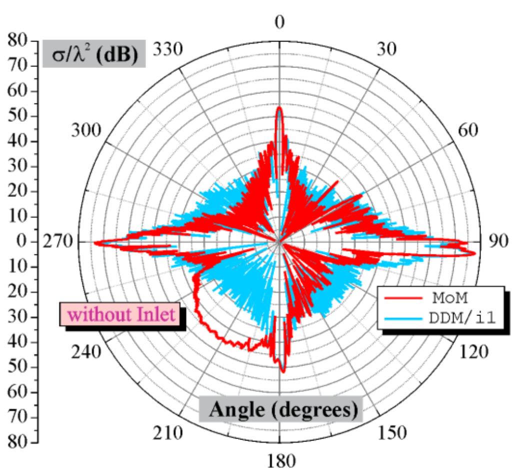

Fig. 3. Monostatic RCS for the model without the inlet, MoM vs. 1st iteration of LSM weighted DDM.

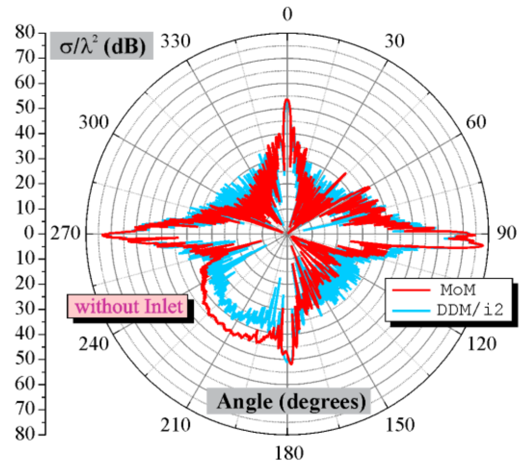

Fig. 4. Monostatic RCS for the model without the inlet, MoM vs. 2nd iteration of LSM weighted DDM.

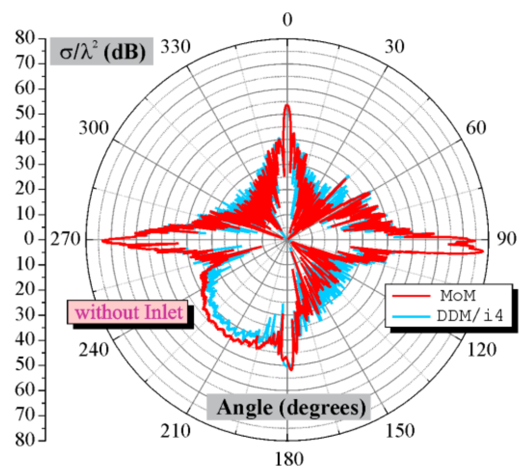

Fig. 5. Monostatic RCS for the model without the inlet, MoM vs. 4th iteration of LSM weighted DDM.

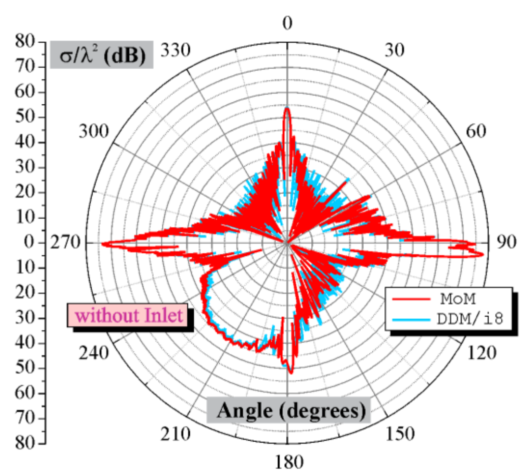

Fig. 6. Monostatic RCS for the model without the inlet, MoM vs. 8th iteration of LSM weighted DDM.

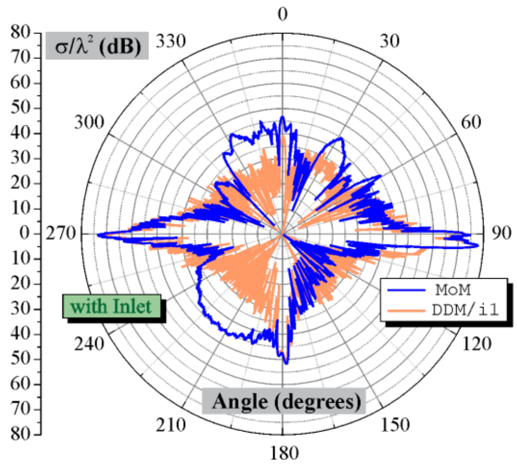

Fig. 7. Monostatic RCS for the model with the inlet, MoM vs. 1st iteration of LSM weighted DDM.

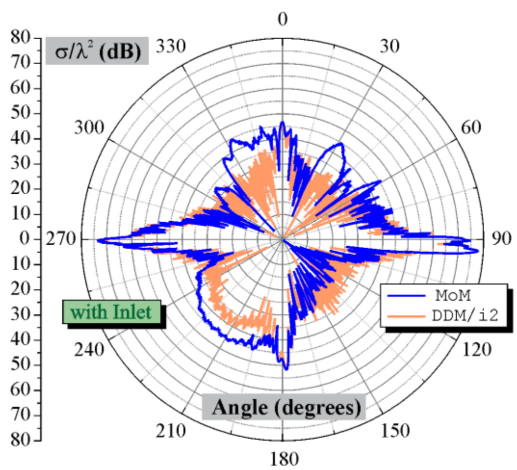

Fig. 8. Monostatic RCS for the model with the inlet, MoM vs. 2nd iteration of LSM weighted DDM.

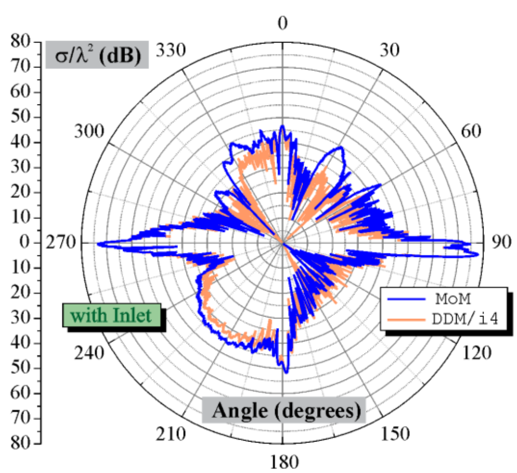

Fig. 9. Monostatic RCS for the model with the inlet, MoM vs. 4th iteration of LSM weighted DDM.

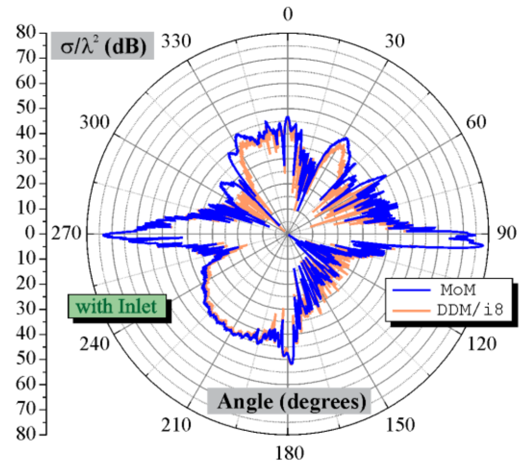

Fig. 10. Monostatic RCS for the model with the inlet, MoM vs. 8th iteration of LSM weighted DDM.

The normalized monostatic RCS for the model without the inlet, obtained by using MoM is compared to the corresponding results after 1, 2, 4, and 8 iterations of LSM weighted DDM in Figs. 3–6. Four narrow lobes (around 0, 90, 190, and 270 degrees) and one wide lobe (approximately between 195 and 240 degrees) can be distinguished. The convergence for the narrow lobes is very fast (only two iterations are needed), whereas about eight iterations are needed for the wide lobe and other zones with the low level RCS.

The normalized monostatic RCS for the model with the inlet, obtained by using MoM is compared to the corresponding results after 1, 2, 4, and 8 iterations of LSM weighted DDM in Figs. 7–10. The inlet cavity has modified RCS in the region of ± 60 degrees around 0 degree (compared to the model with the inlet)—there are few lobes instead of just one, and the convergence for all of them is slower than for the narrow lobes around 90, 190, and 270 degrees. The convergence for the wide lobe between 195 and 240 degrees is essentially unchanged from the model without the inlet. The convergence in the zones with the low level RCS is similar to that of the model without the inlet.

In order to improve convergence, the inlet cavity was manually allocated to the single domain. This is done by using an auxiliary ellipsoid to extract all entities that are in the ellipsoid. Another ellipsoid is used to extract entities around junction of rear wing and the tale of the aircraft (after some numerical experiments, this part of the structure is found responsible for the slower convergence of the wide lobe between 195 and 240 degrees). Prior to the automatic domain decomposition, for each auxiliary ellipsoid a single subdomain is created from the entities that were in the ellipsoid. The rest of the structure is automatically decomposed, as in Fig. 11. In the graphs, this model is referred to as "with Inlet 2".

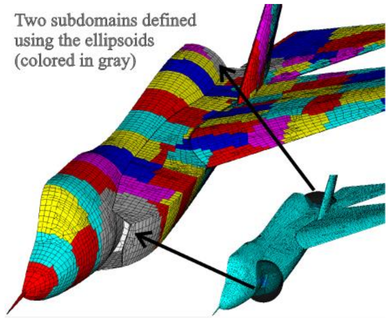

Fig. 11. Model with the inlet, combination of the automatic and manual domain decomposition.

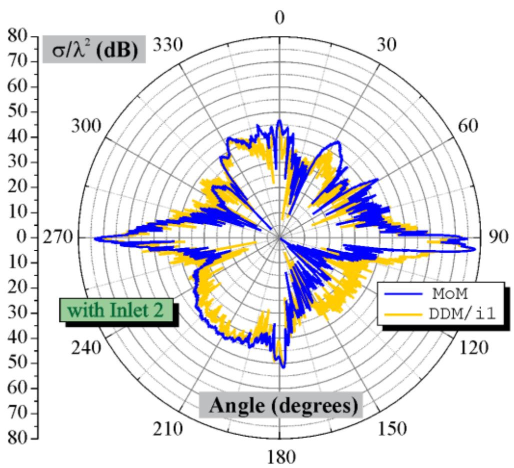

Fig. 12. The monostatic RCS for the model in Fig. 11, MoM vs. 1st iteration of LSM weighted DDM.

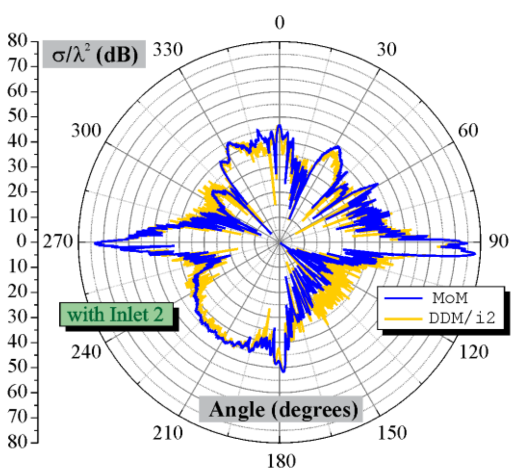

Fig. 13. The monostatic RCS for the model in Fig. 11, MoM vs. 2nd iteration of LSM weighted DDM.

The normalized monostatic RCS for the model with the inlet, obtained by using MoM is compared to the corresponding results after 1, 2, 4, and 8 iterations of LSM weighted DDM (for the model in Fig. 11) in Figs. 12–15.

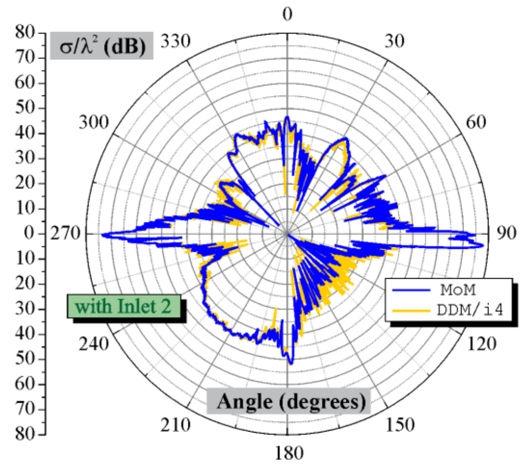

Fig. 14. The monostatic RCS for the model in Fig. 11, MoM vs. 4th iteration of LSM weighted DDM.

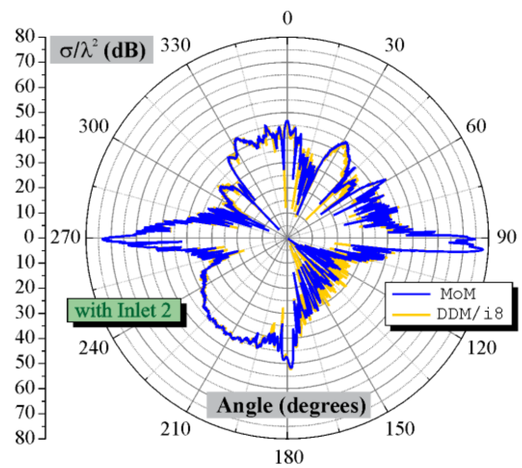

Fig. 15. The monostatic RCS for the model in Fig. 11, MoM vs. 8th iteration of LSM weighted DDM.

The convergence for the wide lobe between 195 and 240 degrees is very fast (essentially, RCS is fully stabilized after four iterations), whereas the convergence in the inlet cavity zone (± 60 degrees around 0 degree) is significantly improved compared to the model without the manually defined subdomains. The convergence in the zones with the low level RCS is slightly improved compared to the model without the manually defined subdomains.

The CPU time and the storage are similar for both models. All analysis were performed using Intel(R) Xeon(R) CPU E5-2650 v4 @ 2.20 GHz (2 processors) with 256 GB RAM.

The MoM analysis CPU time / the MoM matrix storage is ~11 hours / 1860 GB. Note that MoM analysis was optimized for calculations using GPUs. Four NVIDIA GeForce GTX 1080Ti cards are used in the analysis. Estimated CPU time for the MoM analysis without GPUs is ~250 hours.

The CPU time for the LSM weighted DDM after four iterations is ~19 hours, and after eight iterations is ~42 hours. The LSM weighted DDM matrix storage is ~53 GB per iteration. Note that LSM weighted DDM is performed independently in five different zones in the symmetry plane, in order to be able to use the concept of visibility [4]. For that reason, LSM weighted DDM matrix storage for the first iteration is about a half of 53 GB. Note also that MoM analysis is performed with the 1801 excitations (incident EM plane waves), whereas LSM weighted DDM is performed with the 205 excitations—interpolation is then used to obtain RCS for all 1801 excitations. Finally, note that LSM weighted DDM calculations were not optimized for multiple excitations analysis—the primary goal was to improve the convergence the LSM weighted DDM.

IV. CONCLUSION

LSM weighted DDM with the automatic domain decomposition is iterative technique that can be successfully applied to the monostatic RCS problems. Generally, the convergence is good, but automatically decomposed resonant cavities and/or sharp parts of the structure can cause the slower convergence. The convergence can be further improved by the extraction of problematic parts of the structure in separated subdomains, using the advantage of domain decomposition techniques. In the presented case of the aircraft this technique provides very good RCS results in just a few iterations. LSM weighted DDM provides means to solve the problems which electrical size is too large for the standard MoM. In the future work the focus will be on the optimization of the LSM weighted DDM calculations and storage techniques.

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