# Electromagnetic Scattering from a PEMC Circular Cylinder Coated by Topological Insulator (TI)

**A. Shahzad [1], A. Illahi [2], S. Ahmed [1], A. Hussain [1], and Q. A. Naqvi [1]**

[1] Department of Electronics, Quaid-i-Azam University, Islamabad, Pakistan (45320)
gondalanjum@yahoo.com, snfawan@yahoo.com, akhtar_h@yahoo.com, nqaisar@yahoo.com

[2] Research in Modeling and Simulation (RIMS) Group, Department of Physics
COMSATS Institute of Information Technology, Islamabad, Pakistan
ahsanillahi@comsats.edu.pk

***Abstract* —** Scattering of electromagnetic plane wave from a perfect electromagnetic conducting (PEMC) cylinder coated with a topological insulating (TI) material has been presented. The core (PEMC) and cladding material (TI) produce co- and cross-polarized components of electromagnetic field in response to the incident plane wave for a given polarization (TE or TM). When the value of $\theta$ is made zero, TI material becomes ordinary dielectric and the results of PEMC coated with TI with $\theta = 0$ (dielectric material) have been compared with the previously published literature and are found in good agreement. When the coating is removed, same results as that of isolated PEMC circular cylinder have also been reproduced.

***Index Terms* —** Cladding, insulating, isolated, polarization, topological.

## I. INTRODUCTION

Topological insulator's states in 2D and 3D materials were observed theoretically in 2005 and 2007 while their experimental discovery won the 2010 Nobel Prize. Topological insulator material is currently the hottest topic in condensed matter and quantum physics, and is hard to understand. Topological insulator is a type of material that conducts electricity on its surface due to special surface electronic states. The surface states of TI are topologically protected, i.e., they cannot be destroyed by impurities. TIs are made possible due to the combination of time-reversal symmetry and interaction of spin-orbit coupling, which occurs in heavy elements like mercury and bismuth.

TIs are defined by the constitutive relations:

$$\boldsymbol{D} = \epsilon_r \epsilon_0 \boldsymbol{E} - \epsilon_0 \alpha \frac{\theta}{\pi}(c_0 \boldsymbol{B}), \quad (1)$$

$$c_0 \boldsymbol{H} = \frac{c_0}{\mu_0 \mu_r} \boldsymbol{B} + \alpha \frac{\theta}{\pi} \frac{1}{\mu_0 \mu_r} \boldsymbol{B}, \quad (2)$$

where $\epsilon_r$, $\mu_r$ are relative permeability and permittivity and $\epsilon_0$, $\mu_0$ are the permeability and permittivity of free space respectively. $c_0$ is the speed of light in vacuum, $\alpha = e^2/4\pi\epsilon_0\hbar c_0$ is the fine structure constant, $\hbar$ is Plank constant, $e$ is the electric charge and $\theta$ is axion parameter which is uniquely determined by band structure. Only two values of $\theta$ are possible: $\theta = 0$ (i.e., conventional dielectric) and $\theta = \pi$ (i.e., topological insulator), which gives time reversal symmetry. When time reversal symmetry is broken, the $\theta$ is quantized in odd integer values of $\pi$ [1].

Due to these attractive properties many scientists started studying TI. Surface plasmons localized on the topologically nontrivial interface have been studied by Karch [1]. Qi and Zhang studied the theory of topological superconductors in close analogy to the theory of topological insulators [2]. Scattering results from topological insulator cylinders are very few [3-6]. Scattering by TI circular cylinder is discussed in [3]. Scattering from buried TI circular cylinder in a slightly rough surface has been investigated in [4] and buried in a semi infinite medium has been discussed in [5]. In [6], it has been shown that what will happen when a TI cicular cylinder is placed in chiral medium? Electromagnetic scattering from coated cylinder is a more challenging task and is therefore addressed in the present paper.

PEMC is a new class of materials introduced by Lindell and Sihvola [7]. It is the generalization of perfect electric conductor (PEC) and perfect magnetic conductor (PMC) material. It is defined by the boundary conditions:

$$\boldsymbol{n} \times (\boldsymbol{H} + M\boldsymbol{E}) = 0, \quad (3)$$

$$\boldsymbol{n}.(\boldsymbol{D} - M\boldsymbol{B}) = 0, \quad (4)$$

where $M$ denotes the admittance of the PEMC boundary. $M = 0$ for PMC and $M \rightarrow \pm\infty$ for PEC. The circular cylinders are the most basic canonical shape for the study of electromagnetic waves scattering. Cylindrical geometry has a long history in EM problems [8-9, 12-18, 22-26]. Scientists had studied the problems of circular cylinder using composition of different materials, e.g., dielectric, negative refractive index materials (NRM),

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PEMC, chiral, and nihility [10-14, 16, 17, 19-24].

In this paper, an infinite PEMC circular cylinder coated with TI material is considered. The purpose of the study is to explore important scattering characteristics and to provide physical insight of this geometry. Using the large argument approximation of Hankel function, the bi-static echo widths in the far zone are calculated. For the verification of analytical formulation and numerical code, numerical results are compared with the published work. We have used $e^{-i\omega t}$ time dependence which is suppressed throughout the analysis.

In the next few sections, analytical formulation, numerical results and discussion, and conclusions are described.

## II. ANALYTICAL FORMULATION

Geometry of problem is shown in Fig. 1. Inner cylinder is PEMC while the outer cylinder shows the coating layer of TI material. PEMC circular cylinder coated with TI material is of uniform thickness. The cylinder is supposed to be of infinite length along z-axis. Radius of PEMC cylinder is '$a$' while radius of PEMC cylinder coated with TI is '$b$'. The region outside the coating $\rho > b$ is free space and is mentioned as region 0 with wave number $k_0 = \omega\sqrt{\mu_0\epsilon_0}$. The region between $a < \rho < b$ is termed as region 1 with wave number as $k_1 = \omega\sqrt{\mu_1\epsilon_1}$.

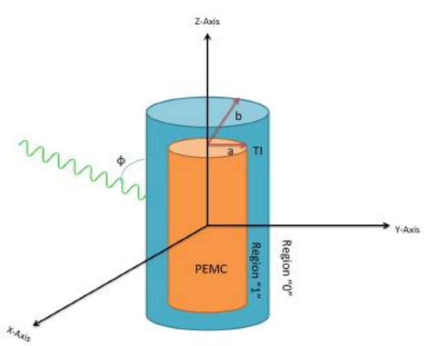


Fig. 1. PEMC Cylinder coated with topological insulator.

The polarization of the incident electric field is taken parallel to the axis of the cylinder. The incident electric field is given by:

$$E^i_{0z} = e^{ik_0\rho cos\varphi}. \quad (5)$$

The incident electric field can be written in terms of cylindrical wave function as:

$$E^i_{0z} = \sum_{n=-\infty}^{\infty} i^n J_n(k_0\rho)e^{jn(\varphi)}. \quad (6)$$

Using Maxwell's equations, the corresponding magnetic field in $\varphi$ direction can be written as:

$$H^i_{0\varphi} = -\frac{E_0}{i\eta_0}\sum_{n=-\infty}^{\infty} J_n'(k_0\rho)e^{jn(\varphi)}. \quad (7)$$

The scattered co-polarized electric field in region 0 can be written as:

$$E^s_{0z} = \sum_{n=-\infty}^{\infty} i^n a^n H_n^{(1)}(k_0\rho)e^{jn(\varphi)}. \quad (8)$$

And the corresponding $\varphi$ component of scattered co-polarized magnetic field can be written as:

$$H^s_{0\varphi} = -\frac{E_0}{i\eta_0}\sum_{n=-\infty}^{\infty} i^n a^n H_n^{(1)'}(k_0\rho)e^{jn(\varphi)}. \quad (9)$$

As the core material of the cylinder is of PEMC, so in addition to co-polarized component cross-polarized component will also appear. The scattered cross-polarized magnetic field in region 0 can be written as:

$$H^s_{0z} = -\frac{iE_0}{\eta_0}\sum_{n=-\infty}^{\infty} i^n b^n H_n^{(1)}(k_0\rho)e^{jn(\varphi)}. \quad (10)$$

And the corresponding $\varphi$ component of scattered cross-polarized electric field can be expressed as:

$$E^s_{0\varphi} = -E_0\sum_{n=-\infty}^{\infty} i^n a^n H_n^{(1)'}(k_0\rho)e^{jn(\varphi)}. \quad (11)$$

Region 1 has two interfaces at $\rho = a$ and $\rho = b$; therefore, co- and cross-polarized electric and magnetic fields in region 1 can be expressed in terms of oppositely traveling cylindrical waves as:

$$E_{1z} = E_0\sum_{n=-\infty}^{\infty} i^n[c^n H_n^{(2)}(k_1\rho) + d^n H_n^{(1)}(k_1\rho)]e^{jn(\varphi)}, \quad (12)$$

$$H_{1\varphi} = -\frac{E_0}{i\eta_1}\sum_{n=-\infty}^{\infty} i^n[c^n H_n^{(2)'}(k_1\rho) + d^n H_n^{(1)'}(k_1\rho)]e^{jn(\varphi)}, \quad (13)$$

$$E_{1z} = -\frac{iE_0}{\eta_1}\sum_{n=-\infty}^{\infty} i^n[e^n H_n^{(2)}(k_1\rho) + f^n H_n^{(1)}(k_1\rho)]e^{jn(\varphi)}, \quad (14)$$

$$H_{1\varphi} = -E_0\sum_{n=-\infty}^{\infty} i^n[e^n H_n^{(2)'}(k_1\rho) + f^n H_n^{(1)'}(k_1\rho)]e^{jn(\varphi)}. \quad (15)$$

In above expressions $J_n(.)$ is the Bessel functions of first kind, while $H_n^{(1)}(.)$ and $H_n^{(2)}(.)$ are the Hankel functions of first and second kinds respectively. Also $a_n$, $b_n$, $c_n$, $d_n$, $e_n$ and $f_n$ are the unknown scattering coefficients. These unknown coefficients can be found by using boundary conditions at the interfaces $\rho = a$ and $\rho = b$.

At $\rho = a$, boundary conditions are:

$$H_{1z} + ME_{1z} = 0 \quad \rho = a, \quad 0 \le \varphi \le 2\pi, \quad (16)$$

$$H_{1\varphi} + ME_{1\varphi} = 0 \quad \rho = a, \quad 0 \le \varphi \le 2\pi. \quad (17)$$

At $\rho = b$, boundary conditions are:

$$H^i_{0\varphi} + H^s_{0\varphi} = H_{1\varphi} - \alpha\frac{\theta}{c_0\pi}E_{1\varphi} \quad \rho = b, \ 0 \le \varphi \le 2\pi, \quad (18)$$

$$H^s_{0z} = H^s_{1z} - \alpha\frac{\theta}{c_0\pi}E_{1z} \quad \rho = b, \ 0 \le \varphi \le 2\pi, \quad (19)$$

$$H^s_{0z} = H_{1z} \quad \rho = b, \ 0 \le \varphi \le 2\pi, \quad (20)$$

$$E^s_{0\varphi} = H_{1\varphi} \quad \rho = b, \ 0 \le \varphi \le 2\pi, \quad (21)$$

where

$$E_{0z} = E^i_{0z} + E^s_{0z}, \quad (22)$$

$$H_{0\varphi} = H^i_{0\varphi} + H^s_{0\varphi}. \quad (23)$$

By the application of above boundary conditions at interface $\rho = a$ and $\rho = b$, a linear matrix is obtained in terms of the unknown scattering coefficients. Solution of this matrix gives unknown scattering coefficients. The values of $a_n$ and $b_n$ give us co- and cross-polarized components of scattered field due to PEMC cylinder coated with TI.

## III. BACK SCATTERING CROSS-SECTIONS (σ)

The ratio of the total power scattered by the scatter to the incident power per unit area on the scatterer is called back scattering cross-section and is given as:

$$\sigma = 2\pi\rho\frac{W^s}{W^i} = 2\pi\rho\frac{|\ E^s\ |^2}{|\ E^i\ |^2}.$$

For parallel polarization, the normalized bi-static echo width (RCS) of the co-polarized and cross-polarized field components is given by:

$$\sigma_{co}/\lambda_0 = \frac{2}{\pi}\left|\ \sum_{n=-\infty}^{n=\infty} a_n e^{in(\varphi)}\ \right|^2,$$

$$\sigma_{cross}/\lambda_0 = \frac{4}{\pi}\left|\ \sum_{n=-\infty}^{n=\infty} b_n e^{in(\varphi)}\ \right|^2,$$

where $a_n$ is the scattering coefficient of co-polarized field and $b_n$ is the scattering coefficient of cross-polarized field. For perpendicular polarization, the duality principle for the above formulation may be used.

## IV. NUMERICAL RESULTS AND DISCUSSION

In this section numerical results are described. The numerical results are based on the above analytical formulations for PEMC coated with TI material. For Figs. 2-8, $k_0a = 1.05$, $k_0b = 2.1$, $e = 1.6*10^{-19}$C, speed of light is $c_0 = 3*10^8 m/s$ and permeability $\mu = 1$.

In Fig. 2 echo width of PEMC circular cylinder coated with TI material has been plotted with observation angle from 0 to $2\pi$ radians. In this figure, $M\eta_1 = 0$, $\epsilon = 9.8$ and $\theta = 0$, which is a case for PMC circular cylinder coated with dielectric material. This result when compared with [24], excellent agreement is found.

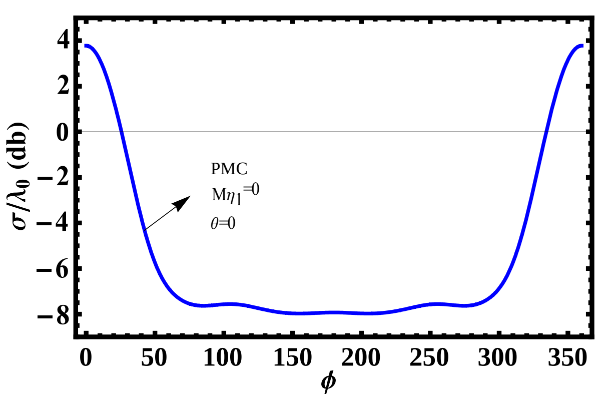


Fig. 2. Echo width from PMC coated with dielectric material with $\epsilon = 9.8$.

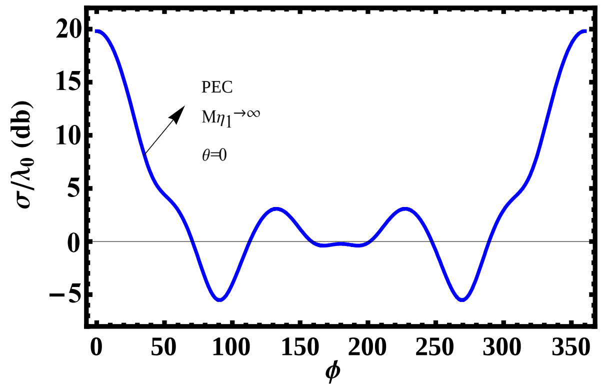


Fig. 3. Echo width from PEC coated with dielectric material $\epsilon = 9.8$.

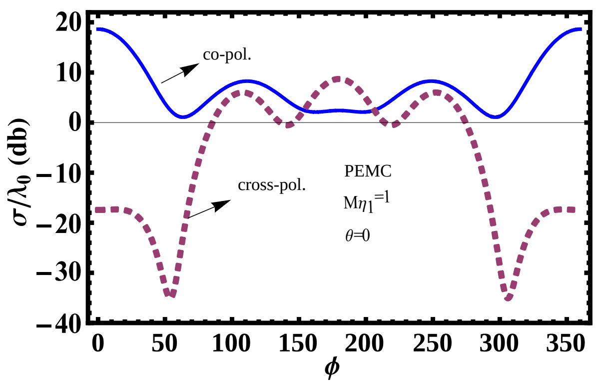


Fig. 4. Echo width of co- and cross-polarized from PEMC coated with dielectric material $\epsilon = 9.8$.

In Figs. 3 and 4, the numerical result is repeated for $M\eta_1 \rightarrow \infty$, $\epsilon = 9.8$ and $\theta = 0$ and $M\eta_1 = 1$, $\epsilon = 9.8$ and $\theta = 0$. Comparison is made for PEC circular cylinder coated with dielectric material. Again, the obtained nice comparison and validated our formulation. With this confidence, the numerical code has been run for different values of $M\eta_1$ and $\theta$, while $\epsilon = 100$ which is the characteristic value of TI material (i.e., $\epsilon$= 50 to 100) and Figs. 5-8 are obtained.

In Figs. 5-8, scattering behavior of PEMC circular cylinder coated with TI material has been highlighted. With these results one can understand the composition of the highly focused material, i.e., TI material with PEMC (which is the most fundamental material for electromagnetic analysis).

Figure 5 represents echo width of co- and cross-polarized components for PMC circular cylinder coated with TI material. In this figure, $M\eta_1 = 0$, $\epsilon = 100$ and $\theta = 41\pi$, which is the case of PMC circular cylinder coated with TI material. In this plot, cross-polarized component has also been appeared along with co-polarized component. This cross-polarized component is due to TI material. Figure 6 shows the case when PEMC circular cylinder coated with TI material has been taken. For this figure, parameters are taken as $M\eta_1 = 1$, $\epsilon = 100$ and $\theta = 41\pi$. On comparing Fig. 5 and Fig. 6, it has been observed co-polarized component is same, while cross-polarized component of Fig. 5 is greater than the cross-polarized component of Fig. 6. This greater contribution in the cross-polarized component is because of PEMC core when coated with TI material.

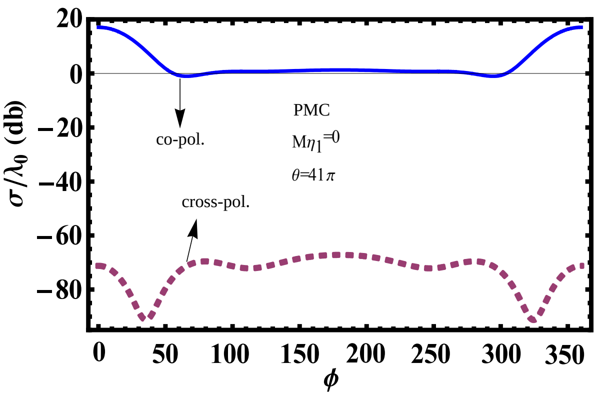


Fig. 5. Echo width of co- and cross-polarized components for PMC with $\epsilon = 100$.

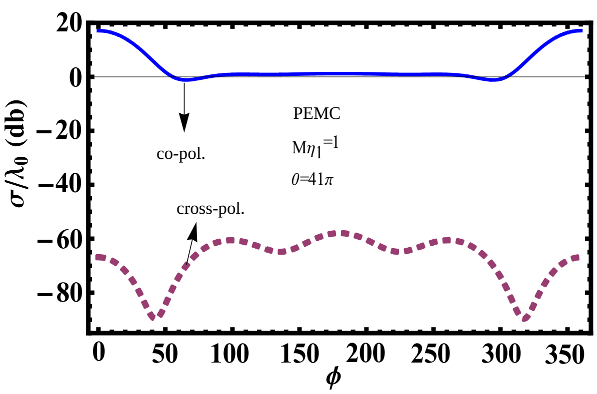


Fig. 6. Echo width of co- and cross-polarized components for PEMC with $\epsilon = 100$.

Figure 7 shows the comparison between co-polarized components for different of $\theta$, i.e., for $\theta = 0$ and $\theta = 41\pi$; when PEMC circular cylinder coated with TI material is considered. It is observed that the behavior of co-polarized component for $\theta = 0$ is same as co-polarized component for $\theta = 41\pi$ near 0-80 and 310-360 but different for 80-310 regardless of the amplitude.

Figure 8 shows the comparison between cross-polarized components for $\theta = 0$ and $\theta = 41\pi$ respectively, when PEMC circular cylinder coated with TI material is considered. It is observed that the behavior of cross-polarized component for $\theta = 0$ is different from co-polarized component for $\theta = 41\pi$ near0-80 and 310-360 but same for 80-310 regardless of the amplitude.

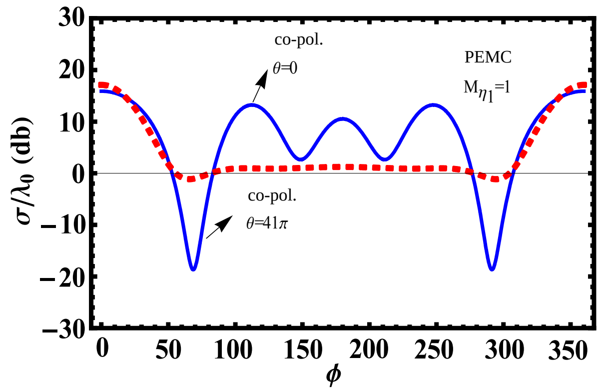


Fig. 7. Echo width of co- and co-polarized components for different values of $\theta$ when core is PEMC with $\epsilon = 100$.

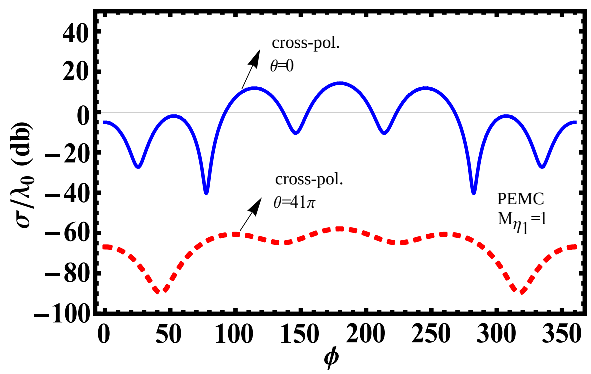


Fig. 8. Echo width of cross- and cross-polarized components for different values of $\theta$ when core is PEMC with $\epsilon = 100$.

## V. NUMERICAL VERIFICATION

In Fig. 2 the code has been verified with the limiting parameters $M\eta_1 = 0$, $\epsilon = 9.8$ and $\theta = 0$ and compared with the literature [24]. The code is further verified in Figs. 3 and 4, the parameters used are: $M\eta_1 \rightarrow \infty$, $\epsilon = 9.8$ and $\theta = 0$ and $M\eta_1 = 1$, $\epsilon = 9.8$ and $\theta = 0$. Comparison is made with [24]. In both the cases excellent agreement is found. The proposed study can also be verified with the help of experiments as well as commercially available simulation software which will be our task.

## VI. CONCLUSIONS

In the present paper analytical formulation of a perfect electromagnetic conducting (PEMC) circular cylinder of infinite length coated with a topological insulating (TI) material has been presented. The core (PEMC) and cladding (TI) material produces co- and cross-polarized components of electromagnetic magnetic field in response to the incident plane wave for a given polarization (TE or TM). By coating TI material on PEMC circular cylinder, again co- and cross-polarized components of the scattered field has been obtained. When the value of $\theta$ is made zero, TI material becomes ordinary dielectric and the results of PEMC coated with dielectric material has been reproduced. When the coating is removed, the same results, as that of isolated PEMC circular cylinder, have been reproduced. Making $M\eta_1 \to \infty$ or 0, PEC and PMC coated with TI material results. By using both aforementioned conditions, results of PEC and PMC coated with ordinary dielectric material have been obtained. Thus, in short, it can be said that the present problem is the most fundamental and generalized which contains all the special cases, i.e., PEC coated with dielectric, PMC coated with dielectric, PEMC coated with dielectric, PEC coated with TI, PMC coated with TI, PEMC coated with TI material can be conveniently achieved. It can also be concluded that the behavior of co- and cross- polarized components vary both in amplitude and shape with variation in geometrical parameters, i.e., $M\eta_1$, $\theta$ and $\epsilon$.

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**Anjum Shahzad** was born in Malakwal Mandi Bauh-ud-Din (Pakistan) in December, 1985. He received his M.Sc. and M.Phil. degrees in Electronics from Quaid-i-Azam University in 2007 and 2010, respectively. He received Gold Medal from the Quaid-i-Azam University. He is the author of three international publications. He is currently working towards his Ph.D. His current research interest includes techniques used in buried landmines detection and stealing, electromagnetic cloaks and scattering from canonical objects.

**Ahsan Illahi** received his Ph.D. degree in Electromagnetics from Quaid-i-Azam University in 2010. Currently he is working as Assistant Professor of Physics in COMSATS Institute of Information Technology (CIIT), Islamabad Pakistan. He is active member of RIMS (Research in Modeling and Simulation) Group at CIIT, Islamabad. His research interests include Modeling and Simulations of physical systems in various fields of science along with the Electromagnetics.

**Shakeel Ahmed** was born in a village Baldher (KPK), Pakistan. He received his initial education from his native town. He graduated from the University of Peshawer and received his Ph.D. from Quaid-i-Azam University, Islamabad (Pakistan). He is author of over twenty (20) international journal publications. His current research interests include electromagnetic scattering from different types of geometries and materials.

**Akhtar Hussain** was born in a small village of district Narowal (Pakistan). He received the degrees of M.Sc. in Physics and Nuclear Engineering from the University of the Punjab and Pakistan Institute of Engineering and Applied Sciences Islamabad, respectively. He received the degree of Ph.D. from Qauid-e-Azam University Islamabad in 2010. He is the author of over a dozen research papers. His research interests include Fractional electromagnetics, Computational electromagnetics Wave guides, Transmission lines and Scattering.

**Qaisar Abbas Naqvi** was born in village of District Narowal (Pakistan). He received his M.Sc., M. Phil. and Ph.D. from Quaid-i-Azam University Islamabad (Pakistan). He has published over one hundred and fifty (150) papers in international journals. He also author of a book and chapters in different books. He has successfully supervised eight Ph.D.'s and more than 80 M. Phil. students after his Ph.D. in 1998. His research interests are: Kobyashi potential, Maslov's Method, GTD, Fractional electromagnetics, buried cylinders, waveguides, slits, strips, antennas and numerical electromagnetics.