A Null Broadening Beamforming Approach Based on Covariance Matrix Expansion

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Abstract **─** In order to improve the performance of antenna array beamforming in the case of jammer motion, a null broadening beamforming approach based on covariance matrix expansion is proposed in this paper. The covariance matrix of the array is expanded through the Kronecker product of an eye matrix and the sample covariance matrix. The steering vector of the array is also expanded. When the expanded covariance matrix is used for beamforming, more linear constraints can be constructed compared with the original sample covariance matrix, so wider and deeper nulls can be obtained with equal number of array elements, or similar performance can be obtained with fewer number of array elements. Necessary numerical procedures are provided and computation complexity is analyzed. The validity of the proposed approach is verified by theoretical analysis and simulation results.

Index Terms **─** Beamforming, covariance matrix expansion, null broadening.

I. INTRODUCTION

Adaptive antenna beamforming has been widely used in radar, sonar, mobile communications and other fields. It helps adaptive arrays improve the reception of the desired signal and suppress interferences by forming nulls at the directions of interferences [1-4]. The performance of the adaptive arrays is severely degraded if the weights of the arrays are not able to adapt sufficiently fast to the changing (nonstationary) jamming situation or to the antenna platform motion. This issue can be handled, however, if a broad null is formed toward the direction of the interference [5-7].

Broad nulls can be formed by the approach of covariance matrix taper (CMT) [8-11], the concept of which is introduced in [10]. The CMT approach does not need prior knowledge of the directions of interferences. Broad nulls will be formed at all the directions of interferences adaptively, and the width of the nulls can be controlled. However, as a price, the depth of the nulls will be reduced.

Quadratic constraint sector suppressed (QCSS) is another approach of null broadening [12, 13]. The QCSS approach needs prior knowledge of the approximate directions of interferences. Broad nulls can be formed around the directions specified, that is to say, the directions of broad nulls can be controlled. In addition, both the width and depth of the nulls can be controlled by the QCSS approach, and the depth of the nulls can be increased. However, the solving process of the QCSS approach is complicated, which is a nonlinear problem. The approach of linear constraint sector suppressed (LCSS) [14] is proposed based on the QCSS approach. The quadratic constraint is transformed into a set of linear constraints, by which the nonlinear problem is transformed into a linear problem and the solving process is simplified. The LCSS approach is an advanced null broadening beamforming method that forms broad nulls through linear constraints, while it can be further improved. In order to obtain wider or deeper nulls, more linear constraints should be constructed by the LCSS approach, which requires more degrees of freedom (DOFs) of an antenna array, so the DOFs of the antenna array is a restrictive factor to the LCSS approach.

Virtual antenna array is an advanced technique that focuses on the methods of forming virtual array elements and transforms the real array into virtual array, which mainly includes the methods of virtual array transformation [15-17] and high order cumulant [18-20]. By using the idea of the virtual antenna array that forms virtual array elements, a null broadening beamforming approach based on covariance matrix expansion (CME) is proposed in this paper. In this proposed approach, the covariance matrix of an array is expanded through the Kronecker product of an eye

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matrix and the sample covariance matrix. The steering vector of the array is expanded as well. Moreover, the LCSS approach is combined. Eventually, the broad nulls are realized. Compared with the LCSS approach, more linear constraints can be constructed by the proposed approach. Therefore, when the numbers of the array elements of the two approaches are equal, wider and deeper nulls can be obtained by the proposed approach and the output signal-to-interference-plus-noise ratio (SINR) can be improved. Besides, similar performance can be obtained by the proposed approach with fewer number of array elements compared with the LCSS approach. The validity of the proposed approach is verified by theoretical analysis and simulation results.

II. SIGNAL MODEL AND THE QCSS APPROACH

Assuming that noncoherent narrowband excitation sources are far away from the antenna array, we consider a uniform linear array (ULA) with N elements and the element space is equal to one-half wavelength. The structure of the ULA antenna is shown in Fig. 1.

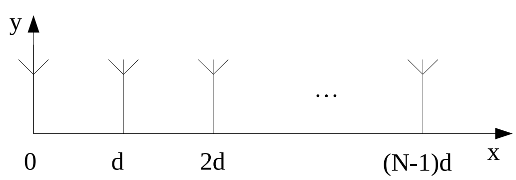

Fig. 1. The structure of the ULA antenna.

The received data $\mathbf{X}(t)$ can be expressed as follows:

$$\mathbf{X}(t)=\mathbf{AS}(t)+\mathbf{N}(t)\,, \qquad (1)$$

where $\mathbf{X}(t)$ is an $N\times 1$ snap data vector, $\mathbf{A}$ is the matrix of steering vectors, $\mathbf{S}(t)$ is the complex signal envelope, and $\mathbf{N}(t)$ is the noise of the antenna array.

If $\mathbf{W}=[\mathbf{w}_1,\cdots,\mathbf{w}_N]^T$ is the weight vector of the antenna array, the output power of the array can be expressed as follows:

$$P_{out}=E\left\{\left|\mathbf{W}^H\mathbf{X}(t)\right|^2\right\}=\mathbf{W}^H\mathbf{R}_{i+n}\mathbf{W}\,, \qquad (2)$$

where $E\{\cdot\}$ denotes expectation, $(\cdot)^H$ denotes conjugate transpose, and $\mathbf{R}_{i+n}$ is the covariance matrix of interference-plus-noise. In practice, this matrix is commonly replaced by the sample covariance matrix with K snapshots as $\hat{\mathbf{R}}=\frac{1}{K}\sum_{k=1}^{K}\mathbf{X}(k)\mathbf{X}^H(k)$.

Assume that the direction of the source of interest, denoted by θ_d, is constant during the observation time $0\le t\le T$. When the interference direction changes around θ_i and the total variation is $\Delta\theta_i$, the optimal weight vector $\mathbf{W}_{opt}$ of the QCSS approach will minimize the output power P_{out}. In addition, the desired signal level should be kept and the average output power within the sector $\Delta\theta_i$ should be lower than a pre-set value η in order to obtain a broad null with certain depth. These can be expressed as follows:

$$\min_{\mathbf{W}}\mathbf{W}^H\hat{\mathbf{R}}\mathbf{W}\quad \text{s.t.}\quad \mathbf{W}^H\mathbf{a}(\theta_d)=1,\ \ \mathbf{W}^H\mathbf{QW}\le\eta\,, \qquad (3)$$

where $\mathbf{a}(\theta_d)$ denotes the steering vector of the desired signal, and $\mathbf{Q}$ is a matrix of $N\times N$ and can be expressed as:

$$\mathbf{Q}=\frac{1}{\Delta\theta_i}\int_{\theta_i-\Delta\theta_i/2}^{\theta_i+\Delta\theta_i/2}\mathbf{a}(\theta)\mathbf{a}^H(\theta)d\theta\,. \qquad (4)$$

The solving process of the optimal weight vector of the array in formula (3) is complicated, which is a nonlinear problem involving the quadratic constraint. To reduce the complexity of the QCSS approach, the LCSS approach was proposed in [14], which replaced the quadratic constraint by a set of linear constraints.

III. THE LCSS APPROACH

Ideally, the goal of null broadening is to obtain a zero power response at the sector $\Delta\theta_i$, that is, $\mathbf{W}^H\mathbf{QW}=0$. Since $\mathbf{Q}$ is a Hermitian matrix, it can be factorized as $\mathbf{Q}=\mathbf{U\Sigma U}^H$, where $\mathbf{U}$ is an $N\times N$ matrix containing the orthonormal eigenvectors of $\mathbf{Q}$, and $\mathbf{\Sigma}$ is an $N\times N$ diagonal matrix containing the eigenvalues of $\mathbf{Q}$ in a decreasing order. Assume that $\mathbf{Q}$ has a rank equal to r, and $\mathbf{U}_r=[\mathbf{u}_1,\cdots,\mathbf{u}_r]$ is the matrix of eigenvectors that correspond to the larger eigenvalues. Then, the quadratic constraint $\mathbf{W}^H\mathbf{QW}=0$ is satisfied if $\mathbf{W}^H\mathbf{U}_r=\mathbf{0}$. Therefore, the quadratic constraint can be transformed into a set of linear constraints, and the optimal weight vector of the array can be obtained as follows [14]:

$$\min_{\mathbf{W}}\mathbf{W}^H\mathbf{RW}\ \text{s.t.}\ \mathbf{W}^H\mathbf{C}=\mathbf{e}_1^T\,, \qquad (5)$$

where $\mathbf{C}=[\mathbf{a}(\theta_d),\mathbf{U}_r]$, and $\mathbf{e}_1=[1,\mathbf{0}^T]^T$. The solution to (5) is:

$$\mathbf{W}_{LCSS}(r)=\mathbf{R}^{-1}\mathbf{C}(\mathbf{C}^H\mathbf{R}^{-1}\mathbf{C})^{-1}\mathbf{e}_1\,, \qquad (6)$$

where the rank r is the number of linear constraints that minimize the average output power of the sector $\Delta\theta_i$. Similar to the QCSS approach, in practice, $\mathbf{R}$ is first replaced with the sample covariance matrix $\hat{\mathbf{R}}$, and then the sample matrix inversion method can be applied where we calculate $\hat{\mathbf{R}}^{-1}$ selectively with or without diagonal loading (DL).

The minimal number r'_{opt} of linear constraints, which needs to be determined to ensure that the average output power of the sector $\Delta\theta_i$ is lower than the pre-set value η, can be expressed as [14]:

$$r'_{opt} = \underset{r'=[1,2,\cdots,N]}{\arg\min}\, q(r') \le \eta , \tag{7}$$

where $q(r') = \mathbf{W}_{LCSS}^H(r')\mathbf{Q}\mathbf{W}_{LCSS}(r')$.

When the DOFs of a real array are more than the optimal rank r'_{opt}, the broad nulls can be achieved by the LCSS approach and the interferences can be suppressed. However, if the DOFs of the real array are less than the optimal rank r'_{opt}, the width or depth of nulls will not be able to reach the requirements. Hence, the performance of the LCSS approach can be improved by increasing the DOFs of the array to construct more linear constraints.

IV. THE CME-LCSS APPROACH

A. The proposed approach

The LCSS approach is an advanced null broadening beamforming method. However, more DOFs of the array will be needed when the performance of the LCSS approach is improved. To solve the problem, the covariance-matrix-expansion-based LCSS (CME-LCSS) approach is proposed in this paper. Compared with the LCSS approach, more linear constraints can be constructed by the CME-LCSS approach and the width and depth of nulls can be improved.

An eye matrix is used to expand the sample covariance matrix $\hat{\mathbf{R}}$. The expanded matrix $\tilde{\mathbf{R}}$ can be expressed as follows:

$$\tilde{\mathbf{R}} = \mathbf{I} \otimes \hat{\mathbf{R}}, \tag{8}$$

where $\mathbf{I}$ is an eye matrix of $N \times N$ order and $\otimes$ denotes the Kronecker product. The steering vector of the antenna array is expanded as:

$$\mathbf{b}(\theta) = \mathbf{a}(\theta) \otimes \mathbf{a}(\theta) . \tag{9}$$

It can be observed from formula (8) that the sample covariance matrix, $\hat{\mathbf{R}}$ (of $N \times N$ order), is expanded to matrix $\tilde{\mathbf{R}}$ (of $N^2 \times N^2$ order). From formula (9), the steering vector, $\mathbf{a}(\theta)$ (of $N \times 1$ order), of the real antenna array is expanded to the steering vector, $\mathbf{b}(\theta)$ (of $N^2 \times 1$ order), of the virtual antenna array. Therefore, virtual antenna array elements are formed and more linear constraints can be constructed.

Ideally, the goal of null broadening is to obtain a zero power response at the sector $\Delta\theta_i$, that is, $\mathbf{W}^H\bar{\mathbf{Q}}\mathbf{W} = 0$. It can be obtained according to the steering vector of virtual antenna array as follows:

$$\bar{\mathbf{Q}} = \frac{1}{\Delta\theta_i}\int_{\theta_i - \Delta\theta_i/2}^{\theta_i + \Delta\theta_i/2} \mathbf{b}(\theta)\mathbf{b}^H(\theta)d\theta , \tag{10}$$

where $\bar{\mathbf{Q}}$ is a matrix of $N^2 \times N^2$ order.

We can factorize it as $\bar{\mathbf{Q}} = \bar{\mathbf{U}}\bar{\mathbf{\Sigma}}\bar{\mathbf{U}}^H$, where $\bar{\mathbf{U}}$ is an $N^2 \times N^2$ matrix containing the orthonormal eigenvectors of $\bar{\mathbf{Q}}$, and $\bar{\mathbf{\Sigma}}$ is an $N^2 \times N^2$ diagonal matrix containing the eigenvalues of $\bar{\mathbf{Q}}$ in a decreasing order. Assume that $\bar{\mathbf{Q}}$ has a rank equal to $\bar{r}$, then $\bar{\mathbf{U}}_{\bar{r}} = [\bar{\mathbf{u}}_1, \cdots, \bar{\mathbf{u}}_{\bar{r}}]$ is the matrix of eigenvectors that corresponding to larger eigenvalues. The quadratic constraint $\mathbf{W}^H\bar{\mathbf{Q}}\mathbf{W} = 0$ is satisfied if $\mathbf{W}^H\bar{\mathbf{U}}_{\bar{r}} = \mathbf{0}$. Therefore, the quadratic constraint can be transformed into a set of linear constraints, and the optimal weight vector of the array can be obtained as follows:

$$\min_{\mathbf{W}} \mathbf{W}^H\tilde{\mathbf{R}}\mathbf{W} \ \text{ s.t. } \mathbf{W}^H\bar{\mathbf{C}} = \bar{\mathbf{e}}_1^T , \tag{11}$$

where $\bar{\mathbf{C}} = [\mathbf{b}(\theta_d), \bar{\mathbf{U}}_{\bar{r}}]$ and $\bar{\mathbf{e}}_1 = [1, \mathbf{0}^T]^T$.

According to the Lagrange multipliers, we define:

$$L(\mathbf{W}) = \tfrac{1}{2}\mathbf{W}^H\tilde{\mathbf{R}}\mathbf{W} + \mu^H(\bar{\mathbf{e}}_1 - \bar{\mathbf{C}}^H\mathbf{W}). \tag{12}$$

The solution of $\partial L(\mathbf{W})/\partial\mathbf{W} = 0$ is $\mathbf{W} = \tilde{\mathbf{R}}^{-1}\bar{\mathbf{C}}\mu$. Substituting $\mathbf{W}$ into $\mathbf{W}^H\bar{\mathbf{C}} = \bar{\mathbf{e}}_1^H$, we can obtain that $\mu = (\bar{\mathbf{C}}^H\tilde{\mathbf{R}}^{-1}\bar{\mathbf{C}})^{-1}\bar{\mathbf{e}}_1$. Finally, μ is substituted into $\mathbf{W}$.

The optimal weight vector of the CME-LCSS approach can be expressed as follows:

$$\mathbf{W}_{CME-LCSS}(\bar{r}) = \tilde{\mathbf{R}}^{-1}\bar{\mathbf{C}}(\bar{\mathbf{C}}^H\tilde{\mathbf{R}}^{-1}\bar{\mathbf{C}})^{-1}\bar{\mathbf{e}}_1 . \tag{13}$$

Similar to the LCSS approach, the optimal number $\bar{r}'_{opt}$ of linear constraints for the CME-LCSS approach, which needs to be determined to ensure that the average output power of the sector $\Delta\theta_i$ is lower than the pre-set value η, can be expressed as:

$$\bar{r}'_{opt} = \underset{r'=[1,2,\cdots,2N-1]}{\arg\min}\, q(\bar{r}') \le \eta , \tag{14}$$

where $q(\bar{r}') = \mathbf{W}_{CME-LCSS}^H(\bar{r}')\bar{\mathbf{Q}}\mathbf{W}_{CME-LCSS}(\bar{r}')$.

DL can be selectively used to improve the stability of the LCSS approach, and the loading to noise ratio (LNR) is $10dB$. The performance of the CME-LCSS approach can also be further improved with DL with the same $LNR = 10dB$.

B. Theoretical analysis

Firstly, the relationship between the sample covariance matrix and the expanded covariance matrix is illustrated. According to Schmidt's orthogonal subspace resolution theory, the sample covariance matrix can be expressed as:

$$\hat{\mathbf{R}} = \hat{\mathbf{U}}\hat{\mathbf{\Sigma}}\hat{\mathbf{U}}^H , \tag{15}$$

where $\hat{\mathbf{U}}$ is a matrix containing the orthonormal eigenvectors of $\hat{\mathbf{R}}$, and $\hat{\mathbf{\Sigma}}$ is a diagonal matrix containing the eigenvalues of $\hat{\mathbf{R}}$ in a decreasing order. The eigenvectors in $\hat{\mathbf{U}}$ correspond to the incident directions of signals and interferences. The eigenvalues in $\hat{\mathbf{\Sigma}}$ correspond to the power of signals and interferences.

Substituting formula (15) into formula (8) and using the properties of Kronecker product [22], we know that:

$$\begin{aligned}\tilde{\mathbf{R}} &= \mathbf{I}\otimes\hat{\mathbf{R}}\\ &= \mathbf{I}\otimes\left(\hat{\mathbf{U}}\hat{\mathbf{\Sigma}}\hat{\mathbf{U}}^H\right)\\ &= \left(\mathbf{I}\otimes\hat{\mathbf{U}}\right)\left(\mathbf{I}\cdot\mathbf{I}^H\otimes\hat{\mathbf{\Sigma}}\hat{\mathbf{U}}^H\right)\\ &= \left(\mathbf{I}\otimes\hat{\mathbf{U}}\right)\left(\mathbf{I}\otimes\hat{\mathbf{\Sigma}}\right)\left(\mathbf{I}\otimes\hat{\mathbf{U}}\right)^H\\ &= \tilde{\mathbf{U}}\tilde{\mathbf{\Sigma}}\tilde{\mathbf{U}}^H\end{aligned}, \quad (16)$$

where:

$$\begin{aligned}\tilde{\mathbf{U}} &= \mathbf{I}\otimes\hat{\mathbf{U}}\\ \tilde{\mathbf{\Sigma}} &= \mathbf{I}\otimes\hat{\mathbf{\Sigma}}\end{aligned}. \quad (17)$$

It can be seen from formula (17) that both $\hat{\mathbf{U}}$ and $\hat{\mathbf{\Sigma}}$ are expanded with an eye matrix. Therefore, the information of the incident directions and the power contained in the expanded matrix $\tilde{\mathbf{R}}$ is the same as that in the sample covariance matrix $\hat{\mathbf{R}}$.

Secondly, the relationship between the steering vectors of the real antenna array and the virtual antenna array is illustrated.

Since the antenna array is an ULA, its steering vector $\mathbf{a}(\theta)=\left[1, e^{j\frac{2\pi}{\lambda}d\sin\theta}, \cdots, e^{j\frac{2\pi}{\lambda}(N-1)d\sin\theta}\right]^T$ is a vector of $N\times1$ order. According to formula (9), we know that $\mathbf{b}(\theta)$ is a vector of $N^2\times1$ order, in which there are $2N-1$ non-repetitive elements. To describe this more clearly, the expansion of steering vector is shown in Fig. 2 as follows.

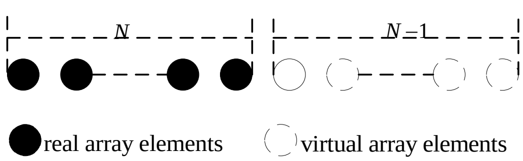

Fig. 2. Schematic diagram of expansion of the steering vector of the antenna array.

It is observed from Fig. 2 that the antenna array is expanded and virtual array elements are formed. There are $2N-1$ non-repetitive array elements, therefore more linear constraints can be constructed compared with the real antenna array. The computational load of the proposed approach is increased as the price. The computation complexity of the LCSS approach is $O(JN^2)+O(N^3)$, where J is the number of sampled points in $\Delta\theta_i$. The computation complexity of the CME-LCSS approach is $O(JN^4)+O(N^6)$, which is greater than that of the LCSS approach. Therefore, the CME-LCSS approach is more applicable to enhancing the situation where the LCSS approach for broadening nulls is not effective because the number of array elements is not large enough.

V. NUMERICAL EXAMPLES AND SIMULATION

Assuming that the noncoherent narrowband excitation sources are far away from the antenna array and the element space is equal to one-half wavelength. The desired signal illuminates on the antenna array in the direction of 0°. Three independent interferences are from the directions of -50°, 20° and 60°. The signal to noise ratio (SNR) is $0dB$. The interferences to noise ratio (INR) is $30dB$. The number of snapshots is 200. With these given conditions, we demonstrate the performance of the CME-LCSS approach and the LCSS approach via computer simulations. In the end, numerical simulation procedures are described.

A. Example 1

The number of array elements is 15. Assume that the widths of broad nulls are set to 10°. Broad nulls are formed around the three directions of -50°, 20° and 60°. To compare the CME-LCSS approach with the LCSS approach, Fig. 3 shows the beam patterns of the two approaches versus the azimuth angle.

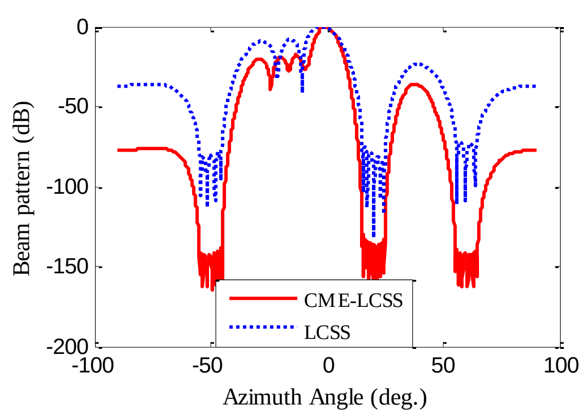

Fig. 3. Beam patterns of the two approaches.

It can be observed from Fig. 3 that broad nulls are formed around the three interferences. The null depth of the CME-LCSS approach is about $-130dB$, while the null depth of the LCSS approach is about $-75dB$. Compared with the LCSS approach, deeper nulls can be obtained by the CME-LCSS approach.

B. Example 2

The number of array elements is 10. Assume that the broad nulls, whose widths are both set to 10°, are formed around the directions of -50° and 60°. To compare the CME-LCSS approach with the LCSS approach when the null widths are equal, Fig. 4 shows the beam patterns of the two approaches versus the azimuth angle.

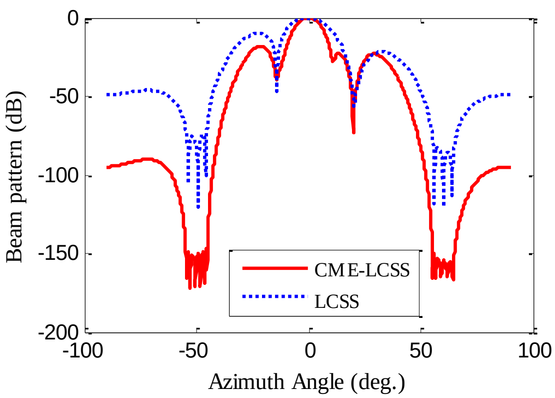

Fig. 4. Beam patterns of the two approaches.

It can be observed from Fig. 4 that broad nulls can be formed around the interferences selectively. Besides, the null depth of the CME-LCSS approach is about $-140dB$, while the null depth of the LCSS approach is about $-80dB$. Compared with the LCSS approach, deeper nulls can be obtained by the CME-LCSS approach.

C. Example 3

Simulation conditions are the same as example 2. While the null width of the CME-LCSS approach is 20° and the LCSS approach is 10°, the performance of the CME-LCSS approach is compared with that of the LCSS approach. Figure 5 shows the beam patterns of the two approaches. Figure 6 shows the output SINR versus the snapshots of the two approaches. Figure 7 shows the output SINR versus the input SNR of the two approaches.

It is observed from Fig. 5 that the null depth of the CME-LCSS approach is about $-110dB$ when the null width is 20°, and the null depth of the LCSS approach is about $-80dB$ when the null width is 10°. Compared with the LCSS approach, deeper and wider nulls can be obtained by the CME-LCSS approach.

From Fig. 6, we know that both approaches need a certain number of snapshots to obtain the stable output SINR, and the output SINR of the CME-LCSS is always higher than that of the LCSS approach.

It can be observed from Fig. 7 that when the input SNR exceeds $10dB$, the output SINR of the two approaches will tend to be stable. Besides, the output SINR of the CME-LCSS approach is always higher than that of the LCSS approach.

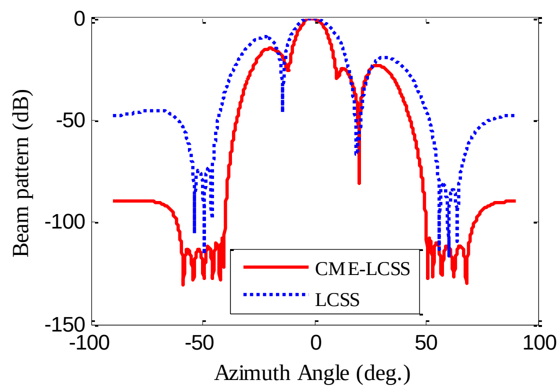

Fig. 5. Beam patterns of the two approaches.

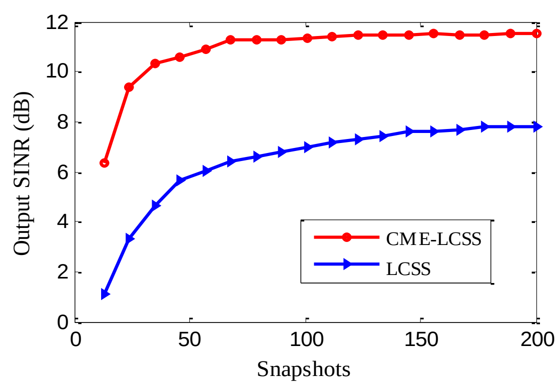

Fig. 6. Output SINR versus snapshots.

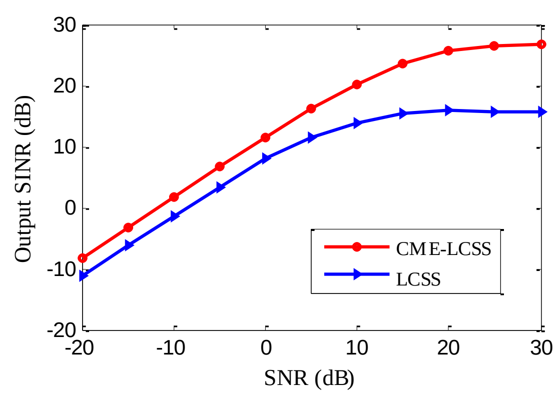

Fig. 7. Output SINR versus input SNR.

D. Example 4

Simulation conditions are the same as Example 3. Both of the CME-LCSS and the LCSS approach are used with DL ($LNR = 10dB$). Figure 8 shows the output SINR versus input SNR of the two approaches.

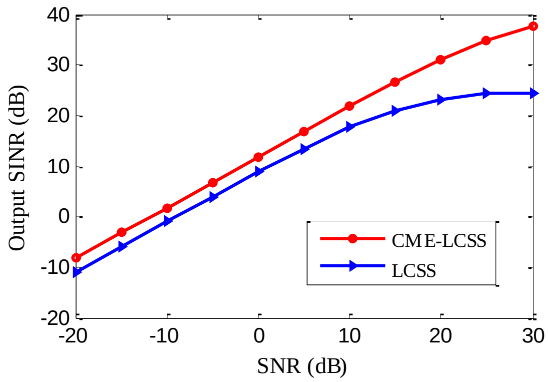

Fig. 8. Output SINR versus input SNR with DL.

It is observed from Fig. 8 that the output SINR of the two approaches will tend to stable when the input SNR exceeds $20dB$, that is to say, the performance of the two approaches are both improved.

E. Example 5

The CME-LCSS approach is used in an array with 10 elements, while the LCSS approach is used in an array with 19 elements. Broad nulls with widths of both 20° , are formed around the directions of -50° and 60° . Figure 9 shows the beam patterns of the two approaches.

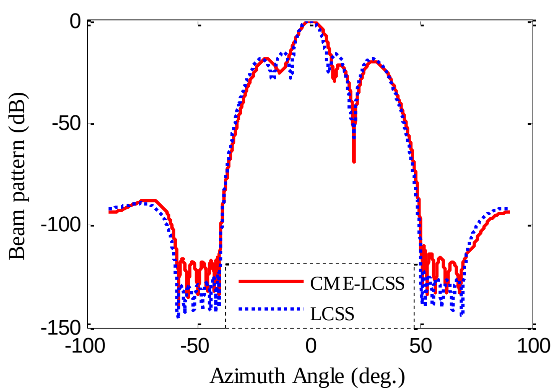

Fig. 9. Beam patterns of the two approaches.

It is observed form Fig. 9 that the beam patterns of the two approaches are similar. Therefore, approximate performance can be obtained by the CME-LCSS approach with fewer array elements compared with the LCSS approach.

F. Numerical simulation procedures

To numerically calculate formula (10), discrete points are taken in $\Delta\theta_i$ with equal spacing, and the integral operation is approximately obtained though the summing operation as follows:

```
theta0=lb:0.01:rb;
 for i=1:length(theta0)
   A=exp(-j*2*pi *[0:N-1]'*sin(theta0(i)) *d /lamda);
   B=kron(A, A);
   Q_bar=B*B'+Q_bar;
 end
```

where 'lb' and 'rb' stand for the boundary of $\Delta\theta_i$, 'kron' stands for the Kronecker product, 'A' stands for $\mathbf{a}(\theta)$ and 'B' stands for $\mathbf{b}(\theta)$. As the number of discrete points increases, the accuracy of $\bar{\mathbf{Q}}$ will increase. However, the computation complexity of the CME-LCSS approach will be greater.

The matrix of linear constraints $\bar{\mathbf{C}}$ in formula (11) can be calculated as follows:

```
[V, D]=eig(Q_bar);
C_bar=[kron(Ad, Ad), V (:,1:lc)];
```

where 'eig' stands for eigen-decomposition, 'V' stands for the eigenvectors matrix of $\bar{\mathbf{Q}}$, 'D' stands for the eigenvalues matrix of $\bar{\mathbf{Q}}$, 'Ad' stands for the steering vector of the desired signal, and 'lc' stands for the number of linear constraints that corresponding to the large eigenvalues of $\bar{\mathbf{Q}}$. Then, the optimal weight vector of the CME-LCSS approach can be obtained using formula (13).

VI. CONCLUSION

A covariance-matrix-expansion-based linear constraint sector suppressed (CME-LCSS) beamforming approach is proposed in this paper. Numerical solution procedures of CME-LCSS are provided and computation complexity is analyzed and compared between CME-LCSS and LCSS. It is verified by simulations that, compared with the LCSS approach, more linear constraints can be constructed by the CME-LCSS approach, so wider and deeper nulls can be obtained with equal number of array elements, or similar performance can be obtained with fewer number of array elements.

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