A Comparative Study of Opposition-Based Differential Evolution and Meta-Particle Swarm Optimization on Reconstruction of Three Dimensional Conducting Scatterers

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Abstract **─** In this paper, opposition-based differential evolution and Meta-particle swarm optimization is applied to reconstruct three dimensional conducting scatterers. Rational Bezier surfaces are utilized to model the shape of the scatterers. The mathematical representation of this surfaces are expanded in terms of Bernstein polynomials. The unknown coefficients of these polynomials depend on a few control points in space. An optimization method is used to find the location of the control points such that a specific measure of the difference between radar cross section (RCS) of the reconstructed and the original target is minimized. Physical optics (PO) approximation is used to find the RCS of a reconstructed scatterer in each iteration of the proposed algorithm. Simulation results show that these algorithms are very stable in the presence of noise.

Index Terms **─** Differential evolution, inverse scattering, particle swarm optimization, physical optics approximation, rational Bezier surfaces.

I. INTRODUCTION

Shape reconstruction of two and three dimensional conducting targets by using electromagnetic scattered field is a typical problem in inverse scattering and has many applications in radar target detection and identification and remote sensing. Several algorithms and approaches have been developed to deal with problem of inverse scattering such as level set schemes [1], linear sampling [2], time reversal [3]. Employing optimization methods to reconstruct the targets is another approach which have been widely used in recent years, mainly because of their simplicity of implementation [4]. Optimization methods are categorized into two main approaches of deterministic and stochastic methods. The main drawback of the deterministic methods is that they may trap in a local minimum. However, in stochastic algorithms, population of a random solution is used and therefore, better solutions help other members to emerge from local minima. Differential evolution (DE), particle swarm optimization (PSO), and genetic algorithm (GA) are the most popular schemes among stochastic algorithms. A comparative study of the performance of DE and PSO, to reconstrcut two dimensional conducting cases is reported in [4]. The performance of the genetic algorithm to reconstrcut two and three dimensional conducting scatterers has been reported in [5-6]. However, there is no report on the performance of DE and PSO for three dimensional conducting scatterers.

The main purpose of this paper is to compare a variant of DE, known as opposition-based differntial evoloution (ODE) and a version of PSO known as Meta-PSO when applied to three dimensional problems. These two algorithms have a good peformance over traditional optimization methods [12-13]. In reconstruction of two and three dimensional scatterers, the shape of the scatterers could be parameterized in terms of some specific polynomials. In the inverse problem [6], the coefficients of these polynomials are optimized such that electromagnetic scattered fields of the reconstructed shape and the original shape approach each other.

One of the few commonly used polynomials in this field are Bernstain polynomials [8]. In this paper, we also use Bernstein polynomials as the basis to expand the surface equation. The coefficients of this polynomial are extracted in the inverse problem using two approaches of ODE and Meta-PSO.

In summary, the novelty of this paper is two folds: (i) in the previous paper DE and PSO methods are discussed and compared for reconstruction of two-

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dimensional objects, but in this paper, we analysize and compare those algorithms to deal with three-dimensional structures, (ii) in this comparison, we use more recent versions of DE and PSO methods, namely ODE and Meta-PSO. These newer approaches are more efficient compared with traditional ones that are discussed in the litreture.

This paper is organized as follows. Section II presents the forward formulation. The inverse formulation is discussed in Section III. The two optimization algorithms are briefly reviewed in Section IV. Simulation results and concluding remarks are represented in Sections V and VI respectively.

II. FORWARD FORMULATION

The back scattered electromagnetic field from a large conducting scatterer can be expressed by physical optics approximation as follows [7]:

$$\vec{E}_s(r) = -\frac{j}{\lambda}\frac{\exp^{-jk_0 r}}{r}(\hat{k}.\vec{I})\vec{E}_0, \vec{I} = \int \hat{n}(r')\exp^{j2\vec{k}.\vec{r}'}ds', \quad (1)$$

where λ is the operating wavelength, $\vec{E}_0$ is the polarization of the incident field, $\hat{k}$ is the wave vector, $\vec{r}'$ is the source point position vector, ds' is the surface differential element, $\hat{n}$ is the surface normal vector at the source, k_0 is the free space wave number and $\vec{I}$ is the physical optics integral. In order to compute the PO integral, the surface geometry is modeled by rational Bezier patches. These patches are parametric and can be expressed in terms of Bernstein polynomials as follows [8]:

$$\vec{r}(u,v) = \frac{\sum_{i=0}^{m}\sum_{j=0}^{n} w_{ij}\vec{b}_{ij}B_i^m(u)B_j^n(v)}{\sum_{i=0}^{m}\sum_{j=0}^{n} w_{ij}B_i^m(u)B_j^n(v)w_{ij}}, \quad (2)$$

where $\vec{b}_{ij} \in R^3$, $i = 0,\ldots,m$, $j = 0,\ldots,n$, are the Bezier patches control points, $w_{ij} \in R$, $i = 0,\ldots,m$, $j = 0,\ldots,n$, are the associated weights, the integers *m* and *n* are degrees of the surface, and *u* and *v* are the parameters that shape the surface. The Bernstein polynomial, $B_i^m(u)$ is expressed as:

$$B_i^m(u) = \frac{m!}{i!(m-i)!}u^i(1-u)^{m-i}, 0 \le u \le 1. \quad (3)$$

Geometrical parameters of Bezier surfaces such as orthogonal and position vectors could be easily calculated in terms of Bernstein polynomials. In the literature, these surfaces are categorized into three groups of singly–curved, doubly–curved and plane patches. The PO integral over these surfaces could be simply evaluated by the stationary phase method and analytical techniques [9], [10] and [11].

III. INVERSE FORMULATION

The objective of the shape reconstruction process is to find the shape of the scatterer such that the difference between radar cross section of the reconstructed and the original shape is minimized. For this purpose, the shape of the scatterer is represented by Bernstein polynomials and the coefficients of these polynomials are calculated through optimization. More precisely, the coefficients of these polynomials are determined such that the cost function is minimized with respect to some specific control points. The cost function is defined as:

$$f(p) = \frac{\sum_{\omega}\sum_{\theta}\sum_{\phi} |\sigma_{\theta\phi\omega}^{true} - \sigma_{\theta\phi\omega}^{rec}|}{\sum_{\omega}\sum_{\theta}\sum_{\phi} |\sigma_{\theta\phi\omega}^{true}|}, \quad (4)$$

where $\sigma_{\theta\phi\omega}^{true}$ and $\sigma_{\theta\phi\omega}^{rec}$ are radar-cross sections of the original and the reconstructed scatterer, respectively. In this paper, opposition-based differential evolution and Meta-particle swarm optimization algorithm are used as the optimization techniques.

A. Opposition-based differential evolution algorithm

In the first step of this algorithm NP parameter vectors of D-dimension are created, that NP is the population of optimization algorithm and D is the number of unknown parameters. Also the opposite of this parameter vecors are produced as follows [12]:

$$ox_{i,j} = a_j + b_j - x_{i,j}, \quad (6)$$

where the minimum and maximum of the j^{th} dimension of the parameter vector are a_j and b_j. Then $ox_{i,j}$ is replaced by $x_{i,j}$ If the cost function of $ox_{i,j}$ is lower than $x_{i,j}$. Next, a mutant vector and a trial vector are created for each target as follows:

$$v_i^{G+1} = x_{r1}^G + F.(x_{r2}^G - x_{r3}^G), \quad (7)$$

$$u_i^{G+1} = (u_{1i}^{G+1}, u_{2i}^{G+1}, \ldots\ldots, u_{Di}^{G+1}), \quad (8)$$

where

$$u_{ji,G+1} = \begin{cases} v_{ji}^{G+1} & if \quad h_j \leqslant H \quad or \quad j = l \\ x_{ji}^{G+1} & if \quad h_j > H \quad and \quad j \ne l \end{cases}. \quad (9)$$

In these equations, *G* is the generation index, r_1, r_2, r_3 are three mutually different integers that also differ from target index *i*, *F* is the mutuant constant that is taken to be 0.8, h_j is a random number in the interval [0,1], $H\varepsilon(0,1)$ is a crossover constant selected by the user, and *l* is a random integer $\in [1,2,...,D]$.

If the cost function of u_i^{G+1} is lesser than x_i^{G+1}, then x_i^{G+1} is changed by the trial vector. In the last step a random number between [0,1] is generated and if it is lower than the preselected jumping rate J_r then $x_{i,j}^G$ is compared with the opposite of it and the one with a lower cost function is selected as the member of the current population.

B. Opposition-based Meta particle swarm algorithm

For simplicity, first, ordinary PSO is explained and then it is generalized to Meta PSO. If a problem has *D* unknown parameters, a group of NP_1 parameter vectors x_i, $i = 1,....,NP_1$, each with dimension D are produced. Each vector has an initial random velocity to search the solution space. This velocity is updated in each iteration of the optimization algorithm as:

$$v_t = w\times v_{t-1} + c_1(pbest_{t-1} - x_{t-1}) + c_2(gbest_{t-1} - x_{t-1}), \quad (11)$$

where *w* is the inertial weight that scale the old velocity, $pbest_{t-1}$ is the previous best solution of each member before iteration *t*, and $gbest_{t-1}$ is the previous best solution of all members before iteration *t*. Moreover, c_1 and c_2 are the two preselected numbers that are usually chosen to be 0.49, 1.49, or 2 [13]. With this velocity, the position of the members are updated in each iteration as:

$$x_t = x_{t-1} + v_t. \quad (12)$$

If the current member has a lower cost function than $pbest_{t-1}$, then, $pbest_{t-1}$ is replaced by this member. The same procedure is used to update $gbest_{t-1}$.

In Meta-PSO, several groups are randomly generated. The velocity of one particle from each group is changed as follows [9]:

$$v_t = w\times v_{t-1} + c_1(pbest_{t-1} - x_{t-1}) + c_2(gbest_{t-1} - x_{t-1}) + c_3(sbest_{t-1} - x_{t-1}), \quad (13)$$

where $sbest_{t-1}$ is the best previous location of all members of all groups. c_1, c_2, and c_3 are three preselected numbers that are selected to be 2 here. In this problem, *NS* groups, where each group has NP_1 members are considered. For the purpose of comparing Meta-PSO with ODE, we choose $NS\times NP_1$ as equal to the population of ODE. Similarly to the previous algorithm a random number between [0,1] is generated and if it is lower than the preselected jumping rate J_r, then $x_{i,j}^G$ is compared with the opposite of it and the one with a lower cost function is selected as the member of the current population.

V. NUMERICAL RESULTS

For the first example, the reconstruction of a perfectly conducting conical curved sector with the height of 1 m, the bottom radius of 1m, and the top radius of 0.5 m is presented. In the reconstruction procedure, the degree of the surface and weight coefficients are selected a-priori. In addition, we assume that the surface curvature is negative. This cone is modeled by 3x2 control points.

The parameters of the Meta-PSO and the opposition-based differential evolution are listed in Table 1 and Table 2. The original cone is compared with the ODE and the Meta-PSO reconstructed cones in Fig. 1 (a) and Fig. 1 (b) respectively. A comparison between the cost function of these two algorithms at various iterations are depicted in Fig. 1 (c). The scattered filed is evaluated at 45 points that are located at $\theta = 60^\circ, 75^\circ, 90^\circ, 105^\circ, 120^\circ$ and at frequencies of $0.4, 0.8, 1.2 GHz$. As shown, ODE converges better than Meta-PSO. The RCS of the original and the reconstructed cones for $45^\circ < \theta < 120^\circ$ and $\phi = 45^\circ$ are presented in Fig. 1 (d). A very good agreement is observed between the RCS of the original cone and the reconstructed one. Also for comparison purpose, the RCS simulated with the moment method is depectied in the same figure. The good agrement between the full wave method and physical optic method can be seen. To measure the accuracy of the reconstruction procedure, the shape error function is defined as [6]:

$$RECP = \frac{1}{N_{vnet}}\left(\sum_{i=0}^{m}\sum_{j=0}^{n}\frac{\|\Delta^{0,1}p_{i,j}^{true} - \Delta^{0,1}p_{i,j}^{reco}\|^2}{\|\Delta^{0,1}p_{i,j}^{true}\|^2} + \frac{\|\Delta^{1,0}p_{i,j}^{true} - \Delta^{1,0}p_{i,j}^{reco}\|^2}{\|\Delta^{1,0}p_{i,j}^{true}\|^2}\right)^{1/2}, \quad (14)$$

where $N_{vnet} = 2mn + m + n$ is the number of elements in the vector net and $\| p_{i,j} \|$ is the Euclidean norm given by:

$$\| p_{i,j} \| = \sqrt{x_{i,j}^2 + y_{i,j}^2 + z_{i,j}^2}. \quad (15)$$

$\Delta^{0,1}p_{i,j}$ and $\Delta^{1,0}p_{i,j}$ are related to the control points as:

$$\Delta^{0,1}p_{i,j} = p_{i,j+1} - p_{i,j} \quad \Delta^{1,0}p_{i,j} = p_{i+1,j} - p_{i,j}. \quad (16)$$

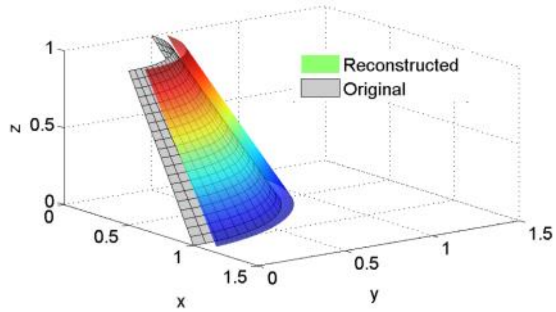
The cost function in this problem is defined on RCS. Because the RCS does not change with the displacement of the shape, we obtain a transformation of the object in the reconstruction. Finally, we define the reconstruction error in a form that does not change with the displacement of the shape.

Table 1: Opposition-based differential evolution parameters

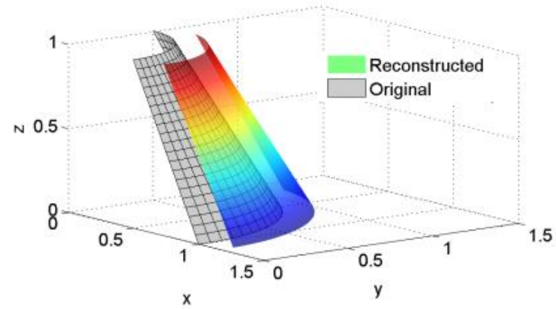
Jumping Rate	Mutant Constant	Crossover Rate
0.5	0.8	0.5

Table 2: Meta-particle swarm optimization parameters

Jumping Rate	w	c1	c2	c3
0.5	0.9-0.4	2	2	2



(a)



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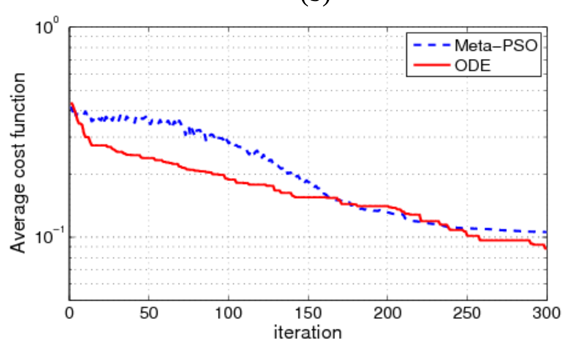

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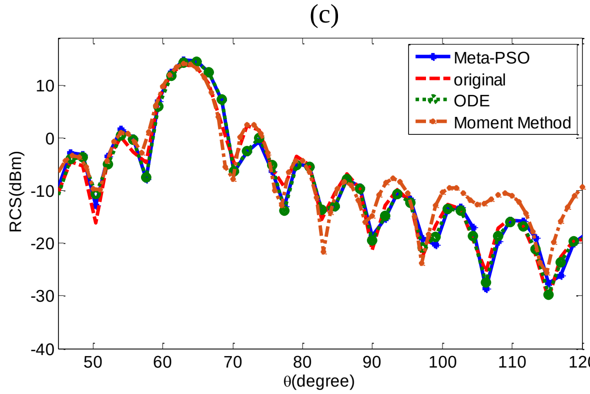


(d)

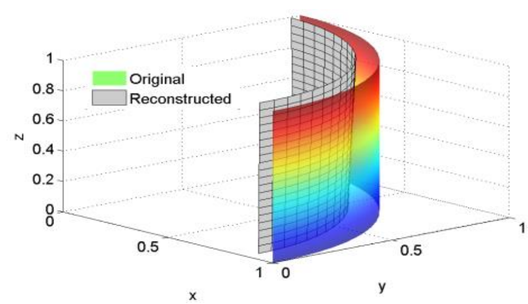
Fig. 1. (a) Comparison between original and reconstructed target by ODE, (b) comparison between original and reconstructed target by Meta-PSO, (c) cost function of ODE and Meta-PSO, and (d) comparison between RCS of reconstructed and original shape at $f = 1.2GHZ$.

In the simulation, the average reconstruction errors for five simulation of ODE and Meta-PSO are 0.16 and 0.07 respectively. The stability of the algorithm is tested in the presence of an additive noise of [6]:

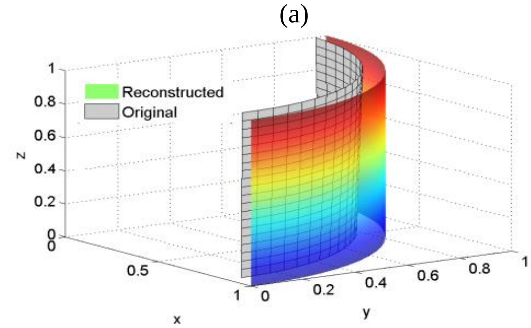
$$\sigma^{n}_{\theta,\varphi,\omega} = \sigma_{\theta,\varphi,\omega} + \sqrt{\sigma^{2}_{\theta,\varphi,\omega}} \times (NL) \times (rand)\,, \qquad (17)$$

where $\sigma_{\theta,\phi,\omega}$ is the RCS of the original shape, *NL* is the noise level, *rand* is a random number between [0,1], and is the root mean square of the original RCS. The reconstruction error with an additive noise level of 0.1 is 0.1764 for ODE and 0.1532 for Meta-PSO. Therefore, given that the reconstruction method is stable in the presence of additive noise in the radar cross-section, it can be concluded that if we use the measurement data for reconstruction, this method is also usable.

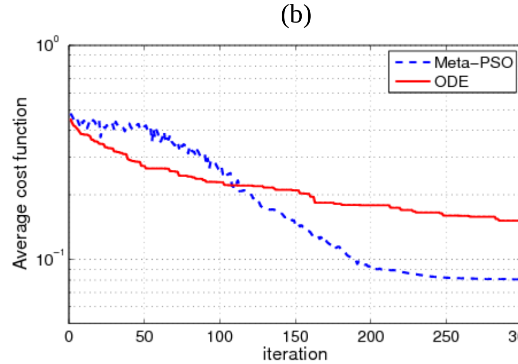
For the second example, the reconstruction of perfectly conducting 90°–cylindrical sector, with the height and radius of 1m is considered. Similarly, Fig. 2 shows the target and the simulation results. The average shape error obtained by ODE is 0.2174 and by Met-PSO is 0.2138. If the number of optimization steps was increased, the radar cross section of the reconstructed body would be closer to the original object.



(a)



(b)



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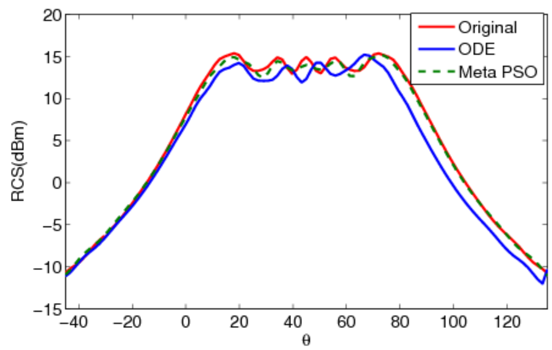

(d)

Fig. 2. (a) Comparison between original and reconstructed target by ODE, (b) comparison between original and reconstructed target by Meta-PSO, (c) cost function of ODE and Meta-PSO, and (d) comparison between RCS of reconstructed and original shape at $f = 1.2GHZ$.

VI. CONCLUSION

ODE and Meta-PSO algorithms are compared for shape reconstruction of three dimensional conducting objectst. In both cases, a good agreement between the reconstructed and the original shape is observed. Bezier surfaces are utilized to model the target and PO approximation is used to compute the scattered field. In addition, the stability of this algorithm in the presence of noise is investigated. Finally from the results we find that Meta-PSO is better than ODE for reconstruction of the target.

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