Efficient Analysis of Object with Fine Structures by Combined MLSSM/MLFMA via Compressed Block Decomposition Preconditioner

Zhao N. Jiang [1,3], Fei Shen [1], Yuan Xu [2], and Xiao Y. Yang [1]

[1] Department of Information Engineering
Hefei University of Technology, Hefei, 230009, China
jiangzhaoneng@hfut.edu.cn

[2] Department of Applied Mathematics
Nanjing University of Science and Technology, Nanjing, 210094, China
xuyuan650708@hotmail.com

[3] State Key Laboratory of Millimeter Waves, Nanjing, 210096, China

Abstract **─** A large dense fine mesh is used to model object with fine structures to guarantee good solution accuracy, and this in turn places an inordinately heavy burden on the CPU in terms of both memory requirement and computational complexity. To analyze the large dense complex linear system efficiently, the combined MLSSM/MLFMA is used to accelerate the matrix-vector multiplication. Multilevel fast multipole algorithm (MLFMA) cannot be used to analyze the box's size of tree structure below 0.2 wavelength, because the "low frequency breakdown" phenomenon would happen. For the large-scale problems, the matrix assembly time of multilevel simply sparse method (MLSSM) is much longer than that of MLFMA. This combined method takes advantage of the virtues of both MLFMA and MLSSM, which is more efficient than either conventional MLFMA or conventional MLSSM. An efficient preconditioning technique based on compressed block decomposition (CBD) is applied to speed up the convergence rate. Numerical results are presented to demonstrate the accuracy and efficiency of the proposed method.

Index Terms **─** Compressed Block Decomposition (CBD) preconditioner, Multilevel Simply Sparse Method (MLSSM), object with fine structures.

I. INTRODUCTION

The increased power and availability of computational resources and acceleration schemes have enabled solution of problems with very large number of unknowns, varying from few thousands to few millions [1]. Another class of problems arises when analyzing structures which require a high local density of unknowns to accurately capture geometric features. This class of problems is referred to as object with fine structures problems exhibit multiple scales in length. For example, small length scale discretizations are required to capture sharp or fine geometric features that are embedded within large and smooth geometries discretized at a coarser length scale. Generally, the characteristic of an object with fine structures problem is the concentration of large number of unknowns in electrically small domains. All these simulations require fast and efficient numerical methods to compute an approximate solution of Maxwell's equations. Numerous authors have derived a variety of methods that are used for computing the electromagnetic problems. The method of moments (MoM) [2-4] is one of the most widely used techniques for electromagnetic problems. However, for large-scale problems, a great number of unknowns are required for modeling objects, which always leads to intensive computation and unaffordable CPU time.

The fast algorithms are developed to reduce computational cost. The most popular fast solution include the multilevel fast multipole algorithm (MLFMA) [5-6], has $O(N\log N)$ (where N denotes the number of unknowns) complexity for a given accuracy. Though efficient and accurate, this algorithm is highly technical. It utilizes a large number of tools, such as partial wave expansion, exponential expansion, filtering, and interpolation of spherical harmonics. However, MLFMA becomes numerically unstable and inefficient when applied to object with fine structures problems. This is a consequence of the fact that Helmholtz MLFMA does not smoothly transition to Laplace MLFMA as frequency tends to zero. Therefore, when the finest level box's size is below 0.2λ (λ indicates the incident wavelength), MLFMA will suffer from "low frequency breakdown" phenomenon [1]. As a result, it cannot be easily applied to analyze the object with fine structures problems.

Submitted On: September 4, 2015
Accepted On: September 9, 2016

In recent years, the matrix decomposition technique has been introduced to analyze the electromagnetic problems, which exploits the well-known fact that for well separated sub-scatterers, the corresponding sub-matrices are low rank and can be compressed [7-8]. In contrast with MLFMA, it is purely algebraic and can be easily interfaced to existing MOM codes. MLSSM is a popular matrix decomposition technique, which has been successfully applied in [9-14] to electromagnetic problems. It has no limit of the box's size and has a memory requirement of $O(N)$ and computational complexity is proportional to $O(N\log N)$. However, for the large-scale problems, the matrix filling time of MLSSM is much longer than that of MLFMA.

In this paper, a hybrid method called combined MLSSM/MLFMA is proposed, which uses the main framework of MLFMA but adopts the MLSSM to deal with the box's size is below 0.2λ. This method takes advantage of both MLFMA and MLSSM and is more efficient than either conventional MLFMA or conventional MLSSM for analyzing the multi-scale problems. For the object with fine structures problems, the matrix condition number is very large due to the mixed discretization. Therefore, the system has poor convergence history and requires urgently a good solver or preconditioner. In this paper, an efficient preconditioning technique based on CBD algorithm [15-17] is applied to improve the property of electric field integral equation (EFIE) formulation.

II. MLSSM ALGORITHM

The impedance matrix filled by MLSSM is carried out based on the same multilevel spatial decomposition of MLFMA. The single level of SSM is presented in [18] and the MLSSM is shown in [9-14]. The structure of the MLSSM representation is given in a multilevel recursion manner [9-14]:

$$Z_l = \hat{Z}_l + U_l Z_{l-1} V_l^H , \tag{1}$$

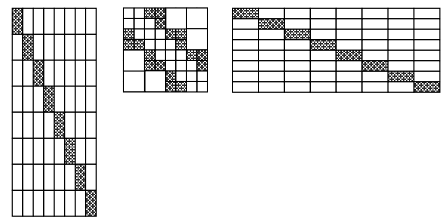
where Z_l is the reduced order impedance matrix and consists of only far interactions at level l+1, which will be compressed in the coarser levels recursively up to level-3. There is no level L+1 near interactions at the finest level L (where L denotes the number of the levels). Thus, Z_L is the impedance matrix Z. In (1), $\hat{Z}_l$ is the sparse matrix containing all near-neighbor interactions at level l of the oct-tree which were not represented at finer level of the oct-tree. U_l and V_l are the new basis and testing function matrices, respectively, which are block diagonal unitary matrices that compress interaction between sources in non-touching groups at level l. The following is the procedure in details. Suppose that the object is decomposed in 4-level oct-tree, the impedance matrix can be expressed as:

$$Z = \hat{Z}_4 + Z_4^{far} = \hat{Z}_4 + U_4 Z_3 V_4^H , \tag{2}$$

where

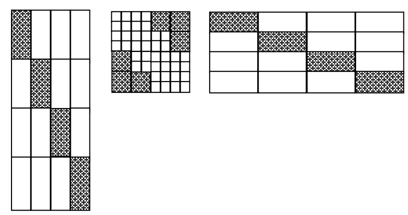
$$Z_3 = \hat{Z}_3 + Z_3^{far} = \hat{Z}_3 + U_3 Z_2 V_3^H . \tag{3}$$

The forms of matrices U_l, Z_{l-1} and V_l^H are shown in Figs. 1 and 2 at levels 4 and 3, respectively.



(a) (b) (c)

Fig. 1. Level 4 SSM matrices: (a): U_4, (b): $\hat{Z}_3$, and (c): V_4^H.



(a) (b) (c)

Fig. 2. Level 3 SSM matrices: (a): U_3, (b): $\hat{Z}_2$, and (c): V_3^H.

The major requirements of the MLSSM memory is to store the matrices $\hat{Z}_l$, U_l, and V_l^H at all levels. The matrix-vector multiplication of MLSSM is very similar to MLFMA in manner, which has $O(N\log N)$ complexity for a given accuracy.

III. COMBINED MLSSM/MLFMA ALGORITHM

A. MLSSM/MLFMA algorithm

MLFMA [5-6] has been widely used to solve the electromagnetic scattering of complex object with the surface integral equation approach. When it is applied to analyze the scattering from the multi-scale objects where dense discretization is necessary to capture geometric features accurately, the memory usage of MLFMA is very large. It is because of that the box's size of tree structure cannot be set to less than 0.2λ, the number of unknowns in the near-field boxes is very large. The near-field matrices of MLFMA is filled by direct method. Therefore, the near-field of MLFMA needs large memory requirement for large-scale problems.

In this section, a hybrid method called combined MLSSM/MLFMA algorithm is proposed, which uses the main framework of MLFMA. The framework of the hybrid method is shown in the Fig. 3, the box's sizes below the dotted line are less than 0.2λ and are filled by MLSSM algorithm, the box's sizes up the dotted line are larger than 0.2λ and are filled by MLFMA.

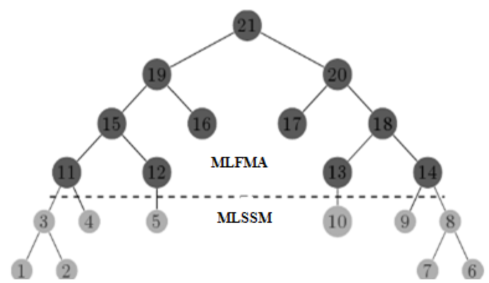


Fig. 3. The framework of combined MLSSM/MLFMA algorithm.

Using the combined MLSSM/MLFMA for filling the impedance matrix Z, a fast matrix-vector production algorithm (MVP) can be obtained as follows:

Subroutine MVP,
(1) The direct MVP algorithm is used to the near interaction impedance matrix;
Begin $l = L:3:-1$,
(2) From $l = L : L_{\mathrm{MLFMA}}+1$,
MLSSM is applied to speed up MVP;
End.
(3) From $l = L_{\mathrm{MLFMA}}$: 3,
MLFMA is used to speed up MVP;
End.
End.

This new method takes advantages of the virtues of both MLFMA and MLSSM, which uses MLFMA to reduce the matrix assembly time of MLSSM and utilizes MLSSM to alleviate the near-field pressure of MLFMA. The efficiency of the method is demonstrated by the numerical results.

B. CBD preconditioner

In order to accelerate the convergence rate of the Krylov iteration, the linear system is transformed into an equivalent one:

$$[M][Z][I]=[M][V], \tag{4}$$

where $[M]$ referred as the preconditioner for the impedance matrix $[Z]$. The product matrix $[M][Z]$ has much better spectral property than original matrix $[Z]$, which leads to a greatly reduced number of iterations. Since the sparse near-field matrix is the best available approximation to the coefficient matrix $[Z]$, it makes sense to use near-field matrix to construct a preconditioner. In this paper, an efficient CBD preconditioner [19] is applied to form the matrix $[M]$.

IV. NUMERICAL RESULTS

In this section, a number of numerical examples are presented to demonstrate the efficiency of the proposed method for analyzing the object with fine structures. All the computations are carried out on a Core-i5 3350P with 3.1 GHz CPU and 4GB RAM in single precision and the MLSSM truncating tolerance is 10^{-3} relative to the largest singular value. In the implementation of the combined MLSSM/MLFMA algorithm, the restarted version of GMRES algorithm [20] is used as the iterative method. The iteration process is terminated when the normalized backward error is reduced by 10^{-3} for all examples.

A. A metal helicopter model

A metal helicopter model with many fine structures is considered in the first example in Fig. 4, which needs large number of unknowns to accurately capture fine structures of the helicopter model. The dimension of the structure is $20.48\,m\times12.37\,m\times6.2\,m$. The incident and scattered angles are $(\theta_i = 0^\circ, \phi_i = 0^\circ)$ and $(0^\circ \le \phi_s \le 180^\circ, \theta_s = 0^\circ)$, respectively. The maximum dimension of the structure is 10.24λ at 150 MHz and the number of unknowns is 74913. The number of the octrees for the combined MLSSM/MLFMA algorithm is L = 5, and L = 4 for the MLFMA. The finest level box's sizes of combined MLSSM/MLFMA algorithm and MLFMA are 0.16λ and 0.32λ, respectively. The bistatic RCS of the proposed method is shown in Fig. 5, and is agreed well with that of FEKO. Table 1 shows the memory storages and the MVP times of the MLSSM/MLFMA and MLFMA. "MVP time" in the table indicates the time of one matrix-vector production. The near-field memory of MLSSM/MLFMA is much less than that of MLFMA, while the total memory consumption of MLSSM/MLFMA is half less than that of MLFMA. The MVP time of MLSSM/MLFMA is also much less than that of MLFMA. The convergence history curves of the MLSSM/MLFMA solved with CBD preconditioner are shown in Fig. 6. It can be found that the proposed method has a much better convergence properties by using the CBD preconditioner.

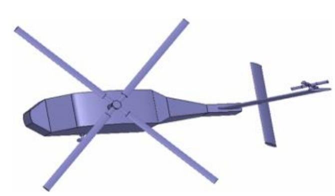


Fig. 4. The configuration of the metal helicopter model.

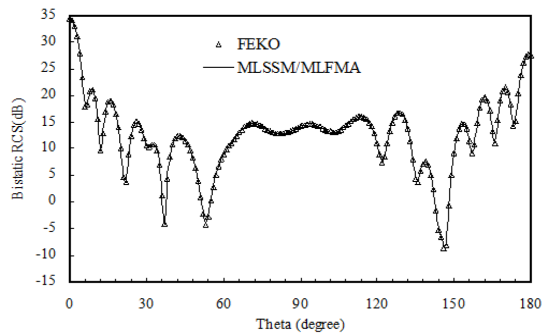


Fig. 5. Bistatic scattering cross section of metal helicopter model.

Table 1: The memory requirements and the MVP times of the proposed method for the metal helicopter model

Algorithms	Near-Field (MB)	Total (MB)	MVP Time (s)
MLFMA	1097	1209	6.07
MLSSM/MLFMA	273	465	2.96

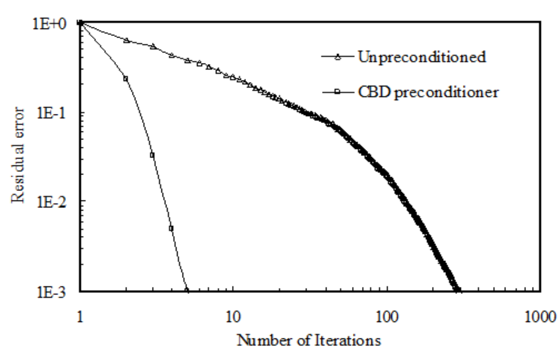


Fig. 6. Convergence histories of the MLSSM/MLFMA solved with CBD preconditioner.

B. A metal missile model

The second multi-scale example is a complex metal missile structure and is analyzed shown in Fig. 7. The dimension of the structure is $6\,m\times 2.56\,m\times 1.32\,m$. The incident and scattered angles are $(\theta_i = 0^\circ, \phi_i = 0^\circ)$ and $(0^\circ \le \phi_s \le 180^\circ, \theta_s = 0^\circ)$, respectively. The maximum dimension of the structure is 6λ at 300 MHz and the number of unknowns is 67420. The number of the octrees for the MLSSM/MLFMA algorithm is L = 4, and L = 3 for the MLFMA. The finest level box's sizes of combined MLSSM/MLFMA algorithm and MLFMA are 0.18λ and 0.37λ, respectively. The bistatic RCS of the metal missile structure is analyzed by the proposed method shown in Fig. 8. It can be observed that the result of the proposed method is agreed well with that of FEKO. The efficiency of the proposed method is analyzed in this example shown in Table 2. It can be found that the memory storage and the MVP time of the MLSSM/MLFMA are both much less than that of the conventional MLFMA. The convergence rate of the CBD preconditioner is shown in Fig. 9. It can be found that the CBD preconditioner is much more efficient than the unpreconditioned GMRES algorithm.

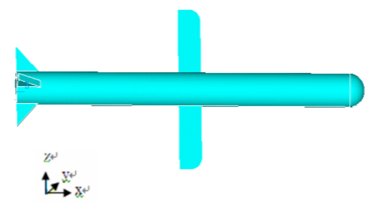


Fig. 7. The configuration of the metal missile model.

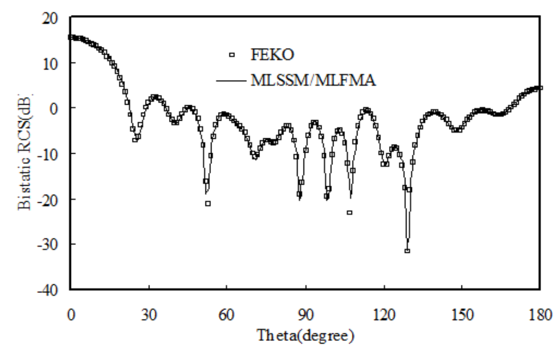


Fig. 8. Bistatic scattering cross section of metal missile model.

Table 2: The memory requirements and the MVP times of the proposed method for the metal missile model

Algorithms	Near-Field (MB)	Total (MB)	MVP Time (s)
MLFMA	871	1045	4.52
MLSSM/MLFMA	209	362	1.67

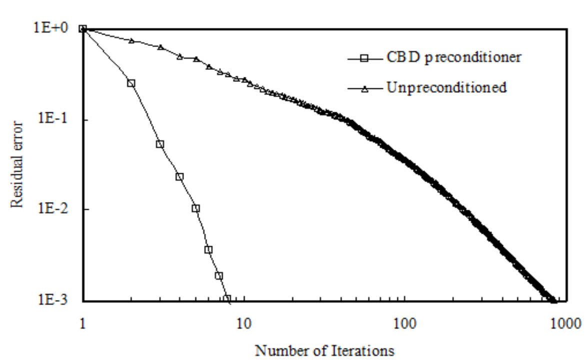


Fig. 9. Convergence histories of the MLSSM/MLFMA solved with CBD preconditioner.

VI. CONCLUSION

In this paper, a novel combined MLSSM/MLFMA algorithm is proposed to solve the electromagnetic scattering of object with fine structures. It takes advantage of the virtues of both MLFMA and MLSSM, which uses MLFMA to reduce the matrix assembly time of MLSSM and utilizes MLSSM to alleviate the near interaction pressure of MLFMA. Since compression of near interactions,the matrix-vector multiplication of MLSSM/ MLFMA is more efficient than that of MLFMA. The CBD preconditioner is used to further speed up the convergence. It can be found that MLSSM/MLFMA combined with CBD preconditioner is very efficient for analyzing the object with fine structures problems.

ACKNOWLEDGMENT

This work is sponsored by Natural Science Foundation of China (61501159), Open Research Program Foundation of State Key Laboratory of Millimeter Waves (K201602), Natural Science Foundation of Anhui Province (1608085MF124).

REFERENCES

[1] W. C. Chew, J. M. Jin, E. Michielssen, and J. Song, *Fast Efficient Algorithms in Computational Electromagnetics*. Boston, MA: Artech House, 2001.

[2] A. F. Peterson, "Accuracy of currents produced by the locally-corrected Nystrom method and the method of moments when used with higher-order representations," *ACES Journal*, vol. 17, no. 1, pp. 74-83, 2002.

[3] S. M. Rao, D. R. Wilton, and A. W. Glisson, "Electromagnetic scattering by surfaces of arbitrary shape," *IEEE Trans. Antennas Propagat.*, vol. AP-30, pp. 409-418, May 1982.

[4] D. Colak, R. J. Burkholder, and E. H. Newman, "On the convergence properties of the multiple sweep method of moments," *ACES Journal*, vol. 22, no. 2, pp. 207-218, 2007.

[5] R. Coifman, V. Rokhlin, and S. Wandzura, "The fast multipole method for the wave equation: A pedestrian prescription," *IEEE Antennas Propag. Mag.*, vol. 35, no. 6, pp. 7-12, June 1993.

[6] M. Vikram and B. Shanker,"An incomplete review of fast multipole methods–from static to wideband–as applied to problems in computational electromagnetics," *ACES Journal*, vol. 24, no. 2, pp. 79-108, 2009.

[7] Z. Jiang, R. Chen, Z. Fan, Y. An, M. Zhu, and K. W. Leung, "Modified adaptive cross approximation algorithm for analysis of electromagnetic problems," *ACES Journal*, vol. 26, no. 2, pp. 160-169, 2011.

[8] F. Canning and J. Shaeffer, "Adaptive cross approximation for MoM matrix fill for PC problem sizes to 157000 unknowns," *ACES International Conference on Wireless Communications & Applied Computational Electromagnetics*, pp. 748-753, 2005.

[9] A. Zhu, R. J. Adams, and F. X. Canning, "Modified simply sparse method for electromagnetic scattering by PEC," *in Proc. IEEE Antennas and Propagation Society Int. Symp*, Washington, DC, vol. 4, pp. 427-430, 2005.

[10] J. Cheng, S. A. Maloney, R. J. Adams, and F. X. Canning, "Efficient fill of a nested representation of the EFIE at low frequencies," *IEEE Antennas and Propagation Society Int. Symp*., pp. 1-4, 2008.

[11] R. J. Adams, A. Zhu, and F. X. Canning, "Sparse factorization of the TMz impedance matrix in an overlapped localizing basis," *Progress in Electromagnetics Research*, *PIER*, 61, pp. 291-322, 2006.

[12] Y. Xu, X. Xu, and R. J. Adams, "A sparse factorization for fast computation of localizing modes," *IEEE Trans. Antennas Propagat.*, vol. 58, no. 9, pp. 3044-3049, 2010.

[13] X. Xu, "Modular Fast Direct Analysis Using Non-Radiating Local-Global Solution Modes," *Ph.D. Dissertation*, Electrical and Computer Engineering, University of Kentucky, 2009.

[14] Z. Jiang, Y. Xu, R. Chen, Z. Fan, and D. Ding, "Efficient matrix filling of multilevel simply sparse method via multilevel fast multipole algorithm," *Radio Science*, 46, RS5011, doi:10.1029/2010RS 004549, 2011.

[15] A. Heldring, J. M. Rius, J. M. Tamayo, and J. Parron, "Compressed block-decomposition algorithm for fast capacitance extraction," *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems*, vol. 27, no. 2, pp. 265-271, Feb. 2008.

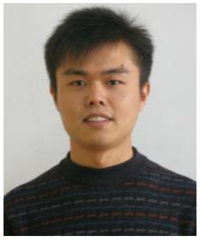
[16] A. Heldring, J. M. Tamayo, J. M. Rius, J. Parron, and E. Ubeda, "Multiscale CBD for fast direct solution of MoM linear system," *IEEE Antennas & Propagation Society International Symposium*, pp. 1-4, 2008.

[17] A. Heldring, J. M. Rius, J. M. Tamayo, J. Parron, and E. Ubeda, "Fast direct solution of method of moments linear system," *IEEE Trans. Antennas Propag*, vol. 55, no. 11, pp. 3220-3228, Nov. 2007.

[18] F. X. Canning and K. Rogovin, "A universal matrix solver for integral equation-based problems," *IEEE Antennas Propag. Mag.*, vol. 45, pp. 19-26, 2003.

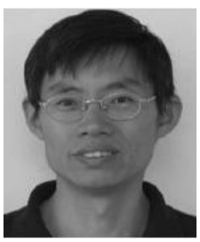
[19] Z. Jiang, R. Chen, Z. Fan, and Y. An, "Modified compressed block decomposition preconditioner for electromagnetic problems," *Microwave and Optical Technology Letters*, vol. 53, no. 8, pp. 1915-1919, Aug. 2011.

[20] Y. Saad, "Iterative methods for sparse linear system," *Studies in Computational Mathematics*, vol. 8, pp. 423-440, 2001.



Zhao Neng Jiang received the B.S. degree in Physics from Huaiyin Normal College, Huaian, China, in 2007, and the Ph.D. degree in Electrical Engineering from Nanjing University of Science and Technology (NJUST), Nanjing, China, in 2013. He worked as a Teacher at the Hefei University of Technology, Hefei, China. His current research interests include computational electromagnetics, antennas and electromagnetic scattering and propagation, electromagnetic modeling of microwave integrated circuits.

Fei Shen received the Ph.D. degree from the University of Electronic Science and Technology of China (UESTC), Chengdu, China, in 2012. He worked as a Teacher at the Hefei University of Technology, Hefei, China. His current research interests include antennas, microwave & millimeter wave devices and techniques



Yuan Xu received the B.S. and M.S. degrees in Mathematics from Suzhou University, Suzhou, China, in 1988 and 1991, respectively, and the Ph.D. degree in Electrical Engineering from Nanjing University of Science and Technology (NJUST), Nanjing, China, in 2001. He worked as a Postdoctoral Scholar at the University of Kentucky, Lexington, in 2006 and 2010. He is currently an Associated Professor with the Electronic Engineering Department, NJUST. His research interests include computational electromagnetics, integral equation methods, and the analysis of large-scale electromagnetics problems.

Xiao Yu Yang received the B.S. degree in Physics from Harbin Normal University, Harbin, China, in 2005, the M.S. degree in Physics from Northeast Normal University, Changchun, China, in 2008, and the Ph.D. degree in Physics from Harbin Institute of Technology, Harbin, China, in 2014. She worked as a Teacher at the Hefei University of Technology, Hefei, China. Her current research interests include properties and device of electromagnetic metamaterials.