Analysis of Truncated Gratings and a Novel Technique for Extrapolating their Characteristics to those of Infinite Gratings

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Abstract **─** Periodic gratings, such as Frequency Selective Surfaces (FSSs) and EBG (electromagnetic band gap) structures, are used in a wide variety of electromagnetic applications and are typically analyzed under the assumption that they are infinite periodic. Since the real-word structures are necessarily finite, and are derived by truncating the corresponding infinite structures, it is of interest to determine how large the finite structure needs to be so that it mimics its infinite counterpart. A related question is how to extrapolate the simulation results of a finite structure to predict the performance of the corresponding infinite structure in a computationally efficient manner. The objectives of this work are to address both of these questions and to present a novel computational technique which hybridizes analytical and numerical techniques to provide the answers. We illustrate the application of the proposed technique by considering the test case of plane wave scattering by a strip grating and investigate the asymptotic behavior of the solution for the current on a truncated periodic grating as we increase its size. The proportionality constant, relating the current distribution on the unit cell of the infinite grating to the corresponding distribution in the truncated grating, is computed, and its asymptotic value is accurately predicted by using an extrapolation algorithm presented in the paper. The required number of strips is estimated such that the current on the finite structure is sufficiently close to that on the infinite one. The results obtained for the current are found to be in excellent agreement with those derived from full-wave simulations.

Index Terms **─** Current density, electromagnetic scattering, extrapolation algorithms, frequency selective surfaces, periodic strip gratings.

I. INTRODUCTION

Frequency selective surfaces (FSSs) composed of periodic arrays of perfectly conducting elements have been extensively used in several applications, *e.g.*, controlling reflection and transmission of electromagnetic waves, electromagnetic band gap (EBG) materials, and metamaterials [1-3]. A review of techniques for the electromagnetic analysis of FSSs is presented in [4]. Established methodologies for the numerical solution of the pertinent boundary value problems entail the application of periodic boundary conditions [1]; the subdomain basis discretization and conjugate gradient techniques [5]; the finite difference time domain (FDTD) method [6]; the finite element method (FEM) [7]; as well as equivalent circuit modeling techniques [8]. Typically, the periodic Green's function (PGF) is used to generate the interaction fields between the elements of the FSS; however, the expression of the PGF is a slowly-convergent infinite series [9].

A representative class of FSSs used to model several applications of the type mentioned above is that of periodic perfectly conducting strip gratings. Electromagnetic scattering by infinite periodic perfectly conducting strip gratings located in free space has been investigated by a number of researchers [10-21], who have implemented both analytical as well as purely numerical methodologies. On the other hand, reported publications dealing with the problem of scattering by truncated periodic gratings, which provide more realistic models for real-world problems, have been relatively sparse and primarily based on the implementation of numerical techniques [22-26].

In this work, we consider a grating composed of periodic perfectly conducting strips as a test example to illustrate the application of a novel solution approach

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of the problem at hand. The approach begins with a truncated ("finite") periodic structure, with a moderate number of cells, and investigates the asymptotic behavior of its solution to develop a novel technique for predicting the performance of the corresponding "infinite" periodic structure. The current induced in the inner region of this truncated grating is computed numerically, and is next utilized as an initial estimate for the computation of the induced current on the corresponding "infinite" grating. We exploit the fact that the current distribution in the inner region of the *truncated* grating with only a moderate number of cells is found to be *proportional* in scale to that of the corresponding induced distribution on the unit cell of the infinite periodic grating, while its shape is essentially the same as that of the periodic one. The proportionality constant is calculated analytically as a sum of a doubly-infinite series, by exploiting the integral representation of the electric field, expanding the unknown current as a Fourier series and then using suitable expressions of the involved functions in the spatial and spectral domains.

By following the above approach, we analyze how the solution of the finite problem converges to that of the infinite one and, also, address the important issue of the minimum number of strips we need in the finite structure in order for it to mimic its infinite counterpart. Examining particular features of the solution of the finite problem is always useful because, realistically, a physical FSS is always finite. The developed approach offers an alternative route for achieving possible speedup in obtaining the solution of the infinite problem by first treating the respective finite problem. Moreover, it circumvents the need to derive the PGF, as well as issues related to convergence of slowly varying infinite series associated with the PGF.

The validity of the derived formulas is tested by examining two limiting cases for which the current is known a *priori*. Numerical results are presented for the computation of the proportionality constant relating the current on the unit cell of the infinite grating to that of the distribution in the inner region of the corresponding finite truncated grating; the latter is computed numerically by an electromagnetic field simulation software. The numerical convergence of this constant with respect to the number of terms retained in the involved series representation is analyzed. It is shown that the limiting value of the constant can be quickly and accurately obtained by using a numerical extrapolation algorithm, which first smoothens the initial oscillations and then predicts the exponentially-decaying behavior of the resulting functions. The current computed by the proposed approach is found to be in excellent agreement with that computed from a full-wave numerical simulation of the infinite periodic structure.

II. ANALYTICAL CONSIDERATIONS

Figure 1 depicts the scattering geometry comprising a grating with period D composed of perfect electric conducting (PEC) strips of width w and illuminated by a normally incident plane wave with electric field given by (under the assumption of $\exp(+j\omega t)$ time dependence):

$$\mathbf{E}^{inc}(z)=E^{inc}(z)\hat{\mathbf{y}}=-\exp(jk_0 z)\hat{\mathbf{y}}, \quad (1)$$

where k_0 is the free-space wavenumber. The developed approach can be generalized to the oblique incidence case with only minor modifications.

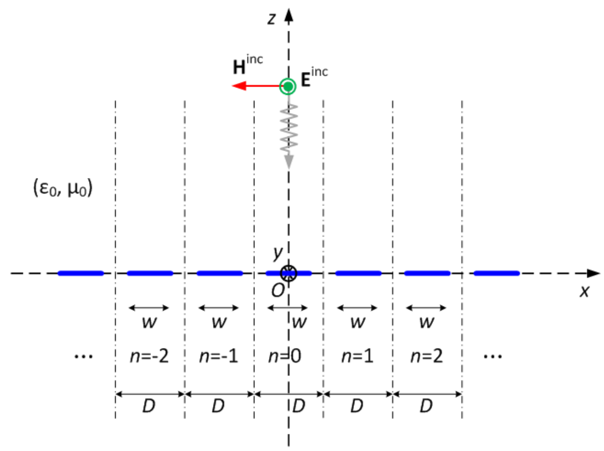


Fig. 1. Geometry of the periodic strip-grating scattering configuration.

First, we consider the *truncated finite* grating comprising of a *moderate* number of cells and solve the problem for the current distribution $\tilde{\mathbf{J}}_0(x)=\tilde{J}_0(x)\hat{\mathbf{y}}$, induced in the center region (*i.e.*, on the unit cell which is indexed as n=0). This distribution is expanded in a Fourier series:

$$\tilde{J}_0(x)=\frac{1}{Z_0 k_0 \frac{w}{2}\sqrt{1-\left(\frac{x}{\frac{w}{2}}\right)^2}}\sum_{\nu=0}^{\infty}(-j)^{\nu}A_{\nu}\cos\left(\tfrac{\nu\pi}{w}\left(x+\tfrac{w}{2}\right)\right), \quad (2)$$

where Z_0 denotes the free-space impedance. The Fourier coefficients A_{ν} are calculated in the standard way and found to be:

$$A_0=\frac{Z_0 k_0}{2}\int_{-\frac{w}{2}}^{\frac{w}{2}}\sqrt{1-\left(\frac{x}{\frac{w}{2}}\right)^2}\,\tilde{J}_0(x)\,\mathrm{d}x$$

$$A_{\nu}=j^{\nu}Z_0 k_0\int_{-\frac{w}{2}}^{\frac{w}{2}}\sqrt{1-\left(\frac{x}{\frac{w}{2}}\right)^2}\,\tilde{J}_0(x)\cos\left(\tfrac{\nu\pi}{w}\left(x+\tfrac{w}{2}\right)\right)\mathrm{d}x,\ \nu=1,2,... \quad (3)$$

In case that $\tilde{J}_0$ is a constant, then the latter expressions are simplified to:

$$A_0 = \frac{Z_0 \pi}{8} k_0 w \tilde{J}_0 \quad , \quad A_{2\nu-1} = 0, \quad A_{2\nu} = Z_0 k_0 w \tilde{J}_0 \frac{J_1(\nu\pi)}{2\nu}, \ \nu = 1,2,... \tag{4}$$

where J_1 denotes the first-order cylindrical Bessel function.

On the basis of past experience with similar grating problems (see, *e.g.*, [19-20]), we postulate that the true current induced on the unit cell of the *infinite* grating structure would simply be $c\tilde{J}_0(x)$, *i.e.*, proportional to the current in the center cell of the finite grating, where c is a complex constant, as yet undetermined. The physical justification of the latter postulate is as follows. In a truncated periodic structure there are reflections from the edges, which decline gradually as we move inwards from the edges. As the size of the grating becomes sufficiently large, the edge effect is substantially reduced in the center region, where only the dominant mode survives. This, in turn, just changes the scale of the field distribution on the unit cell but not its shape, which has presumably stabilized. The scale factor is calculated next, by applying the boundary condition on the unit cell, to eliminate the contributions from the edge reflections.

Next, we show how to determine the scale factor c in a systematic way. The current $c\tilde{J}_0(x)$ is induced on every grating's cell indexed by n (under a shift of the spatial variable x so that it is centered at the center on the n-th cell). Hence, we begin by computing the contributions of the currents:

$$J_n(x) = c\tilde{J}_0(x - nD), \quad nD - \tfrac{w}{2} < x < nD + \tfrac{w}{2}, \tag{5}$$

induced on the cells $n \neq 0$ to the unit cell $n=0$. The electric fields generated by these currents are given by:

$$E_n(x,z) = (-j\omega\mu_0) \int_{nD-\frac{w}{2}}^{nD+\frac{w}{2}} J_n(x') G(x,z;x',0)\,\mathrm{d}x', \tag{6}$$

where G is the free-space Green's function,

$$G(x,z;x',z') = -\frac{j}{4} H_0^{(2)}\left(k_0 \sqrt{(x-x')^2 + (z-z')^2}\right), \tag{7}$$

with $H_0^{(2)}$ denoting the zero-order cylindrical Hankel function of the second kind.

By combining (2)-(7), we get:

$$E_n(x,0) = -\frac{c}{2w} \sum_{\nu=0}^{\infty} (-j)^\nu A_\nu \times \int_{nD-\frac{w}{2}}^{nD+\frac{w}{2}} \frac{\cos\left(\frac{\nu\pi}{w}(x' - nD + \frac{w}{2})\right)}{\sqrt{1 - \left(\frac{x'-nD}{\frac{w}{2}}\right)^2}} H_0^{(2)}\left(k_0 |x - x'|\right)\mathrm{d}x' \ . \tag{8}$$

Then, by performing the change of variables $x' = nD + \xi$ and summing up all the infinite contributions, we can write the *total* electric field induced on the unit cell as:

$$E(x,0) = \sum_{n=-\infty}^{+\infty} E_n(x,0) = -\frac{c}{2w} \sum_{n=-\infty}^{+\infty} \sum_{\nu=0}^{\infty} (-j)^\nu A_\nu \times \int_{-\frac{w}{2}}^{+\frac{w}{2}} \frac{\cos\left(\frac{\nu\pi}{w}(\xi + \frac{w}{2})\right)}{\sqrt{1 - \left(\frac{\xi}{\frac{w}{2}}\right)^2}} H_0^{(2)}\left(k_0 |x - nD - \xi|\right)\mathrm{d}\xi \ . \tag{9}$$

The involved series and integrals in (9) can be handled analytically as follows. First, we recall the well-known Fourier integral expression of the Hankel function:

$$H_0^{(2)}\left(k_0 |x - x'|\right) = \tfrac{1}{\pi} \int_{-\infty}^{+\infty} \frac{e^{-jk_x(x-x')}}{\sqrt{k_0^2 - k_x^2}} \mathrm{d}k_x \ . \tag{10}$$

Now, by substituting (10) in (9), changing the orders of integration and taking into account that (see, *e.g.*, [14]):

$$\int_{-\frac{w}{2}}^{+\frac{w}{2}} \frac{\cos\left(\frac{\nu\pi}{w}(\xi + \frac{w}{2})\right)}{\sqrt{1 - \left(\frac{\xi}{\frac{w}{2}}\right)^2}} e^{jk_x\xi}\mathrm{d}\xi = \frac{1}{4}(-j)^\nu \pi w \left[J_0(\tfrac{1}{2}(\pi\nu - k_x w)) + (-1)^\nu J_0(\tfrac{1}{2}(\pi\nu + k_x w))\right] , \tag{11}$$

where J_0 denotes the zero-order cylindrical Bessel function, we get that,

$$E(x,0) = -\frac{c}{8} \sum_{n=-\infty}^{+\infty} \sum_{\nu=0}^{\infty} A_\nu \int_{-\infty}^{+\infty} \frac{e^{-jk_x(x-nD)}}{\sqrt{k_0^2 - k_x^2}} \left[J_0(\tfrac{1}{2}(\pi\nu + k_x w)) + (-1)^\nu J_0(\tfrac{1}{2}(\pi\nu - k_x w))\right]\mathrm{d}k_x \ . \tag{12}$$

Moreover, we use the following relation (resulting by the Fourier series expression of the Dirac comb):

$$\sum_{n=-\infty}^{+\infty} e^{jk_x nD} = \frac{2\pi}{D} \sum_{n=-\infty}^{+\infty} \delta\left(k_x - n\tfrac{2\pi}{D}\right), \tag{13}$$

to obtain that the total electric field induced on the unit cell takes the form,

$$E(x,0) = -\frac{c}{4}\frac{\pi}{D} \sum_{n=-\infty}^{+\infty} \sum_{\nu=0}^{\infty} A_\nu \frac{e^{-jn\frac{2\pi}{D}x}}{\sqrt{k_0^2 - (n\frac{2\pi}{D})^2}} \times \left[J_0(\tfrac{1}{2}(\pi\nu + \tfrac{2\pi w}{D}n)) + (-1)^\nu J_0(\tfrac{1}{2}(\pi\nu - \tfrac{2\pi w}{D}n))\right] . \tag{14}$$

Finally, we impose the boundary condition on the PEC strip of the unit cell as follows:

$$E(x,0) = \sum_{n=-\infty}^{+\infty} E_n(x,0) = -E^{inc}(0) = 1, \tag{15}$$

to determine the unknown constant c as,

$$c = -\frac{4D}{\pi} \left\{ \sum_{n=-\infty}^{+\infty} \sum_{\nu=0}^{\infty} A_\nu \frac{e^{-jn\frac{2\pi}{D}x}}{\sqrt{k_0^2 - (n\frac{2\pi}{D})^2}} \times \left[J_0(\tfrac{1}{2}(\pi\nu + \tfrac{2\pi w}{D}n)) + (-1)^\nu J_0(\tfrac{1}{2}(\pi\nu - \tfrac{2\pi w}{D}n))\right]\right\}^{-1} . \tag{16}$$

The current distribution on the strip in the unit cell of the infinite grating can now be obtained, once c has been computed and all the subsequent field quantities of interest can be readily determined.

III. LIMITING CASES

The developed approach is validated by examining two limiting cases. The first is a "test" case and corresponds to normal incidence plane wave scattering by an *infinite* PEC plane, which is decomposed into periodic cells as shown in Fig. 2.

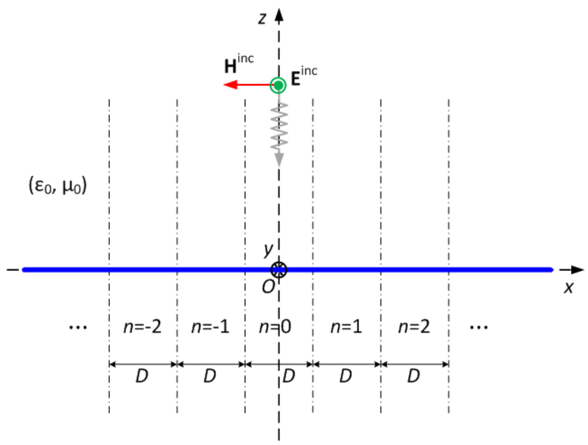


Fig. 2. Geometry of the "test" case corresponding to normal incidence plane wave scattering by an infinite PEC plane.

The *approximate* initial electric current distribution induced on the unit cell n=0 is assumed to be:

$$\tilde{\mathbf{J}}_0 = 2\hat{\mathbf{z}}\times\mathbf{H}^{inc}(0)=2H_x^{inc}(0)\hat{\mathbf{y}} = -2\tfrac{1}{Z_0}\hat{\mathbf{y}}, \qquad (17)$$

where

$$\mathbf{H}^{inc}(z)=H_x^{inc}(z)\hat{\mathbf{x}}=-\tfrac{1}{Z_0}\exp(jk_0 z)\hat{\mathbf{x}},$$

is the magnetic field of the incident plane wave (see (1)). Equation (17) is actually the correct "physical optics" current induced on the infinite PEC plane.

Now, we follow the methodology described in Section II above. The current induced on the unit cell of the infinite structure is taken to be $c\tilde{J}_0 = -2c/Z_0$, with c is yet to be determined. The currents induced on the cells $n\neq0$ are given by:

$$J_n(x)=c\tilde{J}_0(x-nD)=-2c/Z_0,\ (2n-1)\tfrac{D}{2}<x<(2n+1)\tfrac{D}{2},$$

while the total electric field induced on the unit cell is calculated by (6), (7), and (17) as:

$$E(x,0)=\sum_{n=-\infty}^{+\infty}E_n(x,0)=\frac{k_0 c}{2}\sum_{n=-\infty}^{+\infty}\int_{-\frac{D}{2}}^{\frac{D}{2}}H_0^{(2)}\left(k_0\,|x-nD-\xi|\right)\mathrm{d}\xi \cdot \qquad (18)$$

By combining (10) and (18), we find:

$$E(x,0)=c\,\frac{k_0 D}{2\pi}\sum_{n=-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{\operatorname{sinc}(k_x\frac{D}{2})}{\sqrt{k_0^2-k_x^2}}e^{-jk_x(x-nD)}\,\mathrm{d}k_x, \qquad (19)$$

where sinc(z)=sin(z)/z. Then, we apply the boundary condition (15) on the unit cell and determine c as:

$$c=\frac{2\pi}{k_0 D\sum\limits_{n=-\infty}^{+\infty}\left(\int\limits_{-\infty}^{+\infty}\frac{\operatorname{sinc}(k_x\frac{D}{2})}{\sqrt{k_0^2-k_x^2}}e^{-jk_x(x-nD)}\,\mathrm{d}k_x\right)}.$$

The latter is written by means of (13) as:

$$c=\frac{1}{k_0\sum\limits_{n=-\infty}^{+\infty}\frac{\operatorname{sinc}(n\pi)}{\sqrt{k_0^2-(n\frac{2\pi}{D})^2}}e^{-jn\frac{2\pi}{D}x}},$$

which by taking into account that sinc($n\pi$)=0, $n\neq0$, gives the expected result that c=1 (since we have considered in (17) as the initial current on the unit cell the correct current induced on the infinite PEC plane.)

The second limiting case is for $w\to D$, when the infinite periodic grating is reduced to an infinite PEC plane. The approximate initial current distribution induced on the unit cell is taken again to be given by (17). The Fourier coefficients are calculated by (4) as:

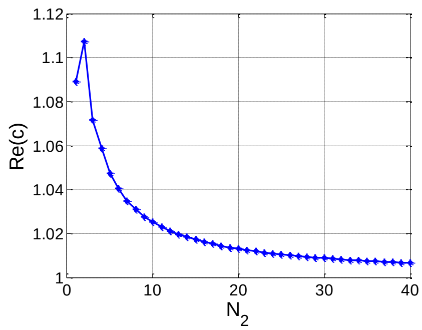
$$\begin{aligned}&A_0=-\tfrac{\pi}{4}k_0 w\\ &A_{2\nu-1}=0,\quad A_{2\nu}=-k_0 w J_1(\nu\pi)/\nu,\ \ \nu=1,2,...\end{aligned}. \qquad (20)$$

By following the methodology of Section II, we find that c is given by (16) where A_ν are given by (20). By letting $w\to D$ in (16), and using (20), we obtain:

$$c=\frac{4}{\pi}\left\{\sum_{n=-\infty}^{+\infty}\frac{e^{-jn\frac{2\pi}{D}x}}{\sqrt{1-(\frac{2\pi}{k_0 D}n)^2}}\times\right. .$$

$$\left.\left\{\frac{\pi}{2}J_0(\pi n)+\sum_{\nu=1}^{\infty}\frac{J_1(\nu\pi)}{\nu}\left[J_0(\pi(\nu+n))+J_0(\pi(\nu-n))\right]\right\}\right\}^{-1} \qquad (21)$$

The series appearing in (21) are computed numerically. Figures 3 and 4 depict the real and imaginary parts of c as computed by (21) vs N_2 for constant N_1=10 and vs N_1 for constant N_2=30, where N_1 and N_2 denote the truncation orders of the series in (21) with respect to the variables n and ν, respectively. In the numerical computations, we take x=0 and tested that other values of x give similar results. It is evident that c converges to 1 as expected because as $w\to D$ the problem is reduced to scattering by an infinite PEC plane for which it is known that (17) gives the correct "physical optics" current induced on the plane. A technique for the fast extrapolation of oscillatory curves, such as the one in Fig. 4 (b), is analyzed in the next section by retaining a smaller number of terms in the series.



(a)

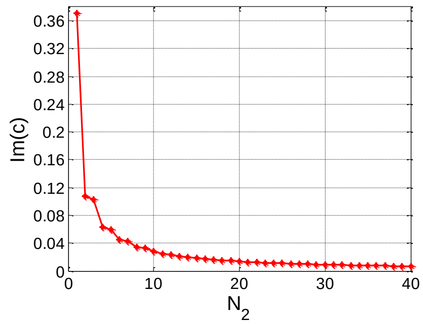


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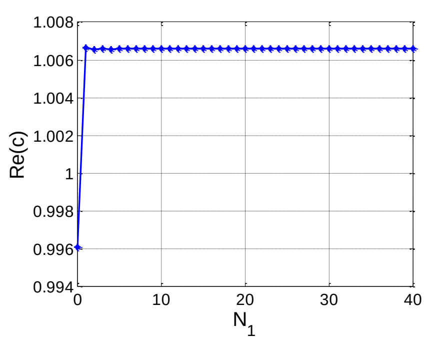
Fig. 3. Real and imaginary parts of c as computed by (21) vs the truncation order N_2 of the series with respect to v for N_1=10 retained terms of the series with respect to n.



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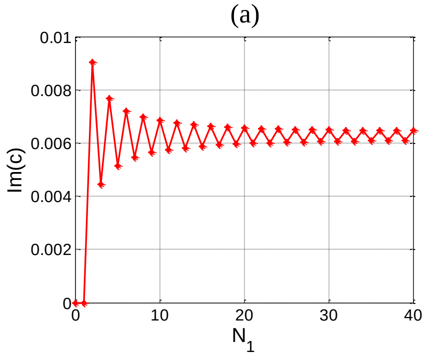
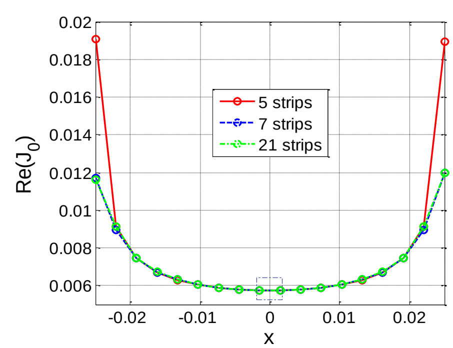


(b)

Fig. 4. Real and imaginary parts of c as computed by (21) vs the truncation order N_1 of the series with respect to n for N_2=30 retained terms of the series with respect to v.

IV. NUMERICAL RESULTS AND DISCUSSION

First, we consider a periodic grating with parameters $D=\lambda/2$ and $w=D/3$ and frequency of the incident field at f=1 GHz. Figure 5 depicts the real and imaginary parts of the current induced on the unit cells, as computed by the commercial code Ansoft HFSS [27], for a finite grating with 5, 7, and 21 strips. Magnifications in the region of the center of the unit cells are depicted in Fig. 6. In addition to these results, we have carried out extensive numerical experiments by increasing the number of strips from 5 to 21 in steps of 2. We have concluded that convergence in the second decimal digit of the induced current is attained for 7 to 9 strips, while we can realize convergence in the third decimal place by using 15 strips. The current induced on the center strip of a truncated grating with 7 strips, shown in Fig. 5, will be considered as the initial approximate current $\tilde{J}_0(x)$ in the numerical algorithm implementing the developed methodology. Figure 7 depicts the real and imaginary parts of c, as computed by means of (16), versus the number N_1 of retained terms in the series of (16) with respect to n for constant number N_2=10 of terms in the series with respect to v.



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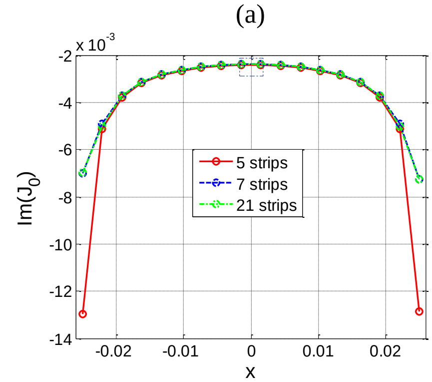
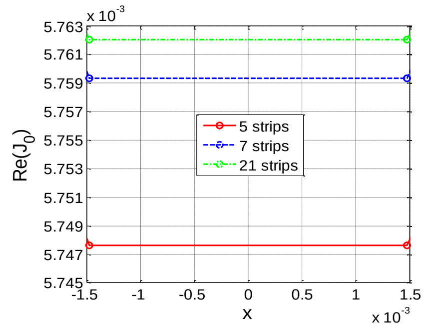


(b)

Fig. 5. Real and imaginary parts of the considered approximate initial current $\tilde{J}_0(x)$ induced on the unit cell of a finite grating with 5, 7, and 21 strips, as computed by the HFSS.



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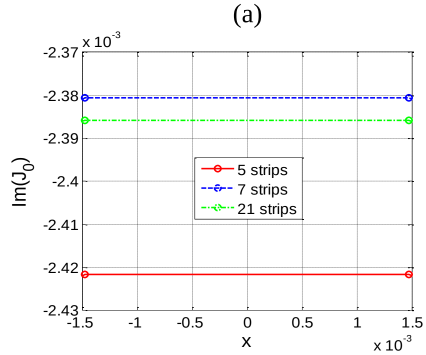


(b)

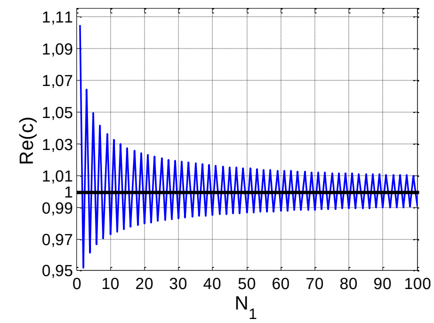
Fig. 6. Magnifications of Fig. 5 in the region of the center of the unit cell.

The limiting values of the oscillatory curves of Fig. 7 can be extrapolated by following the numerical procedure described below. The first two periods of the oscillations are excluded from the extrapolation procedure we follow. Then, from the third period on, we take the averages between successive maximum and minimum values. In this way, we obtain a new set of curves where the initial oscillations are smoothened. The outcomes of this procedure on the oscillating curves of Figs. 7 (a) and 7 (b) are shown in Figs. 8 (a) and (b). By repeating the procedure a second time on the curves of Figs. 8 (a) and 8 (b), we finally obtain the curves of Figs. 8 (c) and 8 (d), where the oscillations are significantly suppressed.

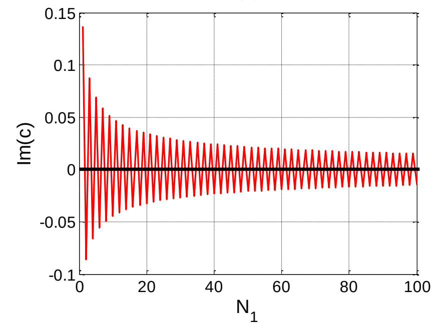
Now, the monotonic behavior of the curves of Re(c) and Im(c) of Figs. 8 (c) and 8 (d) can be predicted in the following way. A two-term exponential model of the form:

$$y(x) = a^* \exp(b^* x) + c^* \exp(d^* x), \tag{22}$$

is fitted to the data yielding the results of Fig. 9, where it is observed that the fitted curves provide very good approximations of the initial data. The obtained limiting value of the complex constant c is very close to the value c=1.007–0.004j, which is obtained by dividing the current induced on the unit cell of the infinite grating over the corresponding current on the truncated grating with 7 strips (the latter quantities were both computed by using HFSS simulations). As shown in Fig. 10, the current computed by the proposed approach is in excellent agreement with that computed from the HFSS simulation of the infinite periodic grating.



(a)



(b)

Fig. 7. Real and imaginary parts of c vs the truncation order N_1 for N_2=10, D=λ/2, w=D/3, and f=1 GHz. The black lines show the mean values of the oscillatory curves.

Furthermore, we are interested in predicting/ extrapolating the behaviors of Re(c) and Im(c) by reducing the total number of terms considered (that is without retaining up to N_1=100, for instance, as in Fig. 9). The variations of Re(c) and Im(c) when only N_1=20 and 30 terms are retained in the series (16) are depicted in Figs. 11 (a), (b) and (c), (d), respectively. The respective fitted curves by means of the two-term exponential model (22) are also depicted. We observe that truncating these exponentially-decaying curves at N_1=20 or 30 and applying the above described extrapolation algorithm can provide quite accurate results for the limiting value of c.

Finally, we consider as a second example, an infinite periodic grating with parameters D=λ and w=D/2 at the same frequency of f=1 GHz. Figure 12 depicts the real and imaginary parts of c, as computed by means of (16), versus N_1 for N_2=10. Also in this case, the obtained result converges to the expected one for the current induced on the unit cell of the infinite grating structure. As in the previously examined example, the obtained curves can be extrapolated by using the algorithm described above to predict their monotonic behavior by considering a relatively small number of N_1 terms.

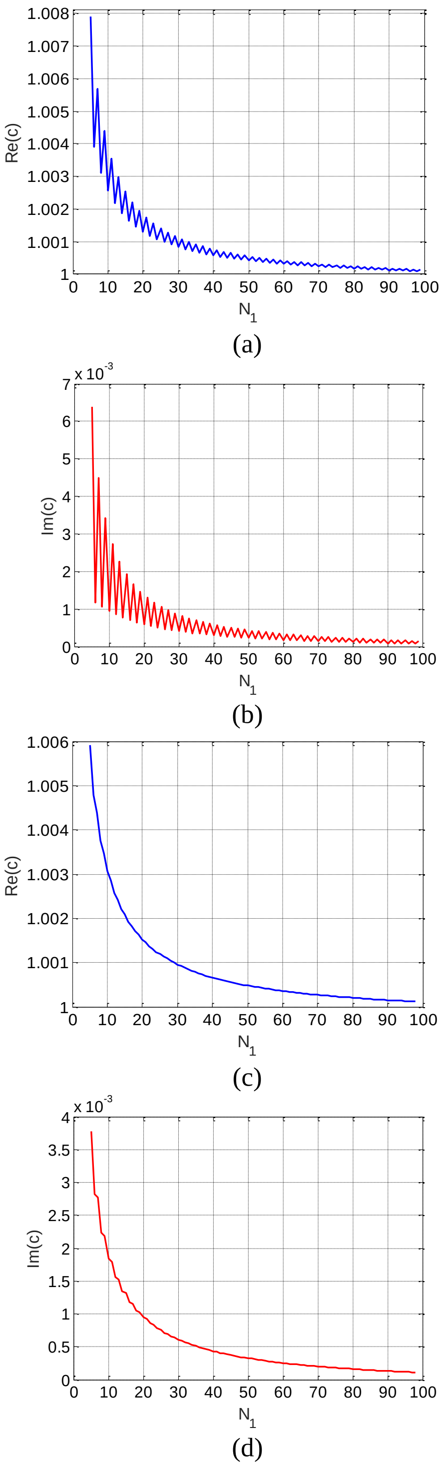


Fig. 8. (a) and (b) Real and imaginary parts of c vs N_1 after implementing the numerical algorithm of smoothening the oscillations on the curves of Figs. 7 (a) and 7 (b). (c) and (d) Corresponding results after implementing a second time the oscillations smoothening algorithm on the previously derived curves of Figs. 8 (a) and 8 (b).

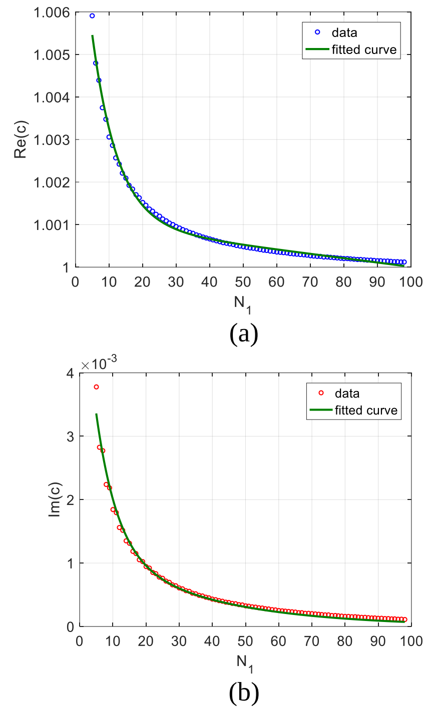


Fig. 9. Real and imaginary parts of c (bullets denoted as "data") for the parameters values of Figs. 8 (c) and 8 (d) together with the respective fitted curves by means of the model (22).

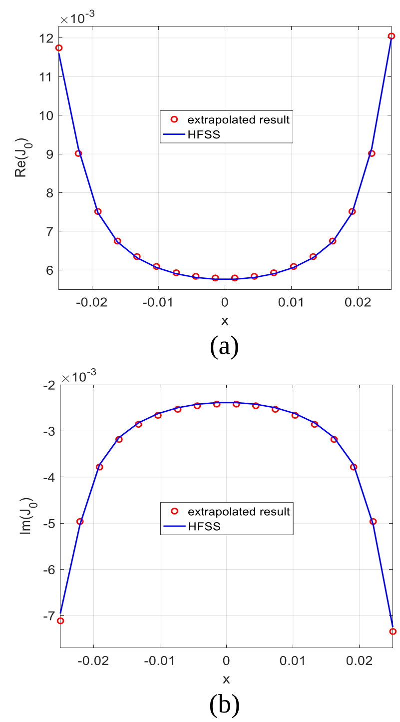


Fig. 10. Current induced on the unit cell of the infinite grating for the parameters of Fig. 5, as computed by the proposed method [*c.f.* model (22)] and by the HFSS.

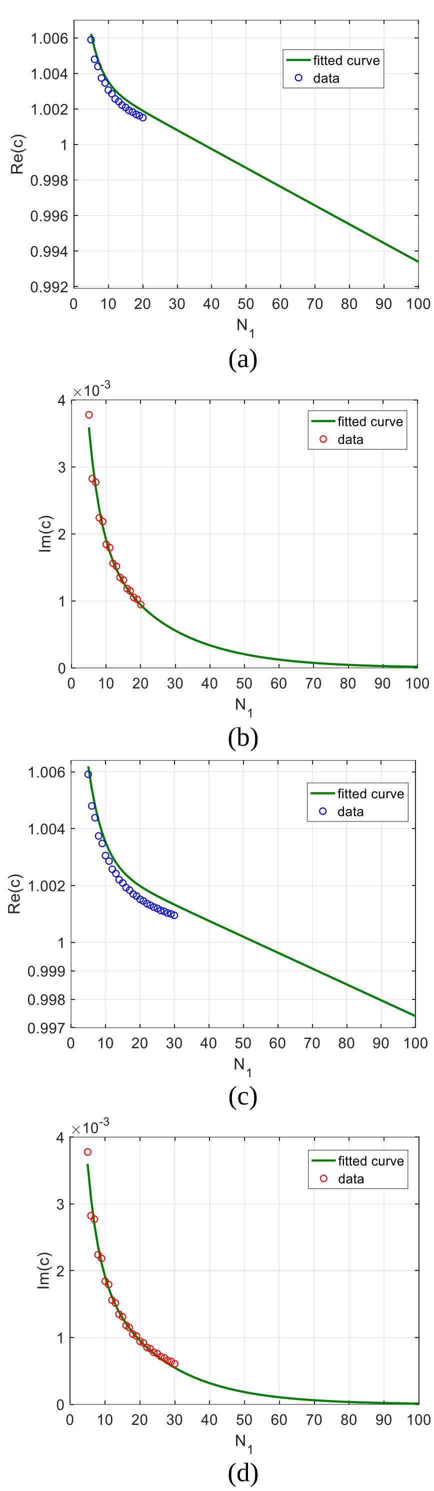


Fig. 11. Real and imaginary parts of c (bullets denoted as "data") for the parameters values of Figs. 8 (c) and 8(d) when only N_1=20 in (a) and (b) and N_1=30 in (c) and (d) terms are retained in the series (16). In every figure, the respective fitted curves by means of the two-term exponential model (22) are also depicted.

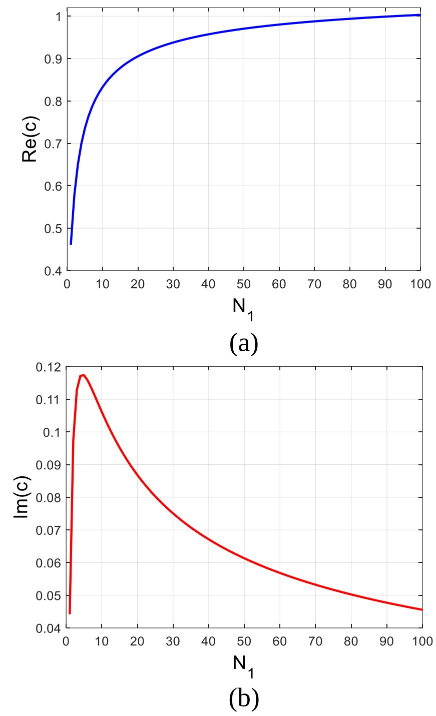


Fig. 12. Real and imaginary parts of c vs the truncation order N_1 for N_2=10, D=λ, w=D/2, and f=1 GHz.

V. CONCLUSIONS

The problem of electromagnetic scattering by a periodic perfectly conducting strip grating was considered. An analytical methodology was developed for extracting the solution for the current induced on the infinite grating from that induced on the respective finite grating with a moderate number of cells. The validity was tested by examining two limiting cases for which the current is known *a priori*. Numerical results were presented for the proportionality constant relating the current on the unit cell of the infinite grating to that of the distribution in the inner region of the corresponding truncated grating. The convergence of this constant with respect to the number of retained terms in the involved series representation was investigated. It was shown that the limiting value of the constant can be quickly and accurately obtained by using a numerical extrapolation algorithm which first smoothens the initial oscillations and then predicts the monotonic behavior of the resulting functions.

The procedure presented in the paper is general. It was applied here to one-dimensional (1-D) gratings with infinite perfectly conducting strips of arbitrary width and can be easily generalized to the oblique incidence case as well as to other similar grating structures, including gratings lying on dielectric substrates (infinite gratings of the latter type have been analyzed by different methodologies; see *e.g.* [28]). The suitable initial number of cells of the truncated finite grating is chosen by examining the field's distribution in the region of the center cell. When this distribution is stabilized, then the current on the center strip of the infinite grating will be the current on the center strip of the corresponding finite grating times a scale factor which has to be determined. By following this approach, we also gain an understanding of the convergence behavior of the current distribution of the finite grating, and how it relates to that of the infinite grating, to determine when the scattering characteristics of the truncated grating are close to those of the infinite one. Additionally, by implementing the proposed approach and examining the aforementioned convergence, we do not need to rely upon commercial codes to validate the results for the current on an infinite grating.

The basic principles of the developed methodology were demonstrated for 1-D FSSs. The methodology can be generalized to two-dimensional (2-D) FSSs, for example phased antenna arrays. The field in the center region of a truncated 2-D antenna array stabilizes to a shape distribution, as one increases the dimensions of the array along the *x*- and *y*-axis, which is the same as the corresponding distribution in the center cell of the infinite 2-D antenna array. This fundamental property was shown in [29]. In this way, we can extend the numerical procedure presented in the paper and compute the scale factor of the current on the center cell of the infinite 2-D antenna array over the respective current of the corresponding truncated 2-D array (composed of that number of elements for which the field's distribution was already found to have stabilized).

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