Wide-Angle Claerbout Scheme for Three-Dimensional Time Domain Parabolic Equation and its CN, ADI, AGE Solutions

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***Abstract* –** The wide-angle Claerbout scheme of three-dimensional time domain parabolic equation (Claerbout-TDPE) is derived in this paper, which can provide accurate results at angles within 25° of the paraxial direction. At first, the Crank-Nicolson (CN) type is introduced to discretize the Claerbout-TDPE. In this way, a three-dimensional EM scattering problem can be divided into a series of two-dimensional ones. Moreover, the alternating direction implicit (ADI) type is utilized to the Claerbout-TDPE. In this way, a three-dimensional EM scattering problem can be further reduced to a series of one-dimensional ones. Furthermore, the alternating group explicit (AGE) type is introduced to the Claerbout-TDPE for higher computational efficiency. Comparisons are made among the CN, ADI and AGE types in the numerical results.

***Index Terms* –** Electromagnetic scattering, time domain parabolic equation, wide angle.

I. INTRODUCTION

The parabolic equation (PE) has been applied to study the wave propagation [1-3] and EM scattering problems [4-7] for several decades. The split-step Fourier-based PE is extremely attractive for its computational efficiency. However, it lacks the flexibility of boundary modeling for complicated targets. Therefore, it is best to use the FD schemes when dealing with problems of complicated boundaries. When applying the finite difference method to the paraxial direction, the PE can be solved in a marching manner. As a result, a series of two-dimensional problems are needed to be computed instead of a three-dimensional problem. Therefore, the computational efficiency can be improved significantly. However, the traditional PE methods are based on finite difference schemes in each transverse plane, which use rectangular meshes. This kind of meshes will result in poor accuracy to approximate complex targets. Moreover, the PE method cannot model the scattering targets with large changes along paraxial direction since the creeping waves cannot be captured. Therefore, some hybrid PE methods have been proposed by us to improve the accuracy and expand the application of the traditional PE method for EM scattering problems [8-11]. Moreover, some novel finite difference schemes and parallel technologies are introduced to accelerate the calculation of PE method [2-3, 11-13]. All of these works are focus on narrow-angle approximation of parabolic equation, which can obtain accurate results within 15° of the paraxial direction. Wide-angle parabolic equation methods were introduced for wider angle EM analysis by using high-order expansions of exponential or square-root functions. The Pade approximation [7, 14-17] is the most popular one, but may result in bad computational efficiency or instability with the Pade order increasing. The right approach is to use rational approximations instead. Therefore, the development of the Claerbout solution has a practical significance. The error for a plane wave propagating at angle α from the horizontal is of the order of $(\sin\alpha)^6$, which makes it acceptable for propagation angles up to 45 degrees or so from the paraxial direction.

In recent years, the transient EM scattering analysis becomes a hot topic. Some numerical methods are used for wide-band analysis, such as the time domain integral equation (TDIE) method, the finite-different time-domain (FDTD) method and so on. However, it is time-consuming to obtain the transient EM properties by these numerical methods. Therefore, the time domain parabolic equation (TDPE) method was proposed to fast solve the transient problems [18-23]. By implementing the finite difference scheme along both the temporal and paraxial directions, the calculation can be taken plane by plane for each time step. It can be found that high efficiency can be obtained by the TDPE method [22]. Moreover, we proposed a marching-on-in-degree solver

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of TDPE in which the weighted Laguerre polynomials are used to expand the electrical fields [23-25]. However, all these methods can only provide accurate results at a small angle of 15° along the paraxial direction.

In this paper, the three-dimensional time domain wide-angle Claerbout parabolic equation (Claerbout-TDPE) is formulated firstly. Accurate results can be obtained at angles within 25° of the paraxial direction by the Claerbout-TDPE method while 15° for the traditional TDPE method. Then three different kinds of finite difference schemes are introduced to solve the Claerbout-TDPE, namely the Crank-Nicolson (CN) scheme, the alternating direction implicit (ADI) scheme and the alternating group explicit (AGE) scheme. For CN scheme, the calculation can be performed in a marching manner along the paraxial direction for each time step. For ADI scheme, the unknowns can be computed row by row or column by column in each transverse plane for any time step, thus the computational efficiency is further improved. For AGE scheme, the reduced transient scattered fields in each transverse plane of any time step are obtained directly without solving any matrix equations. As a result, it has the best performance among CN, ADI and AGE schemes. Moreover, the bistatic RCS result at any observed angle can be obtained by rotating Claerbout-TDPE method. Several numerical examples are given to demonstrate the validity of the proposed method and comparisons are made among different schemes. It can be observed that the AGE scheme of Claerbout-TDPE method can save the computational resources when compared with both the CN and ADI schemes.

II. THEORY

The wide-angle Claerbout TDPE

The standard parabolic equation in frequency domain can be expressed as:

$$\left[\frac{\partial}{\partial x}+ik(1-\sqrt{1+Q})\right]u_\xi^s=0 \qquad \xi=x,y,z, \qquad (1)$$

where u_ξ^s denotes the reduced scattered fields, the x axis is supposed to be the paraxial direction of the parabolic equation. The pseudo-differential operator Q in free space is defined by:

$$Q=\frac{1}{k^2}\left(\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right). \qquad (2)$$

The square-root operator in (1) is expanded with the one order Pade approximation [7], which can be derived as:

$$\frac{\partial u_\xi^s}{\partial x}+\frac{0.25}{k^2}\frac{\partial^3 u_\xi^s}{\partial x\partial^2 y}+\frac{0.25}{k^2}\frac{\partial^3 u_\xi^s}{\partial x\partial^2 z} =ik\left[\frac{0.5}{k^2}\left(\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\right]u_\xi^s \qquad \xi=x,y,z. \qquad (3)$$

Introduce the Fourier transform along the *x*,*y*,*z* axes, which is given by:

$$\Pi_\xi^s(x,y,z,s)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{F}(k)u_\xi^s(x,y,z,k)e^{-iks}dk, \qquad \xi=x,y,z \qquad (4)$$

where $\tilde{F}(k)$ represents the spectrum function, $s=ct-x$ and c is the light speed.

Then the three-dimensional time domain wide-angle Claerbout parabolic equation (Claerbout-TDPE) can be derived by using the forward Fourier transform:

$$4\frac{1}{c^2}\frac{\partial^3\Pi_\xi^s}{\partial t^2\partial x}-2\frac{1}{c}\left(\frac{\partial^3\Pi_\xi^s}{\partial t\partial y^2}+\frac{\partial^3\Pi_\xi^s}{\partial t\partial z^2}\right)=\frac{\partial^3\Pi_\xi^s}{\partial x\partial^2 y}+\frac{\partial^3\Pi_\xi^s}{\partial x\partial^2 z}. \qquad \xi=x,y,z \qquad (5)$$

By introducing the CN scheme to both the temporal and marching steps, the semi-discretized formula for (5) can be obtained:

$$\frac{4}{c^2\Delta x\Delta t^2}\begin{pmatrix}\Pi_{\xi,n+1,l+1}^s-\Pi_{\xi,n,l+1}^s+\Pi_{\xi,n+1,l-1}^s\\ -\Pi_{\xi,n,l-1}^s-2\Pi_{\xi,n+1,l}^s+2\Pi_{\xi,n,l}^s\end{pmatrix} =\left(1+\frac{2}{c\Delta t}\right)\begin{pmatrix}\frac{1}{\Delta y^2}\nabla_y^2\left(\Pi_{\xi,n,l+1}^s-\Pi_{\xi,n,l}^s\right)+\\ \frac{1}{\Delta z^2}\nabla_z^2\left(\Pi_{\xi,n,l+1}^s-\Pi_{\xi,n,l}^s\right)\end{pmatrix}, \qquad \xi=x,y,z \qquad (6)$$

in which, $\Pi^s{}_{\xi,n,l}$ is the transient reduced scattered fields for ξ component, Δx denotes the range step along the paraxial direction, Δt represents the time step, and ∇_y^2,∇_z^2 are the second-order difference operators.

It can be observed from (6) that the calculation can be taken plane by plane along the paraxial direction for any time step. As a result, the computational resources can be saved significantly. The CN, ADI, AGE solutions of Claerbout-TDPE are discussed as follows.

The equations in (6) are coupled with the inhomogeneous boundary conditions. For the conducting targets, the tangential total field is zero on the surface, which yields:

$$\begin{aligned}
&n_x\Pi_y^s(x_b,y_b,z_b,t)-n_y\Pi_x^s(x_b,y_b,z_b,t)=\\
&\qquad -n_xE_y^i(x_b,y_b,z_b,t-x_b/c)+n_yE_x^i(x_b,y_b,z_b,t-x_b/c)\\
&n_x\Pi_z^s(x_b,y_b,z_b,t)-n_z\Pi_x^s(x_b,y_b,z_b,t)=\\
&\qquad -n_xE_z^i(x_b,y_b,z_b,t-x_b/c)+n_zE_x^i(x_b,y_b,z_b,t-x_b/c)\\
&n_y\Pi_z^s(x_b,y_b,z_b,t)-n_z\Pi_y^s(x_b,y_b,z_b,t)=\\
&\qquad -n_yE_z^i(x_b,y_b,z_b,t-x_b/c)+n_zE_y^i(x_b,y_b,z_b,t-x_b/c)
\end{aligned}, \qquad (7)$$

where $\mathbf{E}^\mathbf{i}$ is the incident field, (x_b,y_b,z_b) represents the boundary point, and (n_x,n_y,n_z) denotes the outer normal component.

Moreover, the divergence-free condition is used to ensure the unicity [4]:

$$-\frac{1}{2}\left(\frac{\partial^2\Pi_x^s}{\partial y^2}+\frac{\partial^2\Pi_x^s}{\partial z^2}\right)+\frac{1}{c^2}\frac{\partial^2\Pi_x^s}{\partial t^2}+\frac{1}{c}\frac{\partial^2\Pi_y^s}{\partial y\partial t}+\frac{1}{c}\frac{\partial^2\Pi_z^s}{\partial z\partial t}=0\,. \quad (8)$$

CN solution of the Claerbout-TDPE

By introducing the CN scheme to (6), the full-discretized form of Claerbout-TDPE can be obtained:

$$\begin{aligned}
&\left(\frac{4}{c^2\Delta x\Delta t^2}+\frac{1}{c\Delta t\Delta y^2}+\frac{1}{\Delta x\Delta y^2}+\frac{1}{c\Delta t\Delta z^2}+\frac{1}{\Delta x\Delta z^2}\right)\Pi_{\xi,n+1,l+1}^{s,p,q}-\left(\frac{1}{2c\Delta t\Delta y^2}+\frac{1}{2\Delta x\Delta y^2}\right)\left(\Pi_{\xi,n+1,l+1}^{s,p+1,q}+\Pi_{\xi,n+1,l+1}^{s,p-1,q}\right)\\
&\quad-\left(\frac{1}{2c\Delta t\Delta z^2}+\frac{1}{2\Delta x\Delta z^2}\right)\left(\Pi_{\xi,n+1,l+1}^{s,p,q+1}+\Pi_{\xi,n+1,l+1}^{s,p,q-1}\right)\\
&=\frac{4}{c^2\Delta x\Delta t^2}\left(2\Pi_{\xi,n+1,l}^{s,p,q}-2\Pi_{\xi,n,l}^{s,p,q}+\Pi_{\xi,n,l+1}^{s,p,q}+\Pi_{\xi,n,l-1}^{s,p,q}-\Pi_{\xi,n+1,l-1}^{s,p,q}\right)+\frac{1}{2\Delta x\Delta y^2}\begin{pmatrix}\Pi_{\xi,n+1,l}^{s,p+1,q}+\Pi_{\xi,n+1,l}^{s,p-1,q}-2\Pi_{\xi,n+1,l}^{s,p,q}\\-\Pi_{\xi,n,l}^{s,p+1,q}-\Pi_{\xi,n,l}^{s,p-1,q}+2\Pi_{\xi,n,l}^{s,p,q}\end{pmatrix}\\
&\quad+\frac{1}{2\Delta x\Delta z^2}\begin{pmatrix}\Pi_{\xi,n+1,l}^{s,p,q+1}+\Pi_{\xi,n+1,l}^{s,p,q-1}-2\Pi_{\xi,n+1,l}^{s,p,q}\\-\Pi_{\xi,n,l}^{s,p,q+1}-\Pi_{\xi,n,l}^{s,p,q-1}+2\Pi_{\xi,n,l}^{s,p,q}\end{pmatrix}+\left(\frac{1}{2c\Delta t\Delta y^2}-\frac{1}{2\Delta x\Delta y^2}\right)\left(\Pi_{\xi,n,l+1}^{s,p+1,q}+\Pi_{\xi,n,l+1}^{s,p-1,q}-2\Pi_{\xi,n,l+1}^{s,p,q}\right)\\
&\quad+\left(\frac{1}{2c\Delta t\Delta z^2}-\frac{1}{2\Delta x\Delta z^2}\right)\left(\Pi_{\xi,n,l+1}^{s,p,q+1}+\Pi_{\xi,n,l+1}^{s,p,q-1}-2\Pi_{\xi,n,l+1}^{s,p,q}\right)-\frac{1}{2c\Delta t\Delta y^2}\begin{pmatrix}\Pi_{\xi,n+1,l-1}^{s,p+1,q}+\Pi_{\xi,n+1,l-1}^{s,p-1,q}-2\Pi_{\xi,n+1,l-1}^{s,p,q}\\+\Pi_{\xi,n,l-1}^{s,p+1,q}+\Pi_{\xi,n,l-1}^{s,p-1,q}-2\Pi_{\xi,n,l-1}^{s,p,q}\end{pmatrix}\\
&\quad-\frac{1}{2c\Delta t\Delta z^2}\begin{pmatrix}\Pi_{\xi,n+1,l-1}^{s,p,q+1}+\Pi_{\xi,n+1,l-1}^{s,p,q-1}-2\Pi_{\xi,n+1,l-1}^{s,p,q}\\+\Pi_{\xi,n,l-1}^{s,p,q+1}+\Pi_{\xi,n,l-1}^{s,p,q-1}-2\Pi_{\xi,n,l-1}^{s,p,q}\end{pmatrix}\qquad \xi=x,y,z
\end{aligned} \quad (9)$$

where $\Delta y,\Delta z$ denote the range steps along the y, z directions, $\Pi_{\xi,n,l}^{s,p,q}$ is the ξ component of the transient reduced scattered field for $(n\Delta x, p\Delta y, q\Delta z)$ at the $l\,th$ time step.

It can be seen that the unknowns in the $(n+1)th$ transverse plane for the $(l+1)th$ time step can be computed by the known fields of previous time steps and transverse planes. In this way, the Claerbout-TDPE can be calculated in a marching manner.

ADI solution of the Claerbout-TDPE

The ADI solution of the wide-angle Claerbout PE in frequency domain has been introduced in [16]. Then its time domain counterpart can be derived as:

$$\begin{aligned}
&\begin{bmatrix}\frac{1}{4c\Delta t\Delta y^2} & -\left(\frac{1}{2c\Delta t\Delta y^2}+\frac{1}{\Delta x\Delta t^2c^2}\right) & \frac{1}{4c\Delta t\Delta y^2}\end{bmatrix}\begin{bmatrix}\Pi_{\xi,n+1/2,l+1}^{s,p-1,q}\\\Pi_{\xi,n+1/2,l+1}^{s,p,q}\\\Pi_{\xi,n+1/2,l+1}^{s,p+1,q}\end{bmatrix}\\
&=\begin{bmatrix}-\frac{1}{4c\Delta t\Delta z^2} & \left(\frac{1}{2c\Delta t\Delta z^2}-\frac{1}{\Delta x\Delta t^2c^2}\right) & -\frac{1}{4c\Delta t\Delta z^2}\end{bmatrix}\begin{bmatrix}\Pi_{\xi,n,l+1}^{s,p,q-1}\\\Pi_{\xi,n,l+1}^{s,p,q}\\\Pi_{\xi,n,l+1}^{s,p,q+1}\end{bmatrix}\\
&\quad+\begin{bmatrix}\left(\frac{1}{4\Delta x\Delta z^2}+\frac{1}{4c\Delta t\Delta z^2}\right) & \left(\frac{2}{\Delta x\Delta t^2c^2}-\frac{1}{2c\Delta t\Delta z^2}-\frac{1}{2\Delta x\Delta z^2}\right) & \left(\frac{1}{4\Delta x\Delta z^2}+\frac{1}{4c\Delta t\Delta z^2}\right)\end{bmatrix}\begin{bmatrix}\Pi_{\xi,n,l}^{s,p,q-1}\\\Pi_{\xi,n,l}^{s,p,q}\\\Pi_{\xi,n,l}^{s,p,q+1}\end{bmatrix}\\
&\quad-\begin{bmatrix}\left(\frac{1}{4\Delta x\Delta y^2}-\frac{1}{4c\Delta t\Delta y^2}\right) & \left(\frac{2}{\Delta x\Delta t^2c^2}+\frac{1}{2c\Delta t\Delta y^2}-\frac{1}{2\Delta x\Delta y^2}\right) & \left(\frac{1}{4\Delta x\Delta y^2}-\frac{1}{4c\Delta t\Delta y^2}\right)\end{bmatrix}\begin{bmatrix}\Pi_{\xi,n+1/2,l}^{s,p-1,q}\\\Pi_{\xi,n+1/2,l}^{s,p,q}\\\Pi_{\xi,n+1/2,l}^{s,p+1,q}\end{bmatrix}\\
&\quad-\frac{1}{\Delta x\Delta t^2c^2}\Pi_{\xi,n,l-1}^{s,p,q}+\frac{1}{\Delta x\Delta t^2c^2}\Pi_{\xi,n+1/2,l-1}^{s,p,q}\qquad \xi=x,y,z
\end{aligned} \quad ,(10)$$

$$
\begin{aligned}
&\begin{bmatrix} \frac{1}{4c\Delta t\Delta z^2} & -\left(\frac{1}{2c\Delta t\Delta z^2}+\frac{1}{\Delta x\Delta t^2c^2}\right) & \frac{1}{4c\Delta t\Delta z^2} \end{bmatrix} \begin{bmatrix} \Pi_{\xi,n+1,l+1}^{s,p-1,q} \\ \Pi_{\xi,n+1,l+1}^{s,p,q} \\ \Pi_{\xi,n+1,l+1}^{s,p+1,q} \end{bmatrix} \\
&= \begin{bmatrix} -\frac{1}{4c\Delta t\Delta y^2} & \left(\frac{1}{2c\Delta t\Delta y^2}-\frac{1}{\Delta x\Delta t^2c^2}\right) & -\frac{1}{4c\Delta t\Delta y^2} \end{bmatrix} \begin{bmatrix} \Pi_{\xi,n+1/2,l+1}^{s,p,q-1} \\ \Pi_{\xi,n+1/2,l+1}^{s,p,q} \\ \Pi_{\xi,n+1/2,l+1}^{s,p,q+1} \end{bmatrix} \\
&+ \begin{bmatrix} \left(\frac{1}{4\Delta x\Delta y^2}+\frac{1}{4c\Delta t\Delta y^2}\right) & \left(\frac{2}{\Delta x\Delta t^2c^2}-\frac{1}{2c\Delta t\Delta y^2}-\frac{1}{2\Delta x\Delta y^2}\right) & \left(\frac{1}{4\Delta x\Delta y^2}+\frac{1}{4c\Delta t\Delta y^2}\right) \end{bmatrix} \begin{bmatrix} \Pi_{\xi,n+1/2,l}^{s,p,q-1} \\ \Pi_{\xi,n+1/2,l}^{s,p,q} \\ \Pi_{\xi,n+1/2,l}^{s,p,q+1} \end{bmatrix} \\
&- \begin{bmatrix} \left(\frac{1}{4\Delta x\Delta z^2}-\frac{1}{4c\Delta t\Delta z^2}\right) & \left(\frac{2}{\Delta x\Delta t^2c^2}+\frac{1}{2c\Delta t\Delta z^2}-\frac{1}{2\Delta x\Delta z^2}\right) & \left(\frac{1}{4\Delta x\Delta z^2}-\frac{1}{4c\Delta t\Delta z^2}\right) \end{bmatrix} \begin{bmatrix} \Pi_{\xi,n+1,l}^{s,p-1,q} \\ \Pi_{\xi,n+1,l}^{s,p,q} \\ \Pi_{\xi,n+1,l}^{s,p+1,q} \end{bmatrix} \\
&- \frac{1}{\Delta x\Delta t^2c^2}\Pi_{\xi,n+1/2,l-1}^{s,p,q} + \frac{1}{\Delta x\Delta t^2c^2}\Pi_{\xi,n+1,l-1}^{s,p,q} \qquad \xi = x, y, z
\end{aligned}
\quad . \quad (11)
$$

It can be seen that the unknowns are solved line by line in each transverse plane for any time step. As a result, the computational efficiency can be further improved.

AGE solution of the Claerbout-TDPE

Four asymmetry schemes in Fig. 1 are introduced to (5), and then the AGE solution of the Claerbout TDPE can be expressed as:

$$
\begin{bmatrix} 1+r_1+r_2 & -r_1 & 0 & -r_2 \\ -r_1 & 1+r_1+r_2 & -r_2 & 0 \\ 0 & -r_2 & 1+r_1+r_2 & -r_1 \\ -r_2 & 0 & -r_1 & 1+r_1+r_2 \end{bmatrix} \begin{bmatrix} \Pi_{\xi,n+1,l+1}^{s,p,q} \\ \Pi_{\xi,n+1,l+1}^{s,p+1,q} \\ \Pi_{\xi,n+1,l+1}^{s,p+1,q+1} \\ \Pi_{\xi,n+1,l+1}^{s,p,q+1} \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}, \quad (12)
$$

$$\xi = x, y, z$$

in which,

$$
\begin{aligned}
a &= (r_1+r_2)\left(\Pi_{\xi,n,l+1}^{s,p-1,q}+\Pi_{\xi,n,l+1}^{s,p,q-1}\right)-(3r_1+3r_2-1)\Pi_{\xi,n,l+1}^{s,p,q} \\
&+ r_2\Pi_{\xi,n,l+1}^{s,p,q+1}+r_1\Pi_{\xi,n,l+1}^{s,p+1,q}-(r_1+r_2)\left(\Pi_{\xi,n,l}^{s,p-1,q}+\Pi_{\xi,n,l}^{s,p,q-1}\right) \\
&- \left(2-3r_1-3r_2-\frac{1}{2r_3}\right)\Pi_{\xi,n,l}^{s,p,q}-\left(r_2+\frac{1}{4r_3}\right)\Pi_{\xi,n,l}^{s,p,q+1} \\
&- \left(r_1+\frac{1}{4r_3}\right)\Pi_{\xi,n+1,l}^{s,p+1,q}+\Pi_{\xi,n,l-1}^{s,p,q}+\left(2+r_1+r_2-\frac{1}{2r_3}\right)\Pi_{\xi,n+1,l}^{s,p,q} \\
&+ \left(\frac{1}{4r_3}-r_1\right)\Pi_{\xi,n+1,l}^{s,p+1,q}+\left(\frac{1}{4r_3}-r_2\right)\Pi_{\xi,n+1,l}^{s,p,q+1}-\Pi_{\xi,n+1,l-1}^{s,p,q}
\end{aligned}
,\quad (13)
$$

$$
\begin{aligned}
b &= (r_1+r_2)\left(\Pi_{\xi,n,l+1}^{s,p+2,q}+\Pi_{\xi,n,l+1}^{s,p+1,q-1}\right)-(3r_1+3r_2-1)\Pi_{\xi,n,l+1}^{s,p+1,q} \\
&+ r_1\Pi_{\xi,n,l+1}^{s,p,q}+r_2\Pi_{\xi,n,l+1}^{s,p+1,q+1}-(r_1+r_2)\left(\Pi_{\xi,n,l}^{s,p+2,q}+\Pi_{\xi,n,l}^{s,p+1,q-1}\right) \\
&- \left(2-3r_1-3r_2-\frac{1}{2r_3}\right)\Pi_{\xi,n,l}^{s,p+1,q}+\left(2+r_1+r_2-\frac{1}{2r_3}\right)\Pi_{\xi,n+1,l}^{s,p+1,q} \\
&- \left(r_2+\frac{1}{4r_3}\right)\Pi_{\xi,n,l}^{s,p+1,q+1}+\Pi_{\xi,n,l-1}^{s,p+1,q}+\left(\frac{1}{4r_3}-r_1\right)\Pi_{\xi,n+1,l}^{s,p,q} \\
&+ \left(\frac{1}{4r_3}-r_2\right)\Pi_{\xi,n+1,l}^{s,p+1,q+1}-\left(r_1+\frac{1}{4r_3}\right)\Pi_{\xi,n,l}^{s,p,q}-\Pi_{\xi,n+1,l-1}^{s,p+1,q}
\end{aligned}
,\quad (14)
$$

$$
\begin{aligned}
c &= (r_1+r_2)\left(\Pi_{\xi,n,l+1}^{s,p+2,q+1}+\Pi_{\xi,n,l+1}^{s,p+1,q+2}\right)-(3r_1+3r_2-1)\Pi_{\xi,n,l+1}^{s,p+1,q+1} \\
&+ r_2\Pi_{\xi,n,l+1}^{s,p,q+1}+r_1\Pi_{\xi,n,l+1}^{s,p+1,q}-\Pi_{\xi,n+1,l-1}^{s,p+1,q+1} \\
&- (r_1+r_2)\left(\Pi_{\xi,n,l}^{s,p+2,q+1}+\Pi_{\xi,n,l}^{s,p+1,q+2}\right) \\
&- \left(2-3r_1-3r_2-\frac{1}{2r_3}\right)\Pi_{\xi,n,l}^{s,p+1,q+1}-\left(r_1+\frac{1}{4r_3}\right)\Pi_{\xi,n,l}^{s,p+1,q} \\
&+ \left(2+r_1+r_2-\frac{1}{2r_3}\right)\Pi_{\xi,n+1,l}^{s,p+1,q+1}+\left(\frac{1}{4r_3}-r_2\right)\Pi_{\xi,n+1,l}^{s,p,q+1} \\
&- \left(r_2+\frac{1}{4r_3}\right)\Pi_{\xi,n,l}^{s,p,q+1}+\left(\frac{1}{4r_3}-r_1\right)\Pi_{\xi,n+1,l}^{s,p+1,q}+\Pi_{\xi,n,l-1}^{s,p+1,q+1}
\end{aligned}
,\quad (15)
$$

$$\begin{aligned} d &= \left(r_1 + r_2\right)\left(\Pi_{\xi,n,l+1}^{s,p-1,q+1} + \Pi_{\xi,n,l+1}^{s,p,q+2}\right) - \left(3r_1 + 3r_2 - 1\right)\Pi_{\xi,n,l+1}^{s,p,q+1} \\ &+ r_1\Pi_{\xi,n,l+1}^{s,p+1,q+1} + r_2\Pi_{\xi,n,l+1}^{s,p,q} - \left(r_1 + r_2\right)\left(\Pi_{\xi,n,l}^{s,p-1,q+1} + \Pi_{\xi,n,l}^{s,p,q+2}\right) \\ &- \left(2 - 3r_1 - 3r_2 - \frac{1}{2r_3}\right)\Pi_{\xi,n,l}^{s,p,q+1} + \left(\frac{1}{4r_3} - r_1\right)\Pi_{\xi,n+1,l}^{s,p+1,q+1} - \\ &\left(r_1 + \frac{1}{4r_3}\right)\Pi_{\xi,n,l}^{s,p+1,q+1} + \left(2 + r_1 + r_2 - \frac{1}{2r_3}\right)\Pi_{\xi,n+1,l}^{s,p,q+1} + \Pi_{\xi,n,l-1}^{s,p,q+1} \\ &+ \left(\frac{1}{4r_3} - r_2\right)\Pi_{\xi,n+1,l}^{s,p,q} - \Pi_{\xi,n+1,l-1}^{s,p,q+1} - \left(r_2 + \frac{1}{4r_3}\right)\Pi_{\xi,n,l}^{s,p,q} \end{aligned}, \quad (16)$$

where $r_1 = \dfrac{\Delta x c \Delta t}{4\Delta y^2}, r_2 = \dfrac{\Delta x c \Delta t}{4\Delta z^2}, r_3 = \dfrac{\Delta x}{4c\Delta t}$.

It can be found from (10) that the unknowns can be obtained directly by using the AGE scheme without solving any matrix equations. Therefore, it can achieve the highest computational efficiency.

The Crank-Nicolson scheme is unconditionally stable and it has second order accuracy. The ADI-PE method can be derived directly from the CN-PE method by adding $r_y r_z \delta_y \delta_z / (4ik)^2$ to each side of the equation. In this way, the ADI-PE is also unconditionally stable with second order accuracy as the CN-PE. The AGE method is explicit, second-order accurate, and is unconditionally stable because of using alternative strategies on the boundary grids.

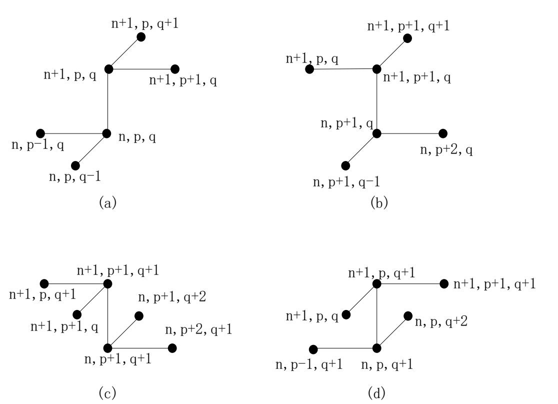

Fig. 1. Four asymmetry schemes for AGE scheme.

III. NUMERICAL EXAMPLES

At first, we consider the transient EM scattering from a PEC sphere with the radius of 5 m. The center frequency of the modulated Gaussian pulse is 300 MHz and its bandwidth is 600 MHz. In this numerical example, the range steps are set to be 0.05 m and there are 800 time steps are needed. The incident wave is fixed at $\theta_{inc} = 90^\circ, \varphi_{inc} = 0^\circ$. As shown in Fig. 2, the forward bistatic RCS results of the CN-Claerbout-TDPE, the ADI-Claerbout-TDPE and the AGE-Claerbout-TDPE at different frequencies are given and compared with the Mie Series. It can be seen that the accuracy can be ensured at the angles of 25°-30° along the paraxial direction. Moreover, as shown in Fig. 3, the transient forward-scattered field of the proposed method is compared with the results for the inverse discrete Fourier transform (IDFT) of the Mie Series. Furthermore, the computational resources are compared in Table 1 among the CN-TDPE, the CN-Claerbout-TDPE, the ADI-TDPE, the ADI-Claerbout-TDPE, the AGE-TDPE and the AGE-Claerbout-TDPE methods. It can be concluded that accurate results can be obtained at wider angles with less computational resources by the proposed AGE-Claerbout-TDPE method than other methods.

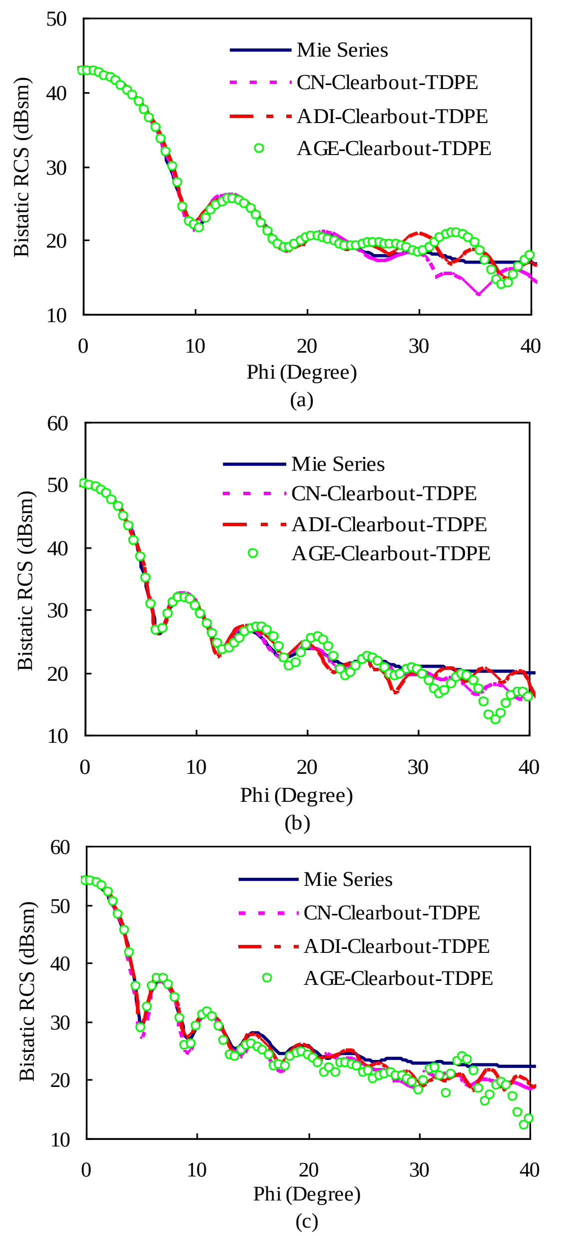

Fig. 2. Bistatic RCS of a PEC sphere: (a) 200 MHz, (b) 300 MHz, and (c) 400 MHz.

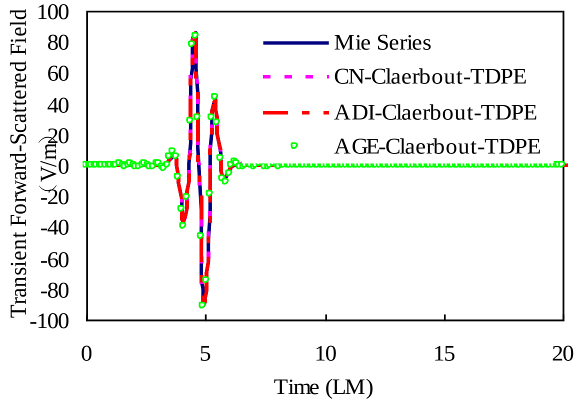

Fig. 3. Transient forward-scattered fields for the PEC sphere.

Table 1: Comparisons of the computational resources for the PEC sphere among different methods

Methods	Number of Discrete Grid	Peak Memory Requirement (MB)	Total CPU Time (s)
CN-TDPE	120*120*120	411	12928
CN-Claerbout-TDPE	120*120*120	487	14432
ADI-TDPE	120*120*120	325	1079
ADI-Claerbout-TDPE	120*120*120	647	1343
AGE-TDPE	120*120*120	335	326
AGE-Claerbout-TDPE	120*120*120	396	394

Secondly, the transient EM scattering from a PEC satellite is analyzed with the incident wave fixed at $\theta_{inc} = 180^{\circ}, \varphi_{inc} = 0^{\circ}$. The center frequency of the modulated Gaussian pulse is 300 MHz and its bandwidth is 600 MHz. The range steps are 0.05 m and there are 600 time steps are needed in this example. As shown in Fig. 4, the complete RCS result of the proposed AGE-Claerbout-TDPE method is compared with the FEKO. It should be noted that the complete RCS result is obtained by five rotating AGE-Claerbout-TDPE runs. There is a good agreement between them. Moreover, as shown in Fig. 5, the stability tests are made for the proposed three solutions. Figure 5 shows that the magnitudes of reduced time-domain scattered fields at the top point of the satellite attenuate exponentially to 1.E-5 V/m. It can be observed that the stability can be guaranteed of the three solutions for $\Delta t = 1/c\sqrt{(1/\Delta x)^2 + (1/\Delta y)^2 + (1/\Delta z)^2}$.

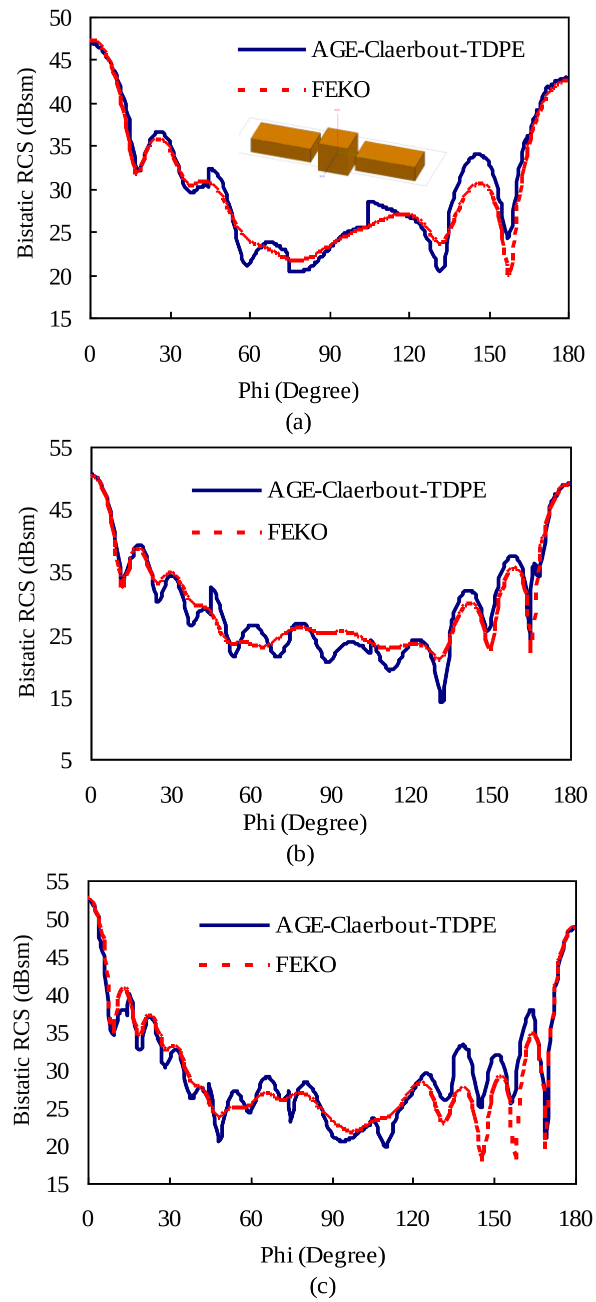

Fig. 4. Bistatic RCS of a PEC satellite: (a) 200 MHz, (b) 300 MHz, and (c) 400 MHz.

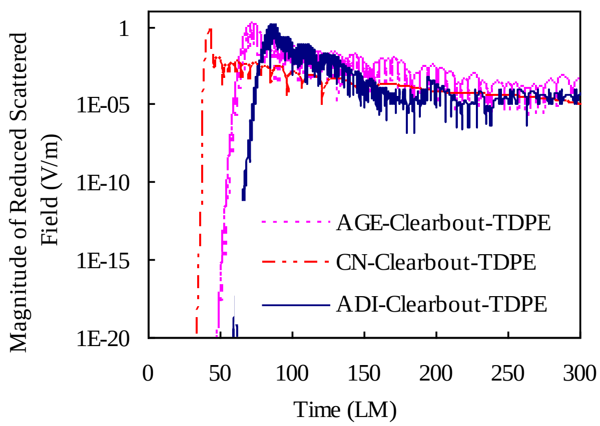

Fig. 5. Stability tests for the PEC satellite among the CN, ADI and AGE solutions.

IV. CONCLUSION

In this paper, a novel wide-angle Claerbout scheme of three-dimensional time domain parabolic equation (Claerbout-TDPE) is proposed to analyze the wide-band EM scattering problems. It can provide accurate bistatic RCS results at wider angle than the traditional TDPE, which is up to 25° along the paraxial direction. The CN, ADI and AGE schemes are used to solve the Claerbout-TDPE and efficiency tests are made among them. Furthermore, the complete bistatic RCS results can be achieved by several rotating Claerbout-TDPE runs.

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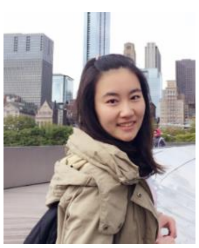

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