Baffle Diffraction in Interferometric Detectors of Gravitational Waves

Giuseppe Pelosi [1], Leonardo Possenti [1], Stefano Selleri [1], and Innocenzo M. Pinto [2]

[1] Department of Information Engineering
University of Florence, Florence, 50139, Italy
giuseppe.pelosi@unifi.it, leonardo.possenti@unifi.it, sefano.selleri@unifi.it

[2] Department of Engineering
University of Sannio, Benevento, 82100, Italy
pinto@sa.infn.it

Abstract **─** This paper presents an efficient high-frequency analysis framework for studying diffraction occurring at irises, or baffles, in the arms of a Fabry-Perot optical interferometer, relevant to the design and operation of interferometric detectors of gravitational waves like LIGO and Virgo.

Index Terms **─** Gaussian beams, gravitational wave detectors, uniform theory of diffraction.

I. INTRODUCTION

Recently interferometric detectors of gravitational waves like LIGO, Virgo and KAGRA provided the first direct observation of gravitational waves [1-2]. These instruments are very long baseline (~5km) Fabry-Perot optical interferometers, where the (quadrupole) spacetime ripples due to a gravitational wave are converted into an intensity modulation of the dark fringe [3-5].

The differential arm length perturbations to be measured are extremely small, of the order of 10^{-21}m, hence requiring extreme precision in the whole optical set-up, and extreme rejection of all possible noise sources. In particular, the (infrared, $\lambda = 1064$nm) laser beam travels the interferometer arms within pipes where high vacuum is created, to minimize light scattering from air molecules. Even if the beam is very narrow, the spot size on the terminal mirrors being a few cm wide, diffraction due to the finite size of the end-mirrors (and other optical components) [6] may cause a small amount of stray light to reach the vacuum pipe walls, that are coupled to environmental noise, creating multipath interference that may result into idiosyncratic noise features, that ultimately hinder the instrument's performance. Absorbing baffles or irises, are accordingly placed along the beam path, to intercept stray light that would eventually reach the pipe walls, and re-couple to the main FP cavity mode [7,8].

However, diffraction from the baffle edges can be itself a source of multipath interference; hence efficient modeling of baffle diffraction is necessary in order to optimize baffle design for present and next generation detectors.

In this paper we apply the uniform theory of diffraction (UTD) to a simple *canonical* baffle problem, using a realistic (Gaussian beam) model for the primary field in the arms, and a fully 3D baffle geometry. The proposed solution is analytic and physically readable, and the UTD-computed diffracted field is projected into the natural Gauss-Laguerre basis describing the light field the FP interferometer arms [4], to obtain an efficient (accurate, readable and easily computable) representation of the diffracted field.

The paper is organized as follows: the needed Gauss-Laguerre and UTD concepts are introduced in Section II and III, respectively. Section IV discusses the geometry of the baffle problem and the proposed UTD solution, with numerical results shown in Section V. Finally, Section VI draws some conclusions and hints for future work.

II. GAUSSIAN BEAMS

Laser beams are usually very narrow-band, and highly directive, so that the beam amplitude drops off rapidly as the angle between the beam axis and the direction of observation increases.

The (scalar) Gaussian beam solution of Helmholtz equation in the paraxial approximation, provides a very useful tool for the mathematical representation of fields which propagate unbounded but confined close to a specific direction, as laser beams [9].

Assuming axial symmetry, and propagation along the positive z direction, the fundamental (scalar) Gaussian beam field is:

$$u(\rho, z) = \frac{w_0}{w(z)} e^{-\frac{\rho^2}{[w(z)]^2}} e^{j\phi_G(z)} e^{-\frac{\pi\rho^2}{\lambda R(z)}} e^{-jkz}. \quad (1)$$

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The function $w(z)$ describes the beam width (distance between the points where the field's amplitude reduces by a factor e^{-1} from its maximum on-axis value); $w_0 = w(0)$ being denoted as the beam waist, as shown in Fig. 1. The larger w_0 the more collimated the beam. $R(z)$ is the wavefront radius of curvature. At the beam waist $z = 0$, $R(0)$ tends to infinity, as one would expect for a plane wave. Further away, $\lim_{z\to\infty} R(z) = z$, that is the radius of curvature of a spherical wavefront originating in $z = 0$. As the Gaussian beam propagates along the z-axis, its phase changes in a way which differs from that of a plane wave. The phase shift is represented by the Gouy phase $\phi_G(z)$ (see [9, 10] for details), resulting in a slightly increased distance between wavefronts.

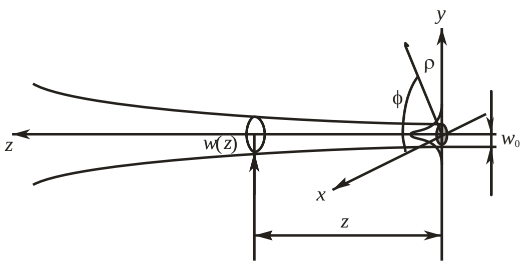

Fig. 1. Reference system and Gaussian beam, showing how the waist $w(z)$ changes along the beam propagation direction.

In real lasers or optical systems, axial symmetry cannot be guaranteed, and higher order modes can be excited. In cylindrical coordinates, these are the Gauss-Laguerre (GL) modes:

$$u(\rho,\phi,z) = \frac{w_0}{w(z)}\left(\frac{\sqrt{2}\rho}{w(z)}\right)^m L_p^m\left(\frac{2\rho^2}{[w(z)]^2}\right)\cdot \\ \cdot e^{-\frac{\rho^2}{[w(z)]^2}} e^{-j\frac{\pi\rho^2}{\lambda R(z)}} e^{-jkz} e^{jm\phi} e^{j\phi_G(z)}. \tag{2}$$

The Laguerre polynomials $L_p^m(x)$ are defined by two integer indices p>0 and m and are of order $N = 2p + |m|$. The related Gouy phase shift $\phi_G(z)$ is larger than that of the fundamental Gaussian beam by a factor $N + 1$.

The two indices p and m describe, respectively, the radial and azimuthal dependence of the beam. In particular, the radial index p denotes the number of nodal rings on a plane perpendicular to the direction of propagation.

The superposition of the (p, m) and (p,-m) GL modes yields the $TEM_{p,m}$ modes shown in Fig. 2.

In the case of interest here, the vector electromagnetic field is linearly polarized, and can be written as follows:

$$\mathbf{u}(\rho,\phi,z) = u(\rho,\phi,z)\hat{\mathbf{p}}\ , \tag{3}$$

where $\hat{\boldsymbol{p}}$ is a constant unit vector.

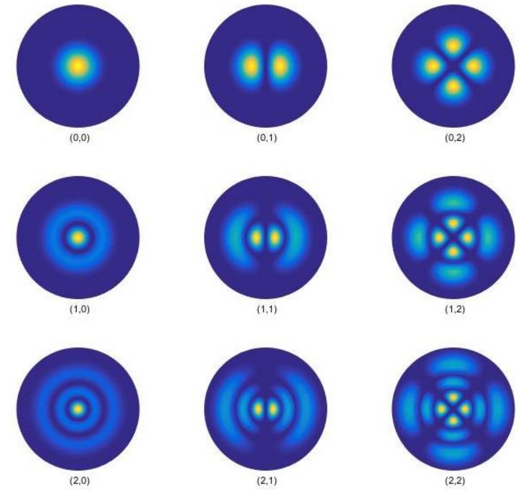

Fig. 2. Transverse intensity distribution of various $TEM_{p,m}$ (GL) beams.

III. UNIFORM THEORY OF DIFFRACTION

In the asymptotic high frequency limit, when the wavelength becomes negligible with respect to the size of the scattering objects, the light field can be computed by ray tracing (eikonal equation, Fermat principle) combined with field transport and energy conservation along the rays, yielding the geometrical optics (GO) solution [11].

The Fermat principle was generalized by Keller to include knife-edge and conical-tip diffracted rays, leading to an improved version of GO known as Geometrical Theory of Diffraction (GTD) [12].

The GTD field, however, may exhibit discontinuities (e.g., at shadow/reflection boundaries) and/or singularities (e.g. at caustics and foci).

The Uniform Geometrical Theory of Diffraction (UTD) was developed to overcome some of these limitations, namely the divergence at shadow boundaries, by the introduction of appropriate, physically motivated transition functions. UTD hence provides continuous field at shadow boundaries, even if it still fails at caustics [13].

In the asymptotic short wavelength limit, diffraction can be considered as a *local* phenomenon and the study of scattering is reduced to that of a few ideal "canonical" problems, the most relevant for our purposes being the "wedge problem" [14, 15].

The field scattered by a wedge is given by the superposition of GO and diffracted (UTD) field: the former takes into account incident and reflected fields in those regions where they exist; the latter guarantees continuity. The diffracted field may be accordingly written as $[\mathbf{E}^d] = [D][\mathbf{E}^i]A(\rho)e^{-jk\rho}$, where $[\mathbf{E}^d]$ and $[\mathbf{E}^i]$ are column matrices holding the components of the diffracted and incident fields respectively, $[D]$ is a square matrix of diffraction coefficients, ρ is the distance from the wedge edge to the observation point and $A(\rho)$ is a spreading factor. Field components are conveniently given in a ray-fixed coordinate system (see [13-15] for details), hence incident and diffracted field are fully described by their parallel and perpendicular components with respect to the incidence and diffraction planes respectively, and $[D]$ is a 2-by-2 matrix. The diffraction matrix is available for perfectly conducting, perfectly absorbing and mixed (impedance) boundary condition wedges [13-15].

IV. FORMULATION

As a toy model for studying baffle diffraction in the beam pipes of a LIGO-like interferometric detector of gravitational waves, we consider the simple geometry depicted in Fig. 3: a planar metal screen placed at $z = z_b$, perpendicular to the pipe/beam axis *z*, with a centered circular aperture of radius *a*.

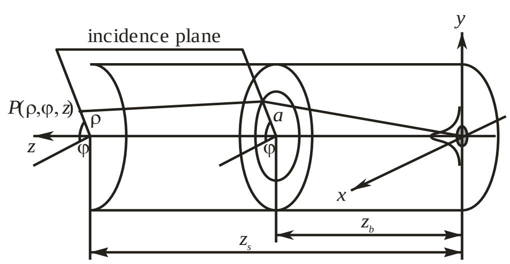

Fig. 3. Geometry for the baffle diffraction problem.

Even if the diffracting edge is a circular rim, its diameter is usually several thousands of wavelengths, so that the edge can be treated as locally straight, and UTD coefficients for the straight wedge can be used.

The baffle is illuminated by a 3D Gaussian beam (3) propagating along the *z*-axis, with the electric field linearly polarized along the *y* direction.

Different polarizations can, of course, be treated in a similar manner.

Table 1: Baffle aperture radius and observing plane used in our simulations

	$z_b = 5a$	$z_s = 2a + z_b$	$z_s = 5a + z_b$
a = 0.05 m	0.25 m	0.35 m	0.5 m
a = 0.15 m	0.75 m	1.05 m	1.5 m
a = 0.5 m	2.5 m	3.5 m	5 m

The diffracted field is evaluated on a plane at $z = z_s$ by resorting to the extended Fermat's principle to locate the diffraction point on the baffle rim for each diffracted ray path.

V. NUMERICAL RESULTS

For the present analysis, a wavelength $\lambda = 1\mu m$ has been considered, with a beam waist $w_0 = 10\lambda_0$, and unit amplitude on the beam axis.

Six different baffle geometries have been considered, whose relevant parameters are collected in Table 1.

The diffracted field has been computed using UTD and is depicted in Fig. 4, in normalized units for easier reading for various values of the baffle's aperture *a* and of the observation plane distance from the baffle's plane.

In all cases, an interference pattern appears. As the aperture becomes larger, the interference pattern tends to vanish.

As the distance between the planes at $z = z_b$ and $z = z_s$ increases, the intensity of the diffracted field also tends to vanish, and no interference pattern shows up.

One of the major numerical issues has been the very low incident field's values on the baffle's edge. Dealing with a wavelength and a beam waist in the order of 10^{-6}m and 10^{-4}m, the beam is extremely collimated and its intensity on the baffle's edge is of the order of 10^{-10}Vm^{-1}. Hence, the evaluation of the diffracted field is critical due to its very low intensity and to the finite machine's precision. Smaller values for the beam waist or an augmented intensity of the incident beam have been used in order to get larger values for the incident field on the baffle's edge, values which are then to be denormalized to attain final results.

Even if UTD overcomes the GTD issues at the shadow boundaries, thanks to the transition function, it is known that, as the distance from the edge increases, the UTD evaluation is more critical [13]. Special care was hence necessary in our case, where the field is computed thousands of wavelength away from the edge, to evaluate the transition function since, again, finite machine precision give rise to non-perfect singularity cancellation in diffraction coefficients very close to the shadow boundaries.

Table 2 shows CPU times. It can be noted how times are not the same, even if the number of points where the diffracted field is computed is the same, due to checks in the evaluation of the transition functions which are necessary as *a* increases and the baffle and the reference section gets farther apart.

Table 2: Times to compute the incident field on the edge, t_{inc}, and the diffracted field on the section, t_d, at two different distances from the baffle

	a = 0.05 m	*a* = 0.15 m	*a* = 0.5 m
t_{inc}	0.017s	0.001s	0.001s
$t_d\ @2a$	79,748s	79,865s	80,393s
$t_d\ @5a$	79,826s	80,393s	81,005s

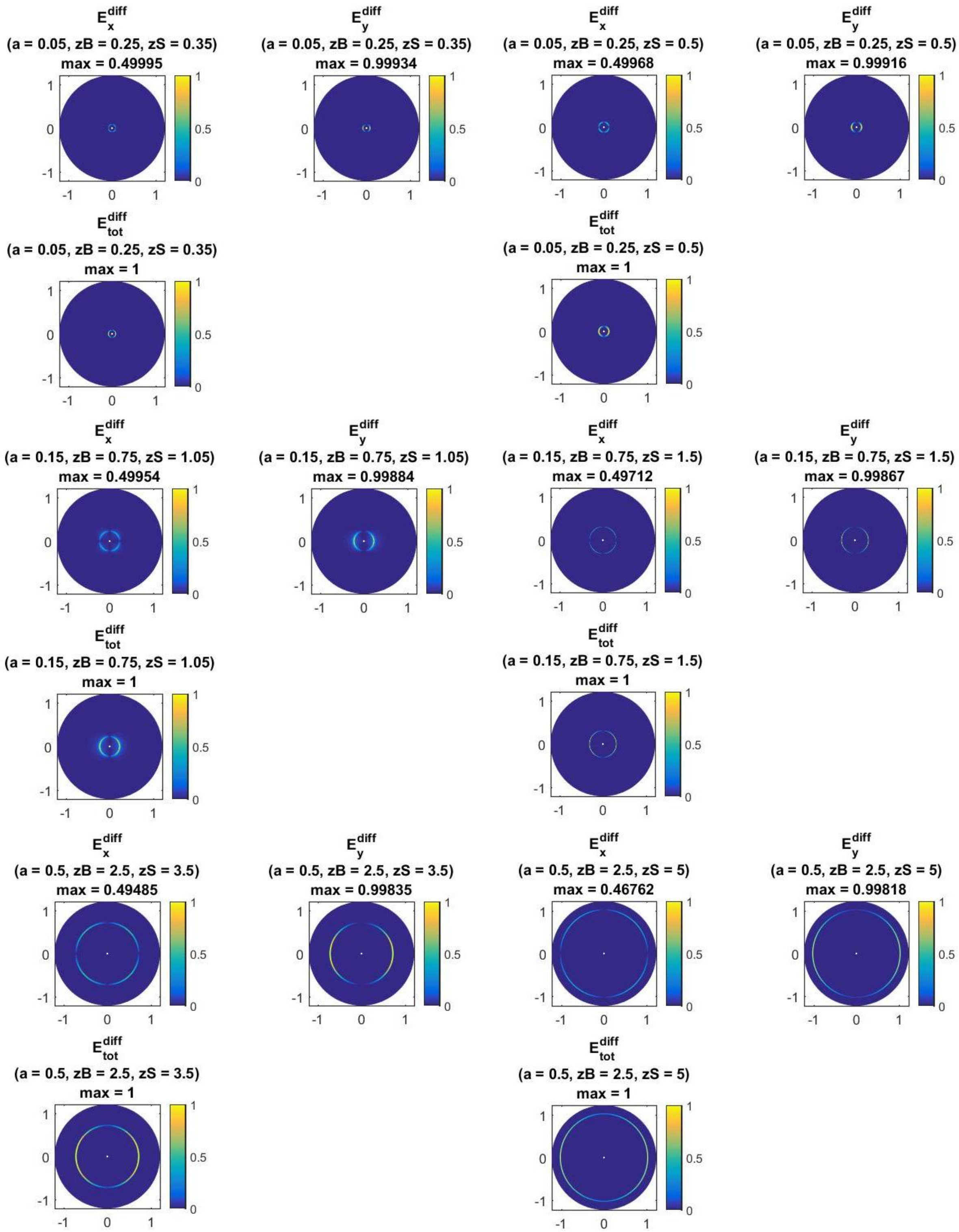

Fig. 4. Normalized transverse electric field components. Left column $z_s = 2a$; right column $z_s = 5a$.

VI. CONCLUSIONS

Light diffraction by a baffle with a circular aperture in a perfectly conducting screen illuminated by a Gaussian beam in a LIGO-like interferometric detector of gravitational waves has been discussed.

Gauss Laguerre modes have been used as a natural representation tool for the fields in a FP cavity, and UTD has been used to compute the scattered fields. The predicted magnitude of the diffracted field is pretty low; yet the extreme sensitivity of gravitational wave detectors needs an accurate analysis of all possible noise sources, including stray light, as discussed here. Further developments will include more realistic geometric and material properties of the baffles, and a study of the impact of baffle-diffracted light on the noise floor of the instrument.

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REFERENCES

[1] B. P. Abbott. et al., "LIGO: The laser interferometer gravitational wave observatory," *Rep. Prog. Phys*., vol. 72, no. 6, pp. 07690-07740, 2009.

[2] T. Accadia, et al., "Virgo: A laser interferometer to detect gravitational waves," *J. Instrum.*, vol. 7, no. 3, pp. 3012-3018, 2012.

[3] Y. Aso, et al., "Interferometer design of the KAGRA gravitational wave detector," *Phys. Rev. D*, vol. 88, no. 8, pp. 043007-043023, 2013.

[4] B. P. Abbott, "Observation of gravitational waves from a binary black hole merger," *Phys. Rev. Letters*, vol. 116, no. 2, pp. 061102-061118, 2016.

[5] B. P. Abbott, "GW151226: Observation of gravitational waves from a 22-solar-mass binary black hole merger," *Phys. Rev. Letters*, vol. 116, no. 6, pp. 241103-241117, 2016.

[6] B. Bhawal, "Diffraction losses of various modes in advanced LIGO arm cavity," *LIGO Document* T050234, 2005.

[7] M. R. Smith, "Stray light control for advanced LIGO," *LIGO Document* G1100232, 2011.

[8] A. Chiummo, "Advanced Virgo stray light control status," *LIGO Document* G1301021, 2013.

[9] A. E. Siegman, *Lasers*. Univ. Science Books, Sausalito (CA), 1986.

[10] A. Javier, "Laser and Gaussian beam propagation and transformation," in *Encyclopedia of Optical Engineering*, Marcel Dekker Inc., New York, 2003.

[11] M. Born and E. Wolf, *Principles of Optics*. Cambridge Univ. Press, Cambridge (UK), 2002.

[12] J. B. Keller, "Geometrical theory of diffraction," *J. Opt. Soc. Amer.*, vol. 52, no. 2, pp. 116-130, 1962.

[13] R. G. Kouyoumjian and P. H. Pathak, "A uniform geometrical theory of diffraction for an edge in a perfectly conducting surface," *Proc. of the IEEE*, vol. 62, no. 11, pp. 1448-1461, 1974.

[14] G. Pelosi, Y. Rahmat-Samii, and J. Volakis, Eds., "High-frequency techniques in diffraction theory: 50 years of achievements in GTD, PTD, and related approaches," [Part I] *IEEE Antennas Propag. Mag*., APM-55 (3), pp. 16-58, June 2013; [Part II] *IEEE Antennas Propag. Mag*., APM-55 (4), pp. 17-79, Aug. 2013.

[15] G. Pelosi and S. Selleri, "The wedge-type problem: The building brick in high-frequency scattering from complex objects," *IEEE Antennas and Propag. Mag.*, vol. APM-55 (3), pp. 41-58, 2013.

Giuseppe Pelosi was born in Pisa, Italy, in 1952. He received the Laurea (Doctor) degree in Physics (summa cum laude) from the University of Florence in 1976. He is currently with the Department of Information Engineering of the same university, where he is currently Full Professor of Electromagnetic Fields. His research activity is mainly focused on numerical techniques for applied electromagnetics (antennas, circuits, microwave and millimeter-wave devices, scattering problems). He was coauthor of several scientific publications on the aforementioned topics, appeared in international refereed journals. He is also coauthor of three books and several book chapters.

Pelosi was elected a Fellow of the IEEE for contributions to computational electromagnetics.

Leonardo Possenti was born in Florence (Italy) in 1991. He received the Laurea degree (cum laude) in Telecommunication Engineering from the University of Florence (Italy). In 2016 he attended a semester at the Tampere University of Technology, in Tampere (Finland). Since November 2016 he has been pursuing his Ph.D. at the University of Florence (Italy). His recent interests are mainly in high-frequency electromagnetic scattering in particular from impedance structures via GTD and UTD solutions and some of their applications in physics and space engineering.

Stefano Selleri was born in Viareggio, Italy, on December 9th, 1968. He obtained his degree (Laurea), cum laude, in Electronic Engineering and the Ph.D. in Computer Science and Telecommunications from the University of Florence in 1992 and 1997, respectively. He is author of over 90 papers on international referred journals and eight books or books chapters. He is currently an Assistant Professor at the University of Florence, where he conducts research on numerical modeling of microwave, devices and circuits with particular attention to numerical optimization.

Innocenzo M. Pinto received the Laurea degree in Electrical Engineering (cum laude) from the University of Naples, Napoli, Italy, in 1976. He became an Assistant Professor of Electromagnetics in 1983, an Associate Professor in 1987, and a full Professor in 1990. In 1998, he joined the University of Sannio, Benevento, Italy, where he has been the University President's Delegate for International Liaisons and Scientific Research, a Member of the Evaluation Board, and the Dean of the Faculty of Engineering. His research interests include Electromagnetics, Gravitational Wave Theory and Experiments, and Applied Mathematics. He has published more than 150 peer-reviewed journal papers and book chapters. Pinto is a Senior Member of the IEEE, a Member of the American Physical Society, and a PI for the LIGO-Virgo Scientific Collaboration and the LCGT-KAGRA Collaboration.