Optimal Design of Elliptical Array Antenna Using Opposition Based Differential Evolution Technique

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Abstract **─** Radiation pattern synthesis of non-uniformly excited planar arrays with the lowest relative side lobe level (SLL) is presented in this paper. Opposition based differential evolution (ODE) scheme, which represents a novel parameter optimization technique in antenna engineering is applied for the parameter optimization of the single and the multi-ring circular array (CA), hexagonal array (HA) and elliptical array (EA) of isotropic elements. To overcome the problem of premature convergence of differential evolution (DE) algorithm, ODE is designed without significantly impairing the fast converging property of DE. Two design examples are presented which illustrate the effectiveness of the ODE based method, and the optimization goal for each example is easily achieved. The design results obtained using ODE are much more improved than those of the results obtained using the state of the art evolution algorithms like particle swarm optimization (PSO), harmonic search (HS) and differential evolution (DE) methods in a statistically significant way.

Index Terms **─** Concentric circular array, concentric elliptical array, concentric hexagonal array, opposition based differential evolution, side lobe level.

I. INTRODUCTION

Uniform circular array [1, 2] has the capability of 360 degree beam scanning without the significance change in SLL or beam width and it can be useful for smart antenna application [3, 4]. The mutual coupling effect is more significant in order to achieve low SLL by reducing the distance between elements in circular arrays. Hexagonal array is presented to overcome the problem of high SLL for smart antenna applications [5]. The comparison between CA and HA shows that the hexagonal array geometry provide deeper nulls and higher gain with the same beam width as circular array [6]. Also, best beam steering ability was found using a uniform hexagonal array of seven patch antennas with a central element [7] which can be applied to the wireless communication of advance generation. Elliptical shaped array and the combinations of elliptical and linear array with array factors are investigated in [8]. The effect of ellipse eccentricity, number of elements and element spacing are also investigated. Array hybridization (mixing two different arrays) approaches can also be used to improve the performance of antenna arrays in terms of SLL and directivity [9].

For optimization of complex, nonlinear and non-differentiable array factor of antenna array, various evolutionary optimization approaches such as firefly algorithm (FFA) [10], particle swarm optimization (PSO) [11], harmonic search (HS) [12], differential evolution (DE) [13] etc., have been widely used. It is accepted that, compared with the other state-of-art optimization techniques, the PSO is a powerful optimization scheme for antenna design problems [14], cluster based wireless sensor network design [15], wiring network diagnosis [16] etc.

Problems with the real valued variables can be solved by DE technique which is one of the finest genetic type process. In DE, mutation operation is used as a primary search mechanism and selection to direct the search toward a more promising region. Selection mechanism is used in genetic algorithm (GA) to generate a population sequence while crossover as principal operation for useful exchange of information of the solutions. This is the fundamental difference between GA and DE. The idea of opposition-based learning (OBL) is introduced in [17] by Tizhoosh. Based on the concept OBL, a new reinforcement learning algorithm is presented in [18, 19] to accelerate the convergence of the algorithm. OBL has been utilized to improve the global search ability and to accelerate the convergence rate of DE in this paper. Therefore, the proposed methodology of opposition based differential evolution (ODE) is used

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to achieve a better estimation for the current candidate solution. In this paper, the array geometry synthesis is first formulated as an optimization problem with the goal of SLL reduction and then is solved using ODE algorithm for optimum current excitations.

II. DESIGN EQUATIONS

A. Array factors

1.1 Single-ring arrays

The general configuration of circular array (CA) with N number elements in x-y plane is shown in Fig. 1. The array factor $AF(\theta,\varphi)$ for CA is given by (1) [2]:

$$AF(\theta,\varphi)=\sum_{n=1}^{N} I_n e^{jkr\sin\theta(\cos\varphi_n\cos\varphi+\sin\varphi_n\sin\varphi)}, \quad (1)$$

where r = radius of circular array; k = wave number; θ = elevation angle; φ = azimuth angle; I_n = excitation coefficient of nth element; angular position of nth element, $\varphi_n = 2\pi(n-1)/N$.

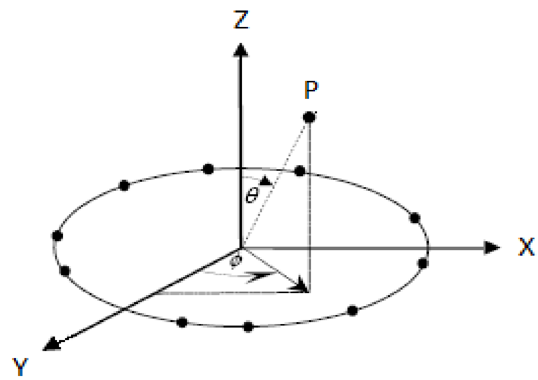


Fig. 1. Circular array (CA) structure.

Figure 2 displays the general configuration of a regular hexagonal array (HA) with 2×N elements (N=6). Here, half (N) of the array elements are situated at the angular points and another half are located at the middle edges of the hexagon.

The far-field pattern of HA can be expressed as array factor AF, given in (2) [4]:

$$AF(\theta,\varphi)=\sum_{n=1}^{N}[A_n e^{jkr_1\sin\theta(\cos\varphi_{1n}\cos\varphi+\sin\varphi_{1n}\sin\varphi)} + B_n e^{jkr_2\sin\theta(\cos\varphi_{2n}\cos\varphi+\sin\varphi_{2n}\sin\varphi)}], \quad (2)$$

where

$$\left.\begin{aligned} r_2 &= r_1\cos(\pi/N) \\ r_1 &= d_e/\sin(\pi/N) \end{aligned}\right\}. \quad (3)$$

The array factor of an N-element elliptical array (Fig. 3) can be derived using (4) [4]:

$$AF(\theta,\varphi)=\sum_{n=1}^{N} A_n e^{jk\sin\theta(a\cos\varphi_n\cos\varphi+b\sin\varphi_n\sin\varphi)}, \quad (4)$$

where a is the semi-major axis and b is the semi-minor axis of the elliptical array.

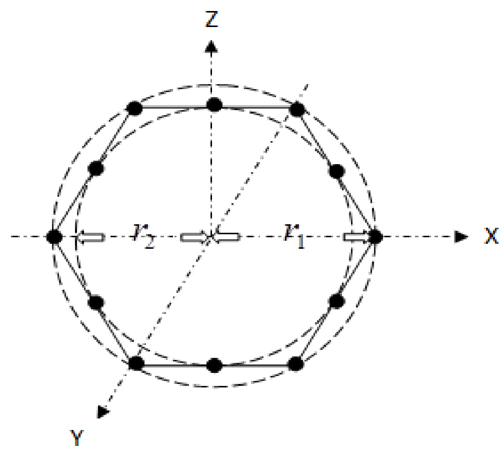


Fig. 2. Hexagonal array (HA) structure.

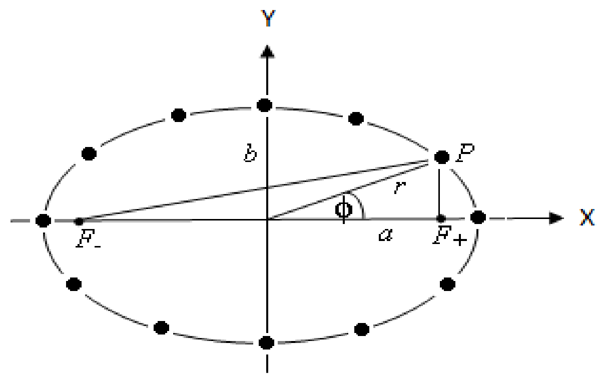


Fig. 3. Elliptical array (EA) structure.

1.2 Concentric-ring arrays

The array factor of M-ring concentric elliptical array (CEA) is given by (5) [4]:

$$AF(\theta,\varphi)=\sum_{m=1}^{M}\sum_{n=1}^{N_m} B_{nm} e^{jk\sin\theta(a_m\cos\varphi_n\cos\varphi+b_m\sin\varphi_n\sin\varphi)}, \quad (5)$$

where N_m is the number of elements lie on m^{th} elliptical ring, B_{nm} is the excitation amplitude and a_m is the semi-major axis and b_m is the semi-minor axis of m^{th} ring. The values of a_m and b_m can be obtained from (6) for a given eccentricity e and inter-ring spacing d:

$$\left.\begin{aligned} a_m &= a+(m-1)d \\ b_m &= a_m\sqrt{1-e^2} \end{aligned}\right\}. \quad (6)$$

In (6), innermost elliptical ring is having the semi-major axis a.

The expression for the array factor of concentric circular array (CCA) in the x-y plane is derived by substituting $a_m = b_m = r_m$ in (5) and is given in (7):

$$AF(\theta,\varphi) = \sum_{m=1}^{M} \sum_{n=1}^{N} B_{nm} e^{jkr_m \sin\theta \cos(\varphi - \varphi_n)} . \quad (7)$$

The array factor of concentric hexagonal array (CHA) can be found by the summation of the array factors of M concentric HAs, given in (8):

$$AF(\theta,\varphi) = \sum_{m=1}^{M} \sum_{n=1}^{N} [A_{nm} e^{jkr_{1m} \sin\theta (\cos\varphi_{1n} \cos\varphi + \sin\varphi_{1n} \sin\varphi)} + , \\ B_{nm} e^{jkr_{2m} \sin\theta (\cos\varphi_{2n} \cos\varphi + \sin\varphi_{2n} \sin\varphi)}] \quad (8)$$

where

$$r_{1m} = r + (m-1)d_h \\ r_{2m} = r_{1m} \cos(\pi / N) . \quad (9)$$

In (9), r is the radius of the smallest circle encompassing the smallest hexagon with the elements lying on its vertices and d_h is the spacing between the hexagons along x-axis.

B. Objective function formulation

For the synthesis of single and concentric ring arrays, cost function *CF* is formulated which is capable of generating pencil beam with lower SLL and higher directivity. *CF* is formulated to meet the corresponding design goal as follows:

$$CF = C_1[SLL_c - SLL_d] + C_2[FNBW_c - FNBW_d] . \quad (10)$$

In (10), SLL_c and $FNBW_c$ are the computed values (corresponding value of I_n=1) of SLL and first null beam width, respectively. SLL_d and $FNBW_d$ refer to the desired values of SLL and first null beam width, respectively, for non-uniform excitation case.

III. OPPOSITION-BASED DIFFERENTIAL EVOLUTION ALGORITHM

A. A brief description of differential evolution (DE) algorithm

For optimization problems over continuous domains, DE algorithm was first developed by R. Storn and K. Price in 1995. Scheme for generating trial parameter vectors is the fundamental idea behind DE algorithm. It produces new parameter vectors by adding the weighted difference vector between two populations. The details of DE algorithm can be found in [13].

B. Concept of opposition-based learning (OBL)

The computation time of any evolutionary optimization method is dependents on the distance between randomly chosen initial solutions (random guesses) and the optimal solution. The improvement of this starting phenomenon can be done with a nearby solution by consecutively testing the opposite solution [17]. Thus, to accelerate the convergence, the nearer of the two guesses (guess and opposite guess) is used as the initial population. The same methodology can be applied continuously to every solution in the current population. The mathematical concept of opposite number and opposition based optimization is stated below:

Let $P = (x_i^1, \cdots\cdots, x_i^d, \cdots\cdots, x_i^n)$ be a point (guess) in *n*-dimensional space,

where

$$\{x_i^1, \cdots\cdots, x_i^d, \cdots\cdots, x_i^n\} \in R ,$$

and

$$x_i \in [A_i, B_i] \, \forall_i \in \{1, \cdots\cdots, d, \cdots\cdots, n\} .$$

The opposite point (opposite guess) $\hat{P} = (\hat{x}_1^1, \cdots\cdots, \hat{x}_1^d, \cdots\cdots, \hat{x}_1^n)$ is defined by its components as stated in (11):

$$\hat{x}_i = A_i + B_i - x_i . \quad (11)$$

Assume $f = (.)$ is a fitness function which is used to measure the candidate's fitness.

Now, for a minimization problem, the point P can be replaced with $\hat{P}$ if $f(\hat{P}) \le f(P)$. Hence, to continue with the appropriate solution, the point and its opposite point are evaluated simultaneously.

C. Opposition-based differential evolution (ODE) algorithm

In this algorithm, opposition-based idea is implanted in DE which is selected as the parent algorithm to accelerate the convergence characteristics with near global optimal solution. The scheme of the OBL [17, 20] is merged in two steps such as initialization and opposition based generation in each iteration. The steps of the suggested ODE are discussed as follow:

Step 1: Generation of opposition based initial population P_0.

for ($i = 0;\ i < S;\ i++$) % Population size S
  for ($j = 0; j < n;\ j++$) % *n*-dimensional space
    $OP_{0_{i,j}} = A_j + B_j - P_{0_{i,j}}$ % initial population P_0 % opposite population OP_0
  end
end

Selection of S suitable solutions from $\{P_0, OP_0\}$.

Step 2: Fitness calculations for each set of particles in the population.

Step 3: Follow the steps of Mutation, Crossover and Selection in DE.

Step 4: Checking for constraints of the problem.

Step 5: Generation jumping (opposition based).

if ($rand(0,1) < J_r$)
  for ($i = 0;\ i < S;\ i++$)

$$\text{for } (\ j=0;\quad j<n;\quad j++)$$

$$OP_{i,j} = \min_j^p + \max_j^p - P_{i,j}$$

end

end

Selection of S suitable solutions from $\{P, OP\}$ as current population, P

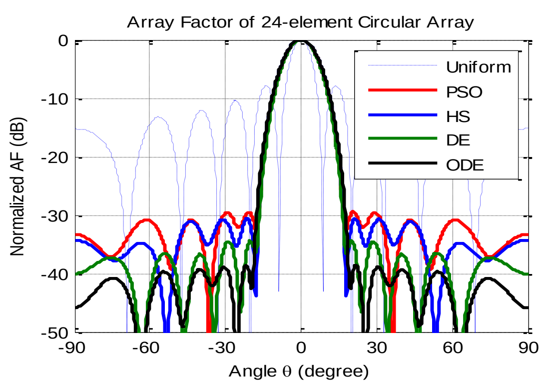
Step 6: Steps 2 to 5 are repeated until the stopping condition is met.

IV. RESULTS AND DISCUSIONS

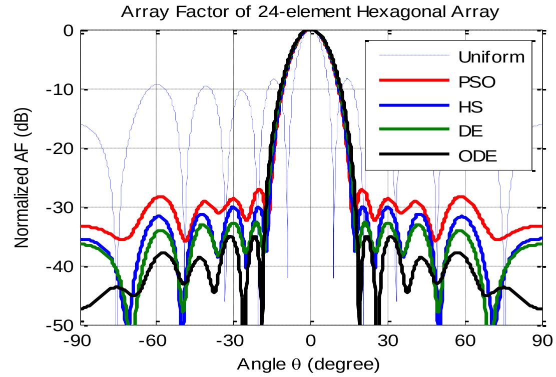
CASE-1: **Simulation results of single ring arrays**

24-element single-ring CA, HA and EA each of which is placed on x-y plane symmetrically with respect to origin are considered. In order to place the elements symmetrically and equally spaced along the edges of the hexagonal geometry, the total number of elements in the array is taken as multiple of six (N=6×4=24). In the case of 24-element HA, 6 elements are placed at the vertices and the remaining 18 elements can be placed along the edges of the hexagon symmetrically. Different values of e (=0.2, 0.4 and 0.6) are considered for elliptical shaped array. So, the total number of arrays taken into account for comparison is five. Circular array can also be defined as elliptical array with eccentricity e=0. Cost function (objective function) *CF* in (10) is utilized in four algorithms in this case. Inter-element spacing *d* in each array is considered as 0.5λ.

A comparison is made among optimized CA, HA and EA in terms of SLL and Directivity (DIR) using PSO, HS, DE and ODE and is given in Table 1. Tables 2-4 present the optimal amplitudes coefficients for these three different array structures. Figures 4 (a) and 4 (b) depict the normalized power patterns for CA and HA, respectively, using PSO, HS, DE and ODE. Figures 4 (c), 4 (d) and 4 (e) represent the normalized power patterns of EA with three different values of eccentricity, respectively. From Table 1, it can be seen that ODE produces the best results as compared with other three state-of-art algorithms for the design of single ring arrays in terms of SLL reduction of approximately 5dB without affecting the directivity considerably.



(a)



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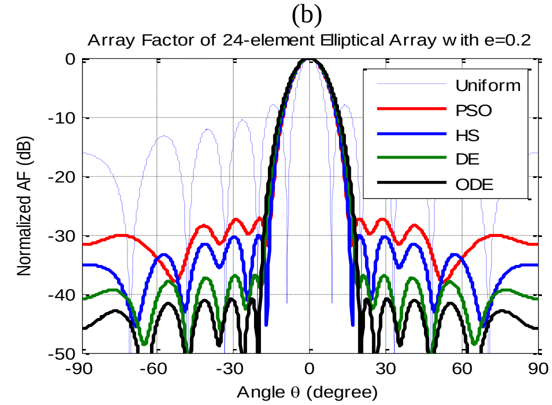

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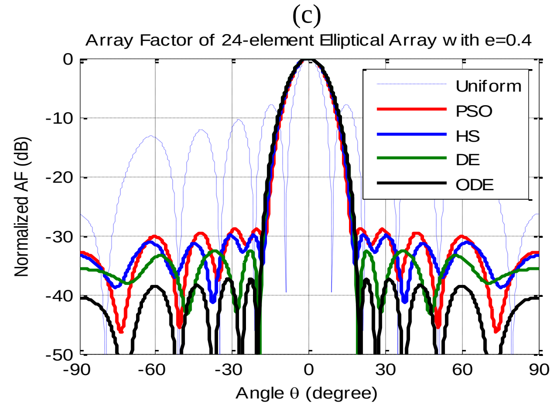

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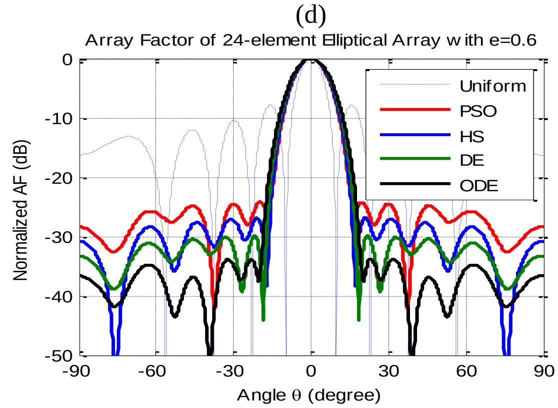


(e)

Fig. 4. Array patterns of: (a) CA, (b) HA, (c) EA with e=0.2, (d) EA with e=0.4, and (e) EA with e=0.6.

***CASE-2:* Simulation results of concentric ring arrays**

CCA, CHA and CEA, each with four concentric rings are considered. Each ring having N_i = (6×i) number of uniformly spaced isotropic elements where i (=1, 2, 3, 4) denotes the ring number counted from the innermost ring. So, each array contains a total of 60 elements. CEA is considered for three different eccentricity values (e=0.2, 0.4, 0.6) where eccentricity value e=0 reveals that the array is the same as CCA. The spacing between two adjacent rings is constant with value d_e=0.5λ. The spacing between two adjacent elements in the innermost ring is also fixed at d_i=0.5λ for each array geometry. Inter-element spacing for the other rings can be calculated from d_e, d_i and N_i. In case of CEA, parameter a_m and b_m can be measured by (6) for a particular value of eccentricity.

Performances of CCA, CHA and CEA in terms of SLL and HPBW (3-dB BW) using PSO, HS, DE and ODE are tabulated in Table 5.

Table 1: Performances of 24-elements CA, HA and EA

Array Configuration		PSO		HS		DE		ODE	
		SLL (dB)	DIR	SLL (dB)	DIR	SLL (dB)	DIR	SLL (dB)	DIR
CA		-29.47	16.1532	-30.61	15.4240	-34.41	15.6591	-38.55	15.4166
HA		-27.09	16.1729	-30.01	14.8225	-32.67	14.9997	-35.00	14.9372
EA	e=0.2	-27.24	15.8365	-30.01	15.7854	-36.87	15.6439	-40.79	14.1663
	e=0.4	-28.83	15.9518	-29.88	15.6905	-32.52	15.5647	-37.35	15.6282
	e=0.6	-24.23	16.7684	-26.91	16.1146	-29.93	16.0594	-33.82	16.1673

Table 2: Excitation amplitude distribution of 24-element CA using three different optimization techniques

Algorithms	Amplitude Distributions	Max. SLL (dB)
PSO	1.0000 1.0000 0.9957 1.0000 0 0.0832 0.2912 0.0450 0.3357 0 0.4797 1.0000 1.0000 1.0000 0.8005 0.3889 0 0 0 0.0039 0.1913 0 0 0.5469	-29.47
HS	1.0000 1.0000 0.5650 0 0.2853 0.0702 0.2348 0 0 0.7194 0.7723 1.0000 1.0000 0.6363 1.0000 0 0.0034 0 0 0 0.1643 0.4550 0 1.0000	-30.61
DE	1.0000 0.6005 1.0000 0 0.2140 0.0000 0.1783 0 0 0.5154 0 1.0000 0.9226 1.0000 0.4431 0.1164 0.2496 0 0.2154 0 0 0.5457 0.7483 0.7388	-34.41
ODE	1.0000 1.0000 0 0.1385 0 0 0.1271 0 0.1580 0.3454 1.0000 0.6367 1.0000 0.9255 0.3543 0.3872 0.1691 0.1933 0 0 0 0.2146 0.7910 0.8686	-38.55

Table 3: Excitation amplitude distribution of 24-element HA using three different optimization techniques

Algorithms	Placement of Elements	Amplitude Distributions	Max. SLL (dB)
PSO	Vertices (A_n)	1 0 0 1 0 0	-27.09
	1st position (B_n)	0.6620 0 0.6761 0.8339 0.8094 0.5046	
	2nd position (C_n)	0.2765 0 0.7325 0.4888 0 0.8614	
	3rd position (D_n)	0 0.3170 0.8608 0.6822 0 1.0000	
HS	Vertices (A_n)	1.0000 0.5811 0 1.0000 0 0.3902	-30.01
	1st position (B_n)	1.0000 0 0.9833 0.9251 0 0.2441	
	2nd position (C_n)	0.2207 0 1.0000 0.9194 0.0035 0.1366	
	3rd position (D_n)	0.0338 0.0026 0.7611 0.5180 0 0.6598	
DE	Vertices (A_n)	1.0000 0.4345 0 1.0000 0 0.0002	-32.67
	1st position (B_n)	1.0000 0 0 0.5435 0.0085 0.3597	
	2nd position (C_n)	0.2094 0.0718 1.0000 1.0000 0 0	
	3rd position (D_n)	1.0000 0 0.7703 0.2914 0.3017 1.0000	
ODE	Vertices (A_n)	1.0000 0.1425 0 1.0000 0 0	-35.00
	1st position (B_n)	1.0000 0 0.4000 1.0000 0 0.2357	
	2nd position (C_n)	0.1017 0.2270 1.0000 0.2146 0.2820 0.8275	
	3rd position (D_n)	0.4004 0.0142 0.6726 0.4989 0 0.6108	

Table 4: Excitation amplitude distribution of 24-element EA using three different optimization techniques for various eccentricities

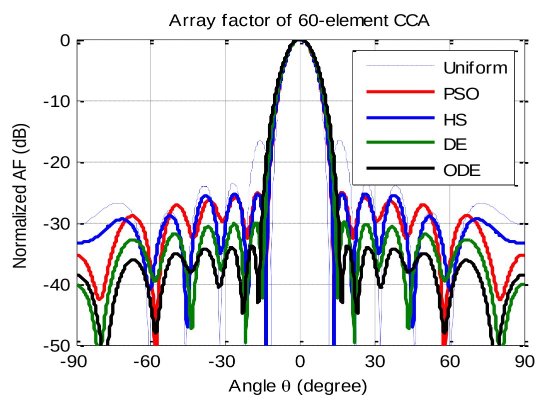
Eccentricity	Algorithms	Amplitude Distributions	Max. SLL (dB)
0.2	PSO	1.0000 0.9594 0.6003 1.0000 0.4613 0.0039 0.3006 0 0 0 0.7405 1.0000 1.0000 0.6549 1.0000 0 0.0000 0 0 0.2649 0.0852 0.5648 0 1.0000	-27.24
	HS	1.0000 0.8396 1.0000 0.1793 0.0255 0.1666 0.0902 0 0.3046 0.5513 0.2263 0.9866 1.0000 0.6980 0.4884 0.4947 0 0 0 0.2471 0.1430 0.1167 0.6210 0.9908	-30.01
	DE	1.0000 0.6269 0.4678 0.4919 0.1234 0.1005 0.0764 0 0 0 0.5070 1.0000 1.0000 0.8321 0.6037 0.3306 0.0595 0 0.1816 0 0.2235 0.3296 0.6330 1.0000	-36.87
	ODE	1.0000 0.7615 0.1217 0 0 0 0.1513 0 0.2165 0.5649 1.0000 1.0000 1.0000 0.9218 1.0000 0.4867 0.1296 0.0780 0.0601 0 0 0 0 0.7373	-40.79
0.4	PSO	1.0000 0.9997 1.0000 0.4158 0.2818 0 0.1337 0 0 0 0 0.5945 1.0000 1.0000 1.0000 0 0.3524 0.0400 0.3626 0.0879 0 1.0000 0.4307 0.9599	-28.83
	HS	1.0000 1.0000 0 0 0.2443 0.0000 0 0 0 0.3575 0.8697 0.5576 1.0000 1.0000 1.0000 0 0 0 0.4252 0 0.3479 1.0000 0.4241 1.0000	-29.88
	DE	1.0000 0.9958 0.9155 0 0 0.0001 0 0.1896 0.2084 0.6724 0.2593 0.7864 1.0000 0.6065 0 0.5220 0 0 0 0.1996 0 0 1.0000 0.9945	-32.52
	ODE	1.0000 0.5842 0 0.5141 0 0.0975 0.0689 0 0.1592 0.0015 1.0000 1.0000 0.8415 0.5841 0 0.2362 0 0.1652 0 0 0.1390 0.2827 1.0000 1.0000	-37.35
0.6	PSO	1.0000 1.0000 1.0000 0.8836 0 0 0.5940 0 0.0243 0 0 0.9582 1.0000 1.0000 1.0000 1.0000 0 0.5579 0 0 0.4713 0.0144 0.6135 1.0000	-24.23
	HS	1.0000 0.9204 0.1960 0.1872 0 0.6382 0 0.0013 0 0.5610 1.0000 1.0000 1.0000 0.8407 0.3550 0 0 0 0.0306 0.5432 0.0597 1.0000 0.9999 1.0000	-26.91
	DE	1.0000 0.9523 0.7445 0.0142 0.1597 0 0 0.2656 0 0.5499 0.3988 0.7088 0.9991 0.9143 1.0000 1.0000 0 0 0 0.5863 0 0 0.1828 1.0000	-29.92
	ODE	1.0000 0.8457 1.0000 0.0008 0 0.0010 0.2668 0 0.3078 0.7505 0.3061 1.0000 0.9968 1.0000 0.4971 0.0578 0 0.3101 0 0 0 0.5414 0.4716 0.6650	-33.82

Table 5: Performances of 60-elements CCA, CHA and CEA using four optimization techniques

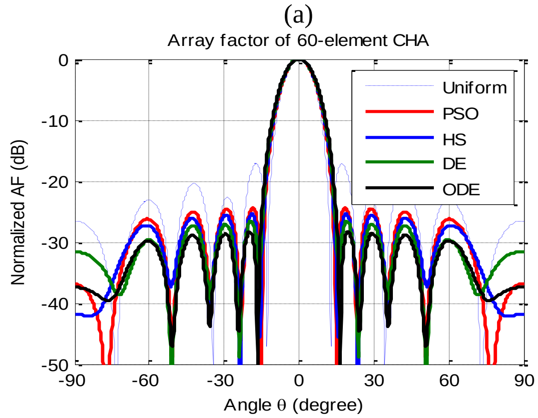
Array Configuration		PSO		HS		DE		ODE	
		SLL (dB)	3-dB BW (degree)	SLL (dB)	3-dB BW (degree)	SLL (dB)	3-dB BW (degree)	SLL (dB)	3-dB BW (degree)
CCA		-25.01	10.6	-25.27	10.8	-30.00	12.2	-33.82	11.8
CHA		-24.39	11.6	-25.43	12.0	-26.60	12.2	-28.27	12.4
CEA	e=0.2	-26.05	11.0	-27.76	11.0	-29.94	11.6	-35.02	12.2
	e=0.4	-26.12	11.6	-28.39	12.0	-30.04	12.0	-31.66	12.0
	e=0.6	-22.85	12.8	-24.91	13.0	-27.66	13.4	-30.37	13.6

Figures 5 (a) and 5 (b) depict the normalized power patterns for CCA and CHA, respectively, using PSO, HS, DE and ODE. Figures 5 (c), 5 (d) and 5 (e) represent the normalized power patterns of CEA with three different values of eccentricity, respectively. From Table 5, it can be seen that ODE produces the best results as compared with other three recently developed algorithms for the design of concentric ring arrays in terms of SLL reduction.

Convergence profiles for all algorithms are also recorded for synthesis of various arrays. The population size and the maximum number of iteration cycles are 100 and 150, respectively.



(a)



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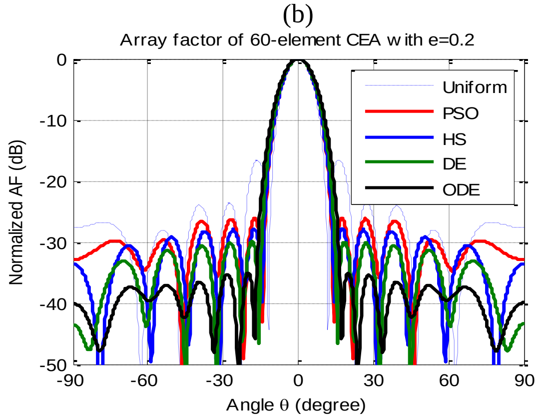

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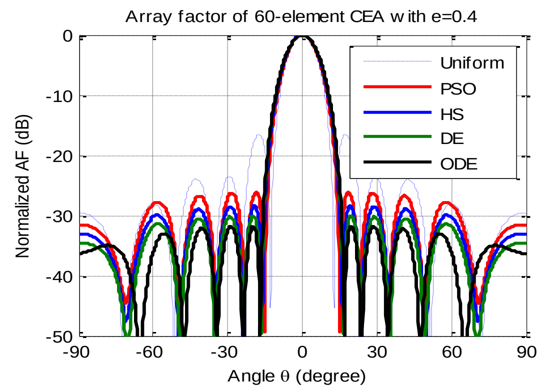

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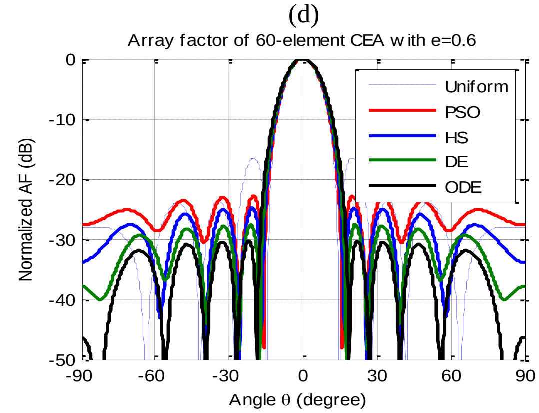

(e)

Fig. 5. Array patterns of: (a) CCA, (b) CHA, (c) CEA with e=0.2, (d) CEA with e=0.4, and (e) CEA with e=0.6.

V. CONCLUSION

ODE proves its better searching ability compared with PSO, HS and DE for the design of various planar arrays in antenna engineering optimization problem. ODE shows its superiority in terms of the best converged solution and convergence speed and efficient SLL reduction. The FNBWs/HPBWs of the synthesized array patterns with fixed inter-element spacing using these algorithms are very close for arrays of same shape and size. From the corresponding tables and figures given in above discussions, it can be observed that ODE produces an array patterns with the reduction of approximately 5 dB in SLL value in compared with PSO, HS and DE in almost all the cases. The other array parameters like directivity or 3-dB beam width of the synthesized array patterns are very close to the arrays of same shape and size. Designing arrays for a low SLL and highly directive pattern, there is no such available direct traditional relation between the SLL and the directivity but it maintains a trade-off relationship. Different array geometries are investigated here rather than the traditional liner or circular array but the combination of them which may explore a new research area in antenna engineering. Consideration of real elements would necessitate a supplementary calculation of coupling effects. Therefore, the judgment of the algorithm would be based on the various structural parameters and resultant patterns.

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