Design of Parabolic Reflector Antenna with Two Directional Cosecant-Squared Pattern Using Curve-Deformation Equations

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Abstract **─** A Ka-band doubly curved reflector antenna for a warning radar system for the road was designed based on the curve-deformation equations. This antenna requires a cosecant-squared radiation pattern in the vertical and horizontal planes. Using curve-deformation equations, the shaped reflector antenna was designed to satisfy the desired beam pattern. The curve-deformation equations include the parameters which determine the shape of the curve, and these parameters are obtained by the optimization process. The simulated result shows the designed antenna satisfies the specifications of the antenna for the road radar system.

Index Terms **─** Cosecant squared pattern, parabolic antenna.

I. INTRODUCTION

Radar is widely used to detect the obstacles and to avoid collisions on the road and airport surfaces [1]-[2]. In order to survey a wide range, the power received from the reflector antenna needs to be uniform for a fixed height. The relationship between the uniform received power and the distance between the object and the antenna can be defined by the cosecant-squared function [3]. The cosecant-squared pattern can be achieved by displacing the reflector surface from the original parabolic shape [4]-[7]. Recently, the analytical regularization method (ARM) [5] was used to produce the cosecant square pattern in the vertical plane. This method solves a two-dimensional integral equation with Dirichlet boundary condition corresponding to the reflector's surface. The geometry of a reflector is generated by continuous segments that are defined using parameterization of different length and bending angles. This contour is a two-dimensional closed curve, from which the cosecant-squared pattern is obtained in the vertical plane. In this paper, the equations of the cosecant-squared patterns in the horizontal and vertical plane will be derived using the ARM's parameterization process. Instead of solving differential equations, the parameter optimization method is used to synthesize the desired reflector. Invasive weed optimization is one method that determines the optimum values of the coefficients of a distribution polynomial using the optimization process [2]. In this optimization process, if the coefficients of the distribution polynomial are given, then the radiation pattern can be obtained using commercial software, and this pattern is compared to the cosecant-squared pattern. This procedure continues until the desired pattern is obtained. Here, the optimization process will be used to acquire the optimum values of the parameters corresponding to the bending angle and length of a curve. These parameters determine the curvature in the direction of the vertical and horizontal planes.

II. DESIGN OF ANTENNA

A. Curve-deformation equations

The formula is based on the parameterization equations of ARM [5]. In order to define the formula concerning the geometry of a cosecant-squared pattern reflector, each interval of the geometrical curve is divided as follows:

$$\frac{2L_k\pi}{L}-\pi \le \theta < \frac{2L_{k+1}\pi}{L}-\pi, \qquad (1)$$

where $L_0 = 0$, $L_1 = 2\,a\tan(\alpha/2)$, $\alpha = \alpha_2 - \alpha_1$, $L_{k+1} = L_k + r_k$, $r_0 = r_1 = 0$, $k = 0, \ldots, N_{max}$, and $L = \sum_1^{N_{max}} L_k$.

As shown in Fig. 1, α_1 and α_2 are the angles between the *x*-axis and the end points of the non-shaped section, and a is the focal length of the parabolic reflector. In (1), N_{max} indicates the number of splitting of $[-\pi, \pi]$ and a parameter r_k determines the length of the k-th section. The non-shaped section is defined as the function of a parameter θ and is as follows:

$$x_1(\theta) = -\frac{2a\cos(\psi)}{1+\cos(\psi)}, \qquad (2)$$

$$y_1(\theta) = \frac{2\,a\sin(\psi)}{1+\cos(\psi)}, \qquad (3)$$

where $\psi = \psi(\theta) = \alpha_2 - \alpha(\theta+\pi)L/(4\pi a\tan(\alpha/2))$, and $\theta \in [-\pi,\ 2L_1\pi/L - \pi]$. Note that, if ψ is cancelled

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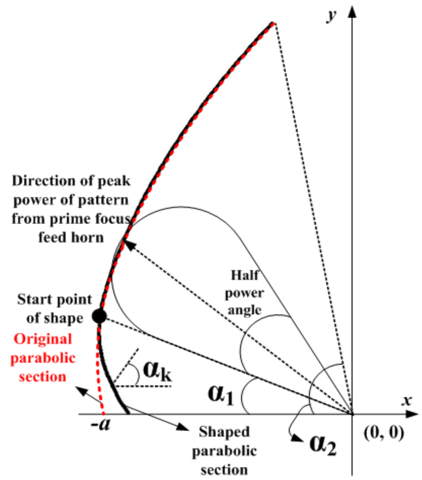
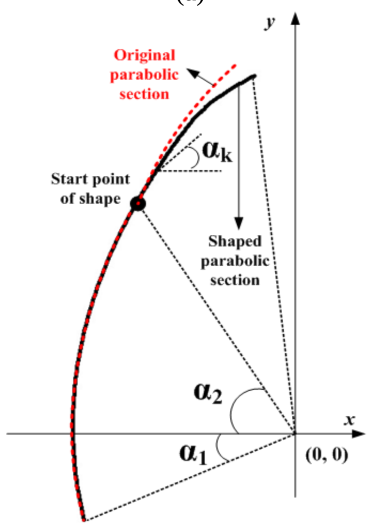
from (2) and (3), then we have the original parabolic equation with respect to parameters x, y:

$$x = \frac{y^2}{4a} - a. \tag{4}$$



(a)



(b)

Fig. 1. (a) Geometry of the shaped reflector with the cosecant-squared pattern directed at a positive angle, and (b) at a negative angle.

The graph of Equation (4) is plotted as indicated by the red dotted line in Fig. 1. The k-th section of the shaped curve of an antenna with cosecant-squared pattern can be represented by a function of parameter ϕ, which is defined on the interval $\phi \in [2L_{k-1}\pi/L - \pi,\ 2L_k\pi/L - \pi]$, and the line when ϕ is varying is as follows:

$$x_k(\phi) = -2a\cos(\alpha_1)/(1+\cos(\alpha_1)) + \left(\frac{(\phi+\pi)L}{2\pi} - L_{k-1}\right)\cos(\alpha_k) + \sum_{i=3}^{k} r_i\cos(\alpha_i), \tag{5}$$

$$y_k(\phi) = 2a\sin(\alpha_1)/(1+\cos(\alpha_1)) - \left((\phi+\pi)L/2\pi - L_{k-1}\right)\cos(\alpha_k) - \sum_{i=3}^{k} r_i\sin(\alpha_i), \tag{6}$$

where $\alpha_k > 0, k = 2, 3, \ldots, N_{max}$, and the last summation $\sum_{i=3}^{k} *$ is zero when $k = 2$.

In Equations (5) and (6), α_k is the bending angle of the k-th section. Using Equations (2) to (6), the shaped reflector of the cosecant-squared pattern in the vertical plane can be obtained by determining α_i and r_i. As shown in Fig. 1 (a), the sections of the shaped parabolic curve are located on the lower part of the non-shaped section. Thus, the cosecant-squared pattern appears at positive angle. However, in order to detect an object on the road, the cosecant beam should be directed at a negative angle. For this purpose, modifications of (5) and (6) are needed. First, the parameters for the translation are defined as:

$$x_{max1} = \max\{x_1(\phi) \mid \phi \in [-\pi,\ 2L_1\pi/L - \pi], y_1(\phi) \geq 0\},$$
$$x_{max2} = \max\{x_1(\phi) \mid \phi \in [-\pi,\ 2L_1\pi/L - \pi], y_1(\phi) < 0\},$$
$$y_{max} = \max\{y_1(\phi) \mid \phi \in [-\pi,\ 2L_1\pi/L - \pi]\},$$
$$y_{min} = \min\{y_1(\phi) \mid \phi \in [-\pi,\ 2L_1\pi/L - \pi]\},$$

and

$$x_{move} = \begin{cases} x_{max1} & \text{if } \alpha_1 \geq 0 \\ x_{max1} - x_{max2} & \text{if } \alpha_1 < 0 \end{cases},$$
$$y_{move} = y_{max} - y_{min}.$$

The function that determines the curve of the reflector with the cosecant-squared pattern directed at a negative angle can be expressed as:

$$x'_k(\phi) = x_2(\phi) + \sum_{i=3}^{k} r_i\cos(\alpha_i) + x_{move}, \tag{7}$$

$$y'_k(\phi) = y_2(\phi) + \sum_{i=3}^{k} r_i\sin(\alpha_i) + y_{move}. \tag{8}$$

Using (7)-(8), the geometry of an elevation direction of reflector can be obtained. The deformation process of an azimuth direction of reflector is carried out on each fraction of partition of the z range. For a fixed point (x_0, y_0) of the shaped curve shown in Fig. 1, the right side of a curve on the transverse section of a $x-z$ plane through y_0 is expressed as a coordinate system (x, z):

For $z \in [z_{j-1}, z_j] \subset [0, z_{max}]$,

$$x = c_j\frac{z^2}{4a} + x_0, \tag{9}$$

where a parameter c_j determines the deformation level of the j-th section, and is less than 1. The left section of the curve can be symmetrically obtained from the right section.

B. Design of reflector using curve-deformation equations

The antenna for the detection radar system requires the specifications presented in Table 1. As shown in Fig. 2, this antenna was installed at a height of 6 m, and was designed to detect within a band of road 20 m wide.

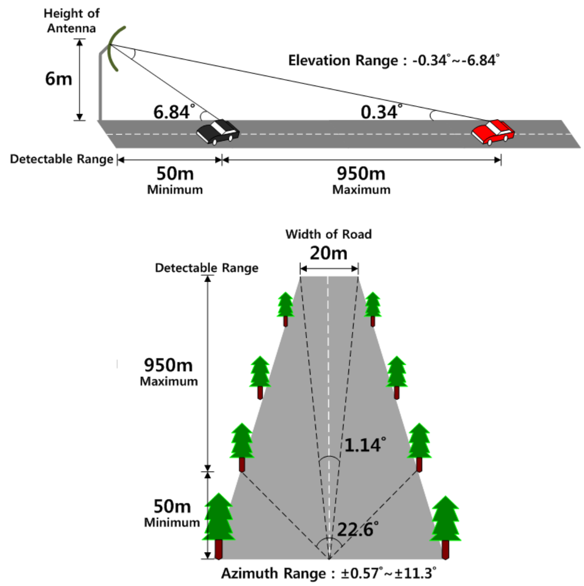


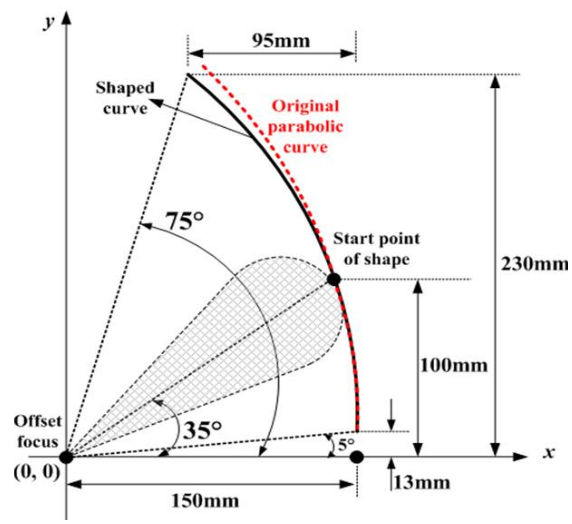
Fig. 2. Range of the detection by the radar system.

Table 1: Specifications of antenna

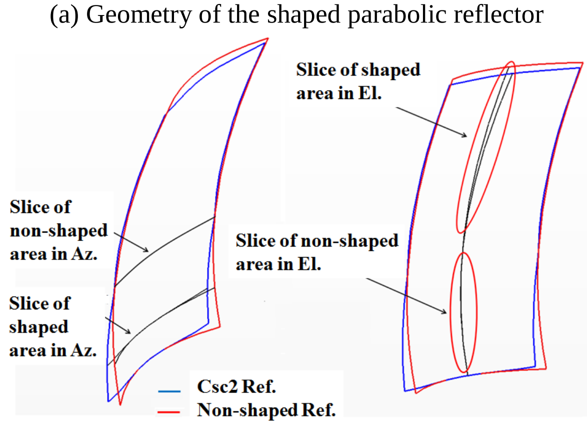
Parameter	Horizontal	Vertical
Frequency	34.5 GHz	34.5 GHz
Maximum gain	32 dBi	32 dBi
Cosecant-squared region	1.14°~7.62°	0.34°~2.29°
Size of antenna	0.20×0.217 m^2	

The measurable distance from the antenna to the target by this radar system is between 150 and 1000 m. In order to obtain the desired cosecant-squared pattern, the values of α_i, r_i, and c_j have to be determined. The first step of the process is to determine the region of the curve that is deformed. Considering the half-power of the radiation pattern of the feed horn, the outside of the half-power angle of the feed pattern was selected as the area of deformation, as shown in Fig. 1.

The shaped curve is approximated by N_{max} sections. In this paper, N_{max} was set to 8. The next step was to select the initial parameters (α_i, r_i, and c_j) randomly. These parameters were inserted into the deformation Equations (7)-(9), and the three-dimensional geometric data was obtained. These data were imported into CATIA (commercial software), and the surface of the reflector was designed and saved as a CATIA file. This surface file was imported into CST (commercial software), and the radiation pattern of the reflector was obtained using CST. Then, this pattern was compared to the desired cosecant-squared pattern. If the difference error between the simulated pattern and the desired pattern is greater than the tolerance limit, then the parameters have to be corrected. The tolerance limit means the acceptable difference error between the desired pattern and the selected pattern. The tolerance limit function can be defined in the same way as introduced in [2]. This process continues until the pattern is well fitted to the desired patterns. Figure 3 (a) shows the geometry of the designed reflector antenna. The focal length was 0.15 m and the offset angle was 35°. The size of the reflector was 0.20×0.217 m^2. Because the cosecant deformation of a reflector is performed in the azimuth and elevation directions, the upper edge of the reflector is inclined toward the aperture along the vertical direction, and the lower edge is bent back along the azimuthal direction, as shown in Figure 3 (b). Table 2 shows the optimized values of the parameter in the curve-deformation equations. The parameter r_i determines the length of shaped curve and, we set the parameter r_i as the same value 0.0189 for all $i > 1$.



(a) Geometry of the shaped parabolic reflector



(b) Comparison with the non-shaped parabolic reflector depicted by CST

Fig. 3. Shaped reflector antenna and comparison with the non-shaped parabolic reflector.

Table 2: Parameters of curve-deformation equations

Parameter	Value [degree]	Parameter	Parameter
α_1	5	c_1	1.0
α_2	37	c_2	0.96
α_3	71	c_3	0.92
α_4	65	c_4	0.88
α_5	62	c_5	0.84
α_6	60	c_6	0.8
α_7	57.5	c_7	0.72
α_8	54.5	c_8	0.64
α_9	53.5	c_9	0.56
α_{10}	51.5	c_{10}	0.48

III. RESULTS OF SIMULATION

A. Measurement of radar system

The newly designed antenna was simulated by CST in order to obtain the radiation patterns. To be agreed with the cosecant-squared pattern using the curve-deformation equations, designing a cosecant antenna is performed in the early stage of the optimization. The results of the radiation patterns of the main beam are displayed at Fig. 4. The beamwidths of the radiation patterns are 2.1° and 4.2° in the azimuthal and elevation directions respectively and the gain is 33.2 dBi. To achieve the desired pattern of Fig. 4, around five to eight optimization procedures are enough. We calculated errors between the desired pattern and the designed pattern using CST. The parameters of the deformation Equations (7)-(9) are corrected to optimized values. But in this case, only the pattern at zero angle was obtained, and in order to get the broad beam, the beamwidth, gain and patterns at each angle must be considered. Moreover, the monopulse beam pattern at each angle also have to be considered because it is used to detect the exact location of the debris on the road. The optimization process have to continue until the desired gain distribution at the detection area as shown at Fig. 5 are obtained, and 8~10 optimization steps were carried out.

The gain of Fig. 5 was obtained proportional to the square of the distance, which means the gain loss of the antenna.

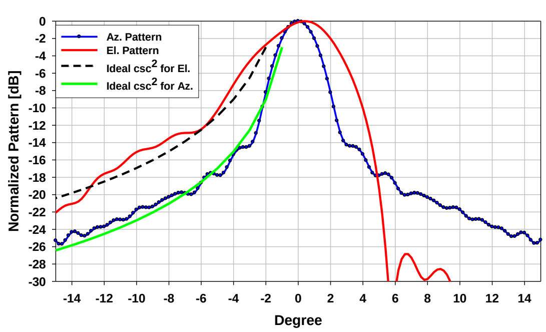


Fig. 4. Radiation pattern obtained from the first optimization step.

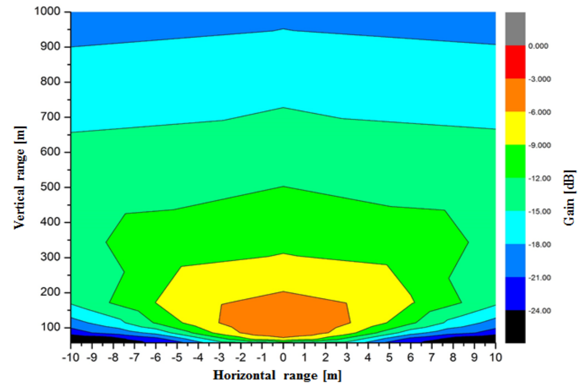
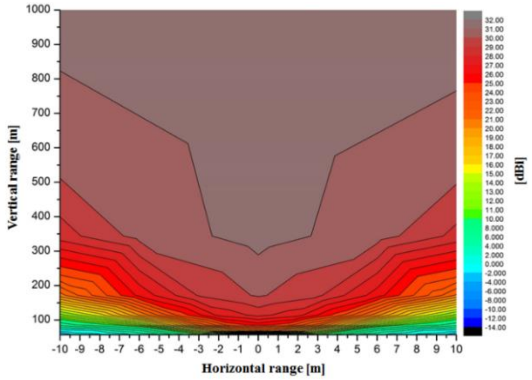


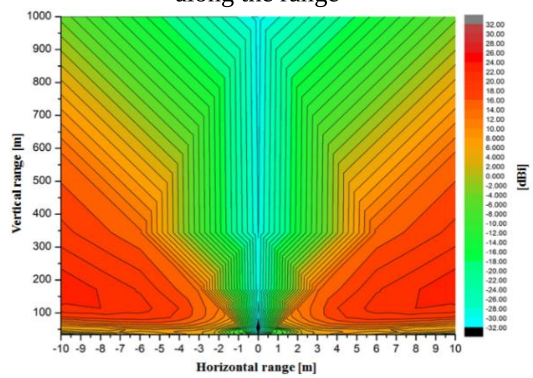
Fig. 5. Gain distribution on the detection area.

Figure 5 shows that the designed antenna has a wide broad cosecant beam directed in a downward direction, which is suitable for detecting objects on the road. Figure 6 displays the power distributions of a sum and a delta pattern of the final designed cosecant antenna. In Fig. 7, these patterns are compared with the desired cosecant-squared patterns. The dual-mode horn antenna for a monopulse feed system was used to obtain the sum and difference beams. The beamwidth used as the edge taper was 14 dB and 18 dB, in the azimuth and elevation planes, respectively. Because the range of the degree satisfying the cosecant-squared pattern is very narrow, the shaped curve was designed to have a wide beamwidth in order to achieve the cosecant-squared pattern. Even though the pattern did not agree well with the cosecant-squared curve, the antenna had higher gain, and the difference monopulse pattern had wider beamwidth, than would an antenna with a pattern well-fitted to the cosecant-squared region over the wide region. As shown in Table 1, the cosecant regions of horizontal and vertical directions were 1.14°~7.62° (=±3.81°) and 0.34°~2.29°, and the radiation patterns of the main beam presented in Fig. 7 satisfied the desired cosecant-squared level. Figure 8 shows the horizontal sections of the radiation pattern of the reflector antenna when the elevation angles varied from 0° to −5°. In the case of the sum patterns, the maximum gain of the main horizontal section was 32 dBi, and the beam peak decreased according to the main vertical section of the pattern. The difference between the maximum gain of 0° sum pattern and that of a 4° sum pattern was only 3 dBi. The sum patterns also maintained the shape and the beamwidth, and satisfied the desired cosecant-squared level. This means that the antenna is designed to be suitable for detecting the debris all over the surface. The difference pattern can be used to determine accurately the angular location of the target. The gap between the null point and the peak point of the difference pattern was almost 5°, and it is sufficient to determine the angular location of the target because the maximum angle in the azimuthal direction within the detection area is ±3.81°.

Figure 9 shows the monopulse curve that represents the difference between the gain of the sum pattern and the gain of the difference pattern.

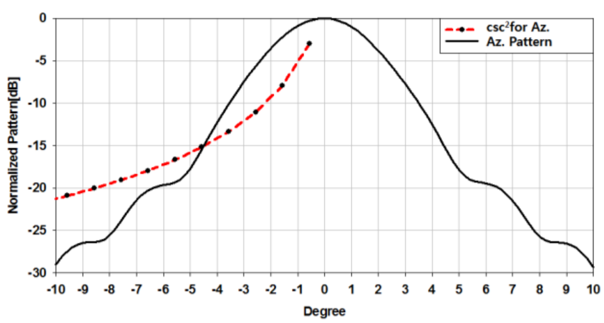


(a) Distribution of sum signals of monopulse antenna along the range

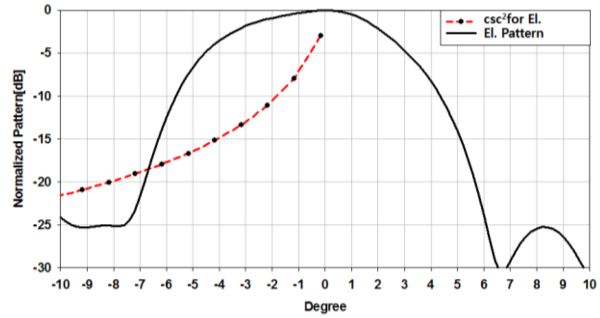


(b) Distribution of delta (or difference) signals of monopulse antenna along the range

Fig. 6. Power distribution of the final designed cosecant antenna along the range of detection.

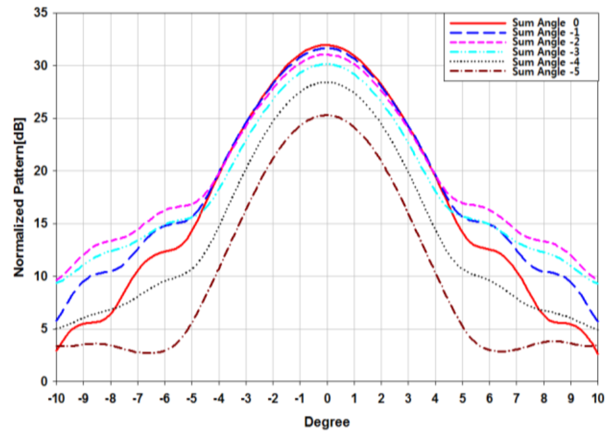


(a) Azimuth radiation pattern

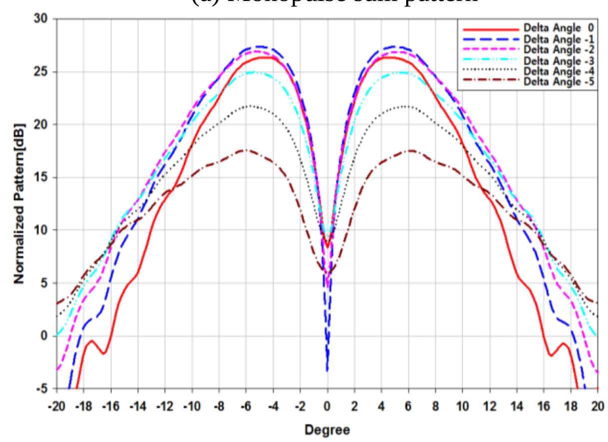


(b) Elevation radiation pattern

Fig. 7. Comparison of a designed pattern with an ideal csc^2 pattern.



(a) Monopulse sum pattern



(b) Monopulse difference pattern

Fig. 8. Azimuthal section of the radiation pattern of the reflector varying elevation angles.

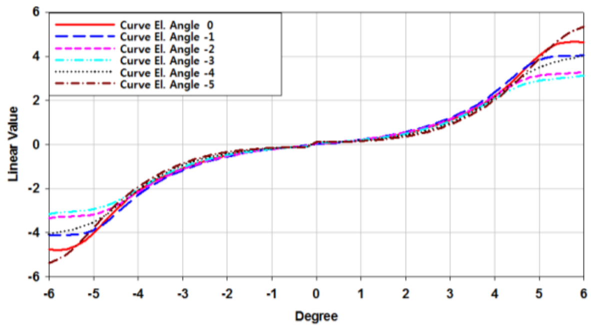


Fig. 9. Monopulse curve.

IV. CONCLUSION

The Ka-band shaped reflector antenna was designed based on a curve-deformation equation. The deformation of the surface was performed in the vertical and horizontal directions. The cosecant-squared patterns in the vertical and horizontal planes were obtained by simulation using the CATIA and CST software packages. These patterns satisfied the specifications of the antenna for a detection radar system for the road. The method herein presented can be used to design antennas for the detection of foreign-object debris on airport surfaces, for surveillance radar, and for the ship-mounted surface radar.

V. ACKNOWLEDGMENT

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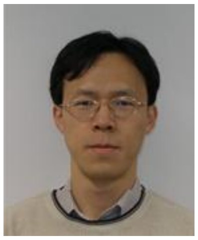

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