A Double Modal Parameter Tracking Method To Characteristic Modes Analysis

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Abstract **─** Mode tracking plays an important role in characteristic modes analysis. However, mode tracking in a proper and efficient way is still a challenging work. Based on modal linear correlation and modal stability, a double modal parameter tracking method is proposed in this paper, which is to track modes by correlating eigenvector and calculating the stability of characteristic angle simultaneously. To eliminate ambiguity case of multi-mode mapping one mode, two sorted mode with max Pearson correlation coefficient are identified same mode by utilizing saving best function. In order to verify accuracy and efficiency of the proposed method, four representative structures are analyzed. It can be observed that the proposed tracking method works better than the traditional methods.

Index Terms **─** Characteristic modes analysis, double modal parameter tracking method, modal linear correlation, modal stability.

I. INTRODUCTION

The Theory of Characteristic Modes (TCM) was first presented by Garbacz in his doctoral dissertation [1] and then refined by Harrington in 1970s [2, 3]. TCM shows deep insight into the nature feature of an object, which is based on the method of moment (MoM). But researchers originally didn't pay much attention to TCM. As the extreme development of computer, TCM has attracted many researchers' interest now. Applying the CMA, Chen an Wang show a UAV platform integrated pattern reconfigurable antenna [4], and a HF band shipboard antenna [5], where the radiation efficiency has been improved. Using the orthogonality between different modes, many antennas with good isolation in MIMO system have been designed by researchers [6-9] .

CMA is generally performed in spectrum. The same mode at different frequency samples needs to be identified firstly, which is called mode tracking. Mode tracking is the basis of characteristic mode application. However, mode tracking in a proper and efficient way is still a challenging work. There are a few of papers involving mode tracking. The tracking methods in these papers are all only tracking one modal parameter: a) Tracking method in paper [10, 11] is based on tracking eigenvalues. b) Tracking method in paper [12-14] is based on correlating eigenvectors. c)Tracking method in paper [15] is based on correlating modal far-field pattern. Tracking methods in a) and b) are both based on one parameter deriving from generalized eigenvalue equation, and no additional calculation is needed. Therefore, these methods have high computational efficiency. However, they can't deal well with complex structures since mode swapping always exists. Tracking method in c) utilizes stability of far-field pattern to obtain better results than a) and b), but a lot of computation has to be added for calculating far-field.

In this paper, a new tracking method named as double modal parameter tracking method (DMPTM) is proposed, which is based on tracking two modal parameters at the same time.

The remainder of this paper is organized as follows. In Section 2, the theory and the formulations in characteristic modes analysis are given briefly. In Section 3, DMPTM is introduced in detail. Four numerical experiments are presented in Section 4 to show the accuracy and efficiency of the proposed method. Section 5 concludes this paper.

II. BRIEF INTRODUCTION TO CMA

Linear combination of a set of characteristic modes can well approximately represent the solution of an electromagnetic problem. These characteristic modes correspond to the inherent properties of electromagnetic objects and have orthogonality between different modes [2]. For the sake of easy reference, the following paragraphs are a brief introduction to characteristic

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modes analysis.

As presented in [2], characteristic modes can be obtained by solving the following generalized eigenvalue problem:

$$XJ_n = \lambda_n R J_n, \tag{1}$$

where X and R are imaginary part and real part of impedance matrix Z, which is derived from the well known method of moment (MoM). J_n and λ_n are named as eigencurrents (or eigenvectors), and eigenvalues λ_n represents the radiation or scattering properties of the corresponding modes. If $\lambda_n > 0$, the mode stores magnetic energy. If $\lambda_n < 0$, the mode stores electric energy. And if $\lambda_n = 0$, the mode is at resonance. It is notable that characteristic modes are independent of a specific source or excitation, and only rely on the shape, size, material and working frequency band of the object.

Because of its large range of eigenvalues [12], characteristic angles α_n are introduced as following:

$$\alpha_n = 180^0 - \arctan(\lambda_n). \tag{2}$$

Obviously, the range of values for α_n is from 90^0 to 270^0. If $\alpha_n = 180^0$, the corresponding modes are at resonance.

In theory, the linear combination of infinite modes is needed to describe the electromagnetic properties of the study object. Fortunately, only several mode with small eigenvalues are needed to descript the electromagnetic behavior for electrically small or moderate objects [16].

III. DOUBLE MODAL PARAMETER TRACKING METHOD

In this section, double modal parameters tracking method is presented in detail.

A. Main ideas

Eigenvalues, eigenvectors and characteristic far fields are three main parameters in CMA, and they have different features: The range of values for eigenvalues is $(-\infty, +\infty)$, and they vary fast with frequencies and are easily affected by numerical accuracy, so eigenvalues are not suitable to be tracked. Characteristic far fields are the most stable among modal parameters, so they can be chosen to track mode. However its calculation cost is expensive and its calculation results are also affected by spatial angular step. The eigenvectors stability is between eigenvalues and characteristic far fields, and they are obtained directly through solving generalized eigenvalue equation, so mode tracking based correlating eigenvectors is a balanced choice between efficiency and accuracy.

However, mode tracking based only one modal parameter [10-15] can work well for simple structure, but they can not deal well with complex object.

This paper proposed a double modal parameter tracking method based modal correlation and modal stability. Its main ideas are:

1) Tracked mode satisfying both model correlation requirement and modal stability requirement can be identified to same mode.

2) To eliminate ambiguity situation where multi-modes map one mode, the pair of modes with the max correlation coefficient are mapped to same mode.

B. Modal correlation requirement

Modal correlation requirement is referred that correlation coefficients of same modes at different frequency samples are larger than specified linear correlation threshold. In this paper, linear correlation is measured by Pearson correlation coefficient, its formulation is followed [14]:

$$r_{m,n} = \left| \frac{\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{N}(x_i - \bar{x})^2}\sqrt{\sum_{i=1}^{N}(y_i - \bar{y})^2}} \right|. \tag{3}$$

In formulation (3), $x = (x_1, x_2, \cdots, x_N)^T$ represents m^{th} eigenvector at frequency f_p, $\bar{x}$ represents the mean value of its components. y and $\bar{y}$ represent n^{th} eigenvector and its mean value at frequency f_q respectively, and $p \neq q$.

The range of values for $r_{m,n}$ is [0,1]. It represents that the two eigenvectors are linearity independent if $r_{m,n}$ equals 0, and it represents that the two eigenvectors are linear correlation if $r_{m,n}$ equals 1. Because eigenvectors are function of frequency, the correlation coefficients between same modes vary with frequency change, so a threshold for linear correlation is needed to be specified. According to our experience, the correlation threshold R_g is set to 0.8～0.9, the default value of R_g is 0.8. It means that sorted mode satisfied correlation requirement if their correlation coefficient is larger than R_g.

C. Modal stability requirement

Modal stability requirement is referred that modal parameters of same mode varying with frequencies are smaller than specified stability threshold. Stability is a universal characteristic in macroscopic physics word, so modal stability is inherent feature. In the aforementioned literatures, modal stability hasn't been paid sufficient attention, and the tracking methods in [10-15] don't make use of the modal stability.

In order to measure modal stability, characteristic angle stability is defined as follows:

$$s = \left|\frac{\Delta\alpha}{\alpha_e}\right| = \left|\frac{\alpha_c - \alpha_e}{\alpha_e}\right|. \tag{4}$$

In formulation (4), α_e represents extrapolated characteristic angle derived from known characteristic angle array, by utilizing Matlab function spline. The α_c represents directly calculated characteristic angle according to formulation (2).

Tracked modes satisfy modal stability requirement if the modal stability of s is smaller than the specified threshold S_g. Because the range of values for characteristic angle is $\left[90^0, 270^0\right]$, so the range of values for s is also large. According to our experiences, S_g is set to 0.2～0.5 which is changed according the complexity of structure. The default value of S_g is 0.5. Figure 1 shows the schematic diagram of modal stability s.

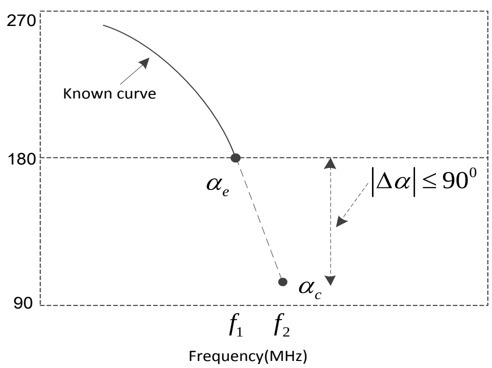


Fig. 1. Schematic diagram of modal stability.

D. The proposed tracking method

The proposed double modal parameter tracking method is divided into three stages: pre-processing stage, mode tracking stage, and post-processing stage.

D1. Pre-processing stage

There are several tasks in the pre-processing stage. Firstly, K eigen-pairs of each frequency, where M frequencies samples are setup at prior, are calculated and ordered in ascending order of eigenvalues. Secondly, Pearson correlation coefficients between eigenvectors of adjacent frequencies are calculated, according to formulation (3). Thirdly, Index-Table array with dimension of $M_{rows} \times K_{columns}$, where each column maps one mode and each row maps one frequency is initialized by its first row filled with $1, 2, \cdots, K$ and others are filled with "NaN". The elements of the first row in Index-Table represent the K modes of the first frequency F1, which does not need to be tracked.

The Matlab function corr is used to calculate Pearson correlation coefficient between every eigenvector at frequency F_i and all K eigenvectors at frequency F_{i-1}, resulting in an array with K elements. After calculating correlation coefficients between all eigenvectors at adjacent frequencies, an array with dimension of $M \times M \times K$ will be obtained.

D2. Mode tracking stage

Mode tracking stage consists 4 tracking function: primary tracking function, rescuing function, opening new mode function, and saving best function, as is shown in Fig. 2. Mode tracking start with K eigenvectors at 2nd frequency sample F_2, then K eigenvectors at 3rd frequency sample F_3, until K eigenvector at last frequency sample F_M. The 4 tracking function will be performed at these frequencies except that primary tracking function, opening new mode function and saving best function are performed at 2nd frequency sample F_2.

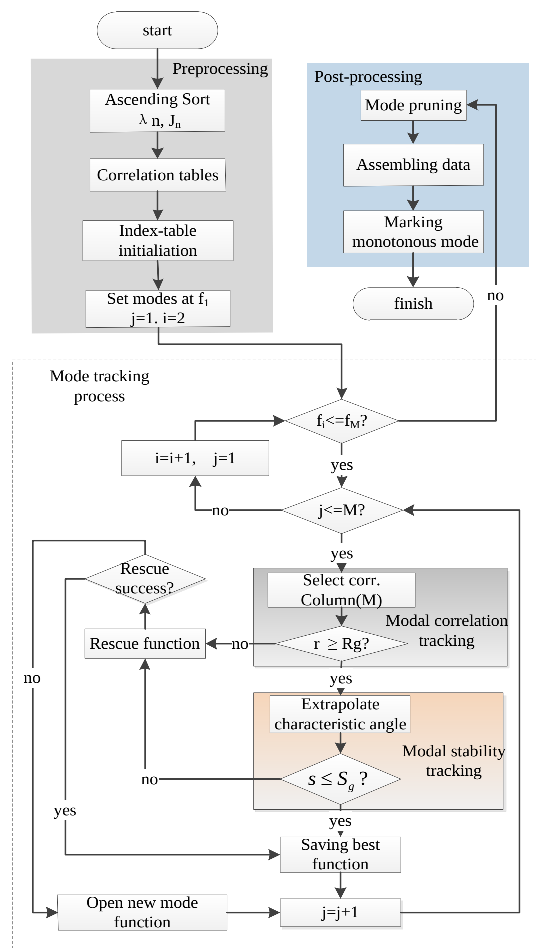


Fig. 2. Flow chart of proposed double modal parameter tracking method.

D2.1. Primary tracking function

Firstly, one determines whether there is any correlation coefficient between the j^{th} eigenvector at F_i (called current mode) and K eigenvectors at F_{i-1} larger than R_g . If there is none of them, tracking process comes to rescuing function. And if there is any, tracking process comes to extrapolate characteristic angle. This is called modal correlation tracking. Then tracking process comes to determine whether the modal stability coefficient is smaller than S_g. If the modal stability requirement is satisfied too, tracking process comes to saving best function, and if not, tracking process comes to rescue function again. This is called modal stability tracking.

D2.2. Rescuing mode function

Firstly, for the sake of convenience, the indicator of previous frequency sample is set as $p=i-2$. Secondly, Pearson correlation coefficients between current eigenvector and K eigenvectors at previous frequency are calculated. Thirdly, modal correlation tracking and modal stability tracking are performed. If both modal correlation tracking and model stability tracking are successful, then rescuing function success, and tracking process comes to saving best function, otherwise tracking process comes to opening new mode function.

D2.3. Opening new mode function

The method of opening new mode function is similar to the one in paper [12]. It is to add a new column at right side of the last column of current Index-Table array. The i^{th} element of the new added column is filled with mode index, and the other elements are filled with "NaN", as is shown in Fig. 3.

D2.4. Saving best function

If tracking process comes to saving best function, it means that both modal correlation requirement and modal stability requirement are satisfied. Without loss of generality, we suppose that j^{th} eigenvector at frequency F_i (called current mode) and m^{th} eigenvector at frequency F_{i-1} do so, and suppose that the mode index of m at frequency F_{i-1} (called previous mode) is kept at 2^{nd} column in Index-Table, as is shown in Fig. 3. If the element of 2^{nd} column corresponding frequency F_i is "NaN", it will be directly replaced by the mode index j. Otherwise, supposing the element is "h", it means that the h^{th} eigenvector at frequency F_i (called kept mode) has been mapped to previous mode. This scenario is known as the ambiguity case of multi-mode mapping one mode. In order to eliminate ambiguity case, one needs to compare the correlation coefficient between current mode and previous mode with the correlation coefficient between kept mode and previous mode, then choose the pair of modes with larger correlation coefficient to map into same mode. Then the failed mode (mode h in Fig. 3.) in competing will be opened new mode if saving best function is performed in rescuing function, or will be put into rescuing function if saving best function is performed in primary tracing function.

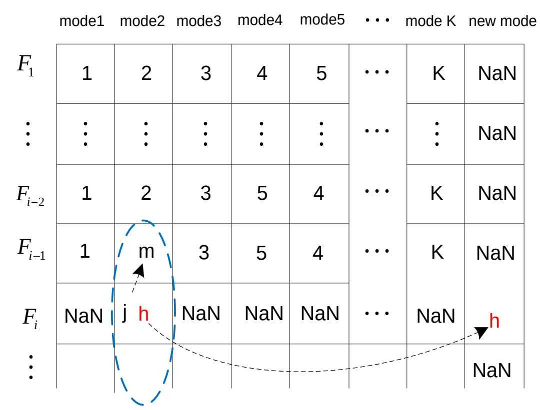


Fig. 3. Example of saving best function and opening new mode function.

D3. Post-processing

There are three works during post-processing in this paper, which are pruning Index-Table array, assembling data of eigen-pairs and marking monotonous modes. These three works are similar to the works in paper [12], it will be briefly described here.

Firstly, pruning Index-Table array means that columns with number of non-nan elements less than three will be cut off.

Secondly, assembling data of eigen-pairs means that the remaining modes in Index-Table after pruning are subsequently fulfilled with the real eigenvalues and eigenvectors, which are corresponding to mode index in Index-Table array.

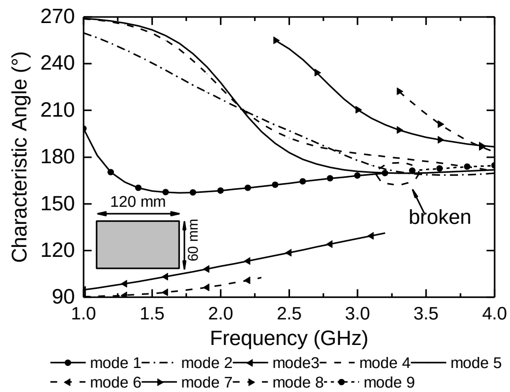
Lastly, monotonous modes means that the modes with negligible meaning within the computed frequency band. It implies that characteristic angle of monotonous modes is smaller than 110^0 or larger than 250^0 within the computed frequency band. So the last work is to find out the index of monotonous modes.

IV. NUMERICAL RESULTS

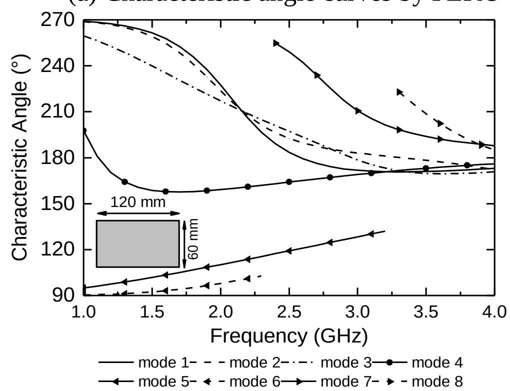
In this section, four examples are presented to demonstrate the capability of the proposed method, the material in each example is perfect conductor, and the same triangle meshes are used for different tracking methods in each example.

A. Mode tracking of rectangular plate

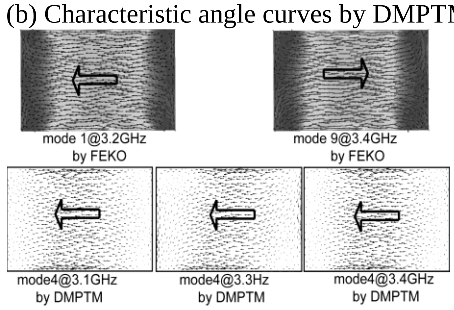
The first example is a rectangular plate with size of 120×60 mm^2, as is shown in Fig. 4. The whole structure is divided into 798 triangles, resulting in 1158 RWG basis functions. The computed frequency band is from 1 GHz to 4 GHz with 0.1 GHz frequency step, and 6 eigen-pairs are required at each frequency sample, and R_g is 0.8 and S_g is 0.5.



(a) Characteristic angle curves by FEKO



(b) Characteristic angle curves by DMPTM



(c) Modal current distribution (big arrows indicate the direction of each model current)

Fig. 4. Characteristic angle curves and modal current distribution of metal rectangular plate.

Figure 4 (a) shows the tracking result by FEKO, and Fig. 4 (b) shows the tracking result by proposed method. It can be seen that the characteristic angle curves are similar except that the order of the modes is different. However, there are still obviously differences between them:

a) The FEKO gives 9 characteristic angle curves for different modes. However the DMPTM gives 8 characteristic angle curves for different modes.

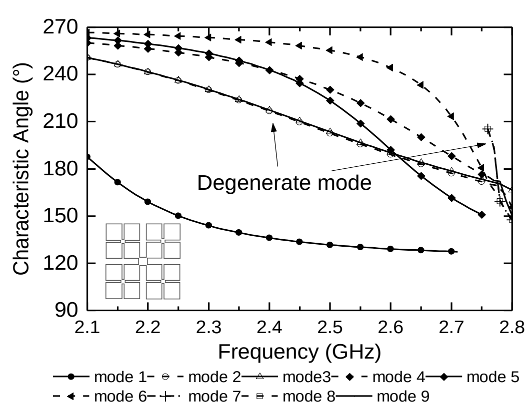
b) The characteristic angle curve of mode 1 in Fig. 4 (a) is closed at 3.2 GHz, and mode 9 is opened at 3.4 GHz. However, the characteristic angle curve of mode 4 in Fig. 4 (b), which corresponds the characteristic angle curve of mode 1 in Fig. 4 (a), is continuous in the entire analysis frequency band.

In order to check tracking results, the mode current distribution is shown in Fig. 4 (c). It is found that the current distribution of mode 1 at 3.2 GHz is the similar to that of mode 9 at 3.4 GHz, which are tracked by FEKO, so they should be the same mode. In contrast, the mode current distribution of mode 4 tracked by DMPTM is similar at 3.1 GHz, 3.3 GHz and 3.4 GHz, so they are the same mode. Therefore, the proposed method in this paper gives the correct tracking results.

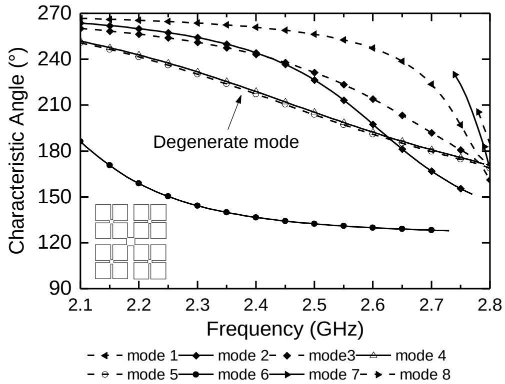
B. Mode tracking of Minkowski fractal structure

The second example is Minkowski fractal structure with outer size of 71.3×71.3 mm^2, as is shown in Fig. 5. The whole structure is divided into 664 triangles, resulting in 856 RWG basis functions. The computed frequency band is from 2.1 GHz to 2.8 GHz with 10 MHz frequency step, and 6 eigen-pairs are required at each frequency sample, and R_g is 0.8 and S_g is 0.5.

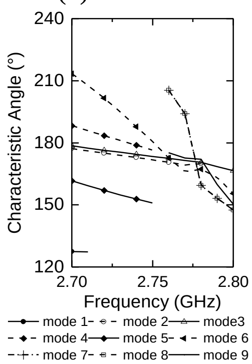
Figure 5 (a) shows the tracking result by FEKO, Fig. 5 (b) shows the tracking result by proposed method, Fig. 5 (c) shows the local enlargement of Fig. 5 (a), and Fig. 5 (d) shows the local enlargement of Fig. 5 (b). Characteristic angle cures in Fig. 5 (a) are very similar to the curves in Fig. 5 (b) except different mode order. It is can been seen from Fig. 5 (c) and Fig. 5 (d) that, there are 2 swapping modes and 2 non-continuous behaviors in Fig. 5 (a), but there are none of them in Fig. 5 (b). This is due to modal stability tracking in the double modal parameters tracking method. Therefore, the proposed tracking method in this paper gives better results than FEKO does in this example.



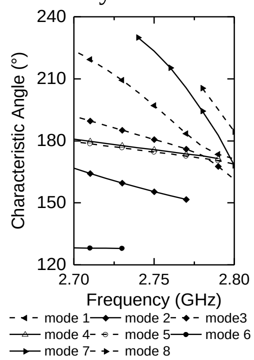
(a) Characteristic angle curves by FEKO



(b) Characteristic angle curves by DMPTM



(c) Local enlargement of Fig. 5 (a)



(d) Local enlargement of Fig. 5 (b)

Fig. 5. Characteristic angle curves of Minkowski fractal structure.

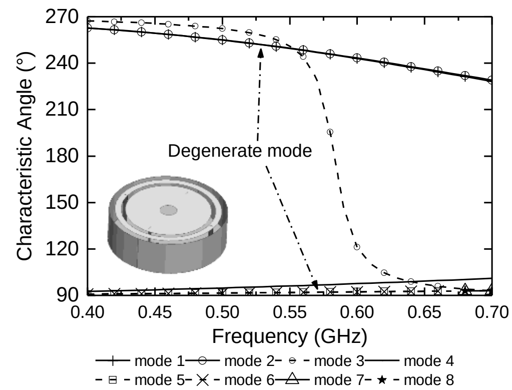
C. Mode tracking of open cavity structure

The third example is open complex cavity structure. The structure is an embedded conformal omni-directional antenna without any feed [17], as is shown in Fig. 6. In the metal cavity of diameter 138.0 mm and height 45.0 mm, there are narrow circular rings with a width 7.0 mm and a wide circular ring with width 40.0 mm. The two rings are located on aperture plane of the cavity. There is one shorting post connecting narrow circular ring and bottom of the cavity at 0^0 and 180^0 respectively, and there is one shoring post connecting wide circular ring and the bottom of the cavity at 90^0 and 270^0 respectively. The four shorting posts form eight "T" junctions.

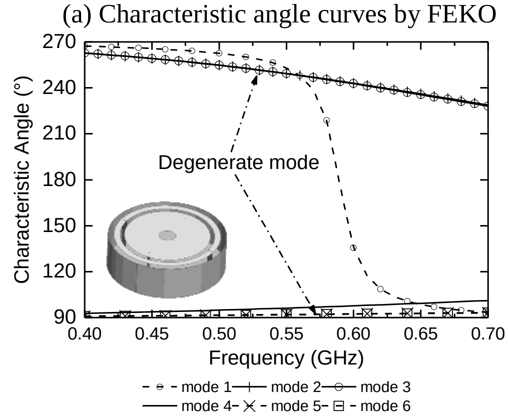
The whole structure is divided into 1289 triangles, resulting in 1834 RWG basis functions. The computed frequency band is from 0.4 GHz to 0.7 GHz with 10 MHz frequency step, and 6 eigen-pairs are required at each frequency sample, and R_g is 0.8 and S_g is 0.5.

Figure 6 (a) shows the tracking result by FEKO, Fig. 6 (b) shows the tracking result by proposed method, Fig. 6 (c) shows the local enlargement of Fig. 6 (a), and Fig. 6 (d) shows the local enlargement of Fig. 6 (b). It can be seen that characteristic angle cures in Fig. 6 (a) are also very similar to curves in Fig. 6 (b) except different mode order. As is pointed in Fig. 6 (a) and Fig. 6 (b), both of them well identify two pairs of degenerate mode. However, the curves of mode 5 and mode 6 in Fig. 6 (c) are broken at 0.67 GHz, resulting that two additional curves of mode 7 and mode 8 are opened at 0.68 GHz. So, there are 2 non-continuous behaviors in Fig. 6 (a), but there are none of them in Fig. 6 (b).

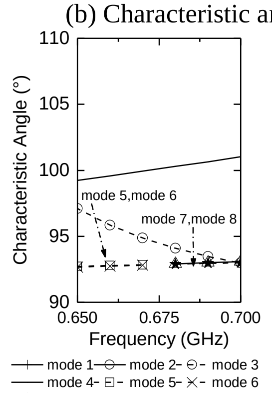
Therefore, proposed tracking method gives better results than FEKO does in this example. This is also due to the double modal parameter tracking method.



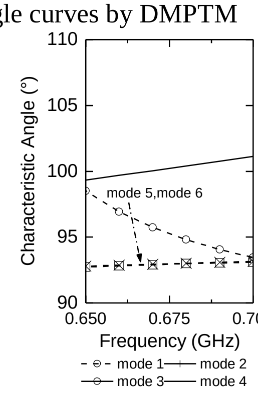
(a) Characteristic angle curves by FEKO



(b) Characteristic angle curves by DMPTM



(c) Local enlargement of Fig. 6 (a)



(d) Local enlargement of Fig. 6 (b)

Fig. 6. Characteristic angle curves of open cavity structure.

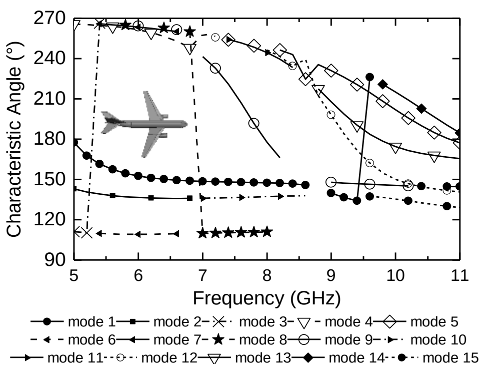
D. Mode tracking of a plane

The fourth example is an approximate model of commercial plane A320, which is with body length of 37 m and wingspan size of 39 meters.

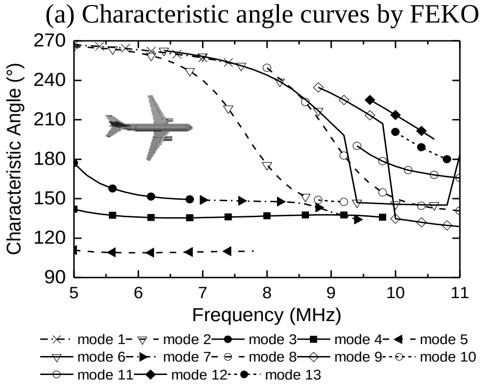
The whole structure is divided into 4180 triangles, resulting in 6270 RWG basis functions. The computed frequency band is from 5 MHz to 11 MHz with 0.2 MHz frequency step, and 6 eigen-pairs are required at each frequency sample, and R_g is 0.8 and S_g is 0.2.

Figure 7 (a) shows the tracking result by FEKO, Fig. 7 (b) shows the tracking result by tracking method based on correlating eigenvector, and Fig. 7 (c) shows the tracking result by proposed method. In this example, the three figures show completely different results: there are 4 swapping modes and 2 non-continuous behaviors in Fig. 7 (a), and there are 2 swapping modes and 2 non-continuous behaviors in Fig. 7 (b), but there are none of them in Fig. 7 (c). Therefore, the proposed tracking method gives better results than FEKO and tracking method based correlating eigenvector.

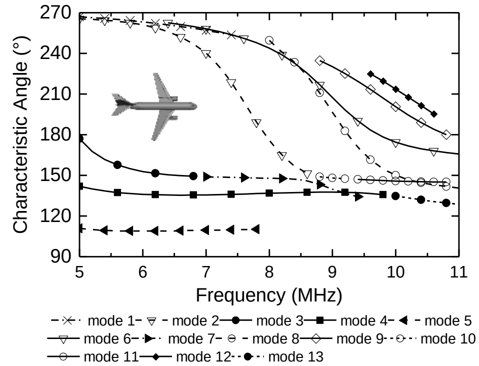
Table 1 shows the detailed comparison of the proposed tracking method and FEKO, including the number of non-continuous behavior and the number of mode swapping. From Table 1, it can be seen that there is no non-continuous behavior and mode swapping for DMPTM in the four examples, but there are some for FEKO in these examples. Therefore, the proposed tracking method gives more correct tracking results than FEKO does.



(a) Characteristic angle curves by FEKO



(b) Characteristic angle curves by correlating eigenvector



(c) Characteristic angle curves by DMPTM

Fig. 7. Characteristic angle curves of A320.

Table 1: Comparison of DMPTM and FEKO

Numeric Example	Number of Mode Swapping		Number of Non-continuous Behavior	
	FEKO	DMPTM	FEKO	DMPTM
Rectangular plate	0	0	1	0
Minkowski fractal structure	2	0	2	0
Open cavity structure	0	0	2	0
A320	4	0	2	0

V. CONCLUSION

In this paper, a double modal parameter tracking method (DMQTM) is proposed, which tracks modes based correlating eigenvectors and calculating the stability of characteristic angle simultaneously. In order to eliminate ambiguity case, saving best function is introduced. To verify accuracy and efficiency of the proposed DMQTM, four examples are analyzed by FEKO, tracking method based on correlating eigenvector and proposed method respectively. The numerical results demonstrate that the proposed method efficiently gives more accurate mode tracking curves than traditional tracking method does. In order to find out the exact resonant frequency for characteristic mode, our future work is to add adaptive frequency.

ACKNOWLEDGMENT

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REFERENCES

[1] R. J. Garbacz, “A Generalized Expansion for Radiated and Scattered Fields,” Ph.D., Ohio State University, 1968.

[2] R. Harrington and J. Mautz, “Theory of characteristic modes for conducting bodies,” *Antennas and*

Propagation, IEEE Transactions on, vol. 19, pp. 622-628, 1971.

[3] R. F. Harrington, J. R. Mautz, and C. Yu, “Characteristic modes for dielectric and magnetic bodies,” *Antennas and Propagation, IEEE Transactions on*, vol. 20, pp. 194-198, 1972.

[4] C. Yikai and W. Chao-Fu, “Synthesis of platform integrated antennas for reconfigurable radiation patterns using the theory of characteristic modes,” in *Antennas, Propagation & EM Theory (ISAPE), 2012 10th International Symposium on*, pp. 281-285, 2012.

[5] C. Yikai and C.-F. Wang, “Shipboard NVIS radiation system design using the theory of characteristic modes,” in *2014 International Symposium on Antennas and Propagation and USNC-URSI Radio Science Meeting*, 2014.

[6] A. Araghi and G. Dadashzadeh, “Oriented design of an antenna for MIMO applications using theory of characteristic modes,” *Antennas and Wireless Propagation Letters, IEEE*, vol. 11, pp. 1040-1043, 2012.

[7] A. Krewski, W. L. Schroeder, and K. Solbach, “Multi-band 2-port MIMO LTE antenna design for laptops using characteristic modes,” in *Antennas and Propagation Conference (LAPC), 2012*, Loughborough, pp. 1-4, 2012.

[8] Z. Miers, L. Hui, and L. Buon Kiong, “Design of multi-antenna feeding for MIMO terminals based on characteristic modes,” in A*ntennas and Propagation Society International Symposium (APSURSI), 2013 IEEE*, pp. 182-183, 2013.

[9] P. Miškovský and A. v. Arbin, “Evaluation of MIMO handset antennas with decorative metal elements using characteristic modes,” in *2014 International Symposium on Antennas and Propagation and USNC-URSI Radio Science Meeting*, 2014.

[10] D. J. Ludick, J. v. Tonder, and U. Jakobus, “A hybrid tracking algorithm for characteristic mode analysis,” 2013.

[11] E. Safin and D. Manteuffel, “Advanced eigenvalue tracking of characteristic modes,” *IEEE Transactions on Antennas and Propagation*, vol. 64, pp. 2628-2636, 2016.

[12] M. Capek, P. Hazdra, P. Hamouz, and J. Eichler, “A method for tracking characteristic numbers and vectors,” in *Electromagnetics Research B*, pp. 115-134, 2011.

[13] B. D. Raines and R. G. Rojas, “Wideband characteristic mode tracking,” I*EEE Transactions on Antennas and Propagation*, vol. 60, no. 7, pp. 3537-3541, 2012.

[14] D. J. Ludick, U. Jakobus, and M. Vogel, “A tracking algorithm for the eigenvectors calculated with characteristic mode analysis,” presented at *The 8th European Conference on Antennas and Propagation (EuCAP 2014)*, 2014.

[15] Z. Miers and B. K. Lau, “Wide band characteristic mode tracking utilizing far-field patterns,” *IEEE Antennas and Wireless Propagation Letters*, 2015.

[16] B. D. Raines, “Systematic Design of Multiple Antenna Systems Using Characteristic Modes,” Ph.D., Philosophy in the Graduate School of the Ohio State University, The Ohio State University, 2011.

[17] H. Qihong, G. Ziping, K. Hengyu, and W. Xianrong, “An embedded omnidirectional conformal antenna based on theory of characteristic modes,” *Journal of Electronics & Information Technology*, vol. 38, pp. 2977-2981, 2016.