An Extension of Cooray-Rubinstein Formula on the Calculation of Horizontal Electric Field from Inclined Lightning Channel

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Abstract **─** An approximate method to calculate the horizontal electric field from inclined lightning channel in frequency-domain was proposed based on the extension of Cooray formula and Cooray-Rubinstein formula. Validations of these two extended formulas were both performed by comparing their calculation results with those obtained by the numerical solution of Sommerfeld's integrals using Bannister approximations. The results have shown that both the accuracy of these two extended formulas are mainly related to the observation distance and the ground conductivity, and will increase with the increasing ground conductivity and observation distance. The extended Cooray formula could predict the horizontal electric field at ground level accurately at the distance beyond 100m. And the extended Cooray-Rubinstein formula could yield an acceptable approximation on the above-ground horizontal electric field with the exception of the observation point at close range under poor ground conductivity less than 0.0001S/m.

Index Terms **─** Approximate calculation, extended Cooray-Rubinstein formula, frequency-domain, horizontal electric field, inclined lightning channel.

I. INTRODUCTION

The calculation of lightning horizontal electric field at a lossy ground is very important in the field-to-line coupling problem. Generally, the solution of horizontal electric field would lead to the so-called Sommerfeld's integrals, which are highly oscillatory and difficult to evaluate numerically. To overcome this difficulty, many simplified approaches have been proposed, especially in frequency domain. An approximate technique known as the wave tilt formula has been used by several researchers to calculate the horizontal field from lightning at ground level [1, 2]. However, the wave tilt formula is only applicable for the calculation of horizontal field at distances beyond a few kilometers. In order to calculate the horizontal field at close range, Cooray [3] introduced an approximate method in frequency domain to calculated the horizontal field at ground level with the help of the ground surface impedance, which is called as Cooray formula and can be applied for the distances as short as 200m. Based on the Cooray formula, Rubinstein [4] presented another approximate formula, known as Cooray-Rubinstein formula, to evaluate the horizontal electric field off the ground at close, intermediate and far distances. Several researches have shown that the Cooray-Rubinstein formula could yield a satisfactory approximation on the above-ground horizontal electric field at different distances [4-6]. General restrictions of validity of the Cooray-Rubinstein approximation were theoretically examined by Wait [7]. However, he mentioned the possibility that the errors due to the violation of these restrictions may not be practically significant. To improve the accuracy of the Cooray-Rubinstein formula, a simple modification was provided by Cooray [8]. Moreover, his calculations showed that the accuracy of the Cooray-Rubinstein formula may become better when propagation effects were included in the magnetic field used in the surface impedance expression [9]. However, it is worthwhile to note that all these above researches are carried out in the case of vertical lightning channel, including the Cooray formula and Cooray-Rubinstein formula. In fact, the lightning channel is usually inclined or tortuous, which has great influence on the lightning electromagnetic field [10]. Due to traditional analytical methods in time domain are difficult to calculate directly the electromagnetic field from the inclined or tortuous channel at a lossy ground, some researchers used the finite difference time domain (FDTD) method to evaluate the effect of the inclined channel [11, 12]. But this method usually takes a long computation time, and is mainly applicable for close and intermediate distances. To calculate the horizontal field from inclined channel at far distance, Wan et al. [13] proposed an approximate method in frequency domain based on the extension of Cooray formula, which is applicable for the horizontal field at ground level. In this paper, in order to calculate the horizontal electric field from inclined lightning channel above a lossy ground, an extension of Cooray-Rubinstein formula for inclined

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lightning channel is derived, which is applicable for close, intermediate and far distances. Its validity is verified by comparing it with the results from the numerical solution of Sommerfeld's integrals using Bannister approximations, the accuracy of which has already been proved by Cooray [14]. Such an extended Cooray-Rubinstein approximation could be useful for analyzing the interaction of lightning electromagnetic fields with power lines or electrical installations due to the inclined or tortuous lightning channel.

Due to that Cooray formula is an important part of Cooray-Rubinstein approximation, the extension of Cooray formula for inclined lightning channel and its validation are firstly provided in this paper. Then, the extension of Cooray-Rubinstein formula for inclined lightning channel is presented. And the influences of the ground conductivity, the observation distance, and the observation height on the extended Cooray-Rubinstein formula for inclined lightning channel are also discussed, respectively.

II. EXTENSION OF COORAY FORMULA FOR INCLINED LIGHTNING CHANNEL

Figure 1 shows the geometry relevant to the calculation of lightning electromagnetic field from inclined lightning channel. According to the vector superposition principle, the inclined current-carrying channel element $I\mathrm{d}\boldsymbol{l}$ can be decomposed into horizontal component and vertical component [15]:

$$\begin{aligned} I\,\mathrm{d}\boldsymbol{l} &= I\,\mathrm{d}l\left(\sin\alpha\boldsymbol{e}_r+\cos\alpha\boldsymbol{e}_z\right) \\ &= I\,\mathrm{d}l\left(\sin\alpha\cos\varphi'\boldsymbol{e}_x+\sin\alpha\sin\varphi'\boldsymbol{e}_y+\cos\alpha\boldsymbol{e}_z\right), \end{aligned} \tag{1}$$

where I is the channel current, α and φ' are the inclined angle and azimuth angle of lightning channel, respectively. $\boldsymbol{e}_x$, $\boldsymbol{e}_y$, and $\boldsymbol{e}_z$ are the unit vectors. Here we will only deal with the differential segment of the inclined lightning channel since superposition assures the applicability of the result to the complete channel.

According to the Maxwell's equations in frequency-domain, we have:

$$\boldsymbol{E} = \nabla\left(\nabla\cdot\boldsymbol{A}\right)-\gamma_0\boldsymbol{A}, \tag{2}$$

$$\boldsymbol{B} = j\omega\varepsilon_0\left(\nabla\times\boldsymbol{A}\right), \tag{3}$$

where $\boldsymbol{E}$ and $\boldsymbol{B}$ are the total electric field and the total magnetic field generated by the inclined channel respectively. ω is the angular frequency, and $\boldsymbol{A} = A_x\boldsymbol{e}_x+A_y\boldsymbol{e}_x+A_z\boldsymbol{e}_z$ is the magnetic vector potential. $\gamma_0 = j\omega\sqrt{\mu_0\varepsilon_0}$, μ_0 and ε_0 are the magnetic permeability and electric permittivity of free space respectively.

We use primed letters for the coordinates of the source points and the unprimed letters for points at which the desired quantities are to be determined. The magnetic vector potential $\mathrm{d}\boldsymbol{A}^x$ generated by the horizontal element $I\,\mathrm{d}x\boldsymbol{e}_x$ can be approximately expressed in the rectangular coordinate system by [16]:

$$\begin{aligned} \mathrm{d}\boldsymbol{A}^x \approx &\frac{I\mathrm{d}x'}{2\pi j\omega\varepsilon_0}\frac{e^{-\gamma_0R}}{R}\frac{\sin\theta}{\sin\theta+\Delta^{-1}\sqrt{1-\Delta^2\cos^2\theta}}\boldsymbol{e}_x \\ &-\frac{I\mathrm{d}x'\cos\theta}{2\pi j\omega\varepsilon_0\gamma_1}\frac{\left(x-x'\right)}{\rho}\frac{e^{-\gamma_0R}}{R^2} \\ &\times\left[\left(1+\gamma_0R\Pi\right)-\frac{\sin\theta}{\gamma_1R}\left(2+2\gamma_0R+\gamma_0^2R^2\right)\right]\boldsymbol{e}_z, \end{aligned} \tag{4}$$

where r, φ and z are the cylindrical coordinates, $R=\sqrt{r^2+r'^2+z'^2-2rr'\cos\left(\varphi-\varphi'\right)}$, $\sin\theta=z'/R$, $\rho=\sqrt{\left(x-x'\right)^2+\left(y-y'\right)^2}$, $\gamma_1=\sqrt{j\omega\mu_0\left(\sigma+j\omega\varepsilon_0\varepsilon_r\right)}$, ε_r is the relative dielectric constant, and σ is the conductivity of the ground. $\Delta=\gamma_0/\gamma_1$, $\Delta_1=\Delta\sqrt{1-\Delta^2\cos^2\theta}$, $w=-\gamma_0R\left(\sin\theta+\Delta\right)^2/2$, $F\left(w\right)=1-j\sqrt{\pi w}e^{-w}erfc\left(j\sqrt{w}\right)$, and $\Pi=\left(\Delta_1F\left(w\right)-\sin\theta\right)/\left(\Delta_1-\sin\theta\right)$.

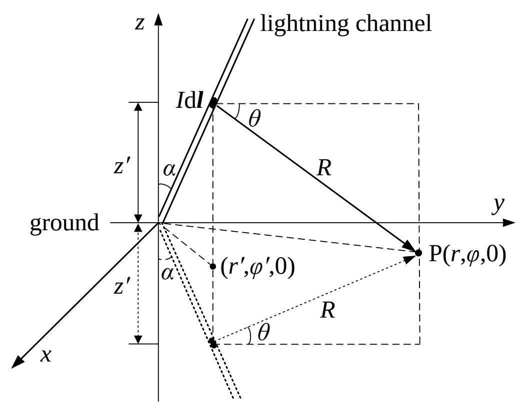


Fig. 1. Geometry relevant to this calculation.

Substituting (4) into (2) and (3), and applying $\left|\Delta^2\right|\ll 1$, we can obtain the horizontal electric field $\mathrm{d}E_r^{hx}$ and azimuthal magnetic field $\mathrm{d}B_\varphi^{hx}$ from the horizontal element $I\,\mathrm{d}x'\boldsymbol{e}_x$ at observation point P as:

$$\begin{aligned} \mathrm{d}E_r^{hx}\left(r,0,j\omega\right)= &-\frac{I\mathrm{d}x'}{2\pi j\omega\varepsilon_0}\frac{\left(x-x'\right)}{\rho}\gamma_0\left\{\gamma_0\Delta_1\left[\frac{\sin^2\theta}{\sin\theta+\Delta_1}\right.\right. \\ &\left.\left.-\frac{\Delta^2}{\sin\theta+\Delta_1}F\left(w\right)\right]\frac{e^{-\gamma_0R}}{R}-\frac{\Delta e^{-\gamma_0R}}{\gamma_1R^3}\left(1+\gamma_0R-\gamma_1R\sin\theta\right)\right\}, \end{aligned} \tag{5}$$

$$\begin{aligned} \mathrm{d}B_\varphi^{hx}\left(r,0,j\omega\right)= &\frac{\mu_0I\mathrm{d}x'}{2\pi}\frac{\left(x-x'\right)}{\rho}\left\{\gamma_0\left[\frac{\sin^2\theta}{\sin\theta+\Delta_1}\right.\right. \\ &\left.\left.-\frac{\Delta_1\Delta}{\sin\theta+\Delta_1}F\left(w\right)\right]\frac{e^{-\gamma_0R}}{R}-\frac{e^{-\gamma_0R}}{\gamma_1R^3}\left(1+\gamma_0R-\gamma_1R\sin\theta\right)\right\}. \end{aligned} \tag{6}$$

Note that if $\left|\Delta^2\right| \ll 1$, $\Delta_1 \approx \Delta$. Then (5) and (6) can be reduced to:

$$\mathrm{d}E_r^{hx}\left(r,0,j\omega\right) = -c\mathrm{d}B_\varphi^{hx}\left(r,0,j\omega\right)\frac{\gamma_0}{\gamma_1}, \tag{7}$$

where c is the speed of light in free space.

Likewise, we can obtain the relationship between the on-ground horizontal electric field $\mathrm{d}E_r^{hy}\left(r,0,j\omega\right)$ and azimuthal magnetic field $\mathrm{d}B_\varphi^{hy}\left(r,0,j\omega\right)$ generated by the horizontal element $I\,\mathrm{d}\,y'\boldsymbol{e}_y$ as:

$$\mathrm{d}E_r^{hy}\left(r,0,j\omega\right) = -c\mathrm{d}B_\varphi^{hy}\left(r,0,j\omega\right)\frac{\gamma_0}{\gamma_1}. \tag{8}$$

The relationship between the on-ground horizontal electric field $\mathrm{d}\,E_r^{v}\left(r,0,j\omega\right)$ and azimuthal magnetic field $\mathrm{d}\,B_\varphi^{v}\left(r,0,j\omega\right)$ generated by the vertical element $I\,\mathrm{d}\,z'\boldsymbol{e}_z$ can be given by the original Cooray formula for vertical lightning channel [3]:

$$\mathrm{d}\,E_r^{v}\left(r,0,j\omega\right) = -c\,\mathrm{d}\,B_\varphi^{v}\left(r,0,j\omega\right)\frac{\gamma_0}{\gamma_1}. \tag{9}$$

Performing the vector superposition to the contributions of all channel elements according to (1), (7), (8) and (9), we can obtain the relationship between the horizontal electric field $E_r\left(r,0,j\omega\right)$ and azimuthal magnetic field $B_\varphi\left(r,0,j\omega\right)$ at ground level generated by the inclined channel as follow:

$$E_r\left(r,0,j\omega\right) = -cB_\varphi\left(r,0,j\omega\right)\frac{\gamma_0}{\gamma_1}. \tag{10}$$

The form of Equation (10) is the same as original Cooray formula for vertical lightning channel. Note, however, that here $B_\varphi\left(r,0,j\omega\right)$ is the azimuthal magnetic field at ground level generated by the inclined lightning channel. Moreover, due to all the above derivations are carried out based on an inclined differential segment of the lightning channel in the space, Equation (10) is suitable for even tortuous lightning channel.

III. VALIDATION OF THE EXTENDED COORAY FORMULA

Cooray [14] has shown that the Bannister approximations could basically provide an accurate description of the lightning generated electric fields over finitely conducting ground. Thus, to verify the reliability of the extended Cooray formula, we have calculated the horizontal electric fields at ground level at distances of 100m, 500m, and 1000m from the return stroke by employing the numerical solution of Sommerfeld's integrals using Bannister approximations, and compared the results with those obtained from (10), as shown in Fig. 2. Note that here, for all the following calculations of the fields in the paper, the lightning channel-base current in [17] and MTLE model are adopted, and the return stroke speed is set to 1.5×10^8m/s. All calculations are performed using Matlab codes.

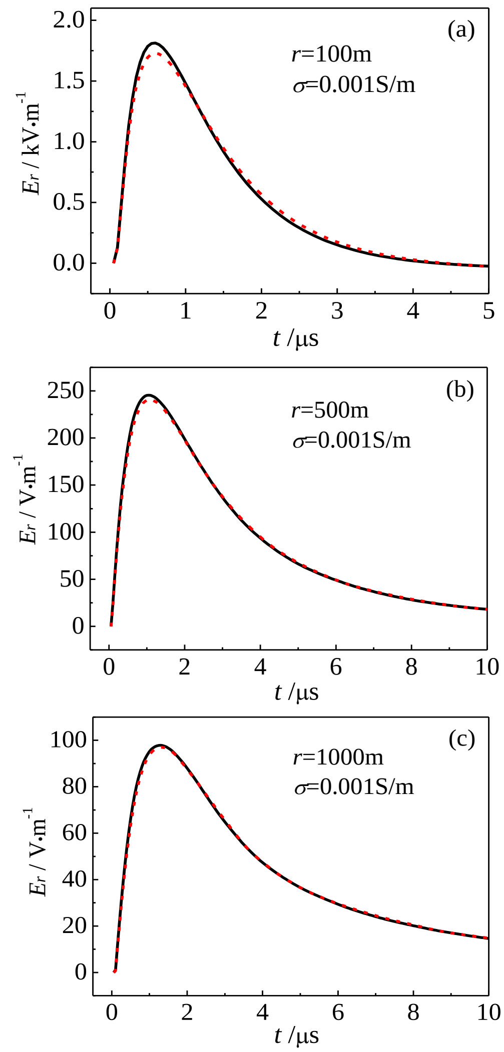


Fig. 2. Comparison of the horizontal fields from the numerical solution of Sommerfeld's integrals (solid line) at ground level with those from the extended Cooray formula (dotted line): (a) r=100m, (b) r=500m, and (c) r=1000m. The inclined angle of lightning channel $\alpha = \pi/6$, the azimuth angle $\varphi = 3\pi/4$, and $\varphi' = \pi/2$.

As follows from Fig. 2, the horizontal electric field waveforms from the extended Cooray formula are essentially coincident with those from the numerical solution of Sommerfeld's integrals beyond 100m. Note that there is only some deviation on the peak value of the waveform for the distance within 100m. The deviation of the peak value is about 4.6% at the distance of 100m.

In order to test the influences of the ground conductivity, the observation distance, the inclined angle of lightning channel and the azimuth angle of observation point on the accuracy of the extended formula, the peak value ratios K of the waveforms from the extended formula to those from the numerical solution of Sommerfeld's integrals are also calculated, as shown in Fig. 3.

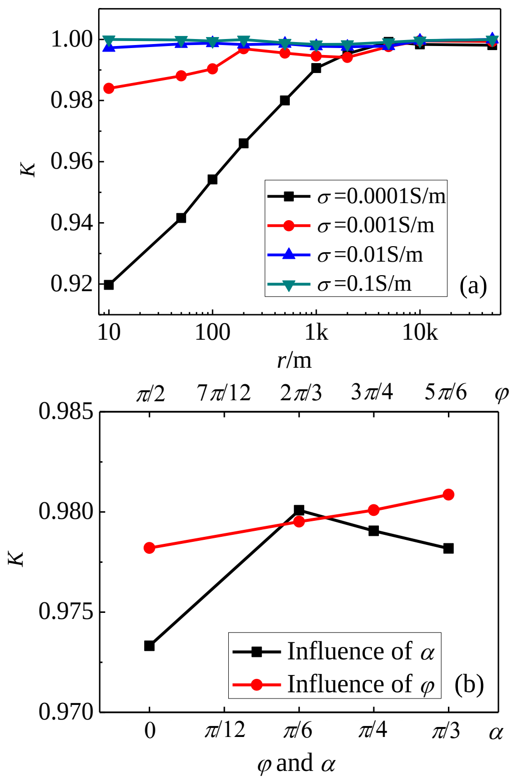


Fig. 3. Influences of the ground conductivity σ, the observation distance r, the channel inclined angle α and the observation azimuth angle φ on the accuracy of the extended Cooray formula. (a) The peak value ratios K of the waveforms under different σ and r, and (b) the peak value ratios K of the waveforms under different α and φ.

As shown in Fig. 3 (a), the accuracy of the extended Cooray formula is related to the ground conductivity and the observation distance. The ratio K will tend to 1.0 with the increasing ground conductivity σ and observation distance r. In other words, the bigger the ground conductivity or the observation distance is, the smaller the deviation of the extended formula is. As shown in Fig. 3 (b), the ratio K is less than 0.9733 for different inclined angles α and azimuth angles φ. That is to say, the deviation of the results calculated by the extended formula from the numerical solution of Sommerfeld's integrals is basically less than 2.67% for different inclined angles of lightning channel and different azimuth angles of observation point, which is very small and can nearly be ignored.

IV. EXTENSION OF COORAY-RUBINSTEIN FORMULA FOR INCLINED LIGHTNING CHANNEL

In this Section, we will present the derivation of the extended Cooray-Rubinstein formula for inclined lightning channel. The geometry relevant to this problem is basically the same as Fig. 1 except that the observation point P(r, φ, z) is off the ground. To calculate the horizontal electric field generated by the inclined channel at a height z=h, the Faraday's equation for a closed path is employed as following:

$$\oint_{l'} \boldsymbol{E}\cdot \mathrm{d}\boldsymbol{l}' = -j\omega\int_S \boldsymbol{B}\cdot \mathrm{d}\boldsymbol{S}, \tag{11}$$

where d$\boldsymbol{l}'$ is a rectangular path of width dr extending from z=0 to z=h, and d$\boldsymbol{S}$ is the surface surrounded by the rectangular path, as shown in Fig. 4. The direction of the surface d$\boldsymbol{S}$ is defined by the direction of the contour l' and the right-hand rule.

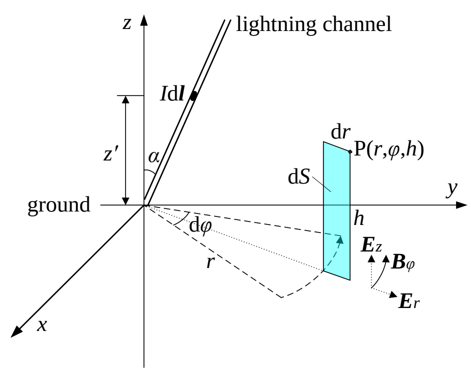


Fig. 4. Geometry relevant to the Faraday's equation shown in (11) for the inclined lightning channel. $\boldsymbol{E}_r$, $\boldsymbol{E}_z$ and $\boldsymbol{B}_\varphi$ are the horizontal electric field, vertical electric field and azimuthal magnetic field generated by the inclined lightning channel, respectively.

Integrating the Faraday's Equation (11), we can obtain:

$$E_r(r,h,j\omega)\mathrm{d}r - E_r(r,0,j\omega)\mathrm{d}r + \int_0^h E_z(r,z,j\omega)\mathrm{d}z - \int_0^h E_z(r+\mathrm{d}r,z,j\omega)\mathrm{d}z = -j\omega\,\mathrm{d}r\int_0^h B_\varphi(r,z,j\omega)\mathrm{d}z, \tag{12}$$

where $E_z(r,z,j\omega)$, and $E_z(r+\mathrm{d}r,z,j\omega)$ are the vertical electric field generated by the inclined lightning channel at radial distance r and r+dr, respectively.

For the case of a perfectly conducting ground, the horizontal electric field at ground level is zero. Thus, we can write (12) as:

$$E_{rp}(r,h,j\omega)\mathrm{d}r + \int_0^h E_{zp}(r,z,j\omega)\mathrm{d}z - \int_0^h E_{zp}(r+\mathrm{d}r,z,j\omega)\mathrm{d}z = -j\omega\,\mathrm{d}r\int_0^h B_{\varphi p}(r,z,j\omega)\mathrm{d}z, \tag{13}$$

where E_{rp}, E_{zp}, and $B_{\varphi p}$ are the horizontal electric field, vertical electric field, and azimuthal magnetic field generated by the inclined lightning channel for the case of a perfectly conducting ground, respectively.

For distances not exceeding several kilometers, several researchers have shown that the perfect ground conductivity assumption is a reasonable approximation for the vertical electric field and the azimuthal magnetic field [4, 5, 10, 18]. Thus, we can approximate the field components B_φ and E_z as:

$$B_\varphi \approx B_{\varphi p}, \text{ and } E_z \approx E_{zp}. \tag{14}$$

Substituting (14) into (12), we can obtain:

$$E_r(r,h,j\omega)\,\mathrm{d}r - E_r(r,0,j\omega)\,\mathrm{d}r + \int_0^h E_{zp}(r,z,j\omega)\mathrm{d}z - \int_0^h E_{zp}(r+\mathrm{d}r,z,j\omega)\mathrm{d}z \approx -j\omega\,\mathrm{d}r\int_0^h B_{\varphi p}(r,z,j\omega)\mathrm{d}z. \tag{15}$$

Comparing equation (13) with (15), we can find that the right sides of these two equations are the same. Thus, Subtracting (13) from (15), we can obtain the total horizontal electric field $E_r(r,h,j\omega)$ generated by the inclined channel at a height z=h as:

$$E_r(r,h,j\omega) = E_r(r,0,j\omega) + E_{rp}(r,h,j\omega). \tag{16}$$

Substituting the extended Cooray formula (10) into Equation (16), we can obtain the extended Cooray-Rubinstein formula as:

$$E_r(r,h,j\omega) = -cB_\varphi(r,0,j\omega)\frac{\gamma_0}{\gamma_1} + E_{rp}(r,h,j\omega). \tag{17}$$

Just like the extended Cooray formula, the form of the extended Cooray-Rubinstein formula for the inclined channel is also the same as that for vertical lightning channel. Note that here $B_\varphi(r,0,j\omega)$ is the azimuthal magnetic field at ground level generated by the inclined lightning channel, and $E_{rp}(r,h,j\omega)$ is the horizontal electric field at a height z=h for the case of a perfectly conducting ground generated by the inclined lightning channel. Moreover, the derivation of the extended Cooray-Rubinstein formula is basically independent of the shape of the lightning channel. Thus, Equation (17) is also suitable for even tortuous lightning channel.

Specially, if the observation point P is at ground level, we have $E_{rp}(r,h,j\omega)=0$ in (17). Then, the extended Cooray-Rubinstein formula will reduce to the extended Cooray formula.

V. VALIDATION OF THE EXTENDED COORAY-RUBINSTEIN FORMULA

To verify the reliability of the extended Cooray-Rubinstein formula, the horizontal electric fields at a height z=10m are calculated by adopting the extended Cooray-Rubinstein formula (17) and the numerical solution of Sommerfeld's integrals using Bannister approximations. Comparisons of the horizontal electric fields from the extended Cooray-Rubinstein formula with those from the numerical solution of Sommerfeld's integrals at different distances are shown in Fig. 5 (for the case of σ=0.0001 S/m) and Fig. 6 (for the case of σ=0.001 S/m). To analyze the influence of the observation distance r on the accuracy of the extended Cooray-Rubinstein formula, the peak time ratios K_t of the waveforms from the extended formula to those from the numerical solution of Sommerfeld's integrals under different distances, as well as the peak value ratios K of the waveforms, are calculated, as shown in Fig. 7.

As follows from Fig. 5 and Fig. 7, the accuracy of the extended Cooray-Rubinstein formula increases gradually with the distance r increases. The closer the distance is, the less the deviations are. For the case of σ=0.0001 S/m, when the observation distance is less than 10 m, there is a visible difference between the horizontal electric field waveforms from the numerical solution of Sommerfeld's integrals and the extended Cooray-Rubinstein formula after the rising edge. At distance r=10m, the deviation of the peak value is about 10.4%, and the deviation of the peak time is about 52.9%. However, when the observation distance exceeds 500m, the horizontal electric field waveforms from the extended Cooray-Rubinstein formula are basically coincident with those from the numerical solution of Sommerfeld's integrals. The deviation of the peak value is less than 6.2%, and the deviation of the peak time is less than 5.3%, as shown in Fig. 7. Thus, it can be concluded that, for the case that the observation height is 10m and the ground conductivity is not less than 0.0001 S/m, the extended Cooray-Rubinstein formula could yield an acceptable approximation on the horizontal electric field at the distance beyond 500m.

Comparing Fig. 5 with Fig. 6, it can be found that the accuracy of the extended Cooray-Rubinstein formula increases with the ground conductivity σ increases. As shown in Fig. 7, the greater the ground conductivity is, the less the deviations are. As shown in Fig. 6, for the case of σ=0.001 S/m, the horizontal electric field waveforms from the extended Cooray-Rubinstein formula are essentially coincident with those from the numerical solution of Sommerfeld's integrals, even at the distance r=10m. As follows from Fig. 7, the deviation of the peak value for the case of σ=0.001 S/m is less than 2.1%, and the deviation of the peak time is less than 6.3%. Especially for the observation distance beyond 500m, there is not any deviation on the peak time between the waveforms from the extended formula and the numerical solution of Sommerfeld's integrals. Thus, it can be concluded that, for the case that the observation height is 10m and the ground conductivity is not less than 0.001 S/m, the extended Cooray-Rubinstein formula could yield a satisfactory approximation on the horizontal electric field at close (10m), intermediate (some kilometers), and far (tens of kilometers) distances.

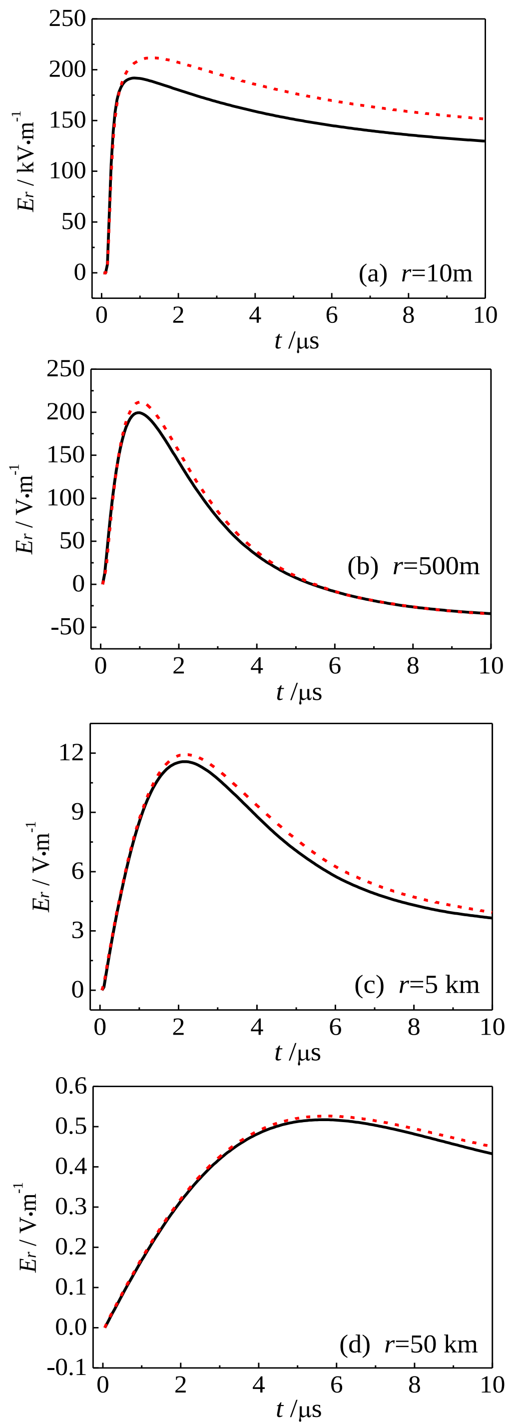


Fig. 5. Comparison of the horizontal field from the numerical solution of Sommerfeld's integrals (solid line) at a height z=10m with that from the extended Cooray-Rubinstein formula (dotted line): (a) r=10m, (b) r=500m, (c) r=5km, and (d) r=50km. The ground conductivity is set to σ=0.0001S/m. The inclined angle of lightning channel $\alpha=\pi/6$, the azimuth angle $\varphi=\pi/2$, and $\varphi'=\pi/4$.

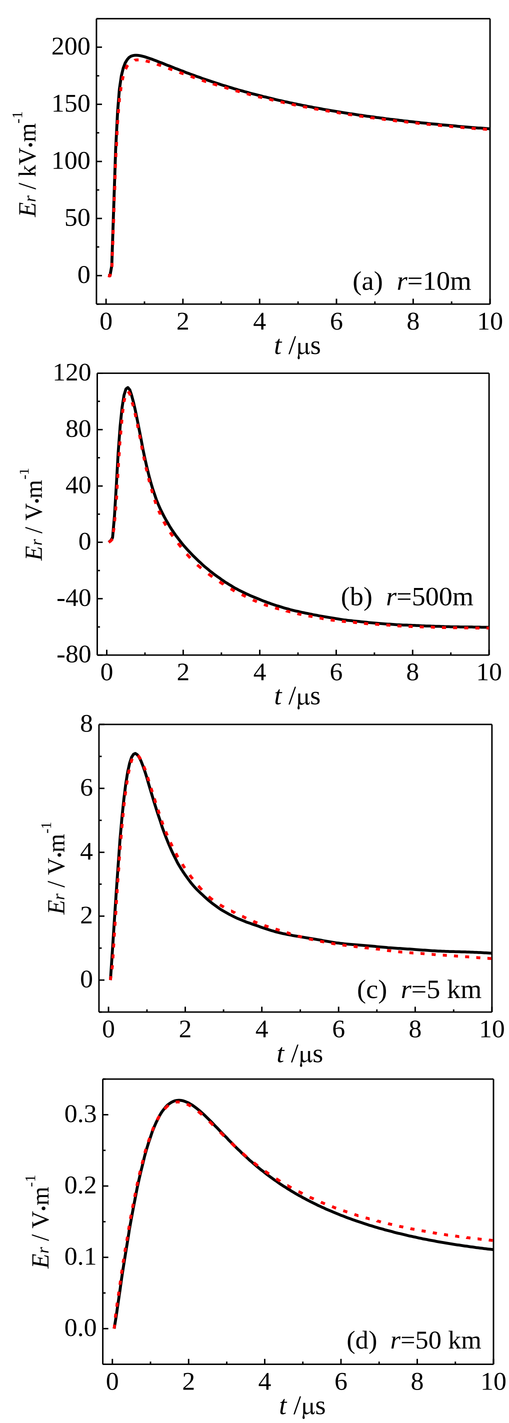


Fig. 6. Comparison of the horizontal field from the numerical solution of Sommerfeld's integrals (solid line) at a height z=10m with that from the extended Cooray-Rubinstein formula (dotted line): (a) r=10m, (b) r=500m, (c) r=5km, and (d) r=50km. The ground conductivity is set to σ=0.001S/m. The inclined angle of lightning channel $\alpha=\pi/6$, the azimuth angle $\varphi=\pi/2$, and $\varphi'=\pi/4$.

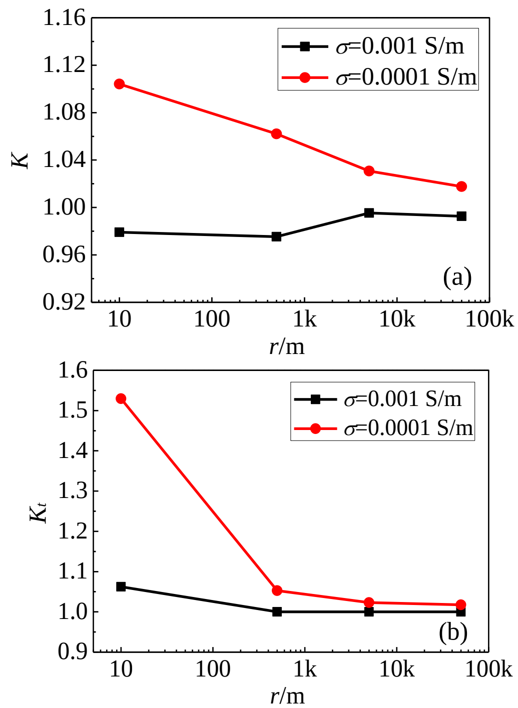
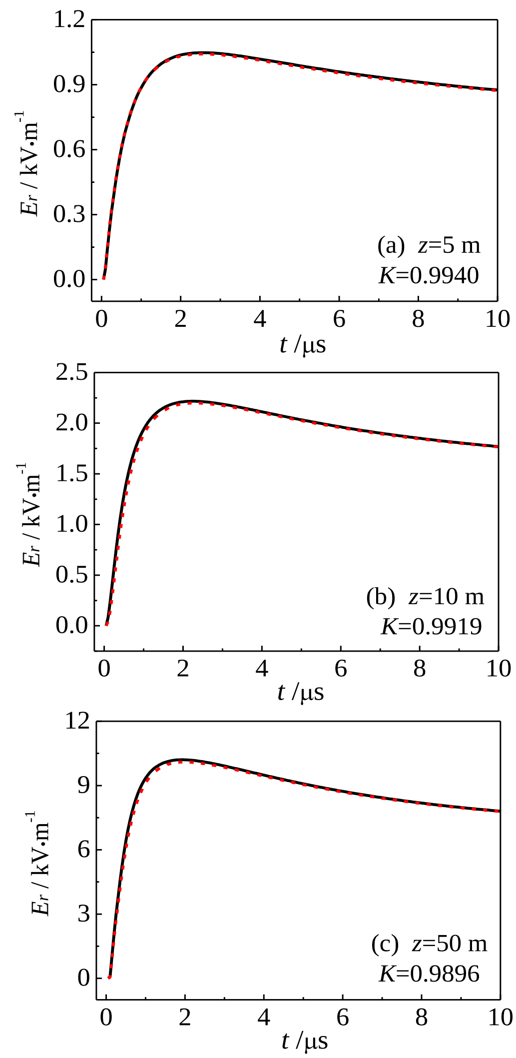


Fig. 7. Influences of the observation distance r on the accuracy of the extended Cooray-Rubinstein formula. (a) The peak value ratios K of the waveforms under different distances, and (b) the peak time ratios K_t of the waveforms under different distances.

Comparing Fig. 5 with Fig. 6, it can be found that the accuracy of the extended Cooray-Rubinstein formula increases with the ground conductivity σ increases. As shown in Fig. 7, the greater the ground conductivity is, the less the deviations are. As shown in Fig. 6, for the case of σ=0.001 S/m, the horizontal electric field waveforms from the extended Cooray-Rubinstein formula are essentially coincident with those from the numerical solution of Sommerfeld's integrals, even at the distance r=10m. As follows from Fig. 7, the deviation of the peak value for the case of σ=0.001 S/m is less than 2.1%, and the deviation of the peak time is less than 6.3%. Especially for the observation distance beyond 500m, there is not any deviation on the peak time between the waveforms from the extended formula and the numerical solution of Sommerfeld's integrals. Thus, it can be concluded that, for the case that the observation height is 10m and the ground conductivity is not less than 0.001 S/m, the extended Cooray-Rubinstein formula could yield a satisfactory approximation on the horizontal electric field at close (10m), intermediate (some kilometers), and far (tens of kilometers) distances.

VI. INFLUENCE OF THE OBSERVATION HEIGHT ON THE EXTENDED COORAY-RUBINSTEIN FORMULA

To evaluate the influence of the observation height on the accuracy of the extended Cooray-Rubinstein formula, the horizontal electric fields at different heights from the extended Cooray-Rubinstein formula and the numerical solution of Sommerfeld's integrals are also calculated here, as shown in Fig. 8. In this calculation, the observation distance is set to r=100m, and the ground conductivity is set to σ=0.001 S/m. Due to that there is basically not any deviation on the peak time between the waveforms from the extended formula and the numerical solution of Sommerfeld's integrals, only the peak value ratios K of the waveforms are calculated here to evaluate the influence of the observation height, which are also shown in Fig. 8.



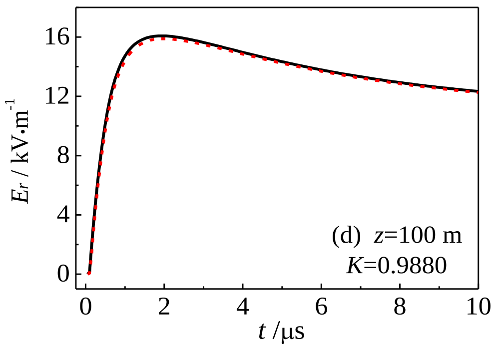


Fig. 8. Comparison of the horizontal field from the numerical solution of Sommerfeld's integrals (solid line) at a distance r=100m with that from the extended Cooray-Rubinstein formula (dotted line): (a) z=5m, (b) z=10m, (c) z=50m, and (d) z=100m. The ground conductivity is set to σ=0.001 S/m. The inclined angle of lightning channel $\alpha = \pi/6$, the azimuth angle $\varphi = \pi/2$, and $\varphi' = \pi/4$.

As follows from Fig. 8 and the calculation results about the peak value ratios K of the waveforms, it can be found that the horizontal electric field waveforms from the extended Cooray-Rubinstein formula are essentially coincident with those from the numerical solution of Sommerfeld's integrals at the height from z=5m to z=100m. According to the calculation results about the peak value ratios K, the deviation of the peak value increases slightly with the observation height increases. However, for the observation point with a height less than 100m, the deviation of the peak value is less than 1.2%, which is very small and can nearly be ignored. Thus, it can be concluded that the influence of the height on the extended Cooray-Rubinstein formula can be negligible while it is used for analyzing the interaction of lightning electromagnetic fields with power lines or electrical installations near the ground.

VII. CONCLUSION

In this paper, an extension of Cooray-Rubinstein formula was presented for the calculation the horizontal electric field from inclined lightning channel in frequency-domain. For this purpose, the Cooray formula was extended firstly for the calculation of horizontal electric field at ground level generated by the inclined channel. The influences of the ground conductivity, the observation distance, the inclined angle of lightning channel and the azimuth angle of observation point on the accuracy of the extended Cooray formula were also analyzed. It was found that the accuracy of the extended Cooray formula increases with the increasing ground conductivity and observation distance, but is not insensitive to the channel inclined angle and the observation azimuth angle. The extended Cooray formula can predict the horizontal field accurately at the distance beyond 100m.

Based on the extended Cooray formula, the Cooray-Rubinstein formula was extended for the calculation of horizontal electric field off the ground generated by the inclined channel. The accuracy of the extended Cooray-Rubinstein formula for inclined lightning channel was also validated by comparing the results with those from the numerical solution of Sommerfeld's integrals. The results showed that the accuracy of the extended Cooray-Rubinstein formula increases with the observation distance and the ground conductivity increase, but is not insensitive to the observation height. It was shown that the extended Cooray-Rubinstein formula could yield an acceptable approximation on the above-ground horizontal electric field with the exception of the observation point at close range under the ground conductivity less than 0.0001S/m.

It is worth noting that this extended Cooray-Rubinstein formula has no limitation regarding the channel shape so that it could be used for the calculation of horizontal electric field generated by any tortuous lightning channel.

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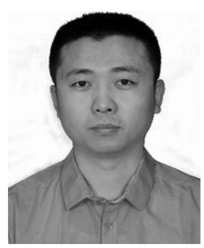

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