Analysis of Transient Scattering from a PEC Coated with Thin Dispersive Dielectric Layer

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Abstract **─** A marching-on-in-time (MOT) based time domain thin dielectric sheet (TDS) method is proposed for analyzing the transient electromagnetic scattering from the perfect electrically conducting (PEC) body coated with thin dispersive material. The sources in the thin dielectric layer are all replaced by the current densities of PEC. A recursive convolution method is utilized to deal with the dispersive properties. The performance of accuracy and stability are investigated numerically.

Index Terms **─** Dispersive dielectric, marching-on-in-time, thin dielectric sheet, transient electromagnetic scattering.

I. INTRODUCTION

Analysis of transient electromagnetic scattering from the thin dispersive dielectric coated perfect electrically conducting (PEC) has been paid considerable attention due to its wide range of applications. For instance, thin dispersive dielectric materials are often utilized to reduce radar cross section (RCS) in military. Many numerical methods are available for such analysis accurately. Compared with differential equation method, time domain integral equation (TDIE) method is a better way for open domain transient scattering analysis because the truncated boundary condition is needless. For the coated PEC structure, time domain volume-surface integral equation (TD-VSIE) [1]-[3] and time domain surface integral equation (TD-SIE) [4]-[5] are both valid. However, both methods will consider the unknowns of dielectric materials, and with the increasing electrical size of target, the available computational resource will be swamped quickly, which is troublesome. When the thickness of the coated dielectric layer is thin enough (in the third part, several numerical results are proposed to demonstrate that reasonable accuracy can be achieved when the thickness of the dielectric is less than 0.09 wavelength at the highest frequency of the incident wave), the TDS approximation is a remarkable choice, all the sources in the thin dielectric coated on the closed PEC part are represents in the terms of current densities on PEC surface. The unknowns of the dielectric parts can be saved, and its efficiency has been proven [6]-[11].

In this paper, a modified MOT based approximation time domain TDS method is proposed. Compare with the conventional method presented in [11], which can only be used to compute the non-dispersive medium. The proposed method extended the applied range to analyze the transient electromagnetic scattering problems from thin dispersive material coated PEC structure. The practicality of the time domain TDS method is improved. Rao-Wilton-Glisson (RWG) basis function and time shifted Lagrange interpolant are chosen for the expansion in space and time domain. The sources in the thin dispersive material layer are all replaced by the current densities on PEC surface, so the spatial unknowns can be only considered on PEC surface. With the number of unknowns reduced markedly, the computational resource will also be saved. Here, a recursive convolution scheme [12] is introduced to handle the dispersive relationship efficiently. The investigation of performance for the proposed scheme is given in numerical results.

This paper is organized as follows. In Section II, the basic theory and formulations of time domain TDS method are presented. Several numerical results are given to demonstrate the accuracy, stability and efficiency of the proposed method in Section III. Conclusions are summarized in Section IV.

II. THEORY AND FORMULATIONS

Consider a transient incident wave $\mathbf{E}^{\text{inc}}(\mathbf{r},t)$ illuminate upon an arbitrarily shaped closed PEC structure coated with an isotropic, nonmagnetic and dispersive thin dielectric layer. The geometry of the

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composite structures is shown in Fig. 1.

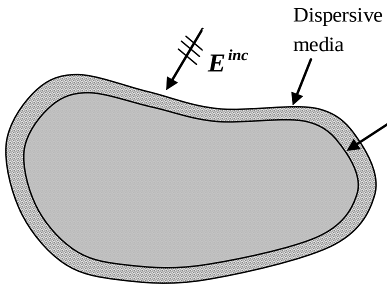


Fig. 1. Geometry of composite structure.

Enforcing the boundary condition on PEC surface *S*, time domain electric field integral equation can be expressed as:

$$\left[\mathbf{E}^{\mathrm{inc}}(\mathbf{r},t)+\mathbf{E}^{\mathrm{sca}}(\mathbf{r},t)\right]_{\tan}=0 \quad \mathbf{r}\in S. \tag{1}$$

Here, the time domain scattering electric field $\mathbf{E}^{\mathrm{sca}}(\mathrm{r},t)$ can be expressed as [11]:

$$\begin{aligned}\mathbf{E}^{sca}(\mathbf{r},t)&=\mathbf{E}^{sca}_{pec}(\mathbf{r},t)+\mathbf{E}^{sca}_{die}(\mathbf{r},t)\\&=-\frac{\mu_0}{4\pi}\frac{\partial}{\partial t}\int_S\frac{\mathbf{J}_s(\mathbf{r}',t-R/c)}{R}ds'-\frac{1}{4\pi\varepsilon_0}\nabla\int_S\frac{\rho_s(\mathbf{r}',t-R/c)}{R}ds'\\&-\frac{\mu_0}{4\pi}\frac{\partial}{\partial t}\int_V\frac{\mathbf{J}_{pol}(\mathbf{r}',t-R/c)}{R}dv'\\&-\frac{1}{4\pi\varepsilon_0}\nabla\left[\begin{aligned}&\int_S\frac{\rho_{pol,down}(\mathbf{r}',t-R/c)}{R}ds'\\&+\int_{S^\Delta}\frac{\rho_{pol,up}(\mathbf{r}',t-R/c)}{R}ds^{\Delta\prime}\end{aligned}\right],\end{aligned} \tag{2}$$

where $\mathbf{E}^{sca}_{pec}$ and $\mathbf{E}^{sca}_{die}$ denote the scattering field generated from PEC and dispersive dielectric, respectively. S^Δ is the upper surface of dielectric, *V* is the volume element of dielectric. $\mathbf{J}_s$ and ρ_s are the surface current and charge on the conductor. ρ_{pol} and $\mathbf{J}_{pol}$ are polarization charge and current of the dielectric layer, respectively.

The relationship between $\mathbf{J}_s$ and ρ_s on the conductor can be deduced from the current continuity condition as:

$$\rho_s(\mathbf{r},t)=-\int_{-\infty}^{t}\nabla\cdot\mathbf{J}_s(\mathbf{r},t')dt' \quad \mathbf{r}\in S. \tag{3}$$

As shown in Fig. 2, the relationship between $\mathbf{J}_{pol}$ and ρ_{pol} at the boundary of different materials can be expressed as:

$$\left[\mathbf{J}_{pol,1}(\mathbf{r},t)-\mathbf{J}_{pol,2}(\mathbf{r},t)\right]\cdot\mathbf{n}(\mathbf{r})=-\partial_t\rho_{pol}(\mathbf{r},t). \tag{4}$$

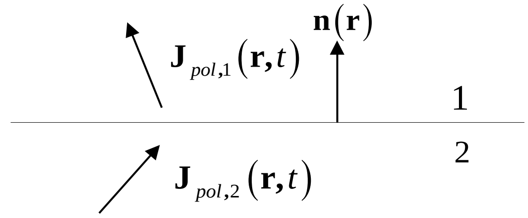


Fig. 2. The interface between two different materials.

If the part 1 is dielectric domain, and part 2 is PEC domain, (4) could change to:

$$\mathbf{J}_{pol}(\mathbf{r},t)\cdot\mathbf{n}(\mathbf{r})=-\partial_t\rho_{pol,down}(\mathbf{r},t). \tag{5}$$

Similarly, if the part 1 is free space, part 2 is dielectric domain, we have that,

$$\mathbf{J}_{pol}(\mathbf{r},t)\cdot\mathbf{n}(\mathbf{r})=\partial_t\rho_{pol,up}(\mathbf{r},t). \tag{6}$$

In the volume of dispersive dielectric, we have:

$$\begin{aligned}\mathbf{J}_{pol}(\mathbf{r},t)&=\partial_t\mathbf{D}(\mathbf{r},t)-\varepsilon_0\partial_t\mathbf{E}(\mathbf{r},t)\\&=\partial_t\mathbf{D}(\mathbf{r},t)-\varepsilon_0\partial_t\left[\gamma(t)\otimes\mathbf{D}(\mathbf{r},t)\right],\end{aligned} \tag{7}$$

where, $\mathbf{D}$ denotes the electric flux density. $\gamma(t)$ is the medium susceptibility and defined in [12]. Substituting the normal boundary condition on PEC surface:

$$\mathbf{n}(\mathbf{r})\cdot\mathbf{D}(\mathbf{r},t)=\rho_s(\mathbf{r},t), \tag{8}$$

to (7) and utilizing the relationship between $\mathbf{J}_s$ and ρ_s mentioned in (3), the relationship between $\mathbf{J}_{pol}$ and $\mathbf{J}_s$ can be obtained as:

$$\begin{aligned}\mathbf{J}_{pol}(\mathbf{r},t)&=-\left[\nabla\cdot\mathbf{J}_s(\mathbf{r},t)\right]\cdot\mathbf{n}(\mathbf{r})\\&+\varepsilon_0\left\{\gamma(t)\otimes\left[\nabla\cdot\mathbf{J}_s(\mathbf{r},t)\right]\right\}\cdot\mathbf{n}(\mathbf{r})\end{aligned}. \tag{9}$$

Utilizing the relationship of (9) to (5) and (6), $\rho_{pol}(\mathbf{r},t)$ is then obtained by following equations:

$$\begin{aligned}\rho_{pol,down}(\mathbf{r},t)&=\int_{-\infty}^{t}\left[\nabla\cdot\mathbf{J}_s(\mathbf{r},t')\right]dt'\\&-\varepsilon_0\left\{\gamma(t)\otimes\left[\int_{-\infty}^{t}\nabla\cdot\mathbf{J}_s(\mathbf{r},t')dt'\right]\right\}\end{aligned}, \tag{10}$$

$$\begin{aligned}\rho_{pol,up}(\mathbf{r},t)&=-\int_{-\infty}^{t}\left[\nabla\cdot\mathbf{J}_s(\mathbf{r},t')\right]dt'\\&+\varepsilon_0\left\{\gamma(t)\otimes\left[\int_{-\infty}^{t}\nabla\cdot\mathbf{J}_s(\mathbf{r},t')dt'\right]\right\}\end{aligned}. \tag{11}$$

According to the TDS theory, the sources of thin dispersive materials are all replaced by the current densities of PEC surface. Substituting (2), (9), (10) and (11) to (1), TD-EFIE can be converted to:

$$\left\{\begin{array}{l} \frac{\mu_0}{4\pi}\int_S \frac{\partial_t \mathbf{J}_s(\mathbf{r},t-R/c)}{R}ds - \frac{1}{4\pi\varepsilon_0}\nabla\int_S\int_{-\infty}^{t-R/c}\frac{\nabla\cdot\mathbf{J}_s(\mathbf{r},t')}{R}dt'ds \\ + \frac{\mu_0}{4\pi}\int_V \left\{\begin{array}{l}\frac{-\partial_t\left[\nabla\cdot\mathbf{J}_s(\mathbf{r},t-R/c)\right]}{R} \\ + \frac{\varepsilon_0\partial_t\left\{\gamma(t)\otimes\left[\nabla\cdot\mathbf{J}_s(\mathbf{r},t-R/c)\right]\right\}}{R}\end{array}\right\}\cdot\mathbf{n}(\mathbf{r})dv \\ + \frac{1}{4\pi\varepsilon_0}\nabla\int_S\int_{-\infty}^{t-R/c}\frac{\nabla\cdot\mathbf{J}_s(\mathbf{r},t')}{R}dt'ds \\ - \frac{1}{4\pi}\nabla\int_S\frac{\left\{\gamma(t)\otimes\left[\int_{-\infty}^{t-R/c}\nabla\cdot\mathbf{J}_s(\mathbf{r},t')dt'\right]\right\}}{R}ds \\ - \frac{1}{4\pi\varepsilon_0}\nabla\int_{S^\Delta}\int_{-\infty}^{t-R/c}\frac{\nabla\cdot\mathbf{J}_s(\mathbf{r},t')}{R}dt'ds^\Delta \\ + \frac{1}{4\pi}\nabla\int_{S^\Delta}\frac{\left\{\gamma(t)\otimes\left[\int_{-\infty}^{t-R/c}\nabla\cdot\mathbf{J}_s(\mathbf{r},t')dt'\right]\right\}}{R}ds^\Delta \end{array}\right\}_{\mathbf{tan}}$$
$$= \mathbf{E}^{inc}_{\mathbf{tan}}(\mathbf{r},t). \tag{12}$$

Here, the recursive convolution scheme [12] is applied. In order to solve (12) conveniently, a new parameter $\mathbf{P}(\mathbf{r},t)$ is defined as:

$$\mathbf{P}(\mathbf{r},t) = \gamma(t)\otimes\mathbf{J}_s(\mathbf{r},t). \tag{13}$$

Now, $\mathbf{P}(\mathbf{r},t)$ and $\mathbf{J}_S(\mathbf{r},t)$ are expanded with spatial and temporal basis functions as follows:

$$\mathbf{P}(\mathbf{r},t) = \sum_{n=1}^{N_S}\sum_{j=1}^{N_t} P_{j,n}T_j(t)\mathbf{f}_n^s(\mathbf{r}). \tag{14}$$

$$\mathbf{J}_s(\mathbf{r},t) = \sum_{n=1}^{N_S}\sum_{j=1}^{N_t} J_{j,n}T_j(t)\mathbf{f}_n^s(\mathbf{r}). \tag{15}$$

Here, RWG [13] basis functions $\mathbf{f}_n^s(\mathbf{r})$ are used as the spatial expansion basis functions. High order time shifted Lagrange interpolant $T(t)$ [14] is used for time expansion, and the order is chosen to be 4. N_S is the number of unknowns. N_t is the number of time steps.

Substituting (13), (14), (15) into (12), and referencing the treatments of convolution operations discussed in [12], after applying the Galerkin and point testing procedures in the space and time domains, respectively, the system of equations can be written as:

$$\tilde{Z}_0 I_i^d = V_i - \sum_{j=1}^{i-1}\tilde{Z}_j I_{i-j}^d + \sum_{j=1}^{i-1} Z_j^p I_{i-j}^p + \breve{Z}\breve{I}_{i-1}, \tag{16}$$

where,

$$I_i^d = \left[J_{i,1},\cdots,J_{i,N_S}\right], \tag{17}$$

$$I_i^p = \left[P_{i,1},\cdots,P_{i,N_S}\right], \tag{18}$$

$$\breve{I}_{i-1} = \left[\breve{P}_{i-1,1},\cdots,\breve{P}_{i-1,N_S}\right], \tag{19}$$

$$\tilde{Z}_{j,mn} = \begin{cases} Z^d_{j,mn} - Z^p_{0,mn}\beta^d_{j,n} & 0\le j\le K \\ Z^d_{j,mn} & j>K \end{cases} \quad (m,n=1,2,3,\cdots,N_S), \tag{20}$$

$$\begin{aligned} Z^d_{i-j,mn} = {} & \frac{\mu_0}{4\pi}\int_S \mathbf{f}_m^s(\mathbf{r})\cdot\left\{\int_S\frac{\sum_{n=1}^{N_S}\mathbf{f}_n^s(\mathbf{r}')\partial_\tau T_j(i\Delta t-R/c)}{R}ds'\right\}ds \\ & -\frac{\mu_0}{4\pi}\int_S \mathbf{f}_m^s(\mathbf{r})\cdot\left\{\int_V\frac{\sum_{n=1}^{N_S}\left[\nabla'\cdot\mathbf{f}_n^s(\mathbf{r}')\right]\partial_\tau T_j(i\Delta t-R/c)}{R}\cdot\mathbf{n}(\mathbf{r}')dv'\right\}ds \\ & -\frac{1}{4\pi\varepsilon_0}\int_S \mathbf{f}_m^s(\mathbf{r})\cdot\left\{\nabla\int_{S^\Delta}\frac{\sum_{n=1}^{N_S}\left[\nabla'\cdot\mathbf{f}_n^s(\mathbf{r}')\right]\partial_\tau^{-1} T_j(i\Delta t-R/c)}{R}ds^{\Delta\prime}\right\}ds \\ & (m,n=1,2,3,\cdots,N_S), \end{aligned} \tag{21}$$

$$\breve{Z}_{mn} = Z^p_{0,mn} \quad (m,n=1,2,3,\cdots,N_S), \tag{22}$$

$$\begin{aligned} Z^p_{i-j,mn} = {} & \varepsilon_0\frac{1}{4\pi\varepsilon_0}\int_S \mathbf{f}_m^s(\mathbf{r})\cdot\left\{\nabla\int_S\frac{\sum_{n=1}^{N_S}\left[\nabla'\cdot\mathbf{f}_n^s(\mathbf{r}')\right]\partial_\tau^{-1} T_j(i\Delta t-R/c)}{R}ds'\right\}ds \\ & -\varepsilon_0\frac{\mu_0}{4\pi}\int_S \mathbf{f}_m^s(\mathbf{r})\cdot\left\{\int_V\frac{\sum_{n=1}^{N_S}\left[\nabla'\cdot\mathbf{f}_n^s(\mathbf{r}')\right]\partial_\tau T_j(i\Delta t-R/c)}{R}\cdot\mathbf{n}(\mathbf{r}')dv'\right\}ds \\ & -\varepsilon_0\frac{1}{4\pi\varepsilon_0}\int_S \mathbf{f}_m^s(\mathbf{r})\cdot\left\{\nabla\int_{S^\Delta}\frac{\sum_{n=1}^{N_S}\left[\nabla'\cdot\mathbf{f}_n^s(\mathbf{r}')\right]\partial_\tau^{-1} T_j(i\Delta t-R/c)}{R}ds^{\Delta\prime}\right\}ds \\ & (m,n=1,2,3,\cdots,N_S), \end{aligned} \tag{23}$$

$$V_{i,m} = \int_S \mathbf{f}_m^s(\mathbf{r})\cdot\mathbf{E}^{inc}(\mathbf{r},i\Delta t)ds \quad (m=1,2,3,\cdots,N_S), \tag{24}$$

K denotes the order of temporal basis function. The parameters β^d and $\breve{I}$ are both the intermediate variables, which are used to represent the material property and recursive convolution, respectively. The definitions of them are both expounded in [12].

III. NUMERICAL RESULTS

In order to demonstrate the validity of the proposed scheme, three numerical examples for Debye, Drude and Lorentz materials will be presented in this section. The incident wave is the modulated Gaussian plane wave and defined as:

$$\begin{aligned} \mathbf{E}^{inc}(\mathbf{r},t) = {} & \hat{\boldsymbol{p}}^{inc}\cos[2\pi f_0(t-\mathbf{r}\cdot\hat{\mathbf{k}}^{inc}/c)] \\ & \times\exp[-0.5(t-t_p-\mathbf{r}\cdot\hat{\mathbf{k}}^{inc}/c)^2/\sigma^2]. \end{aligned} \tag{25}$$

Here, f_0, $f_{\max}$ and f_{bw} denote the center frequency, maximum frequency and bandwidth of the incident wave, respectively. $f_{\max} = f_0 + f_{bw}/2$. $\hat{\mathbf{k}}^{inc}$ and $\hat{\boldsymbol{p}}^{inc}$ are the incident and polarization directions.

To test the accuracy and late-time stability of the proposed method, a Debye material coated PEC sphere is considered. The radius of coated PEC sphere is 0.8m, and the thickness is 0.05m, 0.07m, 0.09m, respectively. The incident wave is modulated with $f_0 = 165\,\text{MHz}$, $f_{bw} = 270\,\text{MHz}$, $\hat{\mathbf{k}}^{inc} = \hat{\boldsymbol{z}}$, $\hat{\boldsymbol{p}}^{inc} = \hat{\boldsymbol{x}}$, $\sigma = 6/(\pi f_{bw})$. The number of total unknowns is 3834. The static relative permittivity $\varepsilon_{r,s}$ of the coated layer is 4.2, the permittivity of the medium at infinite frequency $\varepsilon_{r,\infty}$ is 1, and the relaxation time is $t_0 = 5\times10^{-9}\,s$. The time step Δt is 0.05*Lm*, total time is $3000\Delta t$. In order to verify the precision of the proposed scheme further, root mean square (RMS) error is introduced, the parameter of RMS error is defined as:

$$\text{RMS} = \sqrt{\sum_{i=1}^{n}\left| f_{c,i} - f_{r,i}\right|^2 \Big/ \sum_{i=1}^{n}\left| f_{r,i}\right|^2}\,, \tag{26}$$

where $f_{c,i}$ are the values of bi-static RCS which are computed by the proposed TDS scheme, and $f_{r,i}$ are the reference Mie series values, n is the number of observed angles. The RMS values of different thickness are shown in Fig. 3, it can be found that the accuracy of the proposed method at the most frequencies is ensured when the thickness of coated layer is no more than 0.09 wavelength of $f_{\max}$. The bi-static RCS curves of the coated sphere with 0.07m layer are shown in Fig. 4, which agree well with the exact Mie solutions. From the magnitude of time domain current coefficient shown in Fig. 5, it can be observed that the late-time stability is also ensured.

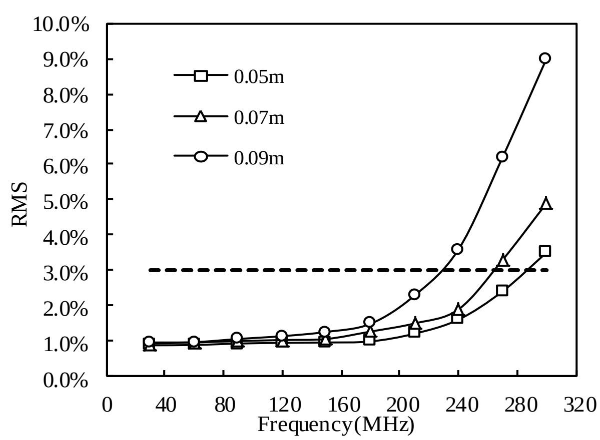


Fig. 3. RMS values of different thickness.

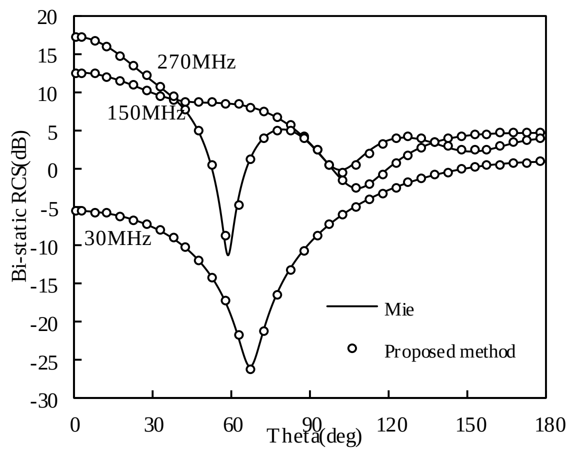


Fig. 4. Bi-static RCS ($\phi = 0^\circ$) of the coated sphere structure at 30, 150, and 270 MHz.

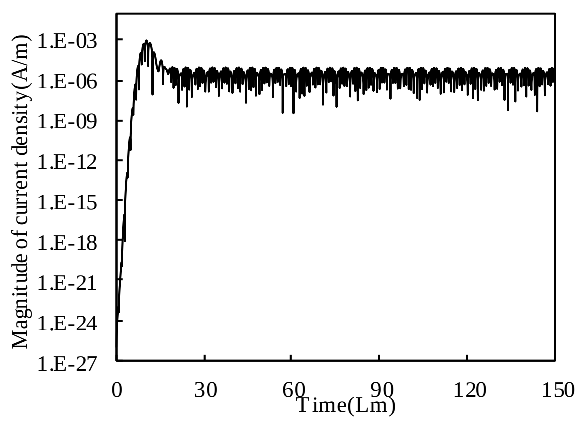


Fig. 5. Magnitude of time domain current coefficient at (0.001971m, -0.797540m, -0.062647m).

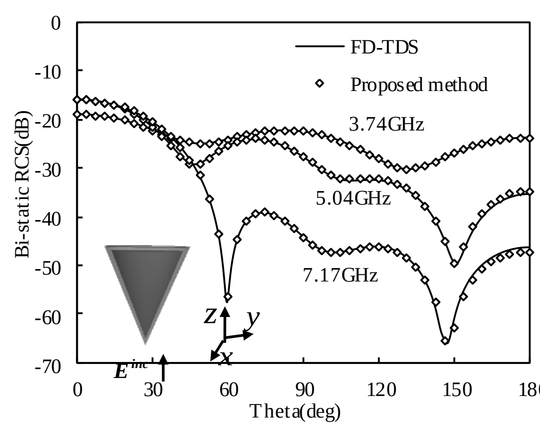


Fig. 6. Bi-static RCS ($\phi = 0^\circ$) of the coated cone structure at 3.74, 5.04, and 7.17 GHz.

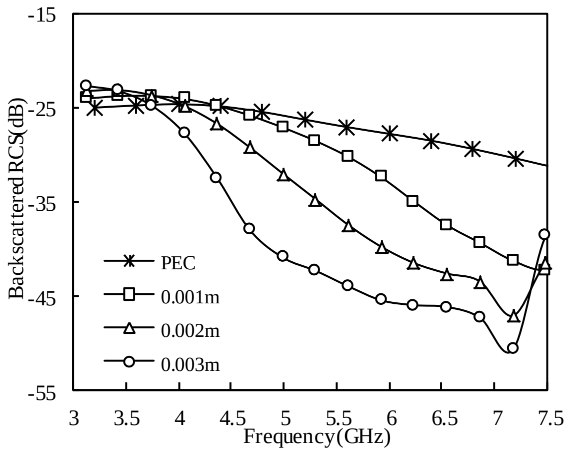


Fig. 7. The comparisons of backscattered RCS with different thickness.

Next, considering the scattering from a coated cone structure, and the coated layer is modeled as a Drude medium with $\varepsilon_{r,\infty}=1$, collision frequency $\delta_p = 8\times10^9 s^{-1}$, plasma frequency $\omega_p = 8\times10^9 s^{-1}$. The radius of inner PEC cone is 0.02m and the height is 0.06m. The thickness of coated dielectric layer is 0.001m, 0.002m, 0.003m, respectively. The center frequency of incident wave is 5.5 GHz, bandwidth is 5 GHz. The time step Δt is $0.01875Lm$, total time is $500\Delta t$. The unknown number of this object is 2376. It is a considerable reduction compared with TD-VSIE [3], which has 15806 unknowns. The bi-static RCS curves of the coated cone with 0.002m layer at different frequencies obtained after the discrete Fourier transform are shown in Fig. 6. Compared with its frequency domain counterpart, it is apparent that there is a good agreement between them. Moreover, the curves of backscattered RCS with different thickness are shown in Fig. 7, the stealth performance of coated structure is evident when compared with the PEC cone, and the performance will be more obvious with the increase of thickness.

At last, an airplane model coated with Lorentz material is considered, as shown in Fig. 8. The dimension of it is $x\times y\times z = 2.56m\times1.32m\times6m$. The coating's thickness is 0.02m. For this composite coated structure, the modeling of the coated layer is troublesome. If the TD-VSIE method is used to analyze it, the modeling of the coated layer is inevitable. However, the proposed method is used, the modeling process is simple. For analyzing the electromagnetic scattering from this composite structure by the proposed method, the time step Δt is $1/20\,Lm$, total time is $600\Delta t$. The incident wave is modulated with $f_0 = 165\,\text{MHz}$, $f_{bw} = 270\,\text{MHz}$, $\hat{\mathbf{k}}^{inc} = -\hat{\boldsymbol{z}}$, $\hat{\boldsymbol{p}}^{inc} = -\hat{\boldsymbol{x}}$, $\sigma = 3/(\pi f_{bw})$ and $t_p = 15\sigma$. The number of total unknowns is 4782. The damping coefficient δ_p of the coated layer is $1\times10^8 s^{-1}$, resonant frequency ω_p is $1.2\times10^8 s^{-1}$, static permittivity $\varepsilon_{r,s} = 2.8$, $\varepsilon_{r,\infty} = 1$. Figure 9 shows the curves of bi-static RCS at different frequencies. Good agreement is obtained again when compared with corresponding frequency domain method, and the ability of analyzing complex target by the proposed method is proven.

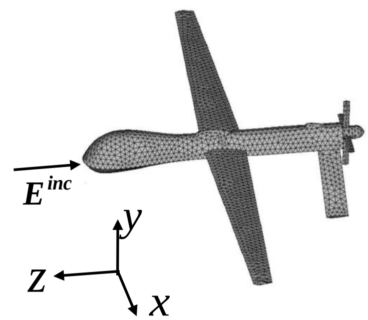
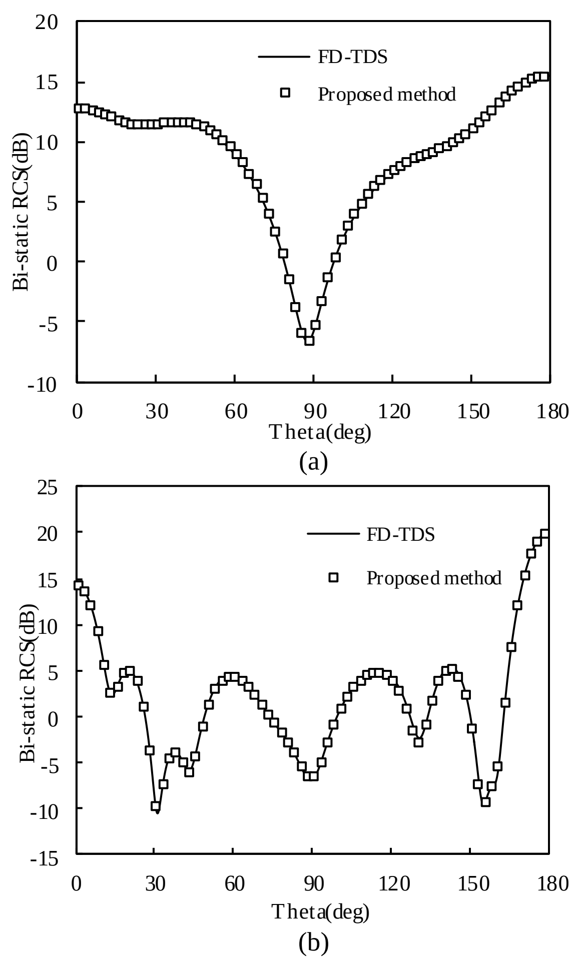
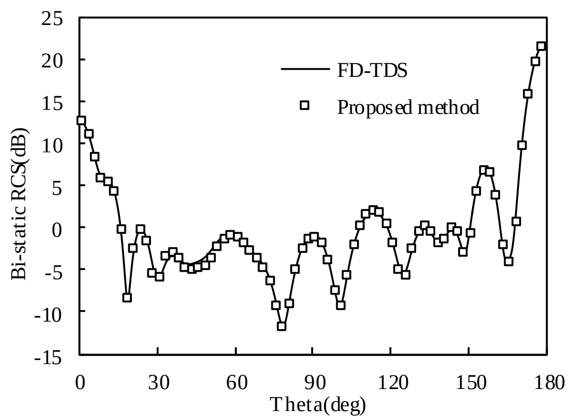


Fig. 8. Geometry of an airplane model.





(c)

Fig. 9. (a) Bi-static RCS $(\phi = 0^\circ)$ of the coated airplane structure at 54 MHz, (b) bi-static RCS $(\phi = 0^\circ)$ of the coated airplane structure at 153 MHz, and (c) bi-static RCS $(\phi = 0^\circ)$ of the coated airplane structure at 252 MHz.

IV. CONCLUSION

In this paper, a modified MOT based TD-TDS method is proposed, it extends the practicality of conventional time domain TDS method to analyze the transient electromagnetic scattering from the conductors coated with thin dispersive dielectric. Because the sources in the thin dispersive material layer are all replaced by the current densities on PEC surface, the number of unknowns will be reduced markedly. From the numerical results, the accuracy and stability of the proposed scheme are ensured, and the ability of analyze the coated stealth structure is demonstrated.

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REFERENCES

[1] K. Aygün, B. Shanker, and E. Michielssen, “Fast time domain characterization of finite size microstrip structures,” *Int. J. Num. Mod Elect. Net. Dev. & Fields,* vol. 15, no. 6, pp. 439-457, 2002.

[2] A. E. Yılmaz, J. M. Jin, and E. Michielssen, “A parallel FFT accelerated transient field-circuit simulator,” *IEEE Trans. Microw. Theory Tech.*, vol. 53, no. 9, pp. 2851-2865, 2005.

[3] Y. L. Hu and R. S. Chen, “Analysis of scattering from composite conducting dispersive dielectric objects by time-domain volume-surface integral equation,” *IEEE Trans. Antennas Propag.*, vol. 64, no. 5, pp. 1984-1989, 2016.

[4] S. M. Rao and T. K. Sarkar, “Numerical solution of time domain integral equations for arbitrarily shaped conductor/dielectric composite Bodies,” *IEEE Trans. Antennas Propag.*, vol. 50, no. 12, pp. 1831-1837, 2002.

[5] B. H. Jung, T. K. Sarkar, and Y. S. Chung, “Solution of time domain PMCHW formulation for transient electromagnetic scattering from arbitrarily shaped 3-D dielectric objects,” *Prog. Electromagn.*, vol. 45, pp. 291-312, 2004.

[6] M. A. Khayat and D. R. Wilton, “Electromagnetic scattering from arbitrarily shaped conducting bodies coated with thin dielectric materials,” *Proc. IEEE APS/URSI Int. Symp.*, Jan. 2004.

[7] I. T. Chiang and W. C. Chew, “A coupled PEC-TDS surface integral equation approach for electromagnetic scattering and radiation from composite metallic and thin dielectric objects,” *IEEE Trans. Antennas Propag.*, vol. 54, no. 11, pp. 3511-3516, Nov. 2006.

[8] J. Hu, L. Lei, Z. Nie, and S. He, “Fast solution of electromagnetic scattering from thin dielectric coated PEC by MLFMA and successive overrelaxation iterative technique,” *IEEE Microw. Wireless Compon. Lett.*, vol. 19, no. 2, pp. 762-764, Dec. 2009

[9] S. He, Z. Nie, and J. Hu, “Numerical solution of scattering from thin dielectric-coated conductors based on TDS approximation and EM boundary conditions,” *Prog. Electromagn.*, vol. 93, pp. 339-354, 2009.

[10] S. F. Tao and R. S. Chen, “Electromagnetic scattering analysis of the conductor coated by multilayer thin materials,” *IEEE Antennas Wirel. Propag. Lett.*, vol. 12, pp. 1033-1036, 2013.

[11] G. S. Cheng, Z. H. Fan, S. F. Tao, and R. S. Chen, “An efficient solution for the transient electromagnetic scattering from a conductor coated by multilayer thin materials,” *IEEE Antennas Wirel. Propag. Lett.*, vol. 14, pp. 1673-1676, 2015.

[12] G. Kobidze, J. Gao, B. Shanker, and E. Michielssen, “A fast time domain integral equation based scheme for analyzing scattering from dispersive objects,” *IEEE Trans. Antennas Propag.*, vol. 53, no. 3, pp. 1215-1226, Mar. 2005.

[13] S. M. Rao, D. R. Wilton, and A. W. Glisson, “Electromagnetic scattering by surfaces of arbitrary shape,” *IEEE Trans. Antennas Propag.*, vol. 30, no. 3, pp. 409-418, May 1982.

[14] B. Shanker, M. Y. Lu, J. Yuan, and E. Michielssen, “Time domain integral equation analysis of scattering from composite bodies via exact evaluation of radiation fields,” *IEEE Trans. Antennas Propag.*, vol. 57, no. 5, pp. 1506-1520, May 2009.