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TABLE OF CONTENTS

A Novel Multibranch-Linear-Linear Basis Function Based Adaptive Accuracy Enhanced Method for MFIE and CFIE From Multiscale PEC Targets Yu Wang, Jie Cheng, Yu Zhang, Lei Yu, Liu-Yang Shen, and Sheng Liu	389
An Efficient MMA-PMA-CBFM for Solving Partial Modification Electromagnetic Scattering Problems Ziang Shen, Xiaoxing Fang, Xinlei Chen, and Zhuo Li	398
Fast and Efficient Analysis of Electromagnetic Radiation from RadomeThermal Protective Enclosure-Antennas Structure Jintong Liu, Pengyuan Wang, Weidong Hu, and Mang He	406
Localized Plane Wave Approximation for Bodies of Revolution in Vegetation Scattering Edward C. Michaelchuck Jr., Roger H. Lang, William O. Coburn, and Samuel G. Lambrakos	413
Comparative Analysis of Microstrip Patch Antenna Performance on Various Dielectric Substrates Fatih Göksel and Saeid Karamzadeh	431
High Efficiency, Low-Power RF Energy Harvesting Tri-Band Rectenna for Mid-Band 5G and Wi-Fi 6E Applications Abdelkrim Belhedri, Boualem Mekimah, Boualem Hammache, Tayeb A. Denidni, Abderraouf Messai, and Mohammed Boulesbaa	440
A Compact High Scanning Rate Leaky-Wave Antenna Based on Spoof Surface Plasmon Polaritons Transmission Line Xiaoyan Zhang, Siyuan Yu, Aiyun Zhan, and Yan Mei	449
A Miniaturized Wilkinson Power Divider with Integrated Phase Shifter Thanapat Chiawchanwattana, Ravee Phomlounsri, Somkuan Srisawat, Niwat Angkawisittpan, and Sivarit Sultornsanee	457
Design of Wideband Filtering Power Divider with High Port Isolation Performance for RF Energy Harvesting Di Wang, Zhenzhong Chen, Hao Li, Na Sun, and Jingqi Yang	473
A Fast LO Leakage Calibration Method for MIMO Transmitter Jianhong Xie, Zhaopeng Fu, Zhengbo Jiang, Jingxin Liu, and Zhangcheng Hao	480

A Novel Multibranch-Linear-Linear Basis Function Based Adaptive Accuracy Enhanced Method for MFIE and CFIE From Multiscale PEC Targets

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Abstract – In this paper, we propose a novel Multibranch-Linear-Linear basis function based adaptive accuracy enhanced (MB-LLB-AAE) method to solve the scattering problem from multiscale perfect electric conductor (PEC) targets. The calculation accuracy of traditional magnetic-field integral equation (MFIE) and combined field integral equation (CFIE) is not as good as that of electric-field integral equation (EFIE) under the same mesh size. The proposal of MBL basis function has solved the accuracy problem of MFIE and CFIE when calculating multiscale PEC targets to a certain extent. However, a MB-LL basis function has two unknowns on a common edge, thus generating a prohibitive computational cost when calculating multiscale targets. Based on this problem, we propose the novel MB-LLB-AAE method, which not only effectively reduces the consumption when calculating multiscale targets, but also maintains an accuracy similar to that of using the MB-LL basis function only. Several numerical examples prove the advantages of the proposed method.

Index Terms – Adaptive accuracy enhanced (AAE) method, combined-field integral equation (CFIE), magnetic-field integral equation (MFIE), multibranch RWG basis function (MB-LL), multiscale objects.

I. INTRODUCTION

With the rapid development of high-power microwaves, the simulation of scattering properties

of targets becomes more and more important [1–3]. The surface integral equation (SIE) method has gradually become a more common method for calculating scattering problems because of its advantages in computing consumption [4–6]. The most widely used equations in SIE are the electric-field integral equation (EFIE) [7], the magnetic-field integral equation (MFIE) [8] and the combined-field integral equation (CFIE) [9]. However, MFIE and CFIE are not as accurate as EFIE when using the same size mesh [10–12]. The main reasons for this problem are two aspects [13]. One is that, although the integral kernel can be calculated accurately in MFIE, the integral kernel has the singularity of high order points [14]. Second, one of the MFIE is a regular integral. When the source triangles and the test triangles are close enough and the integral points are selected on the edges and vertices, a solid angle error will occur in the process of singularity extraction [15]. In order to solve this problem, many solutions are proposed [16–19]. Among these methods, two are relatively representative. One of them uses the Linear-Linear (LL) basis function to improve calculation accuracy [20]. The LL basis function has a higher order than the RWG basis function, thus achieving higher calculation accuracy. However, the unknown quantity on a common edge of the LL basis function is 2. Therefore, if only the LL basis function is used for calculation, it will bring an excessive number of unknowns. Another method is the accuracy enhanced-CFIE (AE-CFIE) method which improves the computational accuracy of CFIE by mixing CFIE with EFIE. Although this

method improves the accuracy of CFIE to a certain extent, it sacrifices convergence and does not consider a multiscale scenario at the same time [21].

The calculation of multiscale targets is also a key issue that needs to be considered. It is difficult to guarantee the accuracy of the calculation if the large mesh is used to mesh the whole target, and a large amount of unknown quantity will be produced if the fine mesh is used. For solving the multiscale problems, a series of methods were proposed [22–25].

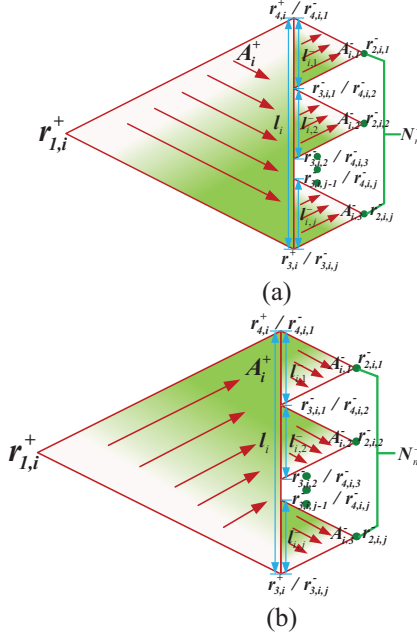


Fig. 1. Schematic diagram of the two kinds of MB-LL basis function.

Recently, a multibranch-RWG (MB-RWG) basis function was proposed to solve the problem of multiscale target computation [26]. The MB-RWG basis function replaces the negative triangle of the RWG basis function to several branches, and guarantees the continuity of the current. By defining the MB-RWG basis function at the junction of the regions divided by different size mesh, the calculation of multiscale targets can be realized effectively, and it is relatively easy to achieve [27]. In [28], we proposed the construction method of multibranch LL basis function (MB-LL), which solved the accuracy problems of MFIE and CFIE when calculating multiscale targets. However, in [28], we only considered using the MB-LL basis function for calculation, which brings a large number of unknowns.

To address both multiscale problems and the issue that the full-domain application of MB-LL basis functions would double the number of unknowns, we extend the AE-CFIE method appropriately. We employ LL and MB-LL basis functions in regions that have a significant

impact on computational accuracy, while using RWG and MB-RWG basis functions in other regions. In this manner, the number of unknowns associated with a single common edge is 2 only in the LL-based regions, whereas it is 1 in the RWG-based regions, thus effectively reducing the total number of unknowns. Based on this idea, in this paper we propose a novel multibranch-Linear-Linear basis function based adaptive accuracy enhanced (MB-LLB-AAE) method. The core innovations of this paper are summarized into three aspects.

- We applied MB-RWG to the AE-CFIE method. Since AE-CFIE is essentially a hybrid of EFIE and CFIE (replacing eligible components with EFIE), the introduction of EFIE degrades the convergence to a certain extent. To tackle this problem, we introduced MB-LL basis functions to further optimize the equation, thus developing the MB-LLB-AAE method. To the best of our knowledge, this is the first attempt to implement the MB scheme in the AE-CFIE framework.
- The proposed method is the first to introduce LL basis functions into the AE-CFIE framework and, combined with the MB scheme, it forms a novel approach more suitable for the calculation of multiscale targets.
- We propose a brand-new replacement strategy for MB-LL basis functions, which is different from the conventional AE-CFIE method. Specifically, for eligible common edges, their corresponding triangle pairs are determined, and all basis functions contained in these triangle pairs are replaced with MB-LL basis functions.

We use several numerical examples to prove the superiority of the proposed method in calculating the scattering problem from multiscale targets.

II. DEFINITION OF THE MB-LL AND THE CONSTRUCTION METHOD OF THE IMPEDANCE MATRIX

Schematic diagrams of the two kinds of $1 - N_n^-$ MB-LL basis functions are shown in Fig. 1. The two kinds of LL basis functions can be expressed as [28]:

$$f_i^{MB-L,1}(\mathbf{r}) = \begin{cases} \frac{l_i}{4A_i^+} (\mathbf{r} - \mathbf{r}_{1,i}^+) \cdot [(\mathbf{r}_{4,i}^+ - \mathbf{r}_{1,i}^+) \times \hat{\mathbf{n}}_i^+] \\ \quad \times (\mathbf{r}_{3,i}^+ - \mathbf{r}_{1,i}^+), & \mathbf{r} \in S_i^+ \\ \frac{l_{i,j}^-}{4A_{i,j}^-} (\mathbf{r} - \mathbf{r}_{2,i,j}^-) \cdot [(\mathbf{r}_{4,i,j}^- - \mathbf{r}_{2,i,j}^-) \times \hat{\mathbf{n}}_{i,j}^-] \\ \quad \times (\mathbf{r}_{3,i,j}^- - \mathbf{r}_{2,i,j}^-), & \mathbf{r} \in S_{i,j}^-, \quad j \in 1, 2, \dots, N_n^- \end{cases} \quad (1a)$$

$$\mathbf{f}_i^{MB-L,2}(\mathbf{r}) = \begin{cases} \frac{l_i}{4A_i^+}(\mathbf{r} - \mathbf{r}_{1,i}^+) \cdot [(\mathbf{r}_{3,i}^+ - \mathbf{r}_{1,i}^+) \times \hat{\mathbf{n}}_i^+] \\ \quad \times (\mathbf{r}_{4,i}^+ - \mathbf{r}_{1,i}^+), & \mathbf{r} \in S_i^+ \\ \frac{l_{i,j}^-}{4A_{i,j}^-}(\mathbf{r} - \mathbf{r}_{2,i,j}^-) \cdot [(\mathbf{r}_{3,i,j}^- - \mathbf{r}_{2,i,j}^-) \times \hat{\mathbf{n}}_{i,j}^-] \\ \quad \times (\mathbf{r}_{4,i,j}^- - \mathbf{r}_{2,i,j}^-), & \mathbf{r} \in S_{i,j}^-, j = 1, 2, \dots, N_n^- \end{cases} \quad (1b)$$

where i indicates that the expression belongs to the i -th MB-LL basis function, j indicates the expression belong to the j -th negative branch of the MB-LL basis function, superscript $+/-$ is intended to demonstrate the expression belongs to positive triangle/negative branch, S is the area of the triangle, A is the domain of the integration, l is the length of the common side, $\mathbf{r}_{1,i}^+$, $\mathbf{r}_{2,i}^+$ and $\mathbf{r}_{3,i}^+$ represent the three vertices of the positive triangle, while $\mathbf{r}_{2,i,j}^-$, $\mathbf{r}_{3,i,j}^-$ and $\mathbf{r}_{4,i,j}^-$ represent the three vertices of the negative branch. The specific positions of the vertices are shown in Fig. 1. $\hat{\mathbf{n}}$ represents the normal vector of a triangle or branch. It should be noted that the current distribution over an MB-LL basis function is non-uniform. The distribution of current intensity is illustrated in Fig. 1, where a darker color indicates a stronger current. In addition, the degrees of freedom (DoF) of an MB-LL basis function are twice those of an MB-RWG basis function. Therefore, the global application of LL and MB-LL basis functions will significantly increase the number of unknowns. For perfect electric conductor (PEC) targets, EFIE and MFIE are defined as:

$$\hat{\mathbf{n}} \times \begin{bmatrix} \mathbf{E}_{inc} - jk\eta \int_S \left(\bar{\mathbf{I}} + \frac{\nabla \nabla}{k^2} \right) \cdot \mathbf{J}(\mathbf{r}') G(R) ds' \\ - \int_S \mathbf{J}(\mathbf{r}') \times \nabla G(R) ds' + \mathbf{H}_{inc} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{J}(\mathbf{r}') \end{bmatrix}, \quad (2)$$

where $G(R) = e^{-jkR}/4\pi R$, $R = |\mathbf{r} - \mathbf{r}'|$ is the distance from the field point to the source point, $\mathbf{r}$ and $\mathbf{r}'$ are the position vectors of the field point and the source point, k is the free space wave number, η is the free space wave impedance, $\mathbf{E}_{inc}$ is the incident wave electric field, and $\mathbf{J}$ is the surface current. $\mathbf{J}$ is expanded by the RWG basis functions and MBRWG basis functions. By linear combination of the EFIE and MFIE in a certain proportion ($a: (1-a), 0 \leq a \leq 1$), CFIE can be constructed as:

$$\mathbf{CFIE} = a \cdot \mathbf{EFIE} + (1-a)\eta \cdot \mathbf{MFIE}. \quad (3)$$

Here, if only LL basis functions and MB-LL basis functions are adopted in the computation, the current density $\mathbf{J}$ needs to be expanded as:

$$\mathbf{J}(\mathbf{r}) = \sum_{n=1}^{N_{LL}} I_n^{LL} \mathbf{f}_n^{LL}(\mathbf{r}) + \sum_{n=1}^{N_{LL}} I_n^{MB-LL} \mathbf{f}_n^{MB-LL}(\mathbf{r}). \quad (4)$$

Substituting (4) into (2) and then applying the Galerkin testing method yields the matrix equation, which can be explicitly written as:

$$\begin{bmatrix} \mathbf{Z}_{LL,LL} & \mathbf{Z}_{LL,MB-LL} \\ \mathbf{Z}_{MB-LL,LL} & \mathbf{Z}_{MB-LL,MB-LL} \end{bmatrix} \begin{bmatrix} \mathbf{I}_{LL} \\ \mathbf{I}_{MB-LL} \end{bmatrix} = \begin{bmatrix} \mathbf{V}_{LL} \\ \mathbf{V}_{MB-LL} \end{bmatrix}. \quad (5)$$

It should be noted that the matrix forms of $\mathbf{Z}$, $\mathbf{I}$, and $\mathbf{V}$ in (9) can be found in [28] and will not be elaborated here. It should be emphasized that (4) and (9) correspond to the case where only LL basis functions and MB-LL basis functions are used for computation. In this paper, current density $\mathbf{J}$ is expanded by employing RWG, MB-RWG, LL, and MB-LL basis functions simultaneously. Accordingly, (4) is extended as:

$$\begin{aligned} \mathbf{J}(\mathbf{r}) = & \sum_{n=1}^{N_{LL}} I_n^{LL} \mathbf{f}_n^{LL}(\mathbf{r}) + \sum_{n=1}^{N_{MB-LL}} I_n^{MB-LL} \mathbf{f}_n^{MB-LL}(\mathbf{r}) \\ & + \sum_{n=1}^{N_{RWG}} I_n^{RWG} \mathbf{f}_n^{RWG}(\mathbf{r}) \\ & + \sum_{n=1}^{N_{MB-RWG}} I_n^{MB-RWG} \mathbf{f}_n^{MB-RWG}(\mathbf{r}). \end{aligned} \quad (6)$$

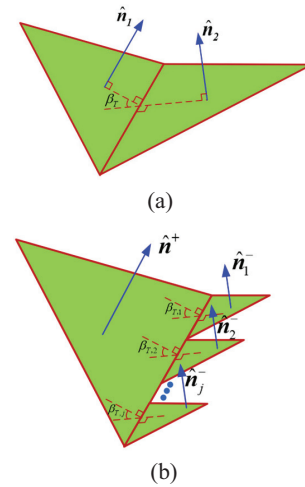


Fig. 2. Schematic diagram of the dihedral angle β_T (a) when using traditional conformal mesh and (b) when using nonconformal mesh.

Similarly, (9) is extended as:

$$\begin{bmatrix} \mathbf{Z}_{LL,LL} & \mathbf{Z}_{LL,MB-LL} & \mathbf{Z}_{LL,RWG} & \mathbf{Z}_{LL,MB-RWG} \\ \mathbf{Z}_{MB-LL,LL} & \mathbf{Z}_{MB-LL,MB-LL} & \mathbf{Z}_{MB-LL,RWG} & \mathbf{Z}_{MB-LL,MB-RWG} \\ \mathbf{Z}_{RWG,LL} & \mathbf{Z}_{RWG,MB-LL} & \mathbf{Z}_{RWG,RWG} & \mathbf{Z}_{RWG,MB-RWG} \\ \mathbf{Z}_{MB-RWG,LL} & \mathbf{Z}_{MB-RWG,MB-LL} & \mathbf{Z}_{MB-RWG,RWG} & \mathbf{Z}_{MB-RWG,MB-RWG} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{I}_{LL} \\ \mathbf{I}_{MB-LL} \\ \mathbf{I}_{RWG} \\ \mathbf{I}_{MB-RWG} \end{bmatrix} = \begin{bmatrix} \mathbf{V}_{LL} \\ \mathbf{V}_{MB-LL} \\ \mathbf{V}_{RWG} \\ \mathbf{V}_{MB-RWG} \end{bmatrix}. \quad (7)$$

Herein, the approach for handling singularities is adopted from [29]. It should be noted that the MB method requires some “special” treatment for meshes of different sizes; that is, the endpoints of the positive shared edges in one basis function must be aligned with those of the negative shared edges. If only the LL-basis functions and MB-LL basis functions are used to expand the current density $\mathbf{J}$, the number of unknowns obtained will be twice that when using the RWG basis functions and MB-RWG basis functions. This is unacceptable for the analysis of large-scale targets.

III. CONSTRUCTION RULES OF THE ADAPTIVE ACCURACY ENHANCED METHOD

To construct the AAE method, we referred to [21]. First, we need to define a dihedral angle β_T . For traditional conformal triangles, we identify the positive and negative triangles corresponding to each common side. Next, we determine the specific angles of β_T using the method employed in Fig. 2 (a).

In the case of using the nonconformal mesh, we need to calculate the angle between the positive triangle and each negative branch as β_T , as shown in Fig. 2 (b).

Next, we need to presuppose a critical value β_0 . In the case of using a traditional conformal triangle, when β_T corresponding to the common side is greater than β_0 , we replace the RWG basis functions corresponding to the common edge and the other four edges which belong to the positive and negative triangles of the common edge by the LL basis function (if the other common edge is open boundary, no replacement is done). Other common edges where β_T is less than β_0 continue to be calculated using the RWG basis function. The specific replacement of the common edges corresponding to the basis function is shown in Fig. 3 (a). In the case of using nonconformal mesh, when the β_T corresponding to the positive triangle and the negative branch is greater than β_0 , the MB-RWG basis function corresponding to this common edge is replaced with the MB-LL basis function. Then we replace the RWG basis functions corresponding to the other two common sides of the positive triangle and the RWG basis functions corresponding to

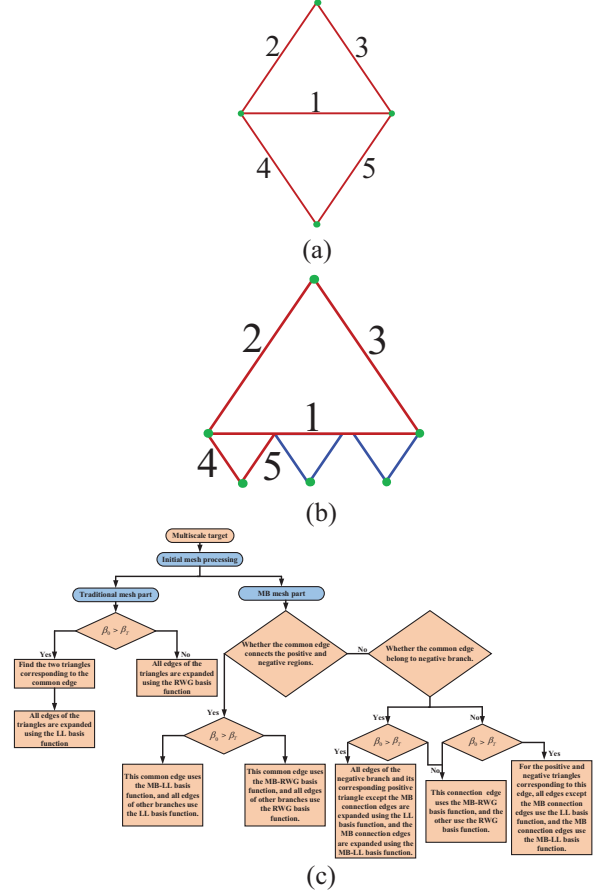


Fig. 3. Specific replacement of the common edges corresponding to the basis function (a) when using the conformal mesh, (b) when using the nonconformal mesh, and (c) the implementation flowchart of the MB-LLB-AAE method.

the other two common edges of this negative branch with LL basis functions (if the other common sides are open boundaries, no replacement is made). The other common edges where β_T is less than β_0 continue to use the RWG basis functions or the MB-RWG basis functions for calculation. The specific common edges corresponding to the replacement RWG basis function is shown in Fig. 3 (b). These are the construction rules of the AAE method. To facilitate readers' understanding of the method, the implementation flowchart of the proposed algorithm is illustrated in Fig. 3 (c).

In actual engineering, only a few computational parts can be very sharp. As a result, the RWG-basis functions and the MB-RWG basis functions are still dominant, only in the sharp parts do we introduce the LL basis functions and the MB-LL-basis functions to improve calculation accuracy. Therefore, compared with using the LL basis functions and the MB-LL basis functions only for calculation, the AAE method can

effectively reduce calculation consumption. At the same time, generally speaking, the sharp parts are the parts that need to be locally refined during our calculations. When using the MB method, a finer mesh can be used to calculate the sharp parts, and a coarser mesh can be used to calculate the smooth parts. This leads to a high possibility of mesh boundaries near the sharp parts. Therefore, it is meaningful to discuss the construction rules of the AAE method in the case of nonconformal meshes. We usually set β_0 to 90° , which is enough to ensure the good performance of AAE.

IV. NUMERICAL RESULTS

The platform used for performing the simulations is a workstation with a 32-core AMD Ryzen Threidripper Pro 3975 WX CPU@ 3.50 GHz and 512 GB RAM. All objects are illuminated by the plane wave propagating in $-z$ direction, and the incident field is x -polarized. The generalized minimum residual method (GMRES) is used to solve the matrix equation, and the residual is 10^{-5} .

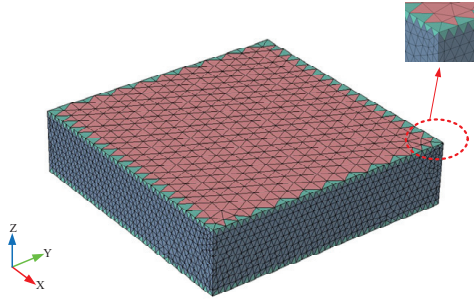


Fig. 4. Mesh schematic diagram of PEC cube for MB method.

A. Cube

In the first example, we use a simple PEC cube model to verify the correctness and accuracy of the proposed algorithm. The size of this cube model is $1 \times 1 \times 0.25$ m. The center of the model is at the origin and its operating frequency is 300 MHz. To verify the advantage of the proposed method in terms of accuracy, we introduce relative error and define it as:

$$e_{rel}(\theta) = \frac{|RCS_a(\theta) - RCS_c(\theta)|}{\max_{\theta} |RCS_a(\theta)|}, \quad (8)$$

where $RCS_a(\theta)$ represents the bistatic RCS result obtained by using a relatively fine mesh and the RWGEFIE method for calculation. Since the EFIE method was used and the mesh was relatively fine, the calculation result is relatively accurate. We take it as the reference standard. $RCS_c(\theta)$ is the result of bistatic RCS calculated by six methods, namely

MB-RWGEFIE method, MB-RWG-MFIE method, MB-LL-CFIE method, MB-LL-MFIE method, MB-AAE-CFIE method, and MB-AAE-MFIE method. For the EFIE method serving as the reference standard, we use triangles with an average side length of $0.01\lambda_0$ to perform meshing on the entire target. For the six methods, we use triangles with an average side length of $0.1\lambda_0$ to perform meshing on two faces of the cube model, and use triangles with an average side length of $0.05\lambda_0$ to perform meshing on the other four faces. The junctions of the different sizes mesh are connected using MB-RWG basis functions or MB-LL basis functions. For the MB-AAE-CFIE method and the MB-AAEMFIE method, we set β_0 to 90° . We present the mesh diagram of the MB method in Fig. 4 and mark the parts in cyan of the AAE method where the MB-RWG basis functions need to be replaced with the MB-LL basis functions. It can be seen that only a small portion of the MB-RWG basis functions have been replaced with MBLL basis functions.

In Fig. 5 (a), we present a comparison chart of the relative error of the six methods. It can be seen that, compared with the MB-RWG-CFIE method and the MB-RWG-MFIE method, the other four methods effectively reduce the calculation error, and the improvement of accuracy by the method based on AAE is not much different from that of the method based on LL basis functions. To further verify the accuracy of the proposed method, we define the root-mean-square errors (RMSE) of the RCS as:

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N |RCS_i - RCS_i^{EFIE}|^2}, \quad (9)$$

where $N = 180$. We use RCS results of $\phi = 0 - 180^\circ$ and $\theta = 90^\circ$ to calculate RMSE. The comparison of RMSE between the MB-RWG-CFIE method, the MB-LLCFIE method, and the MB-AAE-CFIE method are presented in Fig. 5 (b). It can be seen that the RMSE of both methods will gradually decrease with the increase of α , and eventually converge to 0. However, the RMSE of MB-AAE-CFIE method is smaller than that of MB-RWG-CFIE method, and the convergence speed is faster than that of the MB-RWG-CFIE method, and is basically consistent with the MB-LL-CFIE method. Table 1 presents a comparison of the computational resource consumption between the MB-LL method and the MB-LLB-AAE method. It can be observed that, in comparison with the MB-LL method, the MB-LLBAAE method achieves a reduction of approximately 5% in the number of unknowns and a decrease of about 20% in computation time. This result preliminarily validates the superiority of the MB-LLB-AAE method in terms of computational resource savings.

B. Ship

In the second example, we use a complex multiscale ship model to verify the advantages of the proposed method in terms of accuracy and computing resource consumption. The ship model is 23.19 m in length, 2.52 m in width, 4.82 m in height, and operates at a frequency of 300 MHz. This ship model not only has a large-sized hull but also has small-sized naval guns, missile cabins, a cockpit, and high-power microwave weapons distributed on it.

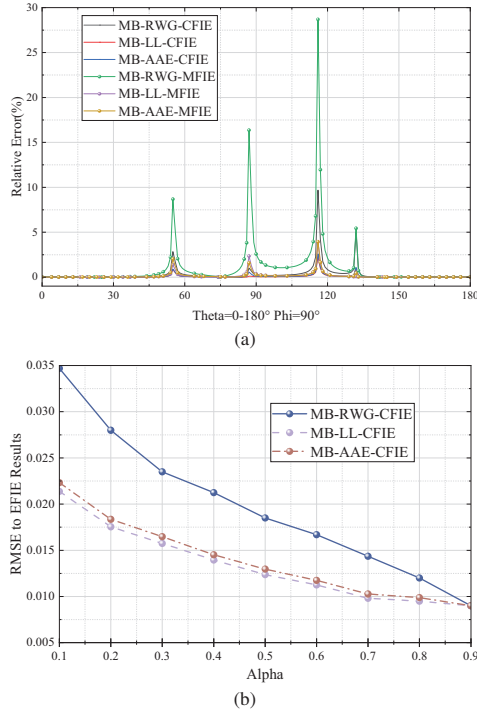


Fig. 5. (a) Relative error of the six methods and (b) RMSE to EFIE results of the MB-RWG-CFIE method, MB-LL-CFIE method, and the MB-AAE-CFIE method.

Table 1: Calculation resource comparison of three methods for analyzing the PEC cube

Method	Number of Unknowns	Memory of Z	CPU Time
MB-LL-CFIE	7302	531 MB	7.7 min
MB-AAE-CFIE	6942	475 MB	6.3 min

This ship model is a typically complex and practically significant multiscale model. For this ship model, we still first use the EFIE method that utilizes fine meshes for calculation and take the calculation results as the reference standard.

We use triangles with an average side length of $0.01\lambda_0$ to mesh the entire ship. Next, we use the

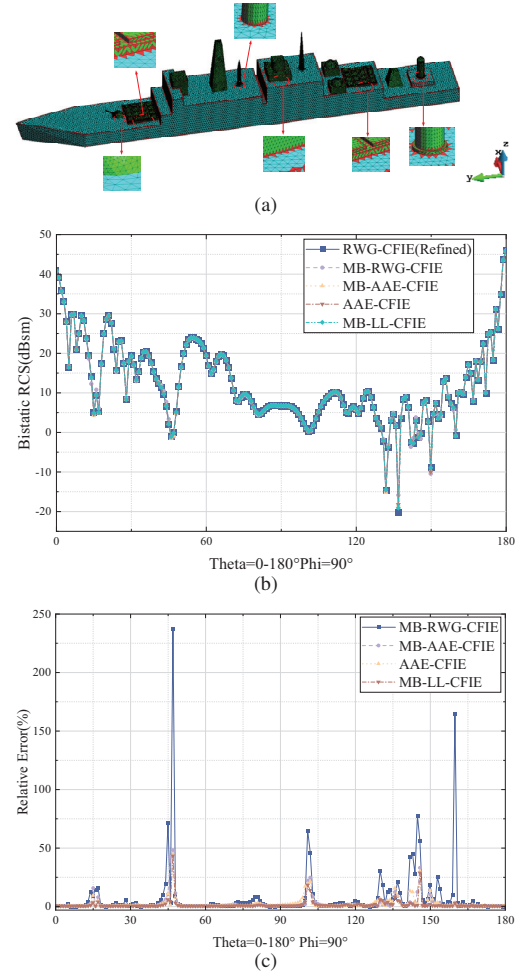


Fig. 6. (a) Mesh schematic diagram of the PEC ship for MB method. (b) Bistatic RCS results for the PEC ship. (c) Relative error results of the MB-RWG-CFIE method, the MB-AAE-CFIE method, the AAE-CFIE method and the MB-LL-CFIE method for the PEC ship.

Table 2: Calculation resource comparison of three methods for analyzing the PEC ship

Method	Number of Unknowns	Memory of Z	CPU Time
AAE-CFIE	113452	126062 MB	656.8 min
MB-LL-CFIE	89502	78456 MB	408.7 min
MB-AAE-CFIE	50482	25416 MB	132.6 min

AAECFIE method, MB-AAE-CFIE method, MB-LL-CFIE method and MB-RWG-CFIE method for calculation. For the MB method, we use a triangle with an average side length of $0.1\lambda_0$ to mesh the hull. The fine structure of the ship is meshed using triangles with an average side length of $0.05\lambda_0$. The connection points

of mesh of different sizes are connected using MB-RWG basis functions or MB-LL basis functions. For the AAECFIE method, the triangles with an average side length of $0.05\lambda_0$ are used to mesh the whole modal and selected β_0 at 90° . In Fig. 6 (a), we present the mesh scheme of the MB method and mark the parts where the MB-RWG basis functions are replaced with the MB-LL basis functions in red.

In Fig. 6 (b), we present the RCS result of the above five methods. To show the accuracy differences more clearly, we present the relative errors of the other four methods and the control group (EFIE with fine mesh) in Fig. 6 (c). It can be seen that the relative error of the MB-AAE-CFIE method is not much different from that of the AAE-CFIE method and the MB-LL-CFIE method, and is much lower than that of the MB-RWG-CFIE method. Meanwhile, in Table 2, we compared the computational consumption of the MB-AAE-CFIE method, the AAE-CFIE method, and the MB-LL-CFIE method. It can be seen that the computational consumption of the MB-AAE-CFIE method is much lower than that of the other two methods in comparison with the MB-LL method, the MB-LLB-AAE method achieves a reduction of approximately 43% in the number of unknowns and a decrease of about 67% in computation time. The proposed method effectively reduces computational cost while improving accuracy, which is very significant for calculating large-scale targets.

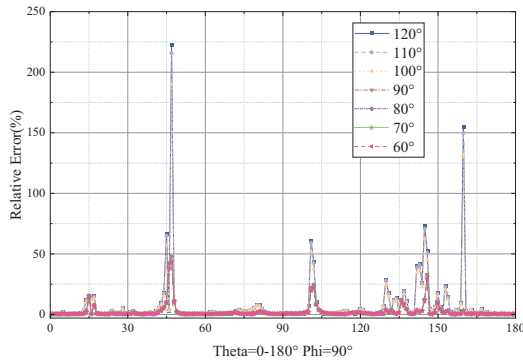


Fig. 7. Comparison of relative errors for the MB-LLBAAE method with different β_0 angles.

To justify the selection of 90° as the criterion for parameter β_0 , a sensitivity study is performed in this work. The parameter α is set to $120^\circ, 110^\circ, 100^\circ, 90^\circ, 80^\circ, 70^\circ$, and 60° , respectively, and all numerical tests are implemented using the MB-LLB-AAE method. The relative errors with respect to the dense-mesh RWGEFIE solution are depicted in Fig. 7, while the number of unknowns and the corresponding reduction percentages relative to the MB-LL method are summarized in Table 3.

Table 3: Comparison of computational resource consumption for the MB-LLB-AAE method with different β_0 angles for analyzing the PEC ship

β_0	Number of Unknowns	Reduction Percentage
120°	44964	49.7%
110°	45226	49.4%
100°	45342	49.3%
90°	50482	43.5%
80°	50722	43.3%
70°	50982	43.0%
60°	51120	42.8%

It can be observed that, as β_0 decreases, the relative error declines gradually, whereas the number of unknowns increases and the reduction ratio decreases accordingly. A distinct turning point is observed at 90° , where the relative error is notably reduced and the unknowns increase significantly. This phenomenon arises because 90° enables the inclusion of right-angle structures, which are the most frequently encountered configurations in practical engineering applications. When β_0 exceeds 90° , the variation in both accuracy and computational cost becomes relatively mild. Conversely, when β_0 is smaller than 90° , typical rightangle edges cannot be covered, and only marginal accuracy improvement can be achieved at the expense of excessive computational resources.

Accordingly, 90° is adopted as the standard value for β_0 , which achieves a favorable balance between improved accuracy and affordable computational overhead. In addition, only the ship model is employed for sensitivity analysis, since the cube model only contains right-angle edges and planar surfaces, which provides limited significance for validation. As a realistic engineering model, the ship model is more suitable to demonstrate the rationality of choosing 90° as the criterion for β_0 .

V. CONCLUSION

In this paper, we propose a novel MB-AAE method for improving the accuracy of MFIE and CFIE. This method only needs to replace a small part of the RWG basic functions and the MB-RWG basic functions with the LL basic functions and the MB-LL basic functions to effectively improve the calculation accuracy. This gives this method an advantage in terms of computing resource consumption. At the same time, this method inherits the advantages of the MB method and can effectively connect mesh of different sizes which is of great significance for calculating multiscale targets. It should

be emphasized that the MB-LLB-AAE method can maintain efficiency even when sharp features ($\beta_0 < 90^\circ$) account for more than 30% of the target. Furthermore, the proposed method can be extended to the computation of dielectric targets in the future to improve the calculation accuracy for dielectric objects. Therefore, the proposed method shows promising prospects in both application and development.

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An Efficient MMA-PMA-CBFM for Solving Partial Modification Electromagnetic Scattering Problems

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Abstract – This paper proposes an efficient algorithm for analyzing electrically large targets after partial modification. After solving the original model using the characteristic basis function method (CBFM), the mesh modification algorithm (MMA) is employed to modify the mesh according to requirements. Subsequently, in the CBFM blocks, only the characteristic basis functions (CBFs) associated with the affected blocks need to be regenerated, and the corresponding data in the reduced matrix are updated to rapidly obtain the solution of the new model by using the partial modification algorithm (PMA). Numerical examples validate the accuracy and efficiency of the proposed method, demonstrating its potential for practical engineering applications.

Index Terms – Characteristic basis function method (CBFM), mesh modification algorithm (MMA), partial modification algorithm (PMA).

I. INTRODUCTION

The characteristic basis function method (CBFM) [1–3] was proposed to reduce the number of unknowns and the matrix size in the traditional method of moments (MoM) [4, 5], thereby overcoming the high memory and computational complexity of MoM when analyzing electrically large targets. The characteristic basis functions (CBFs) in CBFM are not only significantly fewer in number compared to Rao-Wilton-Glisson (RWG) basis functions, but can also be combined with various acceleration algorithms such as adaptive cross approximation (ACA) [6, 7], further reducing the computational resources and enhancing the efficiency. However, in engineering applications, such as optimizing the layout of array antennas, engineers frequently encounter situations where the partial structures need

to be modified repeatedly. The process of remeshing, regenerating CBFs, refilling, and resolving the reduced matrix after each partial modification remains very time-consuming.

To address the challenges of “partial modification problems,” researchers first proposed an “add-on” technique [8, 9] based on the partitioned-inverse formula and the Sherman-Morrison-Woodbury (SMW) formula [10, 11]. Although this technique is limited to adding small structures, it enables the data of unmodified regions to be reused, making the computational cost of the new structure almost negligible. Then, a partial modification algorithm (PMA) [12] was proposed to support more general partial modification operations based on data reuse, including adding, deleting, and modifying partial structure. Subsequently, a mesh modification algorithm (MMA) and the integral equation discontinuous Galerkin (IEDG) method [13, 14] was integrated into PMA [15], enabling mesh modifications during program execution. However, the above methods share a common bottleneck that they all require storing the impedance matrix of the original structure, and the storage complexity of MoM limits the scale of the problem to be solved. Given that add-on technique has already been introduced into CBFM [16], using PMA to reuse the reduced matrix data of CBFM provides an excellent solution for the partial modification of electrically large targets.

This paper proposes the MMA-PMA-CBFM that introduce the PMA [12] with the MMA [15] into the CBFM. First, after solving the original model, block division and CBFs of unaffected blocks are preserved, and then the reduced matrix equation and its solution are updated via the PMA [12]. Second, the MMA [15] is applied, which supports mesh modification of arbitrary shapes through a mesh-based representation and overlap

detection, thereby reducing reliance on preprocessing software. Consequently, the MMA-PMA-CBFM reduces memory and computational cost while enabling automatic mesh modifications and rapid resolving of electrically large targets after partial modifications.

II. METHOD AND FORMULATIONS

A. Conventional CBFM

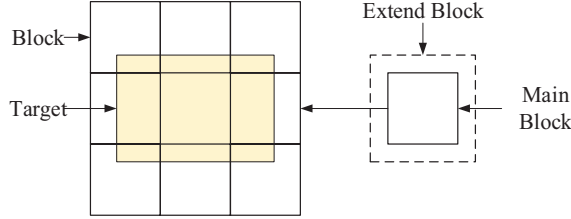


Fig. 1. Schematic diagram of CBFM's partitioning of the computational domain. Main blocks do not overlap with each other, while extended blocks may overlap with other main blocks or extended blocks.

Implementation of the conventional CBFM [3] needs to divide the computational domain into several blocks, called the main blocks. In order to avoid the current singularity caused by blocking, an extended region is added around each main block and referred to as an extended block, as shown in Fig. 1. All RWG basis functions are then assigned to the main blocks and their associated extended blocks. Each RWG basis function belongs to exactly one main block, but may be located within multiple overlapping extended blocks. To ensure the compression ratio of CBFs, the size of the main block is typically selected to be 1 to 2 wavelengths (λ), while the extended size is typically set as 0.1λ to 0.2λ . Further increasing the extended size yields limited effect on improving accuracy, and will increase the computational time of CBF generation.

For the excitation-independent CBFs, the generation for a given block requires solving the MoM equation under multiple planewave excitations at different angles and polarizations:

$$\mathbf{Z}_{i^+} \mathbf{I}_{i^+} = \mathbf{V}_{i^+}, \quad (1)$$

where $\mathbf{Z}$, $\mathbf{I}$, and $\mathbf{V}$ represent the impedance matrix, current vector, and voltage vector, respectively, and i^+ represents all RWG basis functions contained in the main area and extended area of the i -th block. Data corresponding to the extended block are removed from $\mathbf{I}_{i^+}$ to obtain $\mathbf{I}_i$, which only contains data of the main block. However, $\mathbf{I}_i$ still contains redundant information, so it is decomposed using singular value decomposition (SVD):

$$\mathbf{I}_i = \mathbf{U} \Sigma \mathbf{V}^H, \quad (2)$$

and the first several columns of $\mathbf{U}$ are taken as a set of CBFs, represented as $\mathbf{J}_i$. Selection criterion is based on whether the corresponding singular value σ meets the requirements $\sigma \geq \varepsilon \sigma_{\max}$, where the threshold ε is set to 0.001 in this paper. Because the singular values decrease extremely rapidly, the number of CBFs N^{CBF} is far less than the number of RWG basis functions N^{RWG} within the block.

After generating CBFs, the next step is to fill in the reduced impedance matrix $\mathbf{Z}^r$. The reduced mutual impedance matrix $\mathbf{Z}_{ij}^r$ corresponding to the i -th block and j -th block is:

$$\mathbf{Z}_{ij}^r = \mathbf{J}_i^T \mathbf{Z}_{ij} \mathbf{J}_j, \quad (3)$$

where $\mathbf{Z}_{ij}$ is the mutual impedance matrix of the two blocks generated by the RWG basis functions within the two blocks. Since size has been significantly reduced, solving the reduced impedance matrix significantly reduces computational time.

In addition, for two non-overlapping blocks, the filling process can be accelerated by ACA [17, 18]. The mutual impedance matrix can be decomposed by the ACA:

$$\mathbf{Z}_{ij} \approx \mathbf{U}_{ij} \mathbf{V}_{ij}. \quad (4)$$

The size of $\mathbf{Z}_{ij}$ is $N_i^{RWG} \times N_j^{RWG}$, and the size of matrix $\mathbf{U}_{ij}$ and $\mathbf{V}_{ij}$ is $N_i^{RWG} \times k$ and $k \times N_j^{RWG}$, where k is the effective rank solved by the ACA algorithm:

$$k \ll \min(N_i^{RWG}, N_j^{RWG}). \quad (5)$$

$\mathbf{Z}_{ij}^r$ can be expressed as:

$$\mathbf{Z}_{ij}^r = \mathbf{J}_i^T \mathbf{Z}_{ij} \mathbf{J}_j \approx (\mathbf{J}_i^T \mathbf{U}_{ij})(\mathbf{V}_{ij} \mathbf{J}_j). \quad (6)$$

The smaller size of the matrix will reduce the time of matrix multiplication, improving the performance of filling reduced impedance matrix. However, it is still very time-consuming to repeatedly perform this process when solving partial modification problems.

B. PMA-CBFM for solving partial modification problems

In partial modification problems, all modifications are applied to the original model. Even when multiple modifications are performed, each new model is derived from the original, rather than from the previous version. Any partial modification can be divided into two steps. Step 1 is to remove the subtract-structure from the original-structure, leaving what is referred to as the rest-structure. Step 2 is to affix the add-structure to the rest-structure. The key to efficiently solving such problems lies in reusing data obtained from the original model

to avoid redundant computations. Therefore, this paper achieves efficient partial modification analysis based on CBFM by reusing CBF data and reduced matrix data.

Before partial modification, the original structure was solved by the conventional CBFM. After the modifications, it is no longer necessary to re-divide the computational domain. Instead, the original partitioning information can be used.

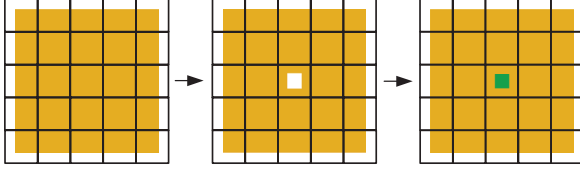


Fig. 2. Schematic diagram of a partial modification. The original-structure is the yellow plate, which is divided into multiple blocks by CBFM and represented by black wireframes. Part of the plate is replaced by a green cube structure.

Taking the partial modification in Fig. 2 as an example, it should be pointed out that if the mesh and RWG basis functions within a block remain unchanged, its CBFs are unaffected. Since partial modifications typically affect only a few blocks, such as in Fig. 2, only one block is modified, which means that most CBFs are reusable. In CBFM, generating CBFs is computationally expensive in the solution process. Therefore, the proposed strategy generates CBFs only for the affected blocks when solving a partial modified model, while reusing the CBFs of all other unaffected blocks. Compared to regenerating CBFs for every block, this approach saves considerable computation time without sacrificing accuracy. The process of generating CBFs is the same as that introduced in section II.A.

In addition to reusing the CBFs, we also introduce PMA [12] to reuse the reduced matrix and its solution of the original model. In step 1 of partial modification, all CBFs of the affected blocks are marked as “subtract,” and CBFs on the unaffected blocks are marked as “rest”. In step 2, new CBFs need to be regenerated and marked as “added” in these affected blocks, unless the block no longer contains any basis functions. We partitioned the reduced equations of two steps as follows:

$$\begin{bmatrix} \mathbf{Z}_{rr} & \mathbf{Z}_{rs} \\ \mathbf{Z}_{sr} & \mathbf{Z}_{ss} \end{bmatrix} \begin{bmatrix} \mathbf{J}_r \\ \mathbf{J}_s \end{bmatrix} = \begin{bmatrix} \mathbf{V}_r \\ \mathbf{V}_s \end{bmatrix}, \quad (7)$$

$$\begin{bmatrix} \mathbf{Z}_{rr} & \mathbf{Z}_{ra} \\ \mathbf{Z}_{ar} & \mathbf{Z}_{aa} \end{bmatrix} \begin{bmatrix} \mathbf{I}_r \\ \mathbf{I}_a \end{bmatrix} = \begin{bmatrix} \mathbf{V}_r \\ \mathbf{V}_a \end{bmatrix}, \quad (8)$$

where subscripts s, r , and a represent subtracted, rest, and added CBFs, respectively. $\mathbf{Z}_{rr}$, $\mathbf{Z}_{ss}$, and $\mathbf{Z}_{aa}$ are

the reduced self-matrices. $\mathbf{Z}_{rs}$, $\mathbf{Z}_{sr}$, $\mathbf{Z}_{ra}$, and $\mathbf{Z}_{ar}$ are the reduced mutual-matrices. $\mathbf{V}_r$, $\mathbf{V}_s$, and $\mathbf{V}_a$ are the reduced voltage vectors. $\mathbf{J}_r$ and $\mathbf{J}_s$ are the reduced current vectors in step 1. $\mathbf{I}_r$ and $\mathbf{I}_a$ are the reduced current vectors in step 2. $\mathbf{V}_r$ will be reused, while $\mathbf{V}_a$ needs to be recalculated based on the partial modification.

According to the method in [12], the solution of (8) can be efficiently calculated as:

$$\mathbf{I}_a = \mathbf{Y}_{aa}(\mathbf{V}_a - \mathbf{Z}_{ar}\mathbf{Z}_{rr}^{-1}\mathbf{V}_r), \quad (9)$$

$$\mathbf{I}_r = \mathbf{Z}_{rr}^{-1}(\mathbf{V}_r - \mathbf{Z}_{ra}\mathbf{I}_a), \quad (10)$$

where

$$\mathbf{Y}_{aa} = (\mathbf{Z}_{aa} - \mathbf{Z}_{ar}\mathbf{Z}_{rr}^{-1}\mathbf{Z}_{ra})^{-1}, \quad (11)$$

$$\mathbf{Z}_{rr}^{-1} = \mathbf{Y}_{rr} + \mathbf{Y}_{rs}(\mathbf{1} - \mathbf{Z}_{sr}\mathbf{Y}_{rs})^{-1}\mathbf{Z}_{sr}\mathbf{Y}_{rr}, \quad (12)$$

$\mathbf{Y}_{rr}$ and $\mathbf{Y}_{rs}$ are the submatrices of the inverse of the impedance matrix of the mother-structure [12]. $\mathbf{1}$ represents the identity matrix.

It can be found that, after solving the original model, $\mathbf{Z}_{rr}^{-1}$ can be reused for multiple modifications. Under the condition that PMA is effective, which means the modified structure is much smaller than the original structure, the number of affected CBFs will also be much smaller than the total CBFs, and computational complexity is reduced considerably. To further illustrate the method presented in this paper, pseudocode is provided here.

```
// Find the computational domain of the original model
and extend it.
Domain = Compute (Original_Mesh);
Domain = Expand (Domain, Distance);
// Divide into blocks and generate CBF
Blocks = Partition (Domain)
for i = 1, BlockNumber
    CBFs(i)=Generate_CBF (Blocks(i))
// Generate the reduced matrix and solve
for i = 1, BlockNumber
    Vr(i) = FillReducedV (CBFs(i))
    for j = 1, BlockNumber
        Zr(i, j) = FillReducedZ (CBFs(i), CBFs(j))
Ir = Solve (Zr, Vr)
// Partial modification
New_Mesh=PartialModify (Original_Mesh,
Subtract_Mesh, Add_Mesh)
// Mark affected blocks & CBFs
for i = 1, BlockNumber
    MarkCBF (Blocks(i), CBFs(i), New_Mesh)
// PMA
PMA_Sub (CBFs_Ori, CBFs_Sub, Zr, Vr, Ir)
PMA_Add (CBFs_Ori, CBFs_Sub, CBFs_Add, Zr, Vr,
Ir)
```

C. MMA for mesh modification

The feasibility of data reuse in partial modification problems stems from the fact that only the mesh of the

affected regions needs to be modified, rather than discretizing the entire model. Conventional approaches, such as “add-on” technique or PMA, impose significant constraints when modifying partial structures. These methods require the new structure to be predefined during preprocessing, and the mesh of all structures must maintain conformity to ensure current continuity. This requirement is often too complex to satisfy, making these traditional approaches suitable only for cases where the structures are separate. To overcome these limitations, the MMA proposed in [15] allows direct mesh modification without preprocessing software, while preserving current continuity across the structures via the IEDG method [13].

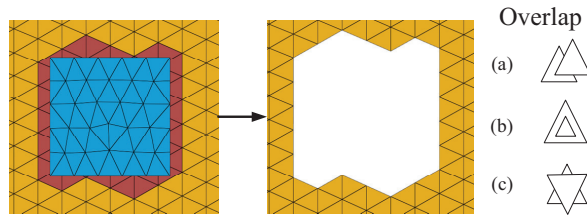


Fig. 3. Schematic diagram of removing the triangles from the original-structure that overlap with the subtract shape. Yellow triangles belong to the original-structure, blue triangles belong to the subtract shape, and red triangles overlap with the subtract shape, which belong to the subtract-structure and will be removed. On the right are three situations that will be considered as overlaps.

Some technical details of the MMA in [15] were not elaborated in detail. In this paper, we supplement some important technical details of the MMA. Continuing with the partial modification shown in Fig. 2 as an example, the program will first remove a square from the plane and then replacing it with a cube. To solve this problem, the steps of the MMA are as follows.

Step 1: Identify triangles located inside the subtract-structure, mark and delete them. When a square needs to be subtracted from the original-structure, as illustrated in Fig. 3, the MMA requires a mesh of corresponding shape at the specified position. It then detects all triangles in the original-structure that overlap with the triangles of the mesh of subtract shape and marks them for removal. The overlap between two coplanar triangles only has three cases, as cataloged in Fig. 3 (right). Accordingly, a node-edge detection method is employed here. The method first checks if each node of a triangle is inside another triangle. For triangle ABC and point P, we have

$$\vec{AP} = u\vec{AC} + v\vec{AB}, \quad (13)$$

when satisfied

$$\begin{cases} 0 \leq u \leq 1 \\ 0 \leq v \leq 1 \\ u + v \leq 1 \end{cases}, \quad (14)$$

it can be considered that point P is inside the triangle. Then check whether the edges of the two triangles intersect. If two line-segments AB and CD intersect in the same plane, they will satisfy

$$\begin{cases} (\vec{AC} \times \vec{AD}) \cdot (\vec{BC} \times \vec{BD}) < 0 \\ (\vec{CA} \times \vec{CB}) \cdot (\vec{DA} \times \vec{DB}) < 0 \end{cases}, \quad (15)$$

and triangles that meet either of the checks are marked for removal. The strategy is able to find triangles whose nodes are not inside the subtract shape but overlap with the shape.

For electrically large targets with a very large number of triangles, traversing all triangles of the subtract shape against all triangles of the original structure would be computationally expensive. To accelerate the overlap detection, we can employ a kd-tree [19] spatial index. The original mesh is partitioned using a kd-tree, and only those triangles within intersecting bounding boxes are tested with triangles of the subtract shape.

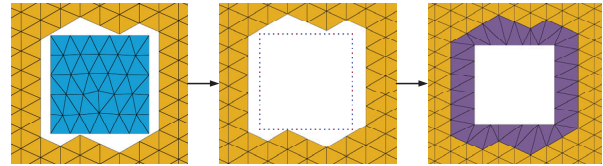


Fig. 4. Diagram of finding the boundary of a subtract shape and moving the nodes of the rest-structure. Blue mesh belongs to the subtract shape, the points in the middle image represent the boundaries of the subtract shape, yellow mesh belongs to the rest-structure, and the triangles in the rest-structure whose nodes have been moved are purple.

Step 2: Move the nodes of rest structure to the square boundary to smooth the jagged edge. As shown in Fig. 4, the MMA first identifies all free edges in the mesh of the subtracted shape, which are defined as edges belonging to only one triangular element and not shared by any adjacent triangles. Subsequently, each node is relocated to the nearest position on the nearest identified edge. Then we uniformly sample points along the edges and move each node to its nearest sample point. This approximation is both fast and effective in improving boundary smoothness.

Step 3: Positioning the add-structure at the modified location and defining RWG and Half-RWG basis functions in the new structure region. As shown in Fig. 5, the

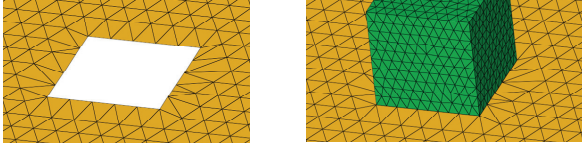


Fig. 5. Diagram showing the addition of add-structure to rest-structure. The boundary of the rest-structure, modified by the MMA, precisely matches that of the add-structure, although the meshes are non-conformal.

green cube is moved to the mesh modification location. Since the different structures are meshed independently, the mesh junctions between them are non-conformal. The current continuity of the non-conformal mesh junctions is guaranteed by the IEDG method [13, 15].

The MMA is based on the principle of removing overlapping triangles and aligning the rest nodes to new boundaries. However, we acknowledge two current limitations. The method is designed for planar subtraction shapes and has not yet been extended to curved surfaces. Additionally, the node relocation step may occasionally generate poorly shaped triangles. Nevertheless, the impact on overall solution accuracy is negligible compared to the significant efficiency gains achieved by avoiding full mesh regeneration.

When integrated with the PMA-CBFM, the MMA enables a powerful solution for partial modification problems. Specifically, the original model is solved using CBFM, and its block partitioning, CBFs and reduced matrix are stored. After the MMA performs a partial mesh modification of arbitrary shape, only the CBFs and reduced sub-matrices related to the affected blocks are regenerated, and the solution of the reduced matrix of the new model is updated via the PMA. This method preserves accuracy while dramatically reducing computational time. Moreover, both the matrix filling and solving processes are inherently parallelizable, further enhancing computational efficiency for electrically large targets. Consequently, the proposed MMA-PMA-CBFM offers a flexible, scalable, and efficient approach for practical engineering applications.

III. NUMERICAL RESULTS

A numerical example is presented to demonstrate the accuracy and efficiency of our method. As illustrated in Fig. 6, the test case involves a perfect electrical conductor (PEC) radar vehicle model. The main vehicle body is about 5 meters long with 26,934 triangles, which is set as the original-structure. The vehicle has a circular platform where a radar can be placed, which has 1056 triangles. In this example, the MMA-PMA-CBFM performs four partial modifications after solving the vehicle. For each modification process, the program will

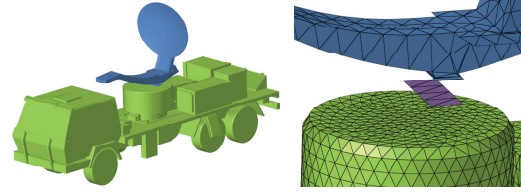


Fig. 6. Schematic diagram of radar vehicle models and meshes, with vehicles in green, radar in blue, and the shape of the bottom of the radar in purple.

rotate the radar 90 degrees and place it on the platform to obtain a new model. This example will calculate the monostatic radar cross section (RCS) results over a theta range of 0 to 90 degrees at 500 MHz. The program was performed on a Dell workstation powered by two Intel Xeon E5-2690v2 CPUs and 512 GB of memory.

Table 1: Comparison of memory between MoM and CBFM

Method	Number of Unknowns	Memory (GB)
MoM	40401	12.2
CBFM	6280	0.3

Table 1 compares the memory required to store the impedance matrix for the original structure using MoM and CBFM. Since the impedance matrix of the original structure must be stored for reuse during partial modifications, and the modified part is typically small, the memory occupied by the original impedance matrix represents the dominant memory requirement during program execution. As shown in Table 1, CBFM significantly reduces the number of unknowns and memory usage compared to MoM. Consequently, the proposed MMA-PMA-CBFM can solve larger problems than the conventional PMA.

Table 2: Comparison of time between conventional CBFM and MMA-PMA-CBFM

Method	CBFM	MMA-PMACBFM	MMA-PMA-CBFM (Parallel)
Original Structure	1359.2 s	1533.8 s	211.0 s
Modify 1	1500.4 s	132.4 s	19.4 s
Modify 2	1464.5 s	164.0 s	24.1 s
Modify 3	1451.0 s	142.6 s	20.9 s
Modify 4	1450.7 s	173.2 s	25.1 s

Table 2 compares the single thread solving time of conventional CBFM and proposed MMA-PMA-CBFM. Because MMA-PMA-CBFM extends the computational domain of the radar structure, its block partitioning

results differ from those of traditional CBFM, leading to differences in computation time. However, these differences are within an acceptable range. Each partial modification replaces approximately 200 to 400 original CBFs with 700 to 800 new CBFs, and the notable reduction in solving time validates the efficiency of proposed MMA-PMA-CBFM in handling partial modifications problems. In addition, we employ OpenMP to parallelize the filling of the impedance matrix and the voltage vector. The parallel solving time of the proposed method also demonstrates excellent parallel performance.

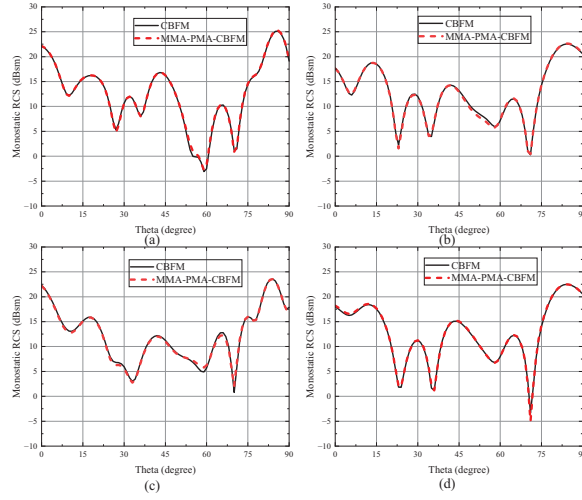


Fig. 7. Comparison of monostatic RCS results of four partial modification processes between conventional CBFM and MMA-PMA-CBFM.

Subsequently, we compare the results of MMA-PMA-CBFM with those of the conventional CBFM, as shown in Fig. 7. Furthermore, the root mean square error (RMSE) of the four modifications in m^2 is presented in Table 3, with the calculation formula given below:

$$\Delta(\theta_i) = RCS_{CBFM}(\theta_i) - RCS_{MMA-CBFM-PMA}(\theta_i), \quad (16)$$

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (\Delta(\theta_i))^2}. \quad (17)$$

It can be seen that the proposed MMA-PMA-CBFM has reliable accuracy performance.

Table 3: RMSE of four modifications

Modification	RMSE
Modify 1	2.2
Modify 2	1.5
Modify 3	1.2
Modify 4	1.3

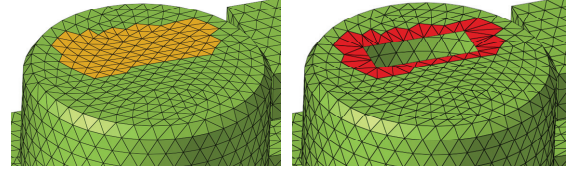


Fig. 8. Result of MMA modifying the mesh of the platform to match radar boundaries. Triangles unchanged are green, triangles overlapped and nodes moved are orange (left), and triangle nodes moved are red (right).

Figure 6 illustrates the different mesh required by MMA during partial modification, including mesh of original structure, modified structure, and subtracted shape. MMA first removes triangles that overlap with the radar bottom on the platform, then modifies the shape of the remaining triangles on the platform to match the radar, and finally places the radar. Figure 8 displays the results of a selected modification. It can be seen that the modified mesh matches the boundary of the radar well.

IV. CONCLUSION

This paper introduces PMA with MMA into CBFM to enhance computational efficiency for solving electrically large partial modification problems. Compared with the conventional CBFM, the proposed MMA-PMA-CBFM can reduce much CPU time for solving partial modification problems. In future work, MMA-PMA-CBFM can be combined with optimization algorithms to achieve rapid optimization of partial structures.

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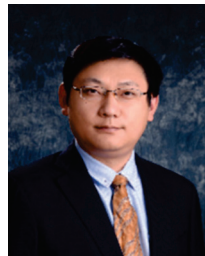
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Fast and Efficient Analysis of Electromagnetic Radiation from RadomeThermal Protective Enclosure-Antennas Structure

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Abstract – In this paper, a novel hybrid method combining the full-wave integral equation and high-frequency approaches is proposed to assess the electromagnetic performance of the radome and thermal protective enclosure (TPE)-enclosed antenna (RTA) structure. By using the approximate equivalence principle twice, the basic difficulty in applying the previously proposed hybrid method to RTA structure is eliminated. The interactions between the antennas and TPE, as well as between the combined antenna-TPE system and the radome, are accounted for iteratively. Numerical results demonstrate that the proposed hybrid method computes the radiation patterns of RTAs with good accuracy and significantly higher efficiency compared to full-wave solutions.

Index Terms – Hybrid method, radiation pattern radome-enclosed antennas, thermal protective enclosure.

I. INTRODUCTION

Radomes are commonly used to protect antennas from adverse environmental effects in modern electronic systems. To meet aerodynamic requirements in various complex environments, a multitude of types of radomes have been developed [1, 2]. During long-distance flight of high-speed vehicles, temperatures within the radome-enclosed space can rise sharply, potentially degrading the performance of communication or radar systems. To mitigate damage risk from excessive heating, an additional dielectric shield, termed a thermal protective enclosure (TPE), is often placed between the antennas and the radome (see Fig. 1). Since the radome itself affects antenna performance [2–4], it is evident that

addition of a TPE will inevitably further distort the radiation pattern. Therefore, it is desired to evaluate the overall electromagnetic (EM) properties of the radome-TPE-antennas (RTAs) in the preliminary stage of system design.

From the viewpoint of computational electromagnetics, the radome-antenna system is typically a large-size, multi-scale structure, making accurate and efficient analysis of its EM performance highly challenging for the existing full-wave methods [3, 5–12]. The addition of TPE further increases the difficulty of full-wave analysis due to increased requirement of computational resources. On the other hand, although high-frequency methods can predict radiation patterns of the radome-antenna systems in some scenarios [13–15], they lack sufficient accuracy for arbitrarily shaped radome. In contrast, the hybrid methods leverage domain-decomposition concept and use suitable solutions in different regions [16–22], thus some of them inherit high efficiency from high-frequency methods while possessing acceptable accuracy. However, the existing hybrid methods seldom account for the mutual coupling between radome and antennas, which limits their applicability in situations where high accuracy is demanded. Recently, a method called VSIE-MSI-MLFMA [23] was proposed to handle large/medium-sized complex radome-antenna structures, which considered the interactions among the antennas and radome in an iterative manner, resulting in high efficiency and accuracy. Nonetheless, for the RTA structures, it is hard to simply consider the radome and TPE as a single multilayered radome, because the gap between them is often large in terms of wavelength, and direct use of the VSIE-MSI-MLFMA method will result in low precision due to poor accuracy of the local planar approximation

used in the MSI process for transmission matrix computation [1, 23].

In this paper, we construct a new mathematical model that not only includes all mutual interactions among the antennas, TPE, and radome in an RTA structure, but also eliminates the main difficulty in applying the VSIE-MSI-MLFMA method to deal with the large air gap between the TPE and the radome. The remainder of the paper is organized as follows. Section II details the derivation of the new mathematical model and its implementation, highlighting improvements over our previous work. In section III, numerical examples are used to verify the accuracy and efficiency of the proposed method. Conclusions are drawn in section IV.

II. FORMULATIONS

A. Approximate equivalence model for EM analysis of RTAs

The configuration of a typical RTA structure is shown in Fig. 1. The entire RTA can be decomposed into three regions: the antenna region, which contains complex antenna structures requiring full-wave methods for accurate solutions; the TPE region, which typically has low permittivity to preserve the EM performance of the antennas while preventing excessive temperature rise; and the radome region, which contains of an arbitrarily-shaped, large-size, multilayer dielectric radome with variable wall-thickness.

First, we use a virtual surface S_v^{TPE} (green dashed line in Fig. 1) to enclose the TPE. According to the equivalence principle, the radiating EM fields ($\vec{E}^{TPE}, \vec{H}^{TPE}$) outside the space bounded by the surfaces which are marked by green solid/dash lines in Fig. 1 are given as the superposition of the fields $\vec{\xi}_l^{TPE}$ (ξ is either E or H , and l is either e, b , or v) produced by the equivalent EM currents on the outer surface S_e^{TPE} , bottom surface S_b^{TPE} , and virtual surface S_v^{TPE} of the TPE

$$\begin{cases} \vec{E}^{TPE} = \vec{E}_e^{TPE} + \vec{E}_b^{TPE} + \vec{E}_v^{TPE} \\ \vec{H}^{TPE} = \vec{H}_e^{TPE} + \vec{H}_b^{TPE} + \vec{H}_v^{TPE} \end{cases} \quad (1)$$

If we keep the TPE structure unchanged and replace the material of TPE by vacuum, the radiating fields outside the same surfaces become

$$\begin{cases} \vec{E}^{TPE_vac} = \vec{E}_e^{TPE_vac} + \vec{E}_b^{TPE_vac} + \vec{E}_v^{TPE_vac} \\ \vec{H}^{TPE_vac} = \vec{H}_e^{TPE_vac} + \vec{H}_b^{TPE_vac} + \vec{H}_v^{TPE_vac} \end{cases}, \quad (2)$$

where $\vec{\xi}_l^{TPE_vac}$ are the partial EM fields produced by the equivalent EM currents on the surfaces S_l^{TPE} of the auxiliary vacuum TPE. Apparently, for vacuum TPE, ($\vec{E}^{TPE_vac}, \vec{H}^{TPE_vac}$) are the same as the radiation

($\vec{E}^{Ant}, \vec{H}^{Ant}$) from the freestanding antennas, i.e.

$$\begin{cases} \vec{E}^{TPE_vac} = \vec{E}^{Ant} \\ \vec{H}^{TPE_vac} = \vec{H}^{Ant} \end{cases} \quad (3)$$

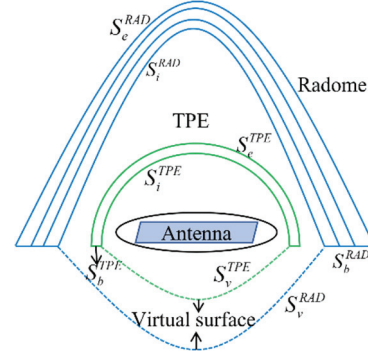


Fig. 1. Schematic diagram of a typical RTA structure.

According to the equivalence principle, the selection of the shape and position of the virtual surface S_v^{TPE} can be arbitrary, so we placed it completely outside the radome-covered space. For typical RTA structures, the ground plane (also a part of the antenna region) of the antennas usually seals the bottoms of TPE and radome, so the field intensities on the virtual surface are much weaker, compared to those in the interior region. Therefore, we assume that the field distributions on S_v^{TPE} do not vary drastically as the materials of TPE change, i.e.

$$\begin{cases} \vec{E}_v^{TPE} \approx \vec{E}_v^{TPE_vac} \\ \vec{H}_v^{TPE} \approx \vec{H}_v^{TPE_vac} \end{cases} \quad (4)$$

If we replace (2)–(4) into (1), an approximate representation of the radiating EM fields ($\vec{E}^{TPE}, \vec{H}^{TPE}$) from the combination of the antenna and TPE regions can be obtained

$$\begin{cases} \vec{E}^{TPE} \approx \vec{E}^{Ant} + (\vec{E}_e^{TPE} + \vec{E}_b^{TPE}) \\ \quad - (\vec{E}_e^{TPE_vac} + \vec{E}_b^{TPE_vac}) \\ \vec{H}^{TPE} \approx \vec{H}^{Ant} + (\vec{H}_e^{TPE} + \vec{H}_b^{TPE}) \\ \quad - (\vec{H}_e^{TPE_vac} + \vec{H}_b^{TPE_vac}) \end{cases}, \quad (5)$$

where $\vec{\xi}_l^{TPE}$ and $\vec{\xi}_l^{TPE_vac}$ ($\xi = E, H$ and $l = e, b$) are transmitted EM fields through the realistic and vacuum TPE, respectively.

To calculate the radiation from the entire RTA, we (Z)nsider the combination of antennas and TPE as a new antenna structure. By enclosing the radome through a virtual surface S_v^{RAD} (blue dash line in Fig. 1) and utilizing the equivalence principle again with the

assumption similar to (3), i.e. the EM fields on the virtual surface S_v^{RAD} remain approximately unchanged with the radomes material, the radiating EM fields ($\vec{E}^{\text{RTA}}, \vec{H}^{\text{RTA}}$) outside the RTA (blue solid/dash lines in Fig. 1) can be calculated in the similar way as those in (1)–(5)

$$\begin{cases} \vec{E}^{\text{RTA}} \approx \vec{E}^{\text{TPE}} + (\vec{E}_e^{\text{RAD}} + \vec{E}_b^{\text{RAD}}) \\ \quad - (\vec{E}_e^{\text{RAD_vac}} + \vec{E}_b^{\text{RAD_vac}}) \\ \vec{H}^{\text{RTA}} \approx \vec{H}^{\text{TPE}} + (\vec{H}_e^{\text{RAD}} + \vec{H}_b^{\text{RAD}}) \\ \quad - (\vec{H}_e^{\text{RAD_vac}} + \vec{H}_b^{\text{RAD_vac}}) \end{cases} \quad (6)$$

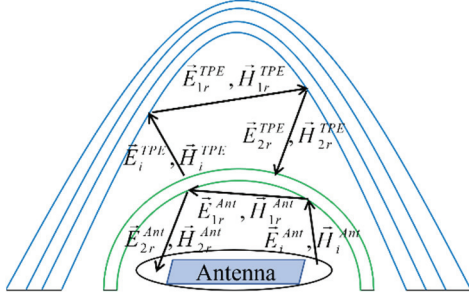


Fig. 2. Field interactions among the antennas, TPE, and radome.

where $\vec{\xi}_l^{\text{RAD}}$ and $\vec{\xi}_l^{\text{RAD_vac}}$ are the partial EM fields produced by the equivalent EM currents on the surfaces S_l^{RAD} of the realistic and auxiliary vacuum radome, respectively. If we substitute (5) into (6), we obtain

$$\begin{cases} \vec{E}^{\text{RTA}} \approx \vec{E}^{\text{Ant}} + (\vec{E}_e^{\text{TPE}} + \vec{E}_b^{\text{TPE}} + \vec{E}_e^{\text{RAD}} + \vec{E}_b^{\text{RAD}}) \\ \quad - (\vec{E}_e^{\text{TPE_vac}} + \vec{E}_b^{\text{TPE_vac}} + \vec{E}_e^{\text{RAD_vac}} \\ \quad + \vec{E}_b^{\text{RAD_vac}}) \\ \vec{H}^{\text{RTA}} \approx \vec{H}^{\text{Ant}} + (\vec{H}_e^{\text{TPE}} + \vec{H}_b^{\text{TPE}} + \vec{H}_e^{\text{RAD}} + \vec{H}_b^{\text{RAD}}) \\ \quad - (\vec{H}_e^{\text{TPE_vac}} + \vec{H}_b^{\text{TPE_vac}} + \vec{H}_e^{\text{RAD_vac}} \\ \quad + \vec{H}_b^{\text{RAD_vac}}) \end{cases} \quad (7)$$

Equation (7) represents a new approximate equivalence model for acquiring the EM radiation of the entire RTA structure.

B. Implementation details of the proposed approach and overcoming drawbacks

As shown in Fig. 2, the EM fields ($\vec{E}^{\text{Ant}}, \vec{H}^{\text{Ant}}$) radiating from the antenna region (including the antenna array and the ground plane) are accurately computed by the full-wave VSIE solver [23], incident on the TPE and then partly reflected multiple times to produce the reflection fields ($\vec{E}_{1r}^{\text{Ant}}, \vec{H}_{1r}^{\text{Ant}}$), ($\vec{E}_{2r}^{\text{Ant}}, \vec{H}_{2r}^{\text{Ant}}$), ... Another part

of ($\vec{E}^{\text{Ant}}, \vec{H}^{\text{Ant}}$) goes through the TPE to form the transmitted fields ($\vec{E}_i^{\text{TPE}}, \vec{H}_i^{\text{TPE}}$), and these waves impinge upon the radome, giving rise to the fields transmitted into free space and the internal multi-reflected fields ($\vec{E}_{1r}^{\text{TPE}}, \vec{H}_{1r}^{\text{TPE}}$), ($\vec{E}_{2r}^{\text{TPE}}, \vec{H}_{2r}^{\text{TPE}}$) within the radome. These reflected fields, in part, transmit back through the TPE and subsequently generate higher-order reflection inside the space enclosed by TPE. In turn, the reflected fields from TPE act on the antenna region and alter the EM currents on them. Ultimately, the modified EM currents update the radiating fields ($\vec{E}^{\text{Ant}}, \vec{H}^{\text{Ant}}$) from antenna region. The above process will be repeated until the current distributions on the antenna region reach final (g) uilibrium. In general, the following inequality is used to quantitatively measure if the iterative process terminates

$$\|I_k^{\text{Ant}} - I_{k-1}^{\text{Ant}}\| / \|I_k^{\text{Ant}}\| \leq \delta, \quad (8)$$

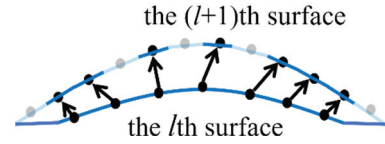


Fig. 3. Locally planar approximation used in transmission matrix computation.

where I_k^{Ant} is the coefficient vector of EM currents on the antenna region during the k th iteration of the MLFMA, δ is the preset threshold, and $\|\cdot\|$ means the 2-norm of a vector.

The above reflection and transmission fields are computed by using the locally planar approximation, ray tracing, and transmission matrix method as employed in the previously proposed VSIE-MSIMLFMA method [23] and in other high-frequency approaches [20–22, 24, 25]. In implementation, each curved surface of the TPE and the radome is discretized into triangular meshes, and the number of triangles on the outer $(l+1)$ th surface is always greater than that on the inner l th surface. Consequently, during the raytracing process, the incident points on the l th surface can always find the exit points on the $(l+1)$ th surface. However, a portion of triangles on the $(l+1)$ th surface invariably lack corresponding exit points (gray points in Fig. 3) corresponding to the incident points (black points in Fig. 3) on the l th surface, resulting in null EM fields on those triangles. This causes the outer $(l+1)$ th surface be in a “broken state” as illustrated in Fig. 3. When the gap between the l th and $(l+1)$ th surfaces is small relative to the wavelength, as in typical multilayer radomes without a TPE, the number of unmatched points is negligible, rendering the “broken state” insignificant. However, in RTA structures, the large air gap (relative to wavelength) between the

Table 1: Material and geometrical parameters of the radome and TPE

Layer	ϵ_r	$\tan\delta$	min/max thickness (mm)
Outer skin	3.8	0.03	1.2/2.4
Core	1.5	0.015	4.8/9.6
Inner skin	3.8	0.03	1.2/2.4
TPE	1.3	0.003	15

TPE and the radome leads to failure in identifying exit points for numerous triangles on the outer surfaces. This renders accurate EM field computation impossible (necessitating zero-field approximation), substantially compromising the algorithm's accuracy. Therefore, for a RTA system, the radome, the TPE, and the air gap between them cannot be treated as a single multilayer dielectric in the previous VSIE-MSI-MLFMA model. In contrast, the new mathematical model, derived in the preceding subsection, successfully decouples the TPE and radome into distinct regions, overcoming the difficulty in computing transmission matrices for RTAs inherent in the previous method.

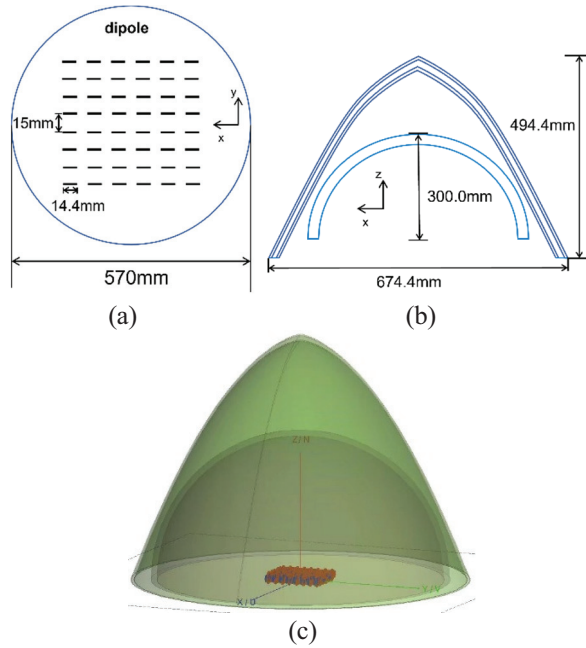


Fig. 4. Radome-enclosed 6×8 dipole array with a hemispherical TPE: (a) layout of the dipole array, (b) sideview of the radome and TPE, and (c) 3-D view of the entire RTA structure.

III. NUMERICAL RESULTS

Numerical examples are presented in this section to validate the proposed method. All computations are

performed on a workstation with a 2.4 GHz 16-core CPU and 384 GB RAM. Unless otherwise specified, the operating frequency is 10 GHz.

A. Radiation of a dipole antenna array hemispherical TPE-A-type sandwiched radome structure

The configuration of a 6×8 dipole array enclosed by a hemispherical TPE and an A-type sandwiched radome is shown in Fig. 4. Each half-wave dipole element has a length of 14.4 mm and the spacing between two adjacent elements is 15 mm. The array is positioned 7.5 mm above a PEC ground plane with a diameter of 570 mm that almost seals the bottom of the TPE. The radome and TPE have the overall dimensions of $674.4 \times 674.4 \times 494.4$ and $600.0 \times 600.0 \times 300.0$ mm³, respectively, and the material properties and thicknesses of the radome and TPE are listed in Table 1.

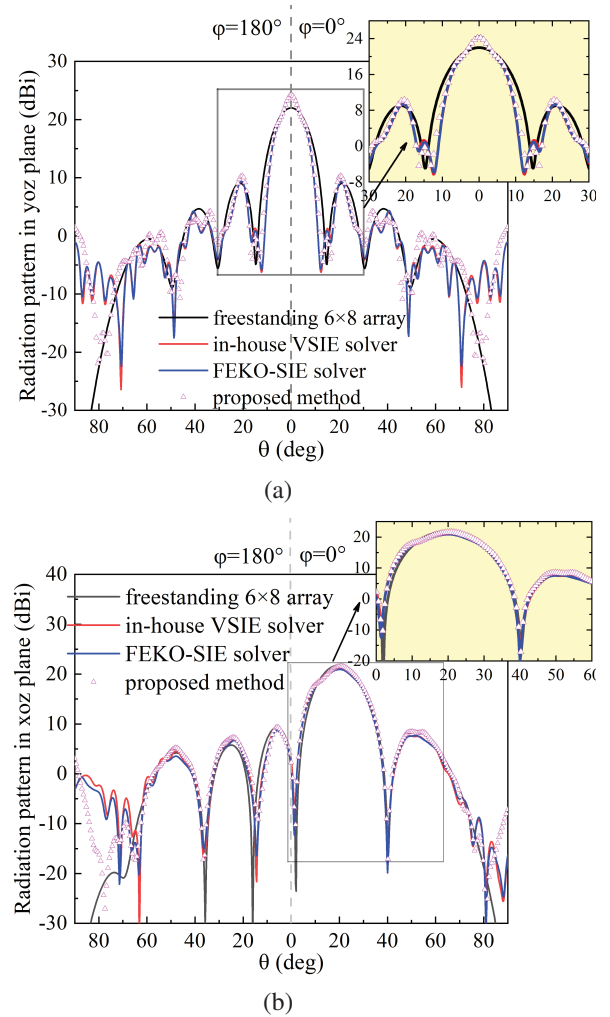


Fig. 5. Radiation patterns of the RTA structure shown in Fig. 4 in (a) yoz-plane (scanning angle is 0°) and (b) xozplane (scanning angle is 20°).

Figures 5 (a) and (b) show the radiation patterns of the RTA in the $yo\phi$ and $xo\phi$ planes when the main beam scans to 0° and 20° , respectively. The proposed method, an in-house full-wave VSIE solver [3], and the commercial FEKO surface integral equation (SIE) solver [26] are applied to calculate the RTA's radiation patterns for comparison purpose. Radiation distortion of the array caused by the radome and TPE is clearly observed in Figs. 5 (a) and (b), and the results of the proposed method agree well with the full-wave ones in the regions of main beam and nearby sidelobes (up to $\pm 60^\circ$) as the scanning angle changes. The accuracy of the proposed method is reduced near the horizontal plane ($\theta = 90^\circ$), which may be caused by the approximation used in (4). Table 2 presents the computational time and memory usage of the methods, compared with the two full-wave solvers, the proposed hybrid method reduces the computational time from 19,000 s to 250 s and the memory cost by more than 95%.

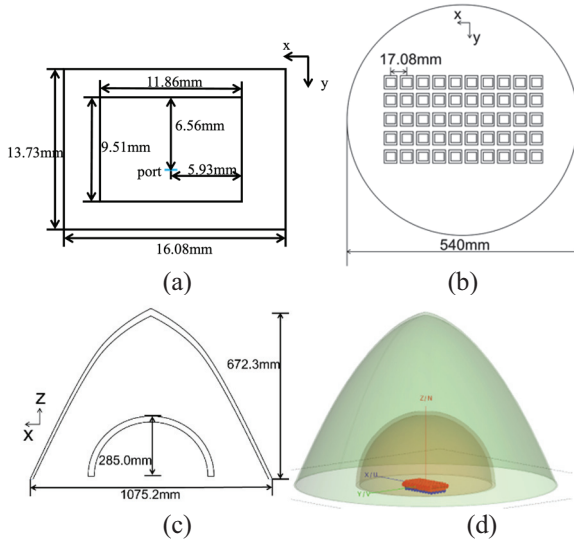


Fig. 6. Radome-enclosed 5×10 microstrip antenna array with a TPE: (a) sizes of the antenna element, (b) layout of the antenna array, (c) side view of the radome and TPE, and (d) 3-D view of the entire RTA.

B. Radiation of a microstrip antenna array hemispherical TPE single-layer radome

The second example is the computation of EM radiation of a 5×10 microstrip array (MSA) enclosed by a single-layer radome and a TPE. The geometry of the antenna element and the layout of the antenna array are shown in Fig. 6. The relative permittivity of the substrate used in the MSA is $2.2 + j0.01$, and the thickness and relative permittivity of the TPE are 15 mm and $2.0 + j0.0002$, respectively. The relative permittivity

Table 2: Comparison of time and memory usage of different methods

	Methods	No of Triangles	No of Tetrahedrons	Time (s)	Mem (GB)
1	VSIE solver	73,169	3,507,620	18,972	251.9
	FEKOSIE	617,080	–	19,413	240.6
	Proposed	1,797,246	–	250	3.1
2	in [23]	1,286,758	11,365	223	2.8
	VSIE solver	68,880	3,915,121	85,824	213.3
	Proposed	1,286,758	11,365	389	5.1

of the radome is $3.3 + j0.08$, and the thickness varies from 12.3 mm at the top to 6.6 mm at the base.

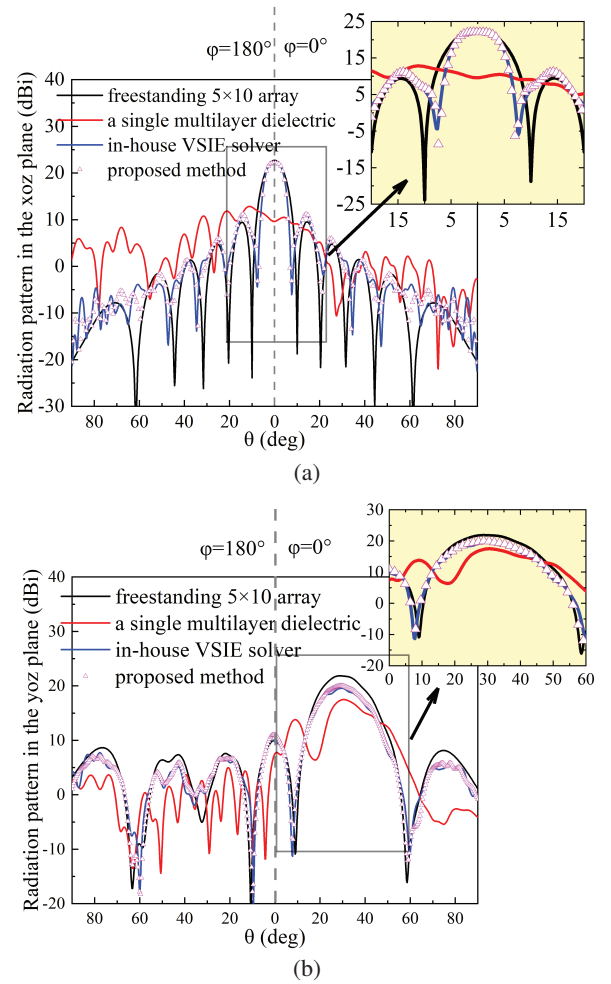


Fig. 7. Radiation pattern of the RTA structure shown in Fig. 6 in (a) $xo\phi$ plane (scanning angle is 0°) and (b) $yo\phi$ plane (scanning angle is 30°).

Figure 7 shows the RTA's radiation patterns calculated by the full-wave VSIE solver and the proposed method. Compared to the freestanding array's radiation pattern, the major effects of the radome and TPE are gain

reduction and increased ripples in the sidelobe regions. For two different scanning angles, the results of the proposed method match the full-wave solution in almost the entire upper half-space. Figure 7 also includes results (red lines) obtained by directly treating the TPE, air gap, and radome as a single multilayer dielectric and applying the method from [23]. As expected, these results significantly deviate from the full-wave solutions. This not only reveals the inherent limitations of the original method in analyzing RTAs but also demonstrates the effectiveness of the new approach. Table 2 confirms that the new method demands significantly less time and memory compared to the full-wave solver.

IV. CONCLUSION

In this paper, a fast and efficient method is proposed for EM analysis of the RTA structures. By applying the approximate equivalence principle twice, a new mathematical model is derived. This model separates the TPE and radome into distinct regions, overcoming the fundamental difficulty in the existing VSIE-MSI-MLFMA method when handling the large air gap between the TPE and radome. The hybrid method takes into account the mutual coupling among the antennas, TPE, and radome in an iterative manner, and all interactions among the three regions can be efficiently computed by virtue of the MLFMA. Numerical results demonstrate the method's good accuracy and significant reductions in computational time and memory requirements compared to full-wave solvers.

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Localized Plane Wave Approximation for Bodies of Revolution in Vegetation Scattering

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Abstract – Large computational electromagnetic problems for scattering from forest canopies in L-band (1–2 GHz) typically require modeling trees by a collection of lossy, dielectric cylinders and disks using Multiple Body of Revolution (MBOR) scattering techniques. MBOR techniques are desired for their computational efficiency compared to the 3-D Method of Moments (MoM). Within the vegetation environment associated with forest canopy, BORs are weakly coupled and, thus, approximations may be made to improve computational efficiency of MBOR scattering. This paper develops an efficient method to calculate the scattered fields from a lossy, finite length, dielectric cylinder, illuminated by a small current source. A small current source is representative of those on an adjacent BOR in MBOR scatter. Computational solutions to this problem exist, but those solutions are complicated and computationally expensive. Using the proposed method, a BOR is discretized into a series of discs such that far field conditions are a function of BOR radius rather than BOR length. These field conditions define the Localized Plane Wave Approximation (LPWA), which provides foundation for a more system specific MBOR scattering methodology, where BORs are harmonically independent of each other. For LPWA validation, the LPWA is compared to both analytical and computational solutions. The approximation shows good agreement within the constraints of the underlying assumptions. Finally,

the method improves computational efficiency by more than an order of magnitude.

Index Terms – Electromagnetic propagation in absorbing media, Method of Moments, microwave propagation, multiple body of revolution, remote sensing, scattering.

I. INTRODUCTION

Understanding microwave propagation through vegetation and forest environments is an important research topic with regards to remote sensing applications, communications, and target discrimination for military applications within these environments. For remote sensing applications, P/L-band frequencies are frequently used due to their forest canopy penetration and ionosphere penetration [1–3]. With respect to communications, the frequency bands can vary from HF to Ka-band.

Within these bands, there are various frequency regimes of interest for vegetative Radio Frequency (RF) scattering where different scattering models are employed. Depending on the application and frequency band of interest, these models generally are of two classes: radiative transfer methods [3, 4] or Distorted Born Theory (DBA) [3, 5]. For transport theory, one uses the scatterer properties to construct an extinction and

phase matrix which are used in the transport equation. In transport theory, scattered powers are added incoherently, and scatterers are assumed to be in the far field of each other. For DBA theory, the scatterer properties are used in conjunction with the Foldy-Lax approximation [3, 6–8] to find the mean field in the scattering medium [9]. From this mean field, an effective medium is derived. The scatterers are then embedded in the effective medium, and the scattered fields are computed and summed coherently. Both single and higher order scattering in the effective medium can be considered.

In particular, L-band is of interest for remote sensing applications that examine backscatter from forest environments such as the Advanced Land Observing Satellite 4 (ALOS-4) launched in July 2025 by the Japanese Space Agency (JAXA) [10] and the NASA/NISAR joint US/India L/C band Synthetic Aperture RADAR (SAR) [11] that was launched on 30 July 2025. Due to the complicated nature of forest environments in L-band and its importance to communications and remote sensing, examination of Lband has been studied extensively [12, 13].

A. Background

Forest environments are often modeled as dielectric, lossy, Bodies of Revolution (BOR) such as cylinders, ellipsoids, and disks [13–15]. These representations cover a wide range of vegetation including tree trunks, needles, and leaves. Classical geometries are specifically chosen due to their relative simplicity and computational efficiency. Additionally, the use of classical shapes, as a discrete representation, provides a path towards simpler representations of forest environments, which by the definition of their nature appear stochastic. Upon examination of this environment and the classical shapes, the problem is well posed to combine both effective medium modeling and Multiple Body of Revolution (MBOR) scattering.

Previously, the DBA model of a singular Caucasian Fir tree was employed by Lang and Landry [16, 17]. The DBA model [18] entails discretizing a tree into cylindrical dielectric scatterers. The tree is then enclosed in a bounding cone, and the average medium dielectric constant is calculated via Foldy-Lax Theory over a series of layers within the cone via the mean field. An example bounding dielectric cone model can be viewed in Fig. 1. The 1st and, if desired, 2nd order scattering is calculated and coherently summed.

For the 1998 IGARSS paper by Lang et al. [16], the simulated monostatic RADAR Cross Section (RCS) results trended with the measured data, but the simulated results under-predicted the monostatic RCS by approximately 4 dB. Due to the RCS underprediction, second order scattering effects were introduced via

Fresnel Double Scattering (FDS) where the scattering cross sections were predicted by the Discrete Dipole Approximation (DDA) [19, 20]. The scattering methods within the FDS-DDA model, however, underpredicted the Caucasian Fir tree RCS as well. Based on previous FEKO 3-D Method of Moments (MoM) simulations, the DDA has limitations when applied to our problem. This motivates an MBOR scattering approach.

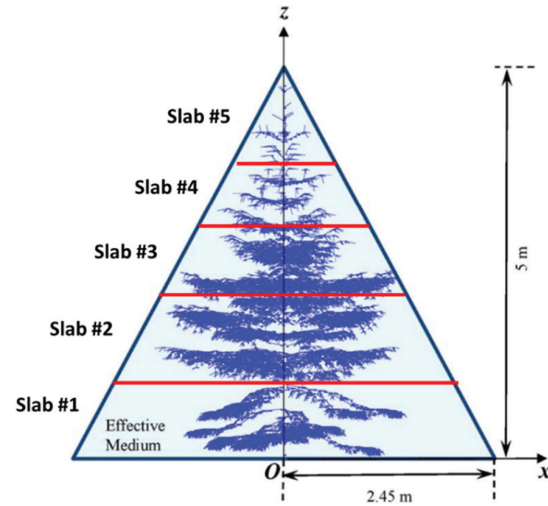


Fig. 1. A 5 m Caucasian Fir tree bounded in a cone that is discretized into layers to calculate the effective medium [19].

Overall, BOR and MBOR problems have been extensively studied over the previous 70 years due to their computational efficiency compared to that of the 3D MoM [21–26]. Recently, MBOR scattering for vegetation has been evaluated by Huang et al. where the Foldy-Lax multiple scattering equations are combined with an MBOR approach, but the model is only applied to thin cylinders where the radius is much less than a wavelength, which is representative of grass and wheat [27].

Additionally, Gu et al. has recently applied a hybrid 3D MoM approach via the combination of the T-Matrix vector cylindrical wave expansions of the scatterers and the Foldy-Lax multiple scattering equation to calculate the total scattered field of multiple trees in close proximity to one another [28]. Due to the dense vegetation environment found in the 5 m Caucasian Fir tree of interest [16, 17], the T-Matrix approach is not ideal, i.e., the cylindrical or spheroidal surfaces of the large aspect ratio cylinders would produce significant overlap regions throughout the tree model. Additionally, the T-Matrix solution does not neatly map within the existing FL/DBA model due to both the effective medium surrounding the scatterers and the multiple rays propagating within the multi-layer medium.

The method for calculating BOR scattered fields, described here, addresses certain computational problems, not well posed for solutions using T-Matrix based methods. First, unlike T-Matrix methods, the addition of spheroidal surfaces, bounding the BOR, are not required for scatterer representation. This reduction in method complexity results by imposing the volumetric condition of non-overlapping BORs, a reasonable approximation for vegetation scattering in the far-field.

Second, the method is structured formally for modeling scattered fields using effective-medium propagation constants. This formal aspect of the methodology, which should be noted for its general application, is not within the scope of this study.

In section IV, constraints are imposed for computational stability of the method, which concerns a range of cylinder aspect ratios, cylinder size to wavelength ratios, and source locations.

B. Objective

Our overarching objective – not the topic of this paper but described here to provide clarity to the reader – is to find the scattering amplitude of two lossy dielectric cylinders that are arbitrarily oriented with respect to each other. Each cylinder has a radius that is small compared to its length; the cylinder radius size can vary up to several wavelengths. The cylinders can be in the near field of each other but are not tightly interacting, i.e., only the first and second order scatter terms are important. The first order scattering amplitude is the bistatic scatter from each individual scatterer. The first term of the second order scattering amplitude is the scatter from the first cylinder onto the second cylinder and vice versa. The computation of the second order scatter consists of two parts: first compute the electric and magnetic surface currents on the first cylinder and, then, decompose the surface currents into dipoles. Finally, find the scattered field from the second cylinder in the presence of a dipole.

The focus of this paper is to develop a computationally efficient method to calculate the scattered fields from a lossy, finite length, dielectric BOR, e.g., a cylinder, that is illuminated by a small current source, i.e., a Hertzian dipole, in support of the goal described above. This modification expands the applicable parameter space compared to the classical thin cylinder approximation. Additionally, the technique avoids using spherical harmonics, the Mautz and Harrington approach [29, 30], by treating the incident field due to the infinitesimal current source as a local plane wave upon each BOR segment. Thus, the method is described as a Localized Plane Wave Approximation (LPWA) for arbitrary wave incidence on BORs. Although studied here in the context of vegetation scattering, the technique may be applied to

any MBOR scattering problem where the BORs are not strongly coupled.

This paper will first develop a derivation for the approximation on a BOR, which also serves as a relatively direct computational encoding for this MoM implementation. An example using an electric Hertzian dipole is derived. Next, the paper validates the results of the LPWA for BORs against the full Fourier series solution, the 3-D MoM in FEKO, and an analytical solution for thin cylinders.

II. BODY OF REVOLUTION METHOD OF MOMENTS

To evaluate the scattered fields from a BOR, the Body of Revolution Method of Moments (BOR-MoM) is first developed to solve for the induced electric and fictitious magnetic surface currents due to an incident field. This derivation was developed by Mautz and Harrington [31]. The software implementation of the BOR-MoM used in this analysis was developed and validated by de Matthaeis and Lang [32, 33].

As described in [34], we restate here, for completeness, the formal mathematical foundation of our development, which is as follows. A BOR is the rotation of a generating curve, planar arc C , about an axis, as shown in Fig. 2. In this study, the axis of rotation will be the z -axis of a Cartesian coordinate system, and the generating curve will only exist in the xz -plane whereby the definition, the curve, C , is rotated 360° around the z -axis. In turn, the surface, S , formed by the BOR, will be the interface separating free space and the scatterer with permittivity and permeability of $\epsilon = \epsilon_r \epsilon_0$ and $\mu = \mu_r \mu_0$, respectively.

The coordinate unit vectors are defined in Fig. 2 by

$$\hat{\tau} = \sin \psi \cos \phi \hat{x} + \sin \psi \sin \phi \hat{y} + \cos \psi \hat{z}, \quad (1)$$

$$\hat{n} = \cos \psi \cos \phi \hat{x} + \cos \psi \sin \phi \hat{y} - \sin \psi \hat{z}, \quad (2)$$

$$\hat{\phi} = -\sin \phi \hat{x} + \cos \phi \hat{y}, \quad (3)$$

where $\hat{\tau}$ is the unit vector tangential to the generating arc, $\hat{n}$ is the unit vector normal to the generating arc, $\hat{\phi}$ is the unit vector in the azimuthal direction, and ψ is the angle between the z -axis and $\hat{\tau}$ vector.

The tangential Electric Field Integral Equations (EFIE) used to evaluate the surface currents along the surface, S , are defined by

$$\begin{aligned} \frac{E_t(\mathbf{r})}{2} &= E_t^i(\mathbf{r}) \\ &- j\omega\mu_0(\mathbf{I} - \hat{n}\hat{n}) \cdot \text{PV} \int_{S^+} \underline{\mathbf{G}}^{(0)}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{J}_S(\mathbf{r}') dS' \\ &- (\mathbf{I} - \hat{n}\hat{n}) \cdot \text{PV} \int_{S^+} \underline{\mathbf{K}}^{(0)}(\mathbf{r}, \mathbf{r}') \times \mathbf{M}_S(\mathbf{r}') dS', \end{aligned} \quad (4)$$

$$\frac{\mathbf{E}_t(\mathbf{r})}{2} = j\omega\mu(\mathbf{I} - \hat{\mathbf{n}}\hat{\mathbf{n}}) \cdot \text{PV} \int_{S^-} \mathbf{G}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{J}_s(\mathbf{r}') dS' - (\mathbf{I} - \hat{\mathbf{n}}\hat{\mathbf{n}}) \cdot \text{PV} \int_{S^-} \mathbf{K}(\mathbf{r}, \mathbf{r}') \times \mathbf{M}_s(\mathbf{r}') dS', \quad (5)$$

where $\mathbf{E}_t(\mathbf{r})$ is the total, tangential, electric field on the surface, $\mathbf{E}_t^i(\mathbf{r})$ is the tangential electric field due to the incident wave, ω is the angular frequency, $\mathbf{I}$ is the unit dyadic, PV denotes a principal value of the integral, S^+ is the outer surface of S , S^- is the inner surface of S , $\mathbf{G}^{(0)}(\mathbf{r}, \mathbf{r}')$ is the electric dyadic Green's function in free space, $\mathbf{G}(\mathbf{r}, \mathbf{r}')$ is the electric dyadic Green's function in the medium, $\mathbf{K}^{(0)}(\mathbf{r}, \mathbf{r}')$ is the magnetic dyadic Green's function in free space, $\mathbf{K}(\mathbf{r}, \mathbf{r}')$ is the magnetic dyadic Green's function in the medium, $\mathbf{J}_s(\mathbf{r}')$ are the induced electric surface currents, and $\mathbf{M}_s(\mathbf{r}')$ are the induced, fictitious magnetic surface currents [23].

The incident fields and induced sources within the EFIE can then be split into τ and φ vector components

$$\mathbf{E}_t^i(\mathbf{r}) = \mathbf{E}_\tau^i(l, \varphi) \hat{\tau} + \mathbf{E}_\varphi^i(l, \varphi) \hat{\varphi}, \quad (6)$$

$$\mathbf{J}_s(\mathbf{r}) = J_\tau(l, \varphi) \hat{\tau} + J_\varphi(l, \varphi) \hat{\varphi}, \quad (7)$$

$$\mathbf{M}_s(\mathbf{r}) = M_\tau(l, \varphi) \hat{\tau} + M_\varphi(l, \varphi) \hat{\varphi}, \quad (8)$$

where $\mathbf{r} \in S$, and the location, $\mathbf{r}$, may be expanded as a function of the azimuthal angle, φ , and the location along the generating arc, l , i.e., $\mathbf{r} = \mathbf{r}(l, \varphi)$.

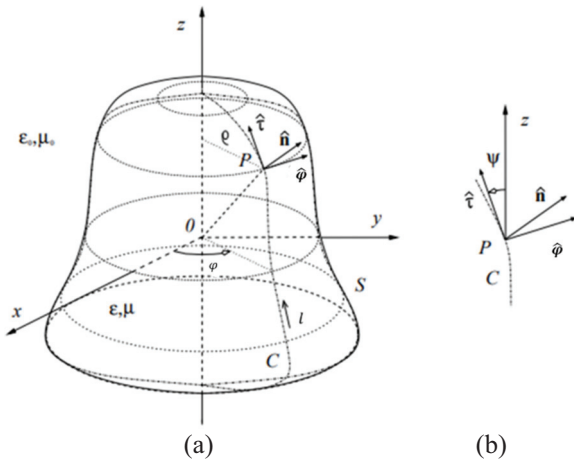


Fig. 2. A body of revolution where (a) is the three-dimensional model with the relevant coordinate system definitions and (b) is the coordinate definitions along the surface of the BOR.

A. Fourier series expansion

To evaluate the induced surface currents on the generating arc, C , the variables are expanded in Fourier series around the z -axis such that (4) and (5) needs to be only evaluated over the generating arc for a series

of harmonics. The resulting incident field and induced sources are

$$\mathbf{E}_{t,p}^i(\mathbf{r}) = \sum_{n=-\infty}^{\infty} \mathbf{E}_{t,p,n}^i e^{jn\varphi}, \quad (9)$$

$$\mathbf{J}_{s,p}(\mathbf{r}) = \sum_{n=-\infty}^{\infty} \mathbf{J}_{s,p,n} e^{jn\varphi}, \quad (10)$$

$$\mathbf{M}_{s,p}(\mathbf{r}) = \sum_{n=-\infty}^{\infty} \mathbf{M}_{s,p,n} e^{jn\varphi}, \quad (11)$$

where $p \in \{\tau, \varphi\}$. The Fourier series coefficients are defined as

$$C_{p,n} = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(\varphi) e^{-jn\varphi} d\varphi, \quad (12)$$

where $C_{p,n} = \mathbf{E}_{t,p,n}^i, \mathbf{J}_{s,p,n}$, or $\mathbf{M}_{s,p,n}$, $f(\varphi) = \mathbf{E}_{t,p}^i, \mathbf{J}_{s,p}$, or $\mathbf{M}_{s,p}$, and $p \in \{\tau, \varphi\}$.

B. Method of Moments expansion

The electric and magnetic surface currents can be discretized via the MoM such that

$$\mathbf{J}_{s,\tau,n}(l) = \sum_{m=1}^M \mathbf{J}_{s,\tau,m,n}(l) = \sum_{m=1}^M \frac{\Lambda_{m,n}^\tau \mathbf{P}_m(l)}{\rho_m}, \quad (13)$$

$$\mathbf{J}_{s,\varphi,n}(l) = \sum_{m=1}^M \mathbf{J}_{s,\varphi,m,n}(l) = \sum_{m=1}^{M+1} \Lambda_{m,n}^\varphi \mathbf{P}_m(l), \quad (14)$$

$$\mathbf{M}_{s,\tau,n}(l) = \sum_{m=1}^M \mathbf{M}_{s,\tau,m,n}(l) = \sum_{m=1}^M \Omega_{m,n}^\tau \mathbf{P}_m(l), \quad (15)$$

$$\mathbf{M}_{s,\varphi,n}(l) = \sum_{m=1}^M \mathbf{M}_{s,\varphi,m,n}(l) = \sum_{m=1}^{M+1} \Omega_{m,n}^\varphi \mathbf{P}_m(l), \quad (16)$$

where $\mathbf{J}_{s,\tau,m,n}, \mathbf{J}_{s,\varphi,m,n}, \mathbf{M}_{s,\tau,m,n}$, and $\mathbf{M}_{s,\varphi,m,n}$ are the surface currents per harmonic for their respective orthogonal and tangential vectors on each discretized segment of the generating arc, m is the segment number, M is the total number of segments, $\mathbf{P}_m(l)$ is the chosen basis function expansion, ρ_m is the distance from the z -axis, and $\Lambda_{m,n}^\tau, \Lambda_{m,n}^\varphi, \Omega_{m,n}^\tau$, and $\Omega_{m,n}^\varphi$ are the basis function coefficients. Note that the basis function coefficients are unique for each harmonic, n , and each segment of the generating arc, m . For this implementation, triangular and rectangular basis functions are chosen [33].

Testing functions are now applied using the symmetric product

$$\langle \chi(l), \alpha(l) \rangle = \int_{-\infty}^{\infty} \chi(l) \cdot \alpha(l) dl. \quad (17)$$

The testing functions chosen are both Dirac Delta functions and rectangular pulse functions [31–33].

Interaction matrix and incident field matrix can be evaluated for a single harmonic on the BOR's generating arc. The equivalent sources are solved by inverting the interaction matrix

$$[I_n] = [Z_n]^{-1} [V_n], \quad (18)$$

where Z_n is the interaction matrix. The induced surface currents per harmonic for both the τ and φ directions are described by

$$I_n = \begin{bmatrix} J_{1,n} \\ J_{2,n} \\ \vdots \\ J_{N,n} \\ M_{1,n} \\ M_{2,n} \\ \vdots \\ M_{N,n} \end{bmatrix}, \quad (19)$$

and the incident, tangential electric and magnetic fields per harmonic and for both the τ and φ directions, are described by

$$V_n = \begin{bmatrix} E_{1,n}^i \\ E_{2,n}^i \\ \vdots \\ E_{M,n}^i \end{bmatrix}. \quad (20)$$

Referring to (18)–(20), the general procedure is such that $[Z_n]^{-1}$ and $[V_n]$ are calculated a priori, and I_n is solved for via (18).

III. INCIDENT FIELD EXPANSION

To generate a matrix formulation for the BOR MoM, the tangential, incident electric field is expanded into Fourier series as

$$\mathbf{E}^i(\mathbf{r}) = \sum_{n=-\infty}^{\infty} \mathbf{E}_n^i(l) e^{jn\varphi}, \quad (21)$$

where n is the harmonic, φ is the azimuthal angle around the z -axis, and $\mathbf{r} = \mathbf{r}(l, \varphi)$ is the coordinate locations of the generating arc. Note that the electric field will be kept as a vector, and the subscript, t , will be dropped from the tangential, electric field vector. These subtle changes simplify the notation throughout this derivation. Using the definition from (12), the Fourier series coefficients are defined by

$$\mathbf{E}_n^i(l) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \mathbf{E}^i(l, \varphi') e^{-jn\varphi'} d\varphi', \quad (22)$$

where l represents the location along the generating arc, the integral bounds are between $[-\pi, \pi]$, and $\mathbf{r} = \mathbf{r}(l, \varphi)$. Note that the integral bounds can also be over $[0, 2\pi]$, but integrating over $[-\pi, \pi]$ allows for the removable singularity at zero to be integrated via the principal value of the integrals.

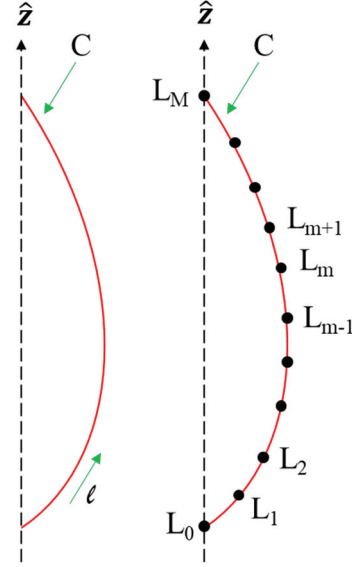


Fig. 3. Discretized generating arc for a BOR.

Furthermore, the generating arc is discretized into a series of segments such that

$$\mathbf{E}^i(\mathbf{r}) = \sum_{m=1}^M \mathbf{E}_m^i(l, \varphi), \quad (23)$$

where $\mathbf{E}_m^i$ denotes the incident field upon a specific segment of the discretized BOR and M is the total number of segments along the generating arc of the BOR, which is shown in Fig. 3. Note that $\mathbf{E}_m^i(l, \varphi)$ is unique for each segment of the BOR. Expanding $\mathbf{E}_m^i(l, \varphi)$ from (23) into a Fourier Series about the φ axis, (21) yields

$$\mathbf{E}_m^i(l, \varphi) = \sum_{n=-\infty}^{\infty} \mathbf{E}_{m,n}^i(l) e^{jn\varphi}. \quad (24)$$

The total incident electric field can then be found by substituting (24) into (23) which yields

$$\mathbf{E}^i(\mathbf{r}) = \sum_{m=1}^M \sum_{n=-\infty}^{\infty} \mathbf{E}_{m,n}^i(l) e^{jn\varphi}. \quad (25)$$

The Fourier series coefficient in (25), $\mathbf{E}_{m,n}^i(l)$, can be determined uniquely for each segment of the BOR generating arc by applying the Fourier Series definition from (12) to (25). Accordingly, the resultant Fourier

series coefficient for a single segment and single harmonic is

$$E_{m,n}^i(l) = \frac{1}{2\pi} \int_{-\pi}^{\pi} E_m^i(l, \varphi') e^{-jn\varphi'} d\varphi'. \quad (26)$$

The symmetric product of (26) with the testing functions results in the Fourier Series coefficients per harmonic in (20).

At this point in the development, an arbitrary incident field is still considered. For the exact method, derived by Harrington and Glisson [29, 30], the known, incident electric field, e.g., an incident, Hertzian dipole, is substituted into (26). The Fourier series coefficient for a single segment of the generating arc is then computed by performing the integral around the BOR z -axis with respect to φ .

Upon examining the structure of the 5 m Caucasian Fir tree, the full expansion method is not required due to the size distribution of the branches and trunk sections. The full expansion method, however, can be applied to the MBOR scattering problem. In this case, the induced surface currents from the 1st BOR would be treated as equivalent electric and magnetic Hertzian dipoles. Then those dipoles would be incident upon the 2nd BOR where the full Fourier Series integration is performed. The Full Expansion method can be used to solve for the scattered fields for any size cylinder with an arbitrary distance between the cylinders, i.e., examination of trees larger than 5 m.

In general, Harrington and Mautz's method is a relatively direct approach using spherical harmonics and reciprocity; however, the method was not designed as an MBOR scattering technique. The method requires integration around the BOR φ -axis to relate the current source to the incident field upon the BOR. The introduction of multiple BORs results in harmonic dependence between the BORs and requires a large number of integrations for MBOR scattering. Thus, the full expansion method suffers a large computational burden for MBOR problem sets.

IV. LOCALIZED PLANE WAVE APPROXIMATION

The following development will derive the LPWA for a vertical Hertzian dipole located on the x -axis and incident on a BOR. Formally, the method can be applied to an arbitrarily located and oriented dipole but, for simplicity's sake, the dipole is oriented vertically on the x -axis. For application to an arbitrarily oriented dipole, the problem can be treated in a straightforward manner using local and global coordinate systems where the same approximations with respect to both the BOR radius and discretization size will still apply.

Let us consider an incident, analytical, spherical wavefront from a vertically oriented, electric, Hertzian

dipole positioned along the x -axis, as shown in Fig. 4, whose source function is described by

$$\mathbf{J}(\mathbf{r}) = J_0 \delta(\mathbf{r} - \mathbf{r}_d) \hat{\mathbf{z}} = J_0 \delta(x - x_d) \delta(y) \delta(z) \hat{\mathbf{z}}, \quad (27)$$

where J_0 is the current density, $\delta(\mathbf{r})$ is the Dirac Delta function, $\mathbf{r}$ is the field location, and $\mathbf{r}_d = x_d \hat{\mathbf{x}}$ is the source location.

To reduce the field complexity, the Hertzian dipole can be represented as a source at the origin in a local coordinate system. Following this approach, the source-function simplifies to

$$\mathbf{J}(\mathbf{r}) = J_0 \delta(\mathbf{r}) \hat{\mathbf{z}}_d, \quad (28)$$

where subscript d denotes that the variable is referenced to the dipole source location.

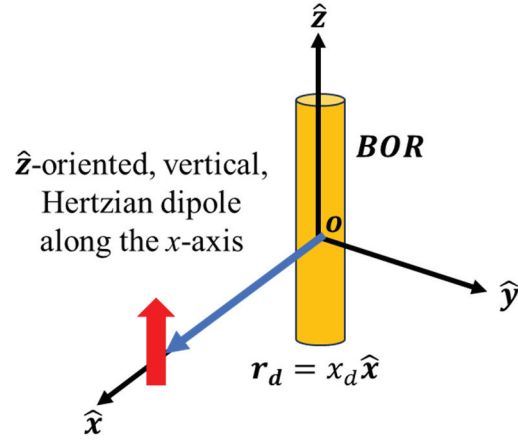


Fig. 4. Vertically polarized, electric, Hertzian dipole centered on the x -axis incident on a BOR.

The electric field due to the source function in (28) has the general form of

$$E_d^i(\mathbf{r}, \mathbf{r}_d) = E_{0,d}^i \frac{e^{-jk_0 R_d}}{R_d}, \quad (29)$$

where

$$\mathbf{E}_{0,d}^i = E_{r_d}^i \hat{\mathbf{r}}_d + E_{\theta_d}^i \hat{\boldsymbol{\theta}}_d, \quad (30)$$

and

$$E_{r_d}^i = \frac{j\eta_0 k_0 J_0}{4\pi} \left(\frac{2}{jk_0 R_d} + \frac{2}{(jk_0 R_d)^2} \right) \cos \theta_d, \quad (31)$$

$$E_{\theta_d}^i = \frac{j\eta_0 k_0 J_0}{4\pi} \left(1 + \frac{j}{jk_0 R_d} + \frac{1}{(jk_0 R_d)^2} \right) \sin \theta_d, \quad (32)$$

where η_0 is the free space wave impedance, k_0 is the free space wavenumber, θ_d is the polar angle in the local, spherical coordinates, and $R_d = |\mathbf{r} - \mathbf{r}_d|$.

A. Expansion over a BOR segment

For the following development, the spherical wave described by (28)–(32) will be adjusted such that the vectors are positional vectors with reference to a BOR segment, m (see Fig. 5). This implies that the vectors can be translated from one coordinate system to another since they are referenced to an (x, y, z) location. This avoids local and global coordinate transformation mathematics within the derivation.

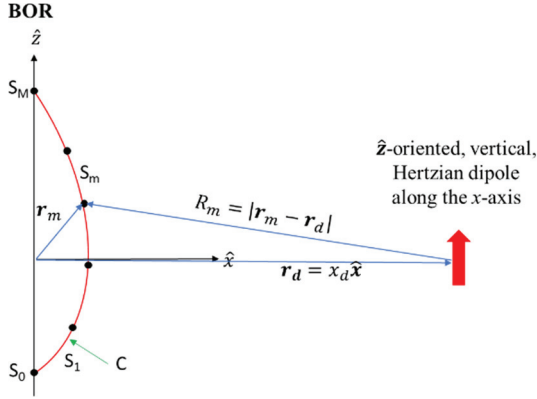


Fig. 5. Diagram depicting a spherical wave source incident on a local BOR segment.

To generate the coefficients in (20) and to use the positional vectors, (29) must first be discretized across the BOR using the definition from (23). Substituting (29) into (23) yields

$$\mathbf{E}_d^i(\mathbf{r}_m, \mathbf{r}_d) = \sum_{m=1}^M \mathbf{E}_{0,m}^i(\mathbf{r}_m, \mathbf{r}_d) \frac{e^{-jk_0 R_m}}{R_m}, \quad (33a)$$

$$\mathbf{E}_{0,m}^i(\mathbf{r}_m, \mathbf{r}_d) = \mathbf{E}_{0,d}^i|_{\mathbf{r}_m}, \quad (33b)$$

where $R_m = |\mathbf{r}_m - \mathbf{r}_d|$.

The BOR discretized segment length can then be characterized as

$$\Delta \mathbf{r}_m = \mathbf{r}_{m+1} - \mathbf{r}_m. \quad (34)$$

For small $\Delta \mathbf{r}_m$, $R(\mathbf{r}_m + \Delta \mathbf{r}_m, \mathbf{r}_d)$ can be expanded as

$$\begin{aligned} R(\mathbf{r}_m + \Delta \mathbf{r}_m, \mathbf{r}_d) &= \sqrt{(\mathbf{r}_m + \Delta \mathbf{r}_m - \mathbf{r}_d) \cdot (\mathbf{r}_m + \Delta \mathbf{r}_m - \mathbf{r}_d)} \\ &= \sqrt{(\mathbf{R}_m + \Delta \mathbf{r}_m) \cdot (\mathbf{R}_m + \Delta \mathbf{r}_m)} \\ &= \sqrt{R_m^2 + (\Delta \mathbf{r}_m)^2 + 2\mathbf{R}_m \cdot \Delta \mathbf{r}_m} \\ &= R_m \sqrt{1 + \left(\frac{\Delta \mathbf{r}_m}{R_m}\right)^2 + \frac{2\mathbf{R}_m \cdot \Delta \mathbf{r}_m}{R_m^2}}. \end{aligned} \quad (35)$$

Assume a small discretization and $|\frac{\Delta \mathbf{r}_m}{R_m}| \ll 1$. Also, note that

$$[1 + w]^{1/2} = 1 + w/2 - w^2/8 + O(w^3), \quad (36a)$$

$$w = \left(\frac{\Delta \mathbf{r}_m}{R_m}\right)^2 + \frac{2\mathbf{R}_m \cdot \Delta \mathbf{r}_m}{R_m^2}. \quad (36b)$$

Using the definition from (36) and assuming $|w| \ll 1$, (35) becomes

$$\begin{aligned} R(\mathbf{r}_m + \Delta \mathbf{r}_m, \mathbf{r}_d) &= R_m \left[1 + \frac{\hat{\mathbf{R}}}{R_m} \cdot \Delta \mathbf{r}_m \right] \\ &= R_m + \hat{\mathbf{R}} \cdot \Delta \mathbf{r}_m + O\left(\frac{\Delta \mathbf{r}_m^2}{R_m}\right), \end{aligned} \quad (37)$$

where $\hat{\mathbf{R}} = \mathbf{R}_m/R_m$. The higher order terms can be neglected if the following conditions are satisfied

$$\frac{k_0 |\Delta \mathbf{r}_m|^2}{2R_m} \sin^2 \alpha \ll 1, \quad (38)$$

where α is the angle between $\hat{\mathbf{R}}$ and $\Delta \mathbf{r}_m$ and $|\frac{\Delta \mathbf{r}_m}{R_m}| \ll 1$. For $N \gg 1$, $\Delta \mathbf{r}_m$ is small and, typically, the BOR-MoM satisfies this condition using standard discretization sizes, i.e., less than or equal to a $\lambda/10$ mesh size.

Substituting (37) into (29) yields

$$\mathbf{E}_d^i(\mathbf{r}_m + \Delta \mathbf{r}_m, \mathbf{r}_d) = \mathbf{E}_0^i \frac{e^{-jk_0(R_m + \hat{\mathbf{R}} \cdot \Delta \mathbf{r}_m)}}{R_m + \hat{\mathbf{R}} \cdot \Delta \mathbf{r}_m}. \quad (39)$$

Note that $R_m + \hat{\mathbf{R}} \cdot \Delta \mathbf{r}_m \approx R_m$ for small $\Delta \mathbf{r}_m$. This condition only applies to the denominator because the incident field phase, $e^{-jk_0(R_m + \hat{\mathbf{R}} \cdot \Delta \mathbf{r}_m)}$, has a high degree of sensitivity, whereas the incident field magnitude scaling, $\frac{1}{R_m + \hat{\mathbf{R}} \cdot \Delta \mathbf{r}_m}$, does not have a high sensitivity to error.

Expanding (39) and recognizing that $\hat{\mathbf{R}} = \hat{\mathbf{k}}_i$ yields

$$\mathbf{E}^i(\mathbf{r}_m + \Delta \mathbf{r}_m, \mathbf{r}_d) = \mathbf{E}_{0,m}^i \frac{e^{-jk_0 R_m}}{R_m} e^{-jk_0 \hat{\mathbf{k}}_i \cdot \Delta \mathbf{r}_m}, \quad (40)$$

where $\hat{\mathbf{k}}_i$ is the incident field unit vector. Equation (40) implies that the spherical source appears as a plane wave across the segment, i.e., the full expansion method is valid so long as (38) is valid. To solve for the coefficients in (20), substitute (40) into (26), yielding

$$\begin{aligned} \mathbf{E}_{m,n}^i(\mathbf{r}_m + \Delta \mathbf{r}_m, \mathbf{r}_d) &= \frac{1}{2\pi} \int_{-\pi}^{\pi} \left[\mathbf{E}_{0,m}^i \frac{e^{-jk_0 R_m}}{R_m} e^{-jk_0 \hat{\mathbf{k}}_i \cdot \Delta \mathbf{r}_m} e^{-jn\varphi} \right] d\varphi. \end{aligned} \quad (41)$$

Recognize that the field coefficients of an arbitrarily sized BOR can be solved for via (41) by integrating around the BOR φ -axis, so long as the BOR generating arc discretization is sufficiently small and the numerical integration converges.

B. Expansion around the BOR φ -axis

At this point within the derivation, an arbitrarily sized BOR may be analyzed by integrating around the BOR φ axis, (41), for each individual segment, m . However, if thin BORs are considered, further approximations may be applied. Examining (41), it is desirable for computational efficiency to extract the spherical wave term, $\mathbf{E}_{0,m}^i \frac{e^{-jk_0 R_m}}{R_m}$, from the integral. This would result in a localized, analytic plane wave solution over the segment, m .

Upon examination of the BOR cross section in Fig. 6, the BOR cross section at the local segment, m , is a circle by definition. The positional vector at the BOR edges can be described by

$$R(\mathbf{r}_m + \Delta\boldsymbol{\rho}_m(\rho, \varphi), \mathbf{r}_d) = |\mathbf{r}_m + \Delta\boldsymbol{\rho}_m(\rho, \varphi) - \mathbf{r}_d|, \quad (42)$$

where $\Delta\boldsymbol{\rho}_m(\rho, \varphi)$ is the vector between $\mathbf{r}_m$ and the BOR edge, and ρ is the radius of the BOR at segment m . A full expansion of $\Delta\boldsymbol{\rho}_m(\rho, \varphi)$ is described in [33].

Only the $\Delta\boldsymbol{\rho}_m(\rho, \varphi)$ term in (42) varies with respect to φ . The $\mathbf{r}_m$ term is a positional vector on the BOR generating arc, segment m . To extract the spherical wavefront from the integration, $\Delta\boldsymbol{\rho}_m(\rho, \varphi)$ must be small, therefore ρ must be small.

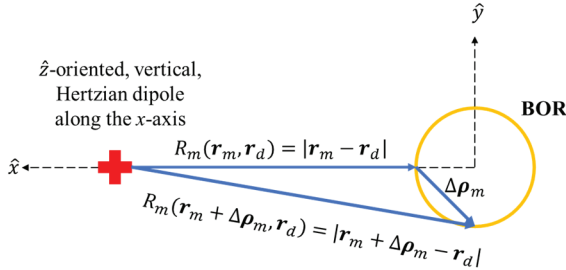


Fig. 6. Spherical wave source incident on a local BOR segment.

Using this assumption, a similar development to (35)(40) can be used, i.e.,

$$\begin{aligned} R(\mathbf{r}_m + \Delta\boldsymbol{\rho}_m, \mathbf{r}_d) &= \sqrt{(\mathbf{r}_m + \Delta\boldsymbol{\rho}_m - \mathbf{r}_d) \cdot (\mathbf{r}_m + \Delta\boldsymbol{\rho}_m - \mathbf{r}_d)} \\ &= \sqrt{(\mathbf{R}_m + \Delta\boldsymbol{\rho}_m) \cdot (\mathbf{R}_m + \Delta\boldsymbol{\rho}_m)} \\ &= \sqrt{R_m^2 + (\Delta\boldsymbol{\rho}_m)^2 + 2\mathbf{R}_m \cdot \Delta\boldsymbol{\rho}_m} \\ &= R_m \sqrt{1 + \left(\frac{\Delta\boldsymbol{\rho}_m}{R_m}\right)^2 + \frac{2\mathbf{R}_m \cdot \Delta\boldsymbol{\rho}_m}{R_m^2}}, \end{aligned} \quad (43)$$

where $R_m = |\mathbf{r}_m - \mathbf{r}_d|$. Note that $\mathbf{r}_m$ is a point along the generating arc and does not vary with respect to φ .

Assume a small discretization and $|\frac{\Delta\boldsymbol{\rho}_m}{R_m}| \ll 1$. Using a similar development to (36)–(38), (43) becomes

$$\begin{aligned} R(\mathbf{r}_m + \Delta\boldsymbol{\rho}_m, \mathbf{r}_d) &= R_m \left[1 + \frac{\hat{\mathbf{R}}}{R_m} \cdot \Delta\boldsymbol{\rho}_m \right] \\ &= R_m + \hat{\mathbf{R}} \cdot \Delta\boldsymbol{\rho}_m + O\left(\frac{\Delta\boldsymbol{\rho}_m^2}{R_m}\right), \end{aligned} \quad (44)$$

with the following condition

$$\frac{k_0 |\Delta\boldsymbol{\rho}_m|^2}{2R_m} \sin^2 \beta \ll 1, \quad (45)$$

where β is the angle between $\hat{\mathbf{R}}$ and $\Delta\boldsymbol{\rho}_m$. Substituting (44) into (41) and noting that $R_m + \hat{\mathbf{R}} \cdot \Delta\boldsymbol{\rho}_m \approx R_m$ for small $\Delta\boldsymbol{\rho}_m$ yields

$$\begin{aligned} \mathbf{E}_{m,n}^i(\mathbf{r}_m + \Delta\boldsymbol{\rho}_m, \mathbf{r}_d) &= \frac{1}{2\pi} \int_{-\pi}^{\pi} \mathbf{E}_{0,m}^i \frac{e^{-jk_0 R_m}}{R_m} e^{-jk_0 \hat{\mathbf{k}}_i \cdot \Delta\boldsymbol{\rho}_m} e^{-jn\varphi} d\varphi \\ &= \mathbf{E}_{0,m}^i \frac{e^{-jk_0 R_m}}{R_m} \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-jk_0 \hat{\mathbf{k}}_i \cdot \Delta\boldsymbol{\rho}_m} e^{-jn\varphi} d\varphi. \end{aligned} \quad (46)$$

Since the $\mathbf{E}_{0,m}^i \frac{e^{-jk_0 R_m}}{R_m}$ term no longer varies with respect to φ , it was extracted from the integration, and (41) now appears as a plane wave across the BOR at segment, m . The plane wave solution for a BOR is analytical and comprised of Bessel functions [23, 33].

At this stage, the coefficients in (20) have been solved for via (46), and (46) is formally a piecewise plane wave approximation over the BOR generating arc where the initial phase and magnitude of the plane wave is the $\mathbf{E}_{0,m}^i \frac{e^{-jk_0 R_m}}{R_m}$ term evaluated at the BOR segment $\mathbf{r}_m$. Thus, the computational complexity of evaluating the incident field source coefficients (20) for MBOR problem set is reduced since plane wave source functions are analytical. Accordingly, (46) is termed the LPWA, allowing for analytical representation of plane waves incident on BORs.

Note that this approximation essentially turns a BOR into a set of thin discs dependent upon the free space wavenumber rather than the material wavenumber. Therefore, the far field/plane wave criteria is a function of the BOR radius rather than BOR length. In turn, the approximation error is dominated by the BOR radius since the BOR can be discretized into thinner discs, i.e., a finer mesh. Additionally, (38) and (45) establish clear criteria for monitoring a software implementation of the approximation.

C. Electric field coordinate expansion

The incident wave may be defined by three orthogonal unit vectors: horizontal polarization, vertical polarization, and the component of the electric field in the

direction of propagation, i.e.,

$$\hat{\mathbf{h}}_i = \frac{\mathbf{k}_i \times \hat{\mathbf{z}}}{|\mathbf{k}_i \times \hat{\mathbf{z}}|} = -\sin \varphi_i \hat{\mathbf{x}} + \cos \varphi_i \hat{\mathbf{y}}, \quad (47)$$

$$\hat{\mathbf{v}}_i = \hat{\mathbf{h}}_i \times \hat{\mathbf{k}}_i = -\cos \theta_i \cos \varphi_i \hat{\mathbf{x}} - \cos \theta_i \sin \varphi_i \hat{\mathbf{y}} - \sin \theta_i \hat{\mathbf{z}}, \quad (48)$$

$$\hat{\mathbf{k}}_i = -\sin \theta_i \cos \varphi_i \hat{\mathbf{x}} - \sin \theta_i \sin \varphi_i \hat{\mathbf{y}} - \cos \theta_i \hat{\mathbf{z}}, \quad (49)$$

where $\hat{\mathbf{h}}_i$ is the horizontal polarization unit vector, $\hat{\mathbf{v}}_i$ is the vertical polarization unit vector, $\hat{\mathbf{k}}_i$ is the unit vector in the direction of incidence, φ_i is the incident, azimuthal angle, and θ_i is the incident, polar angle. The tangential incident field, $\mathbf{E}_m^i$, can be described as

$$\mathbf{E}_m^i = E_{h,m}^i \hat{\mathbf{h}}_i + E_{v,m}^i \hat{\mathbf{v}}_i + E_{r,m}^i \hat{\mathbf{k}}_i. \quad (50)$$

Using the τ and φ components as defined above, (1)–(3), and combining the derived field definitions from de Matthaeis and Lang [33], the incident field becomes

$$E_{\tau,m,h}^i = E_{h,m}^i \sin \psi_m \sin(\varphi_m - \varphi_i) e^{-jk_0 \hat{\mathbf{k}}_i \cdot \mathbf{r}_m}, \quad (51)$$

$$E_{\tau,m,v}^i = -E_{v,m}^i [\cos \psi_m \sin \theta_i + \sin \psi_m \cos \theta_i \cos(\varphi_m - \varphi_i)] e^{-jk_0 \hat{\mathbf{k}}_i \cdot \mathbf{r}_m}, \quad (52)$$

$$E_{\tau,m,r}^i = -E_{r,m}^i [\cos \theta_i \cos \psi_m + \sin \psi_m \sin \theta_i \cos(\varphi_m - \varphi_i)] e^{-jk_0 \hat{\mathbf{k}}_i \cdot \mathbf{r}_m}, \quad (53)$$

$$E_{\varphi,m,h}^i = E_{h,m}^i \cos(\varphi_m - \varphi_i) e^{-jk_0 \hat{\mathbf{k}}_i \cdot \mathbf{r}_m}, \quad (54)$$

$$E_{\varphi,m,v}^i = E_{v,m}^i \cos \theta_i \sin(\varphi_m - \varphi_i) e^{-jk_0 \hat{\mathbf{k}}_i \cdot \mathbf{r}_m}, \quad (55)$$

$$E_{\varphi,m,r}^i = E_{r,m}^i \sin \theta_i \sin(\varphi_m - \varphi_i) e^{-jk_0 \hat{\mathbf{k}}_i \cdot \mathbf{r}_m}. \quad (56)$$

The basis functions chosen in this implementation of the BOR MoM are rectangular and triangular pulses and, thus, performing the integrals for the Fourier series expansion over the basis functions yields

$$E_{\tau,m,h,n}^i = j^n E_{h,m}^i \sin \psi_m T_n^+(u\rho_m) e^{-jn\varphi_i} e^{jvz_m}, \quad (57)$$

$$E_{\tau,m,v,n}^i = j^n E_{v,m}^i [-\cos \psi_m \sin \theta_i J_n(u\rho_m) + j \sin \psi_m \cos \theta_i T_n^-(u\rho_m)] e^{-jn\varphi_i} e^{jvz_m}, \quad (58)$$

$$E_{\tau,m,r,n}^i = j^n E_{r,m}^i [-\cos \psi_m \cos \theta_i J_n(u\rho_m) + j \sin \psi_m \sin \theta_i T_n^-(u\rho_m)] e^{-jn\varphi_i} e^{jvz_m}, \quad (59)$$

$$E_{\varphi,m,h,n}^i = j^{n+1} E_{h,m}^i T_n^-(u\rho_m) e^{-jn\varphi_i} e^{jvz_m}, \quad (60)$$

$$E_{\varphi,m,v,n}^i = j^n E_{v,m}^i \cos \theta_i T_n^+(u\rho_m) e^{-jn\varphi_i} e^{jvz_m}, \quad (61)$$

$$E_{\varphi,m,r,n}^i = j^n E_{r,m}^i \sin \theta_i T_n^+(u\rho_m) e^{-jn\varphi_i} e^{jvz_m}, \quad (62)$$

where ρ_m is the radial distance from the segment m to the z -axis. Equations (57)–(62) are the tangential fields along the generating arc of the BOR for each segment, m , and harmonic, n . The $T_n^+(u\rho)$ and $T_n^-(u\rho)$ functions are described by $J_n(x)$, the Bessel function of the first kind order n , i.e.,

$$T_n^+(x) = \frac{J_{n+1}(x) + J_{n-1}(x)}{2}, \quad (63)$$

$$T_n^-(x) = \frac{J_{n+1}(x) - J_{n-1}(x)}{2}. \quad (64)$$

The remaining variables are described by

$$k_0 \hat{\mathbf{k}}_i \cdot \mathbf{r}_m = -u\rho_m \cos \varphi_m - v z_m, \quad (65)$$

with

$$u = k_0 \sin \theta_i, \quad v = k_0 \cos \theta_i. \quad (66)$$

Given that the incident electric field has been derived, derivation of the magnetic fields follows via duality, where

$$H_{\tau,m,h,n}^i = j^n H_{h,m}^i \sin \psi_m T_n^+(u\rho_m) e^{-jn\varphi_i} e^{jvz_m}, \quad (67)$$

$$H_{\tau,m,v,n}^i = j^n H_{v,m}^i [-\cos \psi_m \sin \theta_i J_n(u\rho_m) + j \sin \psi_m \cos \theta_i T_n^-(u\rho_m)] e^{-jn\varphi_i} e^{jvz_m}, \quad (68)$$

$$H_{\tau,m,r,n}^i = j^n H_{r,m}^i [-\cos \psi_m \cos \theta_i J_n(u\rho_m) + j \sin \psi_m \sin \theta_i T_n^-(u\rho_m)] e^{-jn\varphi_i} e^{jvz_m}, \quad (69)$$

$$H_{\varphi,m,h,n}^i = j^{n+1} H_{h,m}^i T_n^-(u\rho_m) e^{-jn\varphi_i} e^{jvz_m}, \quad (70)$$

$$H_{\varphi,m,v,n}^i = j^n H_{v,m}^i \cos \theta_i T_n^+(u\rho_m) e^{-jn\varphi_i} e^{jvz_m}, \quad (71)$$

$$H_{\varphi,m,r,n}^i = j^n H_{r,m}^i \sin \theta_i T_n^+(u\rho_m) e^{-jn\varphi_i} e^{jvz_m}. \quad (72)$$

Referring to (57)–(72), the field angle of incidence is unique for each segment of the BOR. The incident angle can be characterized geometrically as the angle between the incident source point, $\mathbf{r}_d$ and m^{th} segment of the BOR generating arc.

Overall, the LPWA affected a reduction of computational complexity by means of functions with lookup tables such as Bessel functions. It is interesting to note that (57)–(72) represent a rather direct computational encoding of the LPWA applied to the BOR-MoM for a Hertzian dipole source.

V. HERTZIAN DIPOLE EXAMPLE

The Hertzian dipole derivation has been implemented into a BOR MoM code to modify the incident wave upon the BOR. To validate the LPWA for a BOR, a Hertzian dipole example will be considered as a test

case since it is a classical case with an exact solution, see Fig. 4. The Hertzian Dipole is vertically polarized along the z -axis and located at $x = 0.2$ m, $y = 0$ m, and $z = 0$ m. Thus, the dipole will be located within the reactive near field, Fresnel near field, and far field of the dielectric BOR. The BOR center is located at $x = 0$ m, $y = 0$ m, and $z = 0$ m; see Fig. 7. Note that formally the LPWA applies to magnetic surface currents as well, i.e., magnetic dipoles. For the sake of compactness, only electric dipoles are examined within this paper.

Although there are an entourage of test cases for LPWA validation, only lossy dielectric cylinders representative of a Caucasian Fir tree are examined. Additionally, the Hertzian dipole is placed along the x axis to simplify the analytical solution for a thin cylinder (see Appendix A). Note that the analytical solution relies on the thin cylinder approximation where the phase across the cylinder diameter is assumed constant, i.e., the method is a function of both cylinder radius and dielectric constant [37].

The results are compared to the FEKO 3-D MoM [35] solution, the full integration method [29, 30], which entails integrating the dipole equations around the BOR z -axis for the full Fourier series expansion, and an analytical solution for thin cylinders. The analytical solution is in Appendix A.

A. Case 1: Median branch

Let us consider an incident RF field on a lossy, dielectric cylinder whose statistics are listed in Table 1, at a frequency of $f = 2.0$ GHz. Simulation of MBOR scattering within a 5 m Caucasian Fir tree is of interest; therefore, the cylinder is the approximate size of the median branch of a Caucasian Fir tree [16, 17, 36]. The scattered far fields from the BOR due to the dipole are examined in Cartesian coordinates. The results supporting validation are shown in Figs. 8 and 9. The x position and y -position of the scattered electric field z axis sweep are at $x = 10$ m and $y = 0$ m, respectively.

As expected, all four methods show great agreement for a small cylinder case since the assumptions of the analytical solution and LPWA hold, i.e., $\frac{\rho}{|r_m|}$ and $\frac{|\Delta r_m|}{|r_m|}$ are small.

B. Cases 2 and 3: Largest branch and trunk

To ensure the LPWA method applies to the largest sections within a 5 m Caucasian Fir tree, let us consider an incident RF field on lossy, dielectric cylinders whose statistics are listed in Tables 2 and 3, at a frequency of $f = 2.0$ GHz. The BOR size is consistent with the largest branch and largest trunk sections of a previously vectorized and measured Caucasian Fir tree [16, 17, 36]. The results supporting validation are shown in Figs. 10–13.

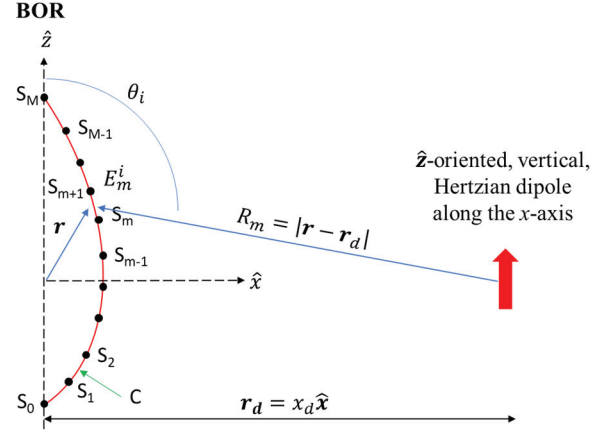


Fig. 7. LPWA BOR validation scenario where a vertically polarized, Hertzian dipole is centered on the x -axis.

Table 1: Case 1 scenario for the LPWA with an incident Hertzian dipole for the median branch in a 5 m Caucasian Fir tree

Quantity	Value
Cylinder Length [m]	0.08
Cylinder Radius [m]	0.001
ϵ_r	$10 - j2.4$
Harmonics	10
Mesh Size	$\lambda_m/40$
$r_d(x, y, z)$ [m]	0.2, 0, 0
Dipole Polarization	$\hat{z}$ (V-Pol)
Dipole Moment [A-m]	1
$\frac{ \Delta r_m }{ r_m }$	0.002
$\frac{\rho}{ r_m }$	0.02

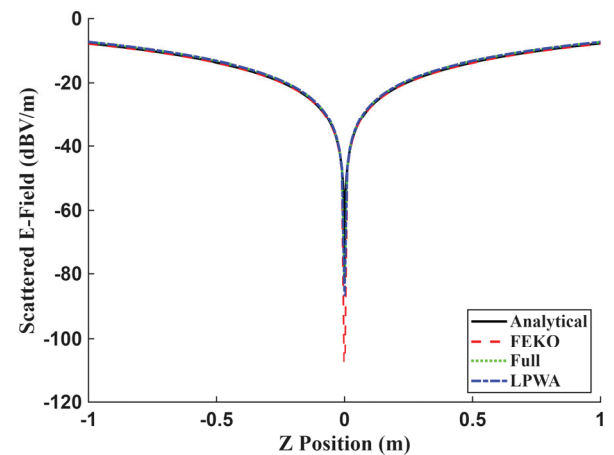


Fig. 8. Median branch – $\hat{x}$ polarized. Magnitude of the bistatic electric field versus z_s for a V-polarized wave with $\hat{x}$ polarized scattering on a dielectric cylinder at $x = 10$ m and $y = 0$ m. The vertically oriented dipole is located at $r_d(x, y, z) = 0.2$ m, 0 m, 0 m.

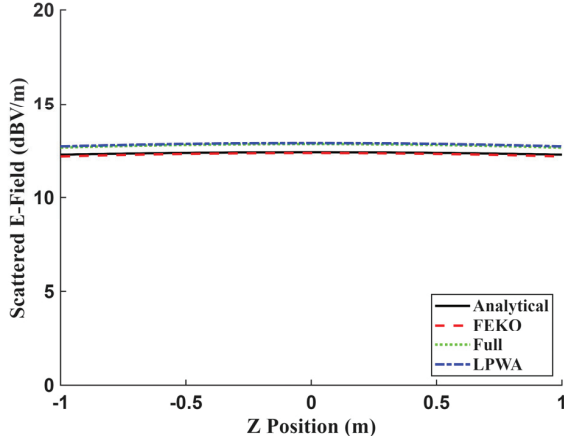


Fig. 9. Median Branch – $\hat{z}$ polarized. Magnitude of the bistatic electric field versus z_S for a V-polarized wave with $\hat{z}$ polarized scattering on a dielectric cylinder at $x = 10$ m and $y = 0$ m. The vertically oriented dipole is located at $\mathbf{r}_d(x, y, z) = 0.2$ m, 0 m, 0 m.

Table 2: Case 2 scenario for the LPWA with an incident Hertzian dipole for the largest branch in a 5 m Caucasian Fir tree

Quantity	Value
Cylinder Length [m]	0.3
Cylinder Radius [m]	0.01
ϵ_r	$10 - j2.4$
Harmonics	10
Mesh Size	$\lambda_m/40$
$\mathbf{r}_d(x, y, z)$ [m]	0.2, 0, 0
Dipole Polarization	$\hat{z}$ (V-Pol)
Dipole Moment [A-m]	1
$\frac{\Delta \mathbf{r}_m}{r_m}$	0.003
$\frac{\rho}{r_m}$	0.05

The results show a relatively small increase in error for the LPWA as the BOR radius increases. However, the solution error is still less than 0.2 dB compared to the Full Expansion solution for the branches present within the Caucasian Fir tree.

The results shown in Figs. 8–13 demonstrate computationally that the analytical, thin cylinder approximation for scattering from a dielectric cylinder is not valid when the BOR radius is large, as expected [37–39]. The analytical solution's error increases significantly as the BOR radius increases, since the cylinders are no longer thin. This implies that the analytical solution is no longer valid when the assumptions underlying the approximations given in Appendix A are no longer satisfied. Thus, traditional thin cylinder approximation methods are not applicable for MBOR scattering of

Table 3: Case 3 scenario for the LPWA with an incident Hertzian dipole for the largest trunk section in a 5 m Caucasian Fir tree

Quantity	Value
Cylinder Length [m]	0.5
Cylinder Radius [m]	0.05
ϵ_r	$23 - j5.2$
Harmonics	10
Mesh Size	$\lambda_m/40$
$\mathbf{r}_d(x, y, z)$ [m]	0.2, 0, 0
Dipole Polarization	$\hat{z}$ (V-Pol)
Dipole Moment [A-m]	1
$\frac{\Delta \mathbf{r}_m}{r_m}$	0.25
$\frac{\rho}{r_m}$	0.05

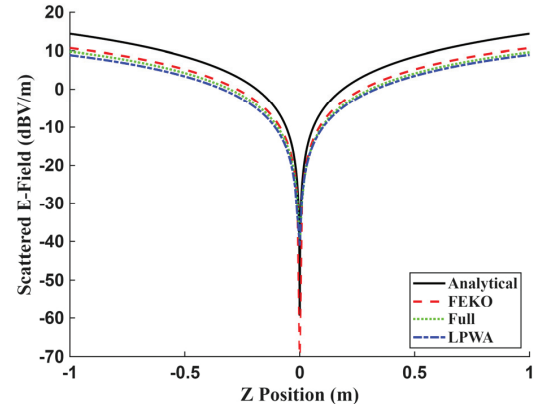


Fig. 10. Largest branch – $\hat{x}$ polarized. Magnitude of the bistatic electric field versus Z_S for a V-polarized wave with $\hat{x}$ polarized scattering on a dielectric cylinder at $x = 10$ m and $y = 0$ m. The vertically oriented dipole is located at $\mathbf{r}_d(x, y, z) = 0.2$ m, 0 m, 0 m.

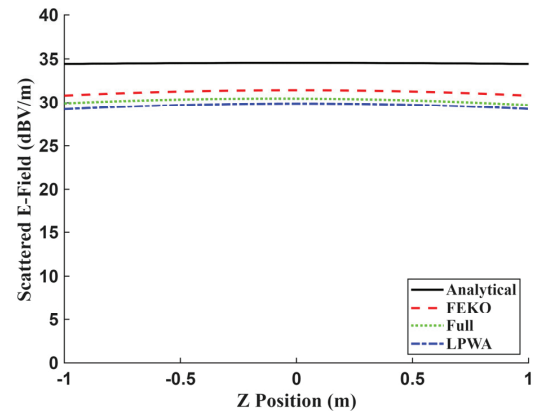


Fig. 11. Largest branch – $\hat{z}$ polarized. Magnitude of the bistatic electric field versus Z_S for a V-polarized wave with $\hat{z}$ polarized scattering on a dielectric cylinder at $x = 10$ m and $y = 0$ m. The vertically oriented dipole is located at $\mathbf{r}_d(x, y, z) = 0.2$ m, 0 m, 0 m.

this class of Caucasian Fir tree. The LPWA, however, does not have the same BOR radius limitation as the analytical solution and, in turn, the LPWA is well suited for solving large MBOR problem sets related to tree scattering. Thus, the LPWA is an ideal method for the class of BORs expected within the 5 m Caucasian Fir tree.

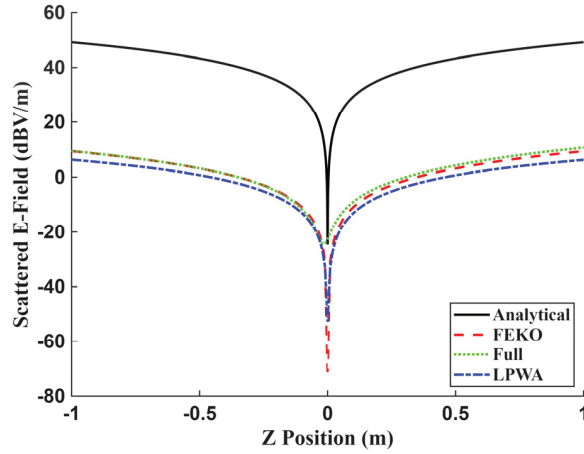


Fig. 12. Largest trunk section – $\hat{x}$ polarized. Magnitude of the bistatic electric field versus z_S for a V-polarized wave with $\hat{x}$ polarized scattering on a dielectric cylinder at $x = 10$ m and $y = 0$ m. The vertically oriented dipole is located at $\mathbf{r}_d(x, y, z) = 0.2$ m, 0 m, 0 m.

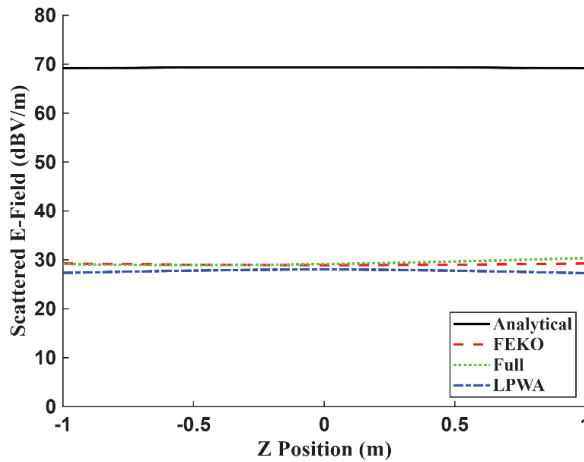


Fig. 13. Largest trunk section – $\hat{z}$ polarized. Magnitude of the bistatic electric field versus Z_S for a V-polarized wave with $\hat{z}$ polarized scattering on a dielectric cylinder at $x = 10$ m and $y = 0$ m. The vertically oriented dipole is located at $\mathbf{r}_d(x, y, z) = 0.2$ m, 0 m, 0 m.

C. Error examination

The following analyses examine LPWA error tolerance. In particular, the graphs show a comparison

between the full expansion method and LPWA for the electric field coefficients.

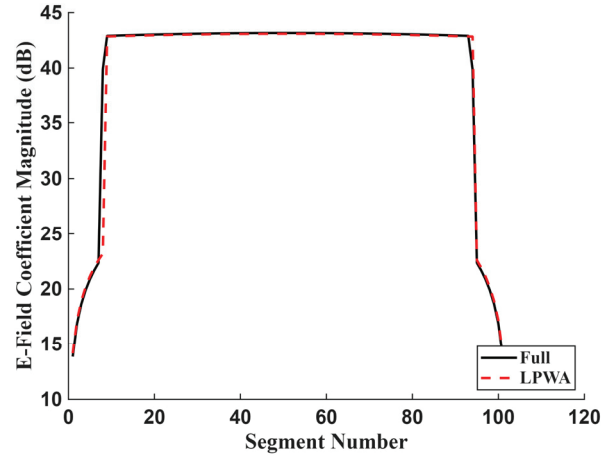


Fig. 14. Incident electric field coefficient, magnitude comparison, for the 0th harmonic, between the full expansion method and LPWA.

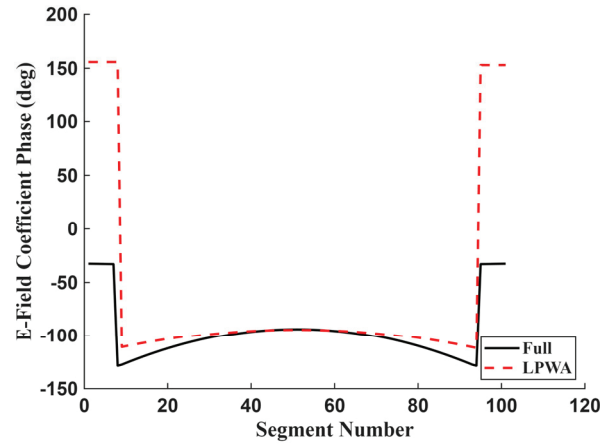


Fig. 15. Incident electric field coefficient phase comparison, for the 0th harmonic, between the full expansion method and LPWA.

A small cylinder is simulated that is 10 cm in length and 4 mm in radius like that of Case 1. The incident field is V-polarized on a dielectric cylinder at a frequency of $f = 3$ GHz. The vertically oriented dipole is located at $\mathbf{r}_d(x, y, z) = 0.2$ m, 0 m, 0 m. Both magnitude and phase of the electric field coefficients at specific harmonics are plotted, shown in Figs. 14–17. Note that only the coefficients for a V-polarized wave are shown here for brevity, but both H-polarized and V-polarized coefficients have been examined.

Overall, the method shows great agreement across the cylinder body for incident electric field coefficient

magnitude. The methods do differ, however, with respect to the coefficient phase. In these validation cases, the phase error primarily occurs at the corner of the cylinder and the upper cylinder segments, where the cylinder segments are perpendicular to the incident electric field.

As it relates to the transverse segment phase error, i.e., the segments on the top and bottom of the cylinder, the transverse segment current magnitudes are on the order of 20 dB lower than that of the segments parallel to the incident electric field. Thus, their induced currents and the resultant contributions to the scattered field are negligible. Additionally, the transverse segment current magnitudes have minimal error compared to the exact solution, which further supports the conclusion that transverse segment phase error is negligible relative to total scattered field error. Since the phase error has negligible impact on the total scattered fields, its source is not explored further in this paper.

To further examine the error due to BOR radius, i.e., the validity and impact of the $\frac{|\Delta\rho_m|}{|R_m|} \ll 1$ assumption, an error plot examining the LPWA versus full expansion method with respect to BOR radius is plotted in Fig. 18. See Table 4 for simulation statistics.

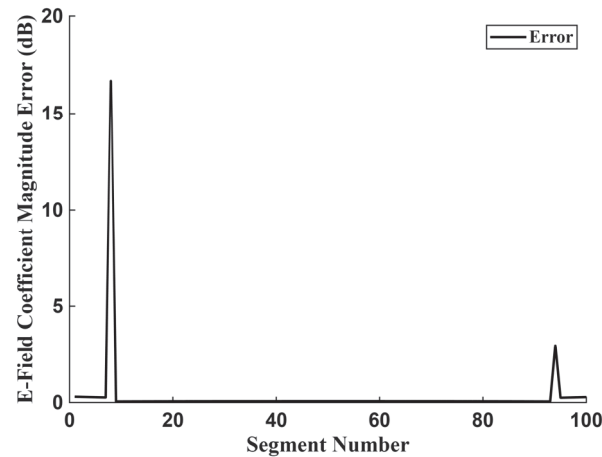


Fig. 16. Incident electric field coefficient magnitude error between the full expansion method and LPWA for the 1st harmonic.

Figure 18 shows the average, difference error magnitude (47) between the full expansion method and LPWA for the scattered electric field in the far field. A cylinder is simulated that is 50 cm in length with various radius values. The incident field is V-polarized on a dielectric cylinder at a frequency of $f = 2$ GHz. The vertically oriented dipole is located at $\mathbf{r}_d(x, y, z) = 0.2 \text{ m}, 0 \text{ m}, 0 \text{ m}$.

As the BOR radius increases and the approximation conditions are no longer satisfied, there is an exponential increase in error (see Fig. 18), which is physically

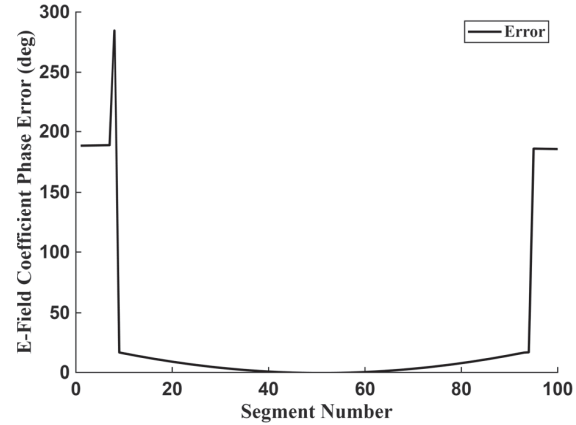


Fig. 17. Incident electric field coefficient phase error between the full expansion method and LPWA for the 1st harmonic.

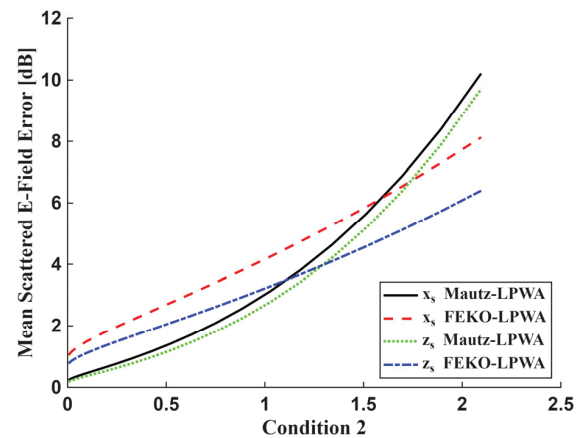


Fig. 18. Far field, average, scattered electric field magnitude error between both the Mautz and LPWA methods and FEKO and LPWA methods versus Condition $2\left(\frac{k_0|\Delta\rho_m|^2}{2R_m}\sin^2\beta \ll 1\right)$ at the scattered field point $x_s = 10 \text{ m}, y_s = 0 \text{ m}, z_s = 0 \text{ m}$ for a V-polarized wave incident on a dielectric cylinder.

Table 4: Error examination scenarios for the LPWA with an incident Hertzian dipole

Quantity	Value
Cylinder Length [m]	0.5
Cylinder Radius [m]	0.001, 0.01, 0.05, 0.1
ϵ_r	10.3-j2.4
Harmonics	10
Mesh Size	$\lambda_m/40$
$\mathbf{r}_d(x, y, z) [\text{m}]$	0.2, 0, 0
Dipole Polarization	$\hat{z}$ (V-Pol)
Dipole Moment [A-m]	1
$\frac{ \Delta\rho_m }{ R_m }$	0.003, 0.003, 0.005, 0.008
$\frac{\rho}{R_m}$	0.005, 0.05, 0.33, 1

consistent and expected. In general, when the conditions specified by (38) and (45) are satisfied, i.e., $\ll 1$, the LPWA error between the exact (Full expansion) method and FEKO 3-D MoM solution is less than 0.2 dB and 1 dB, respectively. With regards to the FEKO 3-D MoM solution, the additional error is not due to the LPWA, but due to error associated with a specific software implementation of this BOR-MoM and its numerical-algorithmic formulation, which typically has a magnitude difference on the order of 1 dB or less compared to the FEKO 3-D MoM. This is due to the limitations of the Mautz and Harrington BOR-MOM solution, as well as the use of low order basis functions not a topic of this paper.

When considering this error, it is important to realize that, for a multi-scatter development, the other branches will be comprised of thousands of segments orthogonal to the trunk [16, 17, 36]. Thus, a majority of the segments, i.e., point sources, will satisfy the $\frac{|\Delta\rho_m|}{|R_m|} \ll 1$ criteria, and only a small number of segments will be susceptible to large errors – a discussion for a future paper.

D. Computational efficiency

With regards to computational efficiency, the BOR-MOM inherently is more efficient than the 3-D MoM; see reference [34] for a breakdown between this BOR-MoM implementation and the FEKO 3-D MoM. The efficiency of the LPWA, however, warrants further discussion.

Typically, MBOR scattering requires mapping currents to spheroidal surfaces, using T-Matrix based procedures, which is not possible for the tree scattering case considered here, as stated in section I. Other model formulations of MBOR scattering require that sources from a BOR couple to adjacent BOR, imposing the need for numerical triple integration for evaluation of the incident field matrix. Thus, a reduction of model complexity should be desirable.

Additionally, calculation of the impedance matrix is typically the slowest calculation component for the MoM. In a multi-scatter model, however, the impedance matrix is calculated once per BOR, while the incident field matrix must be calculated multiple times per BOR. For example, a tree comprised of 19,000 segments would require each BOR's incident field matrix to be calculated 19,000 times, i.e., once per incident plane wave and once per each of the 18,999 BORs illuminating the BOR of interest. Thus, relating scattered fields from adjacent BORs would have excessive computational costs.

To validate the computational benefit of the LPWA, the LPWA incident field matrix computation time is

compared to the Full Expansion integration method in Table 5 for a single harmonic. Overall, Table 5 shows that the LPWA reduces the integration cost of the method by greater than an order of magnitude per harmonic. Also, computation time for the LPWA does not scale proportionately with number of elements M , whereas the full expansion method scales linearly with number of elements.

Table 5: Comparison of the computation time of the LPWA and full expansion methods for a single harmonic

Length	M	Full Expansion Method	LPWA
λ	327	183 ms	9 ms
3λ	526	280 ms	10 ms
6λ	825	483 ms	10 ms
10λ	1223	587 ms	10 ms

Table 6: Comparison of the expected computation time of the LPWA and full expansion methods for multiple BORs with 1000 segments each evaluated over ± 10 harmonics

Number of BORs	Full Expansion Method	LPWA
2	1000 s	20 s
4	6,000 s	120 s
8	28,000 s	560 s
16	120,000 s	2400 s

Notably, the LPWA is formulated to achieve a reduction in computational complexity for MBOR scenarios. Specifically, its formulation does not require integration of the field source, due to the i^{th} BOR around the j^{th} BOR. Table 6 shows a comparison of expected computation times, based on the results of Table 5, for an MBOR scenario, where each BOR has 1000 segments and ± 10 harmonics per BOR. Table 6 shows that even for a two BOR scenario, the computational cost has been reduced by more than an order of magnitude. If the i^{th} BOR is treated as a singular source, rather than a collection of sources, computation time for the LPWA can be decreased even further. This follows in that the full expansion method must treat each segment as a singular source.

The test cases were run on a computer with an Intel i910900k CPU overclocked to 4.9 GHz all core, 64 GB of 3600 MHz DDR4 RAM, and a Samsung 970 Evo Plus NVMe SSD. For this analysis, the software implementation was programmed in MATLAB 2025a, and the code is parallelized and vectorized to maximize computational efficiency. If this program were deployed in C++ rather than MATLAB, it is estimated to improve the computation times by another order of magnitude.

VI. CONCLUSION

This analysis resulted in the derivation and relatively direct computational encoding of an arbitrary wave incident on a BOR for the MoM using the LPWA. Both the incident, tangential, electric, and magnetic fields on both the discretized τ and ϕ segments of the BOR were derived.

The derived equations demonstrated two approaches to the problem including a full expansion method at each segment for arbitrary BOR size, and a LPWA for thin BORs. For the full expansion method, it is assumed that the discretization is small along the generating arc and around the BOR ϕ -axis. When the BOR radius is not large compared to wavelength, we can further simplify the procedure resulting in a source function that is a local plane wave across the whole disk, effectively reducing the far field condition to a function of BOR radius rather than BOR length. Additionally, both of the approximations apply to thicker cylinders than the analytical solution using the thin cylinder approximations. For practical applications, the far field criterion, (38) and (45), can be monitored, and the BOR separation and/or discretization can be adjusted to shrink the parameters as needed.

The derivation was verified and validated using a Hertzian dipole example and compared to the FEKO 3D MoM [35] solution, the full expansion method [29, 30], and the analytical solution for a thin cylinder. The results showed good agreement between the different MoM implementations for scattered field magnitude. In turn, the resulting computational benefit due to the reduced integration complexity of the LPWA resulted in a minimal loss of accuracy to the solution.

Overall, the error induced by the LPWA is acceptable for the specific multi-scatter approach in development for a 5 m Caucasian Fir tree since the trunk sizes, branch sizes, and branch separations examined in this paper are typical of the vectorized Caucasian Fir tree [16, 17, 36]. Additionally, the method is well posed to manage propagation between discrete scatterers when a complex medium is present, e.g., vegetative environments. The next steps are to apply the LPWA to MBOR scattering.

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APPENDIX A

The following development is an abbreviated derivation for the analytical solution of a dipole incident on a thin cylinder.

Following a similar development to section IV, let the axis shifted Hertzian dipole have the following source function in a local coordinate system, see Fig. A.1,

$$\mathbf{J}(\mathbf{r}_d) = J_0 \delta(\mathbf{r} - \mathbf{r}_d) \hat{\mathbf{z}}_d. \quad (\text{A.1})$$

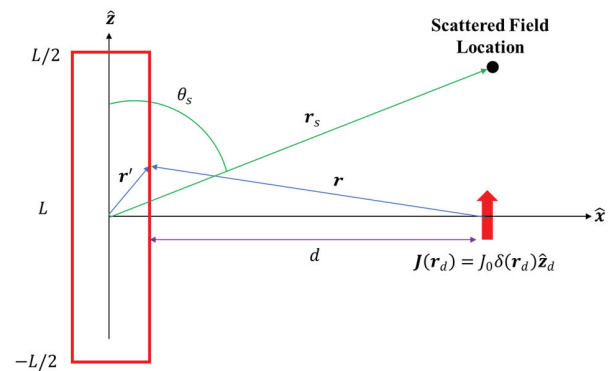


Fig. A.1. Geometry of a Hertzian dipole incident on a cylinder.

Assuming $kr \gg 1$, the electric field due to the source function in (29) has the general form of

$$\mathbf{E}_d^i(\mathbf{r}, \mathbf{r}_d) = E_0 \sin \theta_d \frac{e^{-jk_0 r}}{r} \hat{\boldsymbol{\theta}}, \quad (\text{A.2})$$

$$E_0 = \frac{j\eta_0 k_0 J_0}{4\pi}. \quad (\text{A.3})$$

Since the cylinder is thin, i.e., a is small, the internal electric field may be described by

$$\mathbf{E}_{int} \approx \mathbf{E}_{\parallel} + \frac{2}{1 + \epsilon_r} \mathbf{E}_{\perp}, \quad (\text{A.4})$$

where $\mathbf{E}_{int}$ is the internal field [37–39] and

$$\mathbf{E}_{\parallel} = E_0 \sin^2 \theta_d \frac{e^{-jk_0 R_d}}{R_d} \hat{\mathbf{z}}, \quad (\text{A.5})$$

$$\mathbf{E}_{\perp} = -E_0 \sin \theta_d \frac{e^{-jk_0 R_d}}{R_d} \cos \theta_d \hat{\mathbf{x}}, \quad (\text{A.6})$$

where θ_d is the angle that $\mathbf{r}$ makes with the z-axis of the local coordinate system of the dipole.

Using LeVine et al. [37], the scattered electric field may be described by

$$\mathbf{E}_s(\mathbf{r}_s) = k_0^2 \int_V (\epsilon_r - 1) \underline{\mathbf{G}}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{E}_{int}(\mathbf{r}') d\mathbf{r}', \quad (\text{A.7})$$

where $\underline{\mathbf{G}}(\mathbf{r}, \mathbf{r}')$ is the dyadic Green's function.

Consider a scattered field location along the z-axis (z_s) at a distance x_s with $y_s = 0$. To simplify the integral for the analytical solution, make the following assumptions: $ka \ll 1$, $\frac{z_s}{r_s} \ll 1$, $\frac{x_s}{r_s} \approx 1$, and R_d' is constant over the cross section of a cylinder. Substituting (A.4) into (A.7), and performing the algebraic and geometric

simplifications yields

$$\mathbf{E}_s(\mathbf{r}_s) = \mathbf{E}_x(\theta_s) \hat{\mathbf{x}} + \mathbf{E}_z(\theta_s) \hat{\mathbf{z}}, \quad (\text{A.8})$$

where

$$\mathbf{E}_x(\theta_s) = E_{d0} [P_x^+ I^+(\theta_s) + P_x^- I^-] \frac{e^{-jk_0 r_s}}{r_s}, \quad (\text{A.9})$$

$$\mathbf{E}_z(\theta_s) = E_{d0} [P_z^+ I^+ + P_z^- I^-] \frac{e^{-jk_0 r_s}}{r_s}, \quad (\text{A.10})$$

and

$$P_x^+(\theta_s) = -\frac{1}{2} \sin(2\theta_s), \quad (\text{A.11})$$

$$P_x^-(\theta_s) = \sin^2(\theta_s), \quad (\text{A.12})$$

$$P_z^+(\theta_s) = \cos^2(\theta_s), \quad (\text{A.13})$$

$$P_z^-(\theta_s) = -\frac{1}{2} \sin(2\theta_s), \quad (\text{A.14})$$

$$I^+(\theta_s) = d \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{d^2}{R_d'^3} e^{-jk_0 [R_d' - \sin(\theta_s) z']} dz', \quad (\text{A.15})$$

$$I^-(\theta_s) = \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{d \cdot z'}{R_d'^3} e^{-jk_0 [R_d' - \sin(\theta_s) z']} dz', \quad (\text{A.16})$$

$$R_d = |\mathbf{r}| = \sqrt{d^2 - z^2}, \quad (\text{A.17})$$

$$E_{d0} = \frac{j(\epsilon_r - 1)\eta_0 k_0^3 J_0 A}{(4\pi)^2}, \quad (\text{A.18})$$

where d is the distance between the dipole and cylinder surface, A is the cylinder cross-sectional area, and L is the cylinder length.

Comparative Analysis of Microstrip Patch Antenna Performance on Various Dielectric Substrates

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Abstract – Microstrip patch antennas (MSPAs) have gained significant popularity due to their compact size, ease of fabrication, low cost, lightweight construction, and compatibility with planar and non-planar surfaces. The aim is to identify the optimal substrate for a designed MSPA by comparing the performance of six different substrates. Accordingly, both rectangular MSPA and circular MSPA (CMSPA) have been developed and simulated by using six different substrates: FR4 Glass Epoxy, ROGERS DT5880, ROGERS 4003C, Teflon, Arlon AD300A, and Alumina with thickness of 1.6 mm and 1 mm. Rectangular and CMSPAs have been fabricated and measured to verify the effects of the substrates. The proposed antenna is designed to achieve resonance at the target frequency of 5.8 GHz. This study offers valuable insights into the performance of MSPAs with various substrates, thereby assisting researchers in selecting the most appropriate materials for their designs.

Index Terms – Antenna efficiency, antenna gain, dielectric substrate, impedance bandwidth, microstrip patch antenna.

I. INTRODUCTION

An antenna is a critical component in wireless communication systems, responsible for radiating and receiving electromagnetic waves to enable efficient signal transmission. Its performance is primarily governed by parameters such as operating frequency, impedance matching, radiation efficiency, bandwidth, and physical dimensions. Among various antenna types, microstrip patch antennas (MSPAs) have become increasingly significant due to their low profile, lightweight structure, ease of integration with printed circuit boards, and

cost-effectiveness. MSPAs typically consist of a metallic patch mounted on a dielectric substrate with a ground plane, supporting modes such as TM_{10} for fundamental operation. Despite their advantages, MSPAs exhibit limitations like narrow bandwidth and relatively low gain, which have driven research into techniques such as slotting, stacking, and the use of high-permittivity substrates to enhance performance. The concept of microstrip radiators was first introduced by Deshalbs in 1953 [1], followed by a patent granted to Gutton and Baissinot in France in 1955 [2]. Substantial development occurred after 1970, particularly for aerospace and military applications where size, weight, and reliability were critical [1].

A conventional MSPA comprises a radiating patch, a dielectric substrate, and a ground plane. The dielectric constant of the substrate typically ranges between 2.2 and 12, significantly influencing radiation characteristics, efficiency, and bandwidth. For compact microwave systems, particularly in miniaturized devices, the use of a thin substrate with a high dielectric constant is generally preferred to mitigate undesired radiation and coupling effects while maintaining structural compactness. Microstrip antennas present several inherent advantages, including a low-profile structure, reduced weight, ease of fabrication, conformal adaptability, cost-effectiveness, and seamless integration with microwave circuits. Nevertheless, these antennas exhibit certain limitations, such as reduced radiation efficiency, susceptibility to feed and junction radiation, excitation of surface waves, restricted impedance bandwidth, and increased sensitivity to manufacturing tolerances. Key design parameters include the width of the patch (W_p), length of the patch (L_p), effective permittivity (ϵ_{eff}), inset feed depth (F_i), and the directivity of the antenna $D(\theta, \phi)$ [3, 4]. Additionally, ϵ_r denotes the relative

permittivity of the substrate, while W_f represents the width of the inset feedline, designed to maintain a microstrip line impedance of 50Ω [5].

Several studies have investigated the performance of different substrates utilized in microwave applications. A study investigated a 5G microstrip antenna using FR4, Rogers RO4350B, and Arlon AD255C, focusing on directivity and bandwidth based on permittivity to operate at 60 GHz. The findings suggest that the Rogers RO4350B substrate may exhibit favorable directivity performance at the specified thickness [6]. Another study analyzed five different substrates: Bakelite, FR4, RO4003, Taconic TLC, and RT/Duroid with a uniform substrate thickness of 1.5 mm which focused on a resonant frequency of 10 GHz. Among the substrates evaluated, RT/Duroid demonstrated superior performance, achieving 80% efficiency and 15% bandwidth. Furthermore, RT/Duroid exhibited an impressive return loss of -20.99 dB, positioning it as a favorable choice compared to the other substrates for MSPAs [7]. Additionally, research utilizing Rogers RT/Duroid 5880 with a dielectric constant of 2.2 and a thickness of 0.508 mm focused on designing a dual-band compact antenna for sub-6 GHz 5G applications, simulating performance in both onbody and free-space scenarios. These findings collectively highlight the growing relevance of substrate properties, such as dielectric constant, loss tangent, and thickness, in optimizing antenna performance for emerging technologies like 5G [8].

The growing demand for flexible wireless systems has made antenna design on ultra-thin flexible substrates increasingly significant. This study examines whether substrate thickness and dielectric constant play a critical role during the initial design stage of antennas. Measurements conducted on three flexible antenna prototypes fabricated using materials such as Rogers and polyimide were compared. The results indicate that the substrate is not merely a supporting structure; it must be considered for accurate prediction of feeding structure performance and resonant frequencies. Furthermore, manual mesh refinement is essential to achieve simulation results that closely match measurements, particularly for ultra-thin substrates [9].

A simplified method for estimating dielectric properties, relative permittivity (ϵ_r) and dissipation factor ($\tan\delta$), using a single open-ended microstrip line has been introduced. Unlike traditional dual-line approaches, this technique reduces complexity and cost while providing accurate results for thin substrates. Based on transmission line theory, it incorporates an iterative algorithm for loss estimation without requiring prior knowledge of conductor or radiation losses. Validation on FR-4, Rogers RT6010, and LDPE shows good agreement with reference values for

low- and medium-permittivity materials, though accuracy decreases for high-permittivity, low-loss substrates. The method offers a practical solution for rapid dielectric characterization in RF and microwave design [10].

In [11], the Rogers 4003 substrate is used with a dielectric constant of 3.55 for wide-angle frequency scanning and to increase the gain of the antenna. As a result, the size of the antenna decreased, and the gain was increased to 18.8 dBi.

Six dielectric substrates including FR4 Glass Epoxy, Rogers DT5880, Rogers RO4003C, Teflon, Arlon AD300A, and Alumina were compared for designing a rectangular MSPA operating at 5.8 GHz with a fixed thickness of 1.6 mm. Simulations indicated that substrate properties significantly influence bandwidth, gain, directivity, and return loss, emphasizing the importance of selecting materials compatible with design requirements and validating choices through simulation prior to fabrication [12].

A study investigated the effect of six dielectric substrates-FR4 Glass Epoxy, Rogers DT5880, Rogers RO4003C, Teflon, Arlon AD300A, and Alumina-on the performance of rectangular and circular MSPAs (CMSPAs) designed for 5 G applications at 5.8 GHz using CST Microwave Studio. The antennas were modelled with a substrate thickness of 1.6 mm and copper for the patch and ground. Simulation results revealed that substrate choice significantly influences key parameters such as return loss, bandwidth, gain, and directivity. Teflon achieved the lowest return loss, Rogers DT5880 provided the highest directivity for rectangular MSPA, and FR4 offered the widest bandwidth. Overall, Rogers DT5880 appeared to be the most balanced substrate for both antenna types, highlighting the importance of substrate selection in optimizing MSPA designs for sub-6 GHz 5G technologies [13].

In this work, six different substrates were selected and designed using CST Microwave Studio software to achieve optimal performance at the resonant frequency of 5.8 GHz, which can be utilized sub-6 GHz for 5G communication devices. The substrate thickness was initially set to 1.6 mm and later reduced to 1 mm to examine its impact on the antenna parameters. The antenna is fed using a microstrip line feeding technique, which provides a planar and efficient method for signal transmission. Different parameters of the designed MSPA are analyzed in terms of key performance parameters, including return loss, gain, bandwidth, and its influence on the resonant frequency.

II. MICROSTRIP PATCH ANTENNA DESIGN

The proposed patch antenna was designed using CST Microwave Studio. An inset feedline technique was

Table 1: Dimensions of the MSPA and substrates at 5.8 GHz

Dimensions (mm)	Substrates					
	FR4	DT5880	RO4003	Teflon	Arlon AD300A	Alumina (%99.5)
Wg	25	31	31	31	31	31
Lg	25	31	31	31	31	31
We	25	31	31	31	31	31
Ls	25	31	31	31	31	31
Wp	15	20.4	17.1	20	18.2	11.0
Lp	11	16.4	12.9	16.9	14.1	7.8
wf	3.0	4.92	3.60	5.11	3.98	4
Fi	4.3	5.74	4.73	5.85	5.05	3.12

adopted for the feeding mechanism, offering costeffectiveness and ease of fabrication while ensuring a 50 Ω impedance match and minimal reflection. Both the patch and ground planes were fabricated from copper with a thickness of 0.035 mm.

The proposed rectangular MSPA is optimized for a center frequency of 5.8 GHz, corresponding to the ISM band, by employing rigorous electromagnetic design principles for high-frequency wireless applications. The resonant frequency of the patch is governed by the fundamental TM_{10} mode and can be approximated using the cavity model as:

$$f_r = \frac{c}{2L_{\text{eff}}\sqrt{\epsilon_{\text{eff}}}}, \quad (1)$$

where c is the speed of light in free space, L_{eff} is the effective length of the patch, and ϵ_{eff} is the effective dielectric constant of the substrate. The effective dielectric constant is calculated as:

$$\epsilon_{\text{eff}} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left(1 + 12 \frac{h}{W} \right)^{-1/2}, \quad (2)$$

where ϵ_r is the substrate dielectric constant, h is the substrate thickness, and W is the patch width. The optimization process also considers fringing fields and surface wave suppression to achieve a return loss $S_{11} < -10$ dB, low VSWR, and stable radiation characteristics at 5.8 GHz.

Figure 1 illustrates the configuration and fabricated prototype of the rectangular MSPA optimized for 5.8 GHz operation. As shown in Fig. 1 (a), the antenna consists of a rectangular patch with dimensions of width W_p and length L_p , mounted on a dielectric substrate of thickness h_s , backed by a full ground plane with thickness of copper h_c . The feed mechanism employs an inset-fed microstrip line, where the feedline width (w_f) is designed for 50 Ω impedance, and the inset depth (F_i) is adjusted to achieve impedance matching at the operating frequency. The substrate outline dimensions (W_g , L_g) are chosen to minimize edge

diffraction and maintain stable radiation characteristics. Figure 1 (b) shows the fabricated MSPA prototype with a SubMiniature version A (SMA) connector for measurement, demonstrating the practical implementation of the design. This configuration provides a compact, planar structure suitable for WLAN and ISM band applications, with performance strongly influenced by substrate permittivity and thickness.

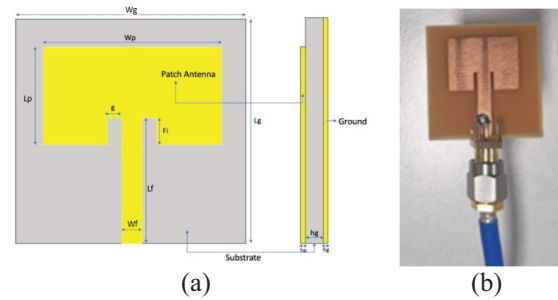


Fig. 1. Configuration of (a) front and side of designed RMSPA and (b) fabricated RMSPA.

The impact of different substrate materials on the performance of a MSPA was analyzed using FR4 Glass Epoxy ($\epsilon_r = 4.3$), Rogers DT5880 ($\epsilon_r = 2.2$), Rogers 4003C ($\epsilon_r = 3.3$), Teflon ($\epsilon_r = 2.1$), Arlon AD300A ($\epsilon_r = 3.0$), and Alumina ($\epsilon_r = 9.9$) with thicknesses of 1.6 mm and 1.0 mm, as summarized in Table 1.

A. Simulation and results of the six substrates for rectangular MSPA

The simulation begins with FR4, followed by Rogers DT5880, Rogers RO4003C, Teflon, Arlon AD300A, and Alumina (99%). Each substrate is analyzed using the MSPA at a frequency of 5.8 GHz. The results of the simulation are presented in Fig. 2, which shows the S_{11} parameters, while Fig. 3 displays the corresponding radiation patterns.

The simulation results indicate that the Teflon substrate exhibits the lowest return loss of -44.2 dB among

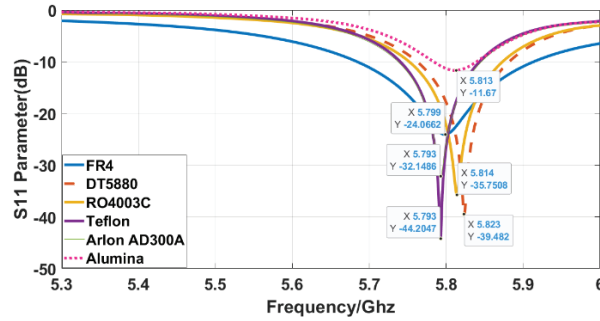


Fig. 2. S_{11} results of the six different substrates at 5.8 GHz.

the six designs evaluated. This suggests that Teflon is the optimal material for researchers aiming to minimize return loss in their antenna projects.

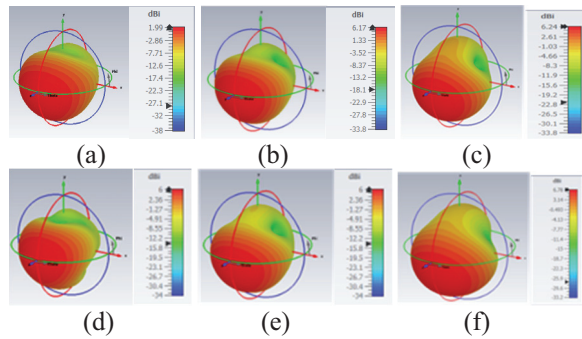


Fig. 3. Radiation pattern of MSPA.

Radiation patterns demonstrate that Fig. 3 (a) FR4 and Fig. 3 (d) Teflon produce wider beams with lower gain, while Fig. 3 (b) Rogers and Fig. 3 (e) Arlon provide higher gain and more directional patterns. Figure 3 (f) Alumina creates a narrow beam, but with moderate gain. Material choice impacts gain, beamwidth, and side lobes. High-frequency laminates generally yield better radiation characteristics.

Figure 4 (a) FR4 and Fig. 4 (c) RO4003C exhibit more scattered currents, indicating higher losses. Figure 4 (b) Rogers and Fig. 4 (e) Arlon have concentrated currents along the patch edges, improving efficiency. Figure 4 (d) Teflon shows moderate distribution, while Fig. 4 (f) Alumina has strong edge currents but limited spread. Material choice affects current flow, efficiency, and radiation behavior.

Higher-directivity substrates are typically lowloss, stable laminates (e.g., Rogers family), which better confine radiation. In comparison of six different substrates, the Rogers DT5880 substrate demonstrates the highest directivity, making it an excellent choice for applications where focused signal transmission is critical. Also, as

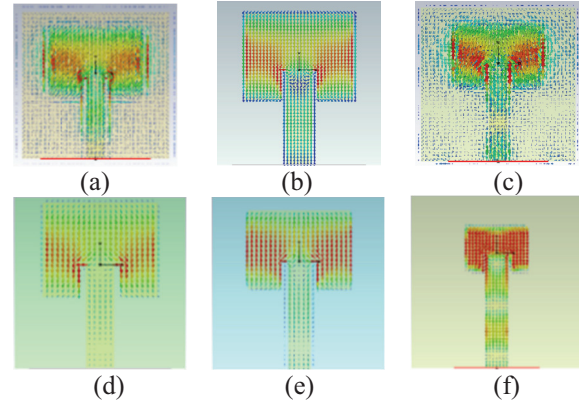


Fig. 4. Surface current of RMSPA.

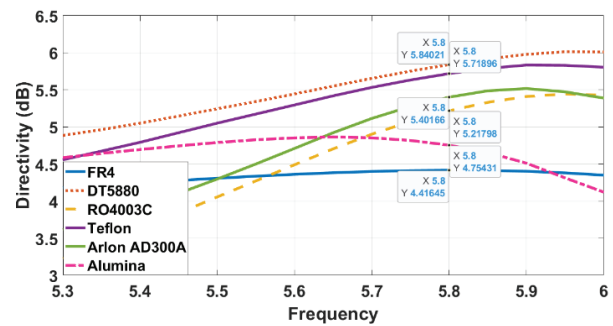


Fig. 5. Directivity of the six different substrates comparison at 5.8 GHz.

illustrated in Fig. 5, for narrow beams at 5.8 GHz, DT5880/RO4003C are preferable to FR4/Arlon.

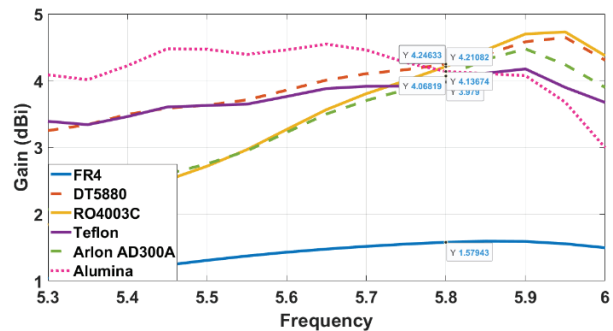


Fig. 6. Gain of the six different substrates comparison at 5.8 GHz.

Rogers 4003C exhibits the highest gain, as shown in Fig. 6, of 6.244 dBi at 5.8 GHz. In contrast, FR4 shows the lowest gain among the six substrates analyzed, with a value of 1.989 dBi. This difference in performance can be attributed to the gain equation, which indicates that gain is influenced by both efficiency and directivity.

At 5.8 GHz, Alumina shows the highest efficiency ($\approx 87\%$), followed by DT5880 ($\approx 80\%$). RO4003C and

Table 2: Dimensions of the MSPA and substrates at 5.8 GHz

Parameters	Results					
	FR4	DT5880	RO4003	Teflon	Arlon AD300A	Alumina (%99.5)
Bandwidth	0.22	0.11	0.12	0.10	0.10	0.055
Return Loss (dB)	-24	-39	-35	-44	-32	-11
Gain (dBi)	1.98	6.17	6.24	5.99	6.09	6.17
VSWR	1.13	1.02	1.03	1.01	1.05	1.70
Directivity (db)	6.4	7.66	7.19	7.5	7.34	6.78

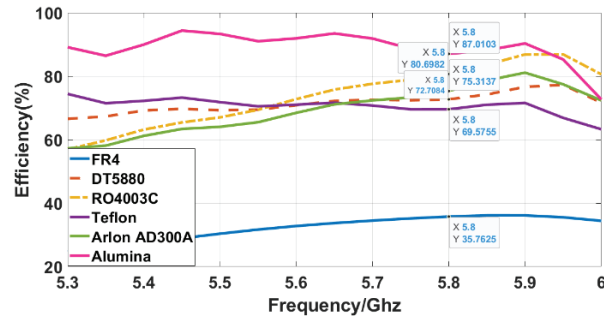


Fig. 7. Efficiency of the six different substrates comparison at 5.8 GHz.

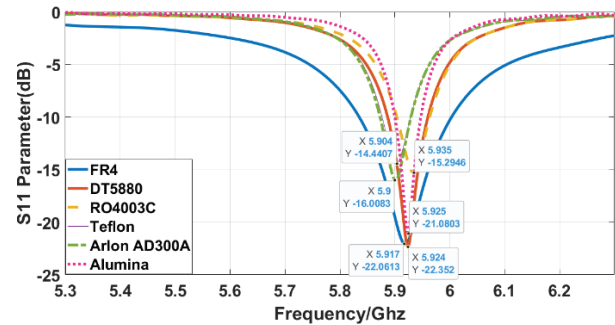
Teflon are mid-tier ($\approx 75\%$ and $\approx 73\%$), while Arlon AD300A is slightly lower ($\approx 70\%$). FR4 remains the least efficient ($\sim 36\%$) due to higher dielectric and conductor losses, as illustrated in Fig. 7. Overall, low-loss, high-quality laminates deliver markedly better radiation efficiency than FR4, as summarized in Table 2.

B. Experimental analysis and results

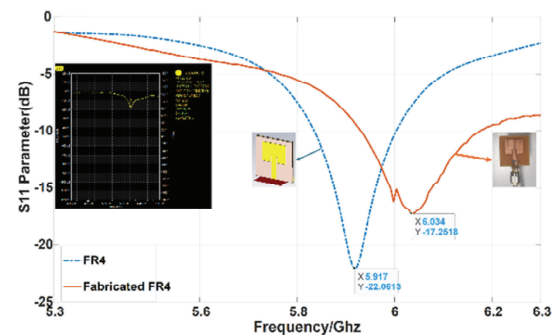
After the simulations, measurements were first performed using a vector network analyzer (VNA) on a rectangular patch antenna fabricated on an FR4 substrate. Furthermore, a CMSPA operating at 5.8 GHz was designed, simulated, and fabricated on the same substrate, and its performance was subsequently measured using the VNA. The goal was to compare the impact of antenna shape on performance at the same frequency. This approach helps determine the optimal geometry and substrate for the desired application.

C. Analysis of the fabricated rectangular microstrip patch antenna

Substrate thickness from 1.6 mm to 1 mm has been varied to evaluate its influence on the MSPA parameters. All other dimensions of the antenna remained constant. FR4, which exhibited the lowest reflection coefficient for the proposed antenna, was selected for fabrication and measurement since it is readily available and offers a cost-effective manufacturing process.

Fig. 8. Reflection coefficient (S_{11}) of the FR4 with thickness of 1 mm substrate on CST Microwave Studio.

The S_{11} plot, as shown in Fig. 8, presents a comparison of return loss for six different substrates at a thickness of 1 mm. All antennas exhibit resonance near 5.8 GHz; however, DT5880 and RO4003C achieve the deepest nulls (~ -23 dB), indicating excellent impedance matching. Teflon, Arlon, and Alumina show slightly higher S_{11} (~ -21 dB), still acceptable. FR4 performs worst (~ -18 dB) with a broader dip, suggesting higher mismatch and losses. Overall, low-loss laminates provide superior matching and bandwidth compared to FR4, as shown in Fig. 9.

Fig. 9. Comparison between simulated and fabricated results of the return loss (S_{11}) of RMSPA.

The simulated antenna achieves its best performance at around 5.918 GHz with an S_{11} of approximately -22.88 dB, whereas the fabricated

antenna resonates near 6.003 GHz with an S_{11} of about -14.08 dB, as summarized in Table 3. There is a noticeable frequency shift and reduced return loss in the fabricated design compared to the simulation. This variation can be attributed to fabrication tolerances and inconsistencies in material properties

Table 3: Comparison of fabricated and simulated rectangular MSPA

Parameters	Substrates	
	FR4	Fabricated FR4
Frequency (GHz)	5.919	6.034
Bandwidth (GHz)	0.168	0.24
Return Loss (dB)	-22.109	-17.251
VSWR	1.097	1.318

D. Simulation and results of the six substrates for CMSPA

A circular patch antenna, which is also frequently utilized in antenna design, was selected for further design and analysis. The dimensions of the circular patch antenna, substrate, feedline, gap, ground, and feed width are calculated using MATLAB code to determine the parameters of the MSPA, as illustrated in Fig. 10 [5].

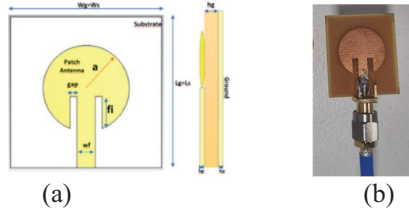


Fig. 10. Configuration of (a) CMSPA front and side view (b) fabricated CMSPA.

The materials evaluated in this section are the same as those used for the rectangular MSPA, including FR4, Rogers DT5880, Rogers RO4003C, Teflon, Arlon AD300A, and Alumina (99%).

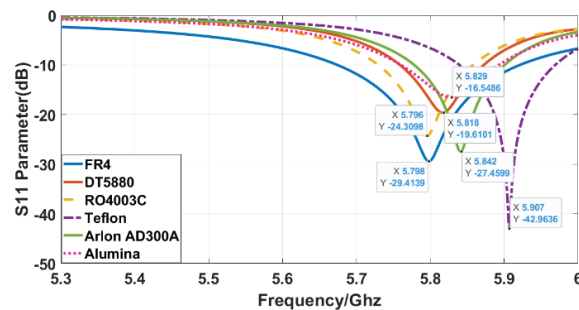


Fig. 11. Results of the return loss (S_{11}) of six different substrates with 1.6 mm thickness.

Based on the results of the simulation of the circular patch antenna, the Teflon substrate has the lowest return loss value of S_{11} , which is -42.96 dB at 5.9 GHz among the six designs, as evidenced in Fig. 11. As mentioned for the rectangular patch antenna, related results were obtained; however, the reflective frequency is 5.9 GHz, not 5.8 GHz. Therefore, we can conclude that FR4 had the lowest return loss at 5.8 GHz. Additionally, in terms of bandwidth performance, the operating frequency ranges from 5.67 GHz to 5.94 GHz, providing the largest bandwidth compared to the other substrates. Researchers can utilize Teflon as the optimal material for minimizing return loss at 5.9 GHz in their projects. However, for 5.8 GHz circular antennas, researchers can choose FR4 for their projects.

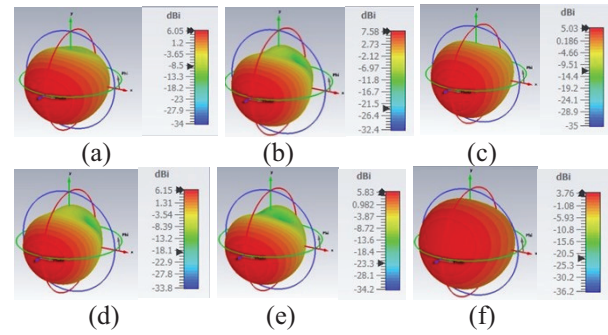


Fig. 12. Radiation pattern of CMSPA.

The color scale indicates gain in dBi, where red regions correspond to higher gain and blue regions to lower gain. Figure 12 (a) FR4 and similar low-cost substrates exhibit broader lobes with moderate gain, while high-frequency laminates and Fig. 12 (f) Alumina shows more directional patterns and slightly higher peak gains. These variations highlight the impact of substrate dielectric properties on antenna radiation behavior.

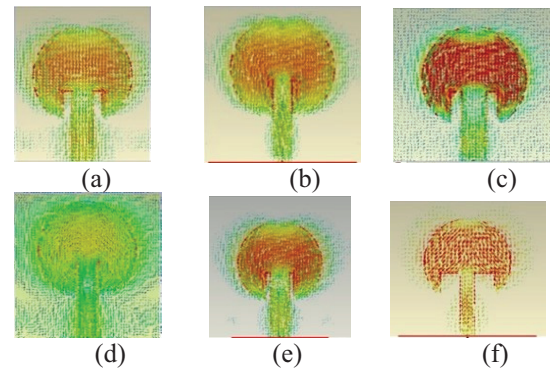


Fig. 13. Surface current of CMSPA.

Figure 13 illustrates the surface current distribution of a circular patch antenna for six different substrates.

The color scale represents current intensity in dB, where red indicates higher current concentration and blue indicates lower levels. Figure 13 (a) FR4 and similar substrates show relatively uniform current spread, while high-frequency laminates and Fig. 13 (f) Alumina exhibit more concentrated currents near the patch edges and feedline. These variations demonstrate how substrate dielectric properties influence current flow and antenna performance.

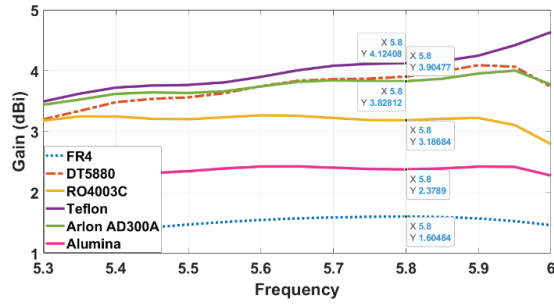


Fig. 14. Gain of the six different substrates comparison at 5.8 GHz.

Alumina shows the highest gain, reaching about 4.24 dBi near 5.8 GHz, followed by Arlon AD300A and RO4003C with gains around 3.9–4.0 dBi. Rogers DT5880 and Teflon exhibit moderate gains between 2.5–3.5 dBi, while FR4 has the lowest gain, staying below 2 dBi throughout the band. As shown in Fig. 14, the trend indicates that high-frequency laminates and ceramic substrates significantly improve antenna gain compared to low-cost FR4. This highlights the importance of substrate selection for optimizing antenna performance.

E. Analysis of the fabricated circular microstrip patch antenna

After analyzing the CMSPA simulation in CST Microwave Studio, the fabricated CMSPA can be compared with the simulated design.

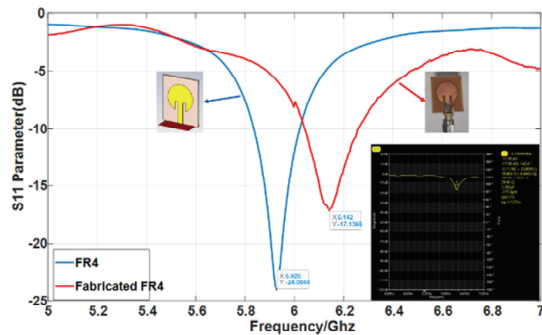


Fig. 15. Comparison between simulated and fabricated results of the return loss (S_{11}) of CMSPA.

In Fig. 15, the blue dashed line represents the simulation, showing a deep resonance at 5.928 GHz with an S_{11} of about -24 dB. The fabricated antenna (red line) resonates at 6.142 GHz with an S_{11} of approximately -17.09 dB, indicating a frequency shift and reduced return loss compared to the simulation. This discrepancy is likely due to fabrication tolerances, connector effects, and material property variations. Overall, the antenna maintains acceptable performance despite these differences.

For both RMSPA and CMSPA, the same measurement techniques and conditions were used. We also observe similar discrepancies between the measured and simulated results depicted in the S_{11} graph. However, these two different shapes of the MSPA are suitable for sub-6 GHz devices and applications. In Fig. 15, the results from the fabricated antenna closely align with the simulated results. One potential reason for the shift in the resonance frequency to a higher range could be attributed to the soldering effect of the SMA connector.

Table 4: Comparison of fabricated and simulated CMSPA

Substrates		
Parameters	FR4	Fabricated FR4
Frequency (GHz)	5.928	6.142
Bandwidth (GHz)	0.186	0.282
Return Loss (dB)	-24	-17.09
VSWR	1.134	1.325

During the analysis of the fabricated CMSPA assembled on an FR4 substrate, the bandwidth is larger than the simulated one, as presented in Table 4. However, the return loss of the fabricated antenna has decreased by approximately 0.857 dB due to the measurement process. This discrepancy can be attributed to the manufacturing process or limitations of the measurement method. These small errors in the fabricated antenna can contribute to shifts in the results. Based on the VSWR value, we can conclude that there is minimal distortion in the standing wave. Furthermore, when comparing the fabricated antenna with the simulation, both the return loss and VSWR results indicate that the simulated antenna performs better than the fabricated one. However, these differences do not mean that we cannot use the fabricated antenna. On the contrary, we can use the fabricated antenna for our applications and devices that operate sub-6 GHz.

III. CONCLUSION

This study investigates the design and performance of microstrip patch antennas (MSPAs) using six different substrates (FR4 Glass Epoxy, Rogers DT5880,

Rogers 4003C, Teflon, Arlon AD300A, and Alumina) through simulations in CST Microwave Studio. Both rectangular and CMSPAs' configurations were analyzed, with copper patches and ground planes. Simulation results revealed substrate-dependent performance variations: RMSPA achieved a maximum gain of 6.244 dBi with Rogers 4003C, while FR4 provided the widest bandwidth (0.229 GHz). Rogers DT5880 exhibited the highest directivity (7.665 dB), Teflon delivered the best return-loss (-44.20 dB), and Alumina demonstrated superior efficiency. To experimentally validate the simulation results, FR4-based antenna prototypes with a substrate thickness of 1 mm were fabricated and characterized using a VNA. In the simulation stage, both 1 mm and 1.6 mm substrate thicknesses were considered to analyze the impact of substrate thickness on antenna performance. Measurements indicated a resonance frequency shift and reduced return loss, primarily due to manufacturing tolerances and SMA connector soldering effects. The findings highlight the significant influence of dielectric constant, substrate thickness, and fabrication conditions on MSPA performance, providing practical design guidelines for sub-6 GHz wireless communication systems.

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High Efficiency, Low-Power RF Energy Harvesting Tri-Band Rectenna for Mid-Band 5G and Wi-Fi 6E Applications

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Abstract – In this paper, a new tri-band rectenna is presented for radio frequency energy harvesting (RFEH) deployment. The proposed RFEH system integrates two primary components: an antenna and a rectifier. First, the antenna consists of a CPW-fed circularly-shaped patch incorporating three slots. Each slot is strategically introduced to reject a specific band. Second, the rectifier circuit is structured to ensure an efficient RF-to-DC conversion over three bands. High efficiency peaks are achieved at 78%, 76%, and 60% respectively at 3 GHz, 4 GHz, and 6.5 GHz, under 5 dBm input power, with overall efficiency average exceeding 71%. Peak DC output voltages of 2.52 V, 2.47 V, and 2.20 V are obtained at the same frequencies under the same conditions. The results confirm that the proposed rectenna effectively spans both the 5G mid-band and Wi-Fi 6E band, enabling the simultaneous and reliable powering of low-power devices. Furthermore, the antenna exhibits high frequency repeatability, achieved through the independent tuning of each band via strategically integrated slots.

Index Terms – Power conversion efficiency (PCE), radio frequency energy harvesting (RFEH), rectifier circuit, Schottky diode, tri-band rectenna.

I. INTRODUCTION

Radio frequency energy harvesting (RFEH) has emerged as a vital solution for empowering low-power wireless electronic devices, including sensors, remote controls, RFID tags, Internet of Things (IoT), and

biotelemetry systems. By harnessing ambient RF signals, RFEH enables sustainable, battery-free operation, reducing maintenance costs and supporting seamless deployment of next-generation smart and connected technologies.

In the literature, many works on RFEH, have been proposed with the aim of improving the power conversion efficiency (PCE), and optimizing the utilization of harvested energy in both single-band and multi-band configurations [1–17]. Actually, single-band operation and low PCE of RFEH systems significantly restrict their ability to harvest enough energy. Therefore, a multiband rectenna is a pivotal solution in wireless RFEH, attracting considerable attention for their capability to harvest radio frequency (RF) energy across multiple bands simultaneously [1, 3, 4, 8–10]. Such a rectenna significantly outperforms traditional designs, enabling higher cumulative power generation. It also offers improved adaptability to diverse RF environments, and greater potential for powering next generation low-power electronic devices and IoT systems. However, the increase in the number of operating bands decreases substantially the PCE, posing therefore a huge issue in multi-band rectenna scenarios. In such architectures, the PCE obtained per band is typically lower than that of single-band rectennas, which results in an overall PCE average lower than the values reported for single-band counterparts. This behavior is attributed to the power split across multiple paths, which leads to an accumulation of ohmic and mismatch losses associated with each path. For instance, the single-band rectennas in [2, 6, 7] exhibit PCE peaks of 61% at an input power

of 10 dBm, 67.3% at $P_{in} = 18$ dBm, and 67.16% at $P_{in} = 5$ dBm, respectively. Conversely, the reduced PCE observed in multi-band rectennas [1, 3, 8, 9] remains a significant challenge in the field. According to the work in [1], a tri-band rectenna shows a relatively low PCE average of three bands of about 28.8% at $P_{in} = -10$ dBm. Another contribution in [3] shows PCE of dual-band rectenna of 54.3% at 2.4 GHz and 52.7% at 5.8 GHz under input power of $P_{in} = 0$ dBm, resulting in a PCE average of 53.5%. Also, a dual-band rectenna in [8] shows PCE peaks of 54% at 2.4 GHz and 44.4% at 5.8 GHz under 14 dBm input power, which yields a PCE average of 49.2%. In [9], a dual band rectenna offers peak conversion efficiencies of 61% at 1.75 GHz and 72% at 2.45 GHz with an input power of 0 dBm, giving a PCE average of 66.5%.

Among the rectennas reported in the literature, the proposed design combines high efficiency with multi-band capability, achieving a greater PCE average through the appropriate selection of the rectifier topology and the careful optimization of the matching network. Additionally, the incorporation of slot technique in the antenna geometry yields efficient frequency tuning, achieved simply by adjusting the length of each slot, offering precise and repeatable frequency. The proposed rectenna is designed exclusively to cover the standard 802.11ax [18], which is developed for unlicensed operations of the 5G and extended Wi-Fi 6E band of 5.925–7.125 GHz. This last offers high throughput, higher reliability, and low-latency [19]. Within its operating band, the proposed rectenna exhibits high capability for integration into advanced technologies.

II. ANTENNA DESIGN

The proposed antenna, as depicted in Fig. 1, is designed using a rogers RO4003C substrate of a permittivity of 3.38, a loss tangent of 0.0027, and a height of 1.524 mm. A CPW-fed circularly-shaped patch antenna is proposed including three slots within the feeding line and radiating element to reject three unwanted bands. The CST Microwave studio suite simulator tool version 2024 [20] is used for designing the antenna.

Figure 2 portrays the reflection coefficient magnitude in dB against frequency in GHz of various configurations including: no slot, one slot, two slots and three slots. The final optimized configuration corresponds to the design incorporating two C-shaped slots and one U-shaped slot. In this design, the surface current is concentrated around the extremity of each slot, creating an impedance mismatch that stops the radiation at the associated frequency. The guided wavelength of the C-shaped slot is given as

$$\lambda_g = \frac{\lambda_0}{\sqrt{\epsilon_{eff}}}, \quad (1)$$

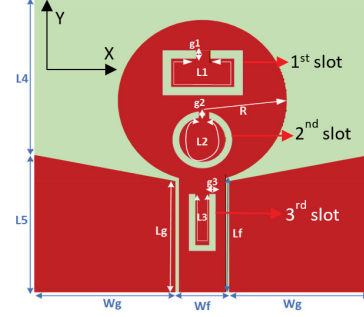


Fig. 1. Proposed CPW-fed circularly-shaped antenna. $L1=29.6$, $L2=16$, $L3=12.4$, $L4=21$, $L5=15$, $Lf=13.13$, $Lg=12.75$, $Wg=17.10$, $R=10$, $Wf=3.8$, $g1=0.5$, $g2=0.72$, $g3=0.5$ (all dimensions in mm).

where λ_0 is the free-space wavelength and $\epsilon_{eff} = (\epsilon_r + 1)/2$ is the effective permittivity. The length of each slot, denoted L_{slot} , is around $\lambda_g/2$, which gives high capability for frequency shifting. Indeed, the notched frequency, denoted f_{notch} , is given as [21]

$$f_{notch} = \frac{c}{2L_{slot}\sqrt{\epsilon_{eff}}}. \quad (2)$$

In the case of the U-shaped slot, the notched frequency is given as [22]

$$f_{notch} = \frac{c}{2(L_{slot} + W_{slot})\sqrt{\epsilon_{eff}}}, \quad (3)$$

where L_{slot} and W_{slot} are the length and the width of the U-shaped slot respectively.

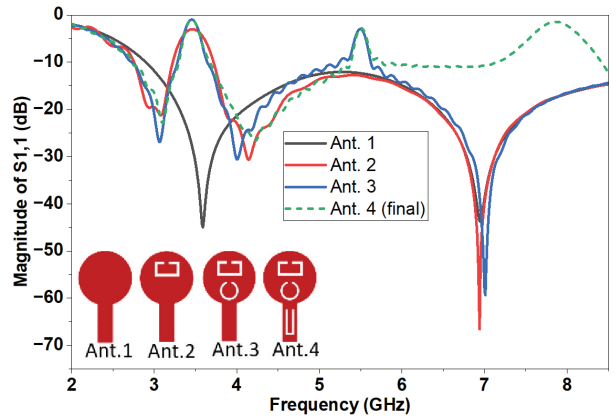


Fig. 2. Magnitude of $S_{1,1}$ in dB against frequency in GHz of proposed configurations.

Figure 3 illustrates the realized gain (in dBi) as a function of frequency (in GHz) for several design configurations. These configurations include the baseline design with no slot, as well as designs incorporating one slot, two slots, and three slots. The final configuration

represents the three-slot design, which exhibits a deliberate reduction in gain at the three rejected frequency bands.

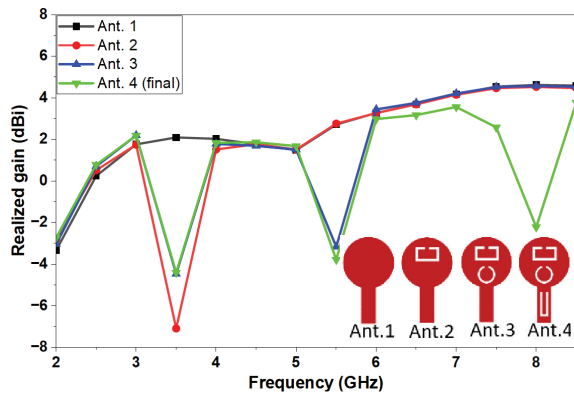


Fig. 3. Realized gain in dBi against frequency in GHz of proposed configurations.

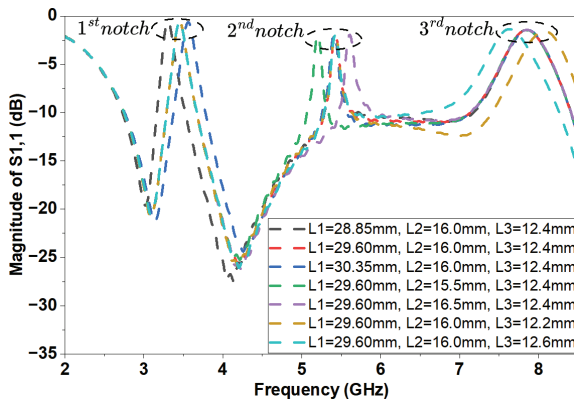


Fig. 4. Slot length optimization, including L1, L2, and L3.

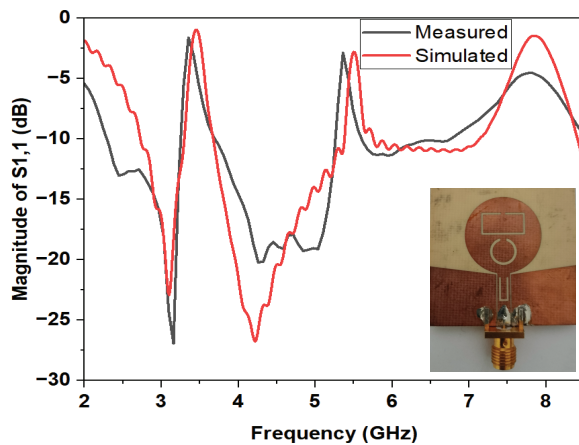


Fig. 5. Magnitude of $S_{1,1}$ in dB against frequency in GHz of proposed antenna.

Figure 4 demonstrates the effect of slot length on the reflection coefficient, where each slot length is varied across three values, as indicated in the figure legend. The increase in slot length decreases the frequency notch as expected, which is consistent with the behavior predicted by equations (2) and (3).

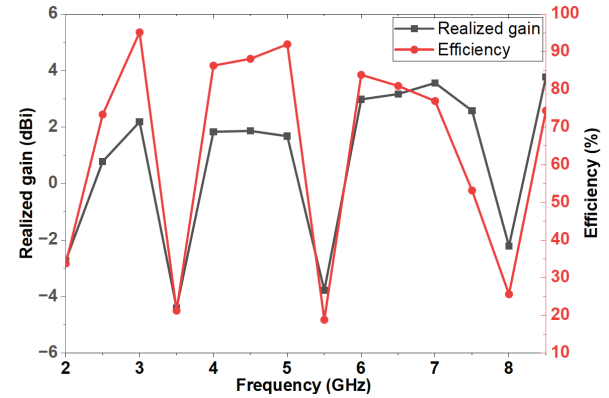


Fig. 6. Realized gain (in dBi)/efficiency (in %) against frequency (in GHz) of the proposed antenna.

In Fig. 5, the reflection coefficient magnitude (in dB) is plotted against frequency (in GHz) for both simulation and measurement results. The plot demonstrates excellent impedance matching across the operating bands, with strong agreement between measured and simulated results. A slight discrepancy is observed due to the influence of the SMA connector.

The Agilent 8722ES network analyzer is used in measuring the reflection coefficient. The calibration is done by applying short, open, load, and through (SOLT) standards. Figure 6 presents both the realized gain (in dBi) and efficiency (in %) as a function of frequency (in GHz) for the optimized configuration. This latter corresponds to the three-slot design, which demonstrates significant reduction in both gain and efficiency at the three rejected frequency bands. Across the operating bands, the antenna maintains excellent radiation efficiency, reaching its maximum value of 95% at 3 GHz, with adequate gain.

Figure 7 shows the radiation patterns of the proposed antenna at three operating frequencies of 3 GHz, 4 GHz, and 6.5 GHz. The radiation pattern is evaluated in both the E- and H-planes (XZ- and YZ-planes). The antenna exhibits stable bidirectional radiation at all frequencies with adequate values.

Figure 8 demonstrates the anechoic chamber where the proposed prototype is measured. The measurement distance between the probe and the antenna under test (AUT) is 1.524 m. The double-ridged horn antenna, model AH-118, is used as a probe with an input power of 300 Watts.

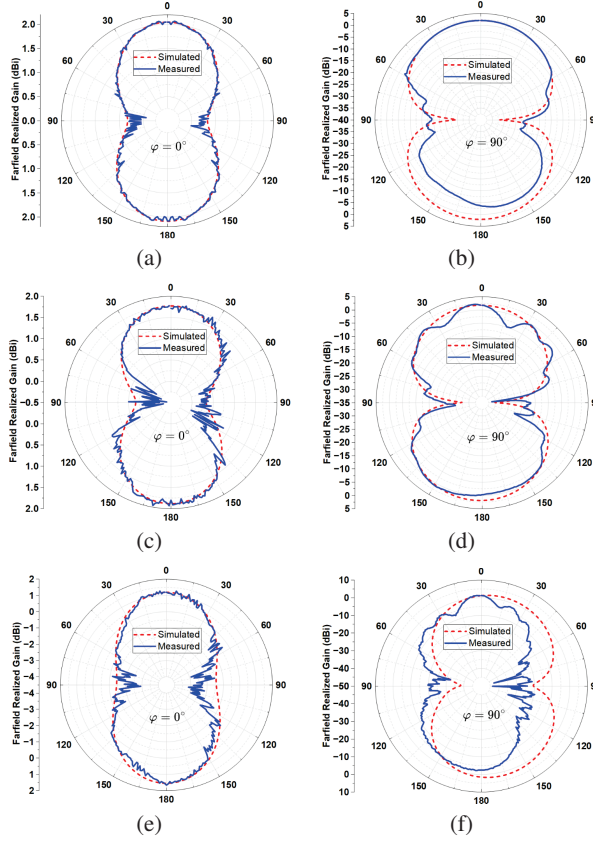


Fig. 7. Radiation patterns of proposed antenna at (a,b) 3 GHz, (c,d) 4 GHz, and (e,f) 6.5 GHz.

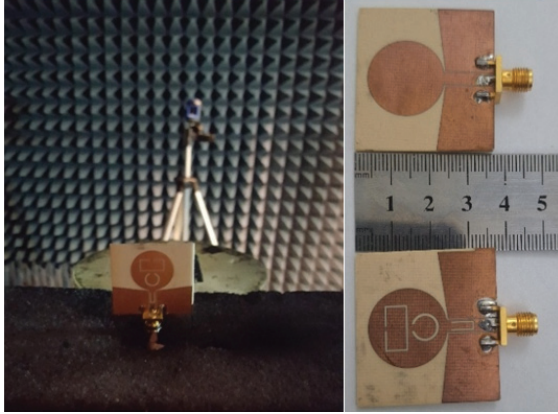


Fig. 8. Anechoic chamber and proposed prototype.

III. A TRI-BAND RECTIFIER DESIGN

The rectifier section, as presented in Fig. 9, consists of four main blocks: a matching network that ensures proper impedance matching between the antenna and the rectifier circuit; the rectifier circuit, which converts the incoming RF signal into DC power; a DC-low pass filter that suppresses higher harmonics orders to

maintain signal purity; and finally, the load, representing the device to be powered. The design of the rectifier is obtained by using advanced design system (ADS), simulator version 2020 [23].

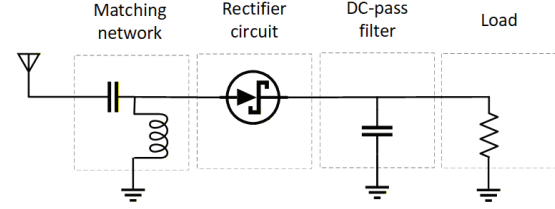


Fig. 9. Rectifier synoptic scheme.

The PCE is the crucial parameter in RFEH, which measures the power ratio between DC and RF signals. It is expressed as [8]:

$$PCE(\%) = \frac{P_{DC}}{P_{in}} \times 100\% = \frac{V_L^2}{R_L P_{in}} \times 100\%, \quad (4)$$

where P_{DC} is the DC power delivered to the load R_L . P_{in} is the input power injected at the rectifier input. Actually, the input power is linearly related to the rectenna gain by

$$P_{in} = \frac{P_d \lambda^2}{4\pi} G_r, \quad (5)$$

where P_d denotes the incident power density (in W/m^2), G_r denotes the receiving antenna gain, and λ denotes the free-space wavelength. To obtain a highly sensitive rectenna, the antenna gain should be increased. The received power at the rectenna, denoted P_r , is expressed as follows

$$P_r = P_t + G_t + G_r + 20 \log_{10} \left(\frac{\lambda}{4\pi d} \right), \quad (6)$$

where P_t denotes the transmitted power (horn antenna power); G_t and G_r are the measured gains of the standard horn antenna and the rectenna under test, respectively. The parameter λ indicates the operating wavelength in free-space, and d is the distance separating the two antennas. The equivalent circuit model of the Schottky diode is portrayed in Fig. 10 [24].

The equivalent impedance of the Schottky diode denoted Z_d is given, according to its ECM in Fig. 10, by

$$Z_d = jL_p \omega + \frac{\left(R_s + \frac{R_j}{1+jR_j C_j \omega} \right) \times \frac{1}{jC_p \omega}}{\left(R_s + \frac{R_j}{1+jR_j C_j \omega} \right) + \frac{1}{jC_p \omega}}. \quad (7)$$

The input impedance of the voltage-doubler rectifier, denoted Z_{in} , is given according to Fig. 11 by

$$Z_{in} = \frac{1}{jC_1 \omega} + Z_d // \left\{ Z_d + \left(\frac{1}{jC_2 \omega} // R_L \right) \right\}. \quad (8)$$

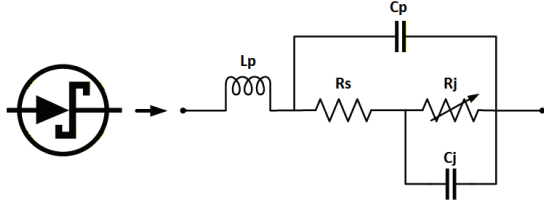


Fig. 10. Schottky diode equivalent circuit model (ECM). SPICE parameters of the used SMS7621 series are: $L_p = 0.7$ nH, $C_p = 0.25$ pF, $R_s = 12$ Ω , $C_{j0} = 0.1$ pF, $I_s = 0.04$ μ A, $N=1.05$, $TT = 10^{-11}$ s, $M = 0.35$, $E_g = 0.69$ eV, $XTI = 2$, $F_c = 0.5$, $B_V = 3$ V, $I_{BV} = 0.01$ mA, and $V_j = 0.51$ V.

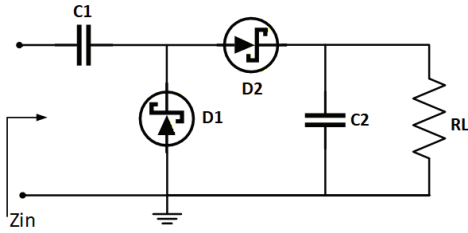


Fig. 11. One stage Dickson voltage-doubler rectifier using Schottky diode SMS7621 series.

The input impedance of the rectifier, denoted Z_{in} , is plotted in Fig. 12 as a function of frequency for different input power levels. The input impedances of the rectifier, at input power of 5 dBm, are $10.263 - j70.659$ Ω at 3 GHz, $6.509 - j49.822$ Ω at 4 GHz, and $1.887 - j21.531$ Ω at 6.5 GHz, respectively.

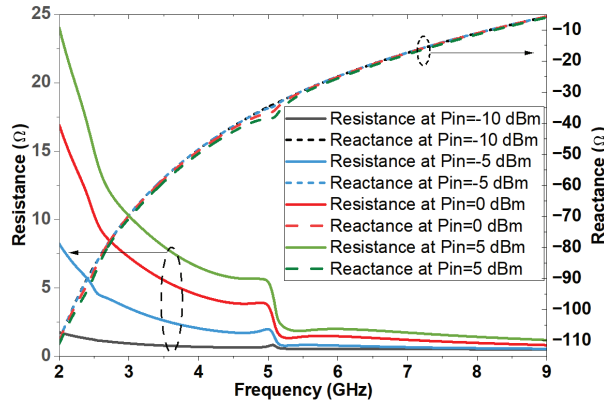


Fig. 12. Rectifier input impedance in Ω versus frequency in GHz by applying different values of input power P_{in} .

Figure 13 displays the proposed tri-band energy harvesting rectifier circuit, including three L-section matching networks. Each circuit is calculated using the Smith Chart in Fig. 14. The calculated values for each band are applied and slightly tuned under $P_{in}=5$ dBm input power

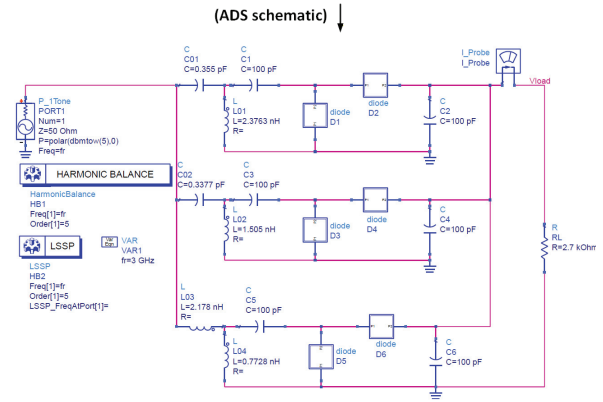
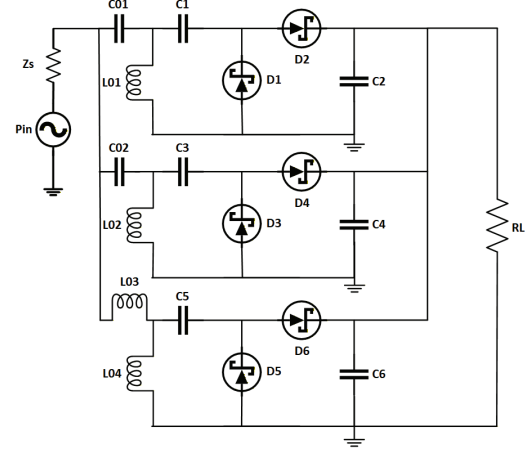


Fig. 13. Proposed rectifier circuit for tri-band RF energy harvesting deployments using Schottky diode SMS7621, $C_1 = C_2 = 100$ pF, $R_L = 2.7$ k Ω .

to maximize the PCE, satisfying the matching condition $Z_{in} = Z_s^*$. Each L-section circuit converts the 50 Ω source impedance to the conjugate input impedance of each stage. The power delivered from the source encounters, in fact, three different paths, and the current flows through the path that is matched for the given frequency.

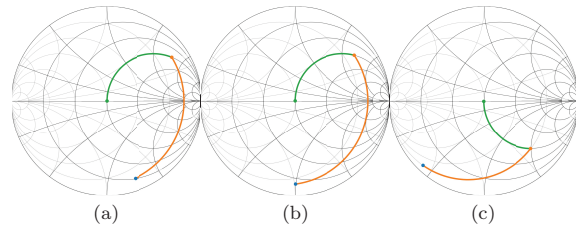


Fig. 14. Impedance matching between the antenna and rectifier using Smith Chart at (a) 3 GHz, (b) 4 GHz, and (c) 6.5 GHz at $P_{in} = 5$ dBm.

After the rectification process, the Schottky diode output is not purely DC. Instead, it contains a DC component along with the fundamental frequency and various higher-order harmonics. This spectral composition

arises from the nonlinear behavior of the diode, which distorts the input signal. Therefore, a low-pass DC filter is crucial to suppress these harmonics [25]. The Schottky instantaneous diode current is expressed as follows

$$I_d = I_s \left(e^{\frac{V_d}{nV_T}} - 1 \right), \quad (9)$$

where I_s is the saturation current, V_d is the diode voltage, n is the ideality factor, and $V_T = kT/q$ is the thermal voltage.

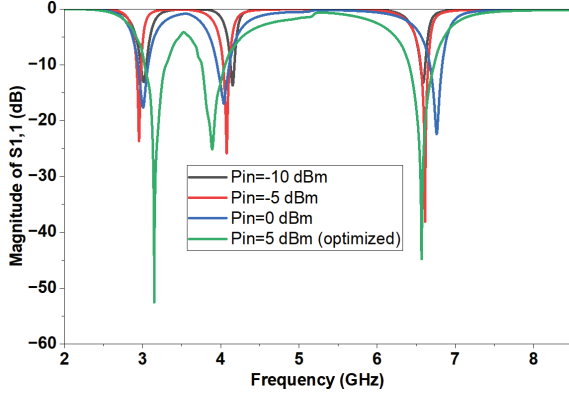


Fig. 15. Magnitude of the reflection coefficient in dB versus frequency in GHz of the proposed rectifier circuit by applying different input power levels.

Figure 15 shows the magnitude of reflection coefficient in dB as a function of frequency in GHz for the proposed rectifier under input power levels of -10 dBm, -5 dBm, 0 dBm, and 5 dBm. The best impedance matching is obtained at 5 dBm, which exhibits the lowest reflection coefficient across the three widest bandwidths centered at 3 GHz, 4 GHz, and 6.5 GHz.

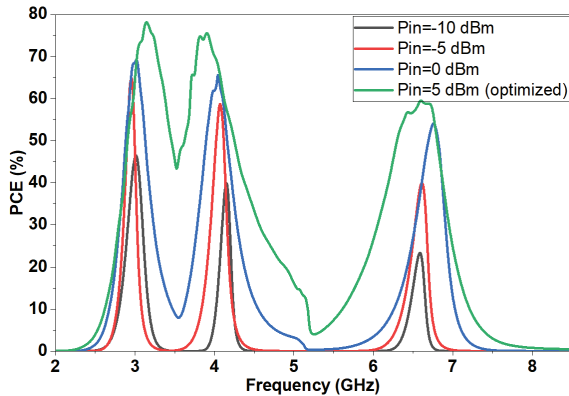


Fig. 16. Power conversion efficiency (PCE) in % versus frequency in GHz for the proposed rectifier circuit by applying different input power levels.

Figures 16 and 17 present the PCE in % and the DC output voltage (V_L) in V as a function of frequency

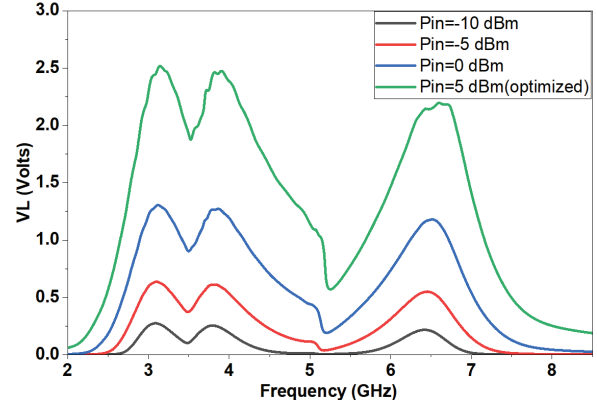


Fig. 17. DC output voltage (V_L) in V against frequency in GHz for the proposed rectifier circuit.

in GHz, for the proposed rectifier circuit at different input power levels, including -10 dBm, -5 dBm, 0 dBm, and 5 dBm. The highest values of PCE and DC output voltage are obtained by applying an input power level of 5 dBm (3 mW). Reached peak values of the PCE are 78% , 76% , and 60% respectively at 3 GHz, 4 GHz, and 6.5 GHz. The corresponding peak DC output voltages of 2.52 V, 2.47 V, and 2.20 V are obtained at the same frequencies under 5 dBm input power.

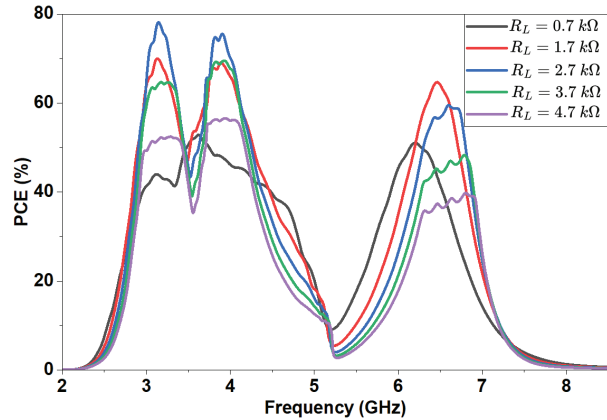


Fig. 18. Power conversion efficiency (PCE) against frequency at different values of R_L for the proposed rectifier circuit at 5 dBm.

Figure 18 displays the PCE against frequency for the proposed rectifier circuit at different values of R_L including 0.7 kΩ, 1.7 kΩ, 2.7 kΩ, 3.7 kΩ, and 4.7 kΩ. The optimized PCE is achieved with a load of $R_L = 2.7$ kΩ, as expected, since this value is considered in the matching network.

Figure 19 shows the PCE and DC output voltage (V_L) against input power (P_{in}) for the proposed rectifier circuit at 3 GHz, 4 GHz, and 6.5 GHz. The peak value of PCE is obtained at 5 dBm input power, while the DC

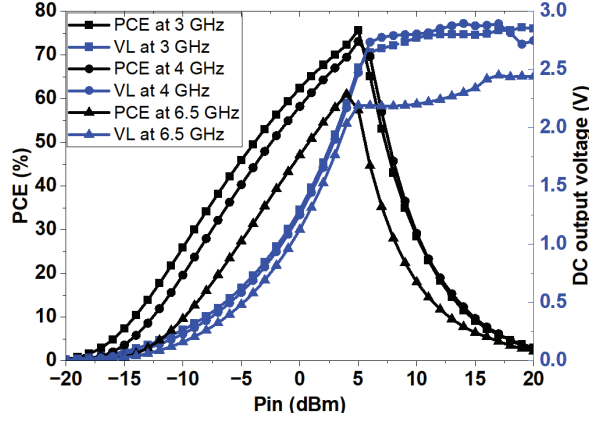


Fig. 19. Power conversion efficiency (PCE) and DC output voltage (V_L) against input power (P_{in}) for the proposed rectifier circuit at 3 GHz, 4 GHz, and 6.5 GHz.

output voltage continues to increase with input power. A merit factor is proposed here to perfectly characterize a multiband rectifier, referred to as the efficiency average, and given by

$$P\hat{C}E = \frac{1}{k} \sum_{i=1}^k PCE_i, \quad (10)$$

where k is the total number of bands and PCE_i is the i^{th} band PCE.

Table 1: Performance comparison with the literature

Ref.	Year	Number of bands/ P_{in}	Max. PCE/ PCE Ave.	Design Complexity
[1]	2025	3/−10 dBm	36.8%/29%	High
[2]	2025	1/18 dBm	61%/NA	Medium
[3]	2025	2/0 dBm	54.3%/53.5%	High
[5]	2023	1/17 dBm	>50%/NA	High
[6]	2024	1/18 dBm	67.3%/NA	Medium
[7]	2024	1/5 dBm	67.16%/NA	Low
[8]	2025	2/14 dBm	54%/44.4%	Medium
[9]	2025	2/0 dBm	61%/66.5%	Medium
[10]	2025	3/−20 dBm	33.3%/23.3%	Medium
Proposed		3/5 dBm	78%/71%	Low

Max. = maximum, Ave. = average, NA = not applicable.

Table 1 compares the proposed rectenna to those recently reported in the literature. The proposed design demonstrates clear superiority in terms of the efficiency average, number of operating bands, and design simplicity. In contrast, most existing works [1–3, 5–10] exhibit non-repeatable configurations and/or low efficiencies, restricting their ability in harvesting energy efficiently. The proposed rectifier shows a higher PCE of 78% with higher efficiency average of 71%, outperforming the majority of designs in the literature [1–3, 5–10].

IV. CONCLUSION

In this paper, a tri-band rectenna has been proposed and tailored for harvesting low-power RF energy from different bands. The power conversion efficiency (PCE) has peaked at 78%, 76%, and 60% respectively at 3 GHz, 4 GHz, and 6.5 GHz, yielding a PCE average exceeding 71% across the three bands. The proposed rectifier covers the mid-band 5G and Wi-Fi 6E deployments. A high simplicity of the proposed rectenna has been shown, which makes it highly repeatable in frequency with efficient AC-to-DC power conversion. Furthermore, incorporating the slot technique into the antenna design yields a highly tunable and repeatable frequency response. By combining high efficiency with multiband operation across independently tunable frequencies, the proposed rectenna delivers stable and sustained power output suitable for energy harvesting, particularly in low-power wireless devices.

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A Compact High Scanning Rate Leaky-Wave Antenna Based on Spoof Surface Plasmon Polaritons Transmission Line

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Abstract – A leaky-wave antenna with wide scanning angle and high scanning rate based on spoof surface plasmon polaritons transmission line (SSPPs-TL) is proposed. A tilted unit structure is introduced to flexibly adjust the dispersion characteristics, which not only reduces the cut-off frequency of the antenna but also enhances the electric field confinement. In addition, in order to achieve faster beam scanning, a row of periodic metal patches are placed on one side of the SSPPs slow wave structure, while periodic modulation metal strips are introduced on the other side. The proposed design converts slow waves into fast waves while broadening the scanning angle, thereby increasing the scanning rate. The effectiveness of the proposed scheme has been verified by the designed SSPP leaky-wave antenna in this paper. The measured results demonstrate that the designed antenna achieves a total scanning range of 41° to 30° and a maximum gain of 9.31 dBi in the operating frequency band of 5.8–8.4 GHz. Moreover, this antenna features a low profile and a compact structure and can be applied in communication systems.

Index Terms – Asymmetric transmission structure, beam scanning, high scanning rate, leaky-wave antenna, spoof surface plasmon polaritons (SSPPs).

I. INTRODUCTION

In recent years, leaky-wave antennas [1] have received extensive attention due to their characteristics, including frequency beam scanning, strong directivity, compact structure and low cost. Compared with traditional array antennas [2], leaky-wave antennas do not require a complex feed network or a large number of array elements to control beam scanning. Therefore, they offer significant advantages, such as light weight, small size, and low cost, making them highly suitable for radar sensors and wireless communication systems [3].

The radiation of leaky-wave antennas is typically generated by the gradual leakage of electromagnetic energy along the structure [4–6]. These structures can be categorized into two types: uniform leaky-wave antennas [7, 8] and periodic leaky-wave antennas [9]. Uniform leaky-wave antennas are usually designed based on patch array structures, which results in a large longitudinal size. In contrast, periodic leaky-wave antennas do not rely on antenna arrays and can excite radiation modes solely through the transmission structure of periodic sine modulation. However, they have the disadvantage of large horizontal dimensions. To address the limitations of leaky-wave antenna structures, various methods have been introduced into the design in recent years, including Goubau lines [10], composite right/left-handed metamaterial transmission line [11], substrate integrated waveguide [12, 13] and spoof surface plasmon polaritons transmission lines (SSPPs-TLs) [14–16]. An SSPPs-TL can emulate the optical properties of surface plasmon polaritons (SPPs) at microwave frequencies, exhibiting characteristics such as short wavelength, field focusing, and strong dispersion. Thus, it has become an important structure for facilitating the design of beamforming antennas.

In the early stages of SSPP antenna research, wider beam scanning angles attracted more attention. For example, in 2016, Yin et al. designed a patch-type SSPP leaky-wave antenna operating at 3–6 GHz with a scanning angle of 55° [17]. In 2017, Guan et al. extended the scanning angle to 66° by enhancing the coupling strength of the SSPP antenna with a double-layer structure [18]. In 2018, a 16 GHz bandwidth was utilized to achieve a remarkable 90° scanning range [19]. However, the above designs face similar problems of a large size or a low scanning efficiency (because of the large frequency band). Since then, the study of compact SSPP antennas with narrow bandwidths and relatively wide scanning angles has attracted increasing interest.

In 2018, an SSPP antenna by loading periodic sinusoidal modulation strips was proposed. Its bandwidth was only 2 GHz, but its scanning range reached from 30° to 51° [20]. However, its size remained relatively large ($9.3 \times 1.1\lambda_{\min}^2$). In 2019, a similar technique was used to reduce the size of an SSPP antenna to $3.5 \times 0.8\lambda_{\min}^2$ with a scanning angle of 64.6° and a bandwidth of 4.5 GHz [21]. The reason for the low scanning rates of SSPP antennas is that most of their structures are periodic, which may result in the Bragg effect. Bragg resonance can significantly reduce radiation efficiency and may even completely suppress electromagnetic wave radiation. This phenomenon is referred to as open stopband (OSB). In 2024, an impedance-discontinuous SSPP periodic structure was introduced to suppress OSB, which enables the antenna to achieve a scanning angle of 84° while maintaining a relatively small size ($5.8 \times 0.6\lambda_{\min}^2$) and bandwidth (5 GHz) [22]. In summary, the primary challenge for SSPP antennas lies in balancing a narrow bandwidth and a compact size while achieving a wide scanning angle.

In this paper, a wide-angle high scanning rate SSPP antenna is proposed. The tilted straight strips are utilized as the periodic structure of the SSPPs-TL, which can reduce the lateral size of the antenna, lower the cutoff frequency, and enhance the electric field bonding ability. On one side of the SSPPs-TL, radiation patches are introduced to leak electromagnetic energy from the SSPPs-TL into free space, while on the other side, periodic modulation metal ripples are introduced to form an asymmetric SSPPs transmission structure. The asymmetric structure disrupts the Bragg reflection condition and reduce the energy reflection, thus weakening the influence of the OSB, expanding the scanning range of the antenna and improving its scanning rate, which is crucial for spectrum conservation and improving frequency band utilization efficiency, as it can cause continuous beam angle changes within a limited spectral range, reduce the directional dwell time of the beam, and thus reduce the probability of signal interference through spatiotemporal signal distribution.

II. DESIGN CONFIGURATION AND APPROACH

A. Overall design of the antenna

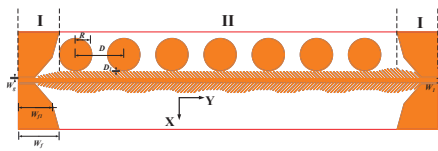


Fig. 1. Overall structure of the proposed SSPPs leaky-wave antenna.

The overall structure of the proposed antenna is illustrated in Fig. 1. It has a size of $181 \times 42 \text{ mm}^2$. The substrate of the antenna is an F4B dielectric with a thickness of 0.8 mm and a relative dielectric constant of 2.25 (loss tangent of 0.001). The coppers on the substrate have a thickness of 0.035 mm. This antenna consists of two parts: I and II. Part I comprises a coplanar waveguide and two transition structures at both ends, which are designed for impedance matching between the coplanar waveguide and the SSPPs-TL. Part II consists of double-sided asymmetric SSPPs strips and seven equidistant circular metal radiation patches. These asymmetric periodic strips effectively suppress side-lobes and enhance wide-edge radiation. Furthermore, to enhance the gain of the leaky-wave antenna, the height of the strips is gradually increased by ΔH . Utilizing this method effectively lowers the cut-off frequency of SSPPs-TL, thereby enabling a tighter confinement of electromagnetic energy. Subsequently, when radiation patches are incorporated, a greater amount of energy is coupled into the radiation and subsequently radiated into free space, which leads to an enhanced gain. Ultimately, the electromagnetic waves propagate in parts I and II and radiate into free space by coupling with radiation patches.

B. Design of SSPPs-TL

The structures of the SSPPs-TL are shown in Fig. 2. To reduce the longitudinal size of the antenna, tilted unit structures are employed. The metal strips on the upper and lower sides of the transmission line exhibit distinct configurations. Except for the ones used for impedance matching at both ends, the strips on the upper side of the transmission line are evenly distributed in parallel, while the others on the lower side exhibit periodic wave like distribution characteristics.

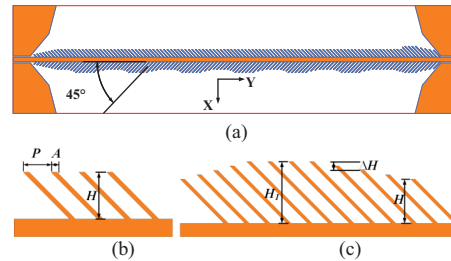


Fig. 2. Structures of the SSPPs-TL: (a) overall structure, (b) unit structures on the upper and (c) lower side of SSPPs-TL.

The dispersion characteristics of the metal strips shown in Fig. 2 (c) are given in Fig. 3. The data is obtained using the CST Microwave Studio's eigenmode

solver. The variation in wave number k_p defined as

$$k_p = 2\pi f / v_p, \quad (1)$$

is observed. Here, f is the frequency of the leaky wave and v_p is its phase velocity. The relationship between k_p and the parameters of the strips is

$$k_p = k_0 \sqrt{1 + \frac{A^2}{P^2} \tan^2(k_0 H)}, \quad (2)$$

where k_0 is the wave number in free space, A and P represent the width and period of the strips (see Fig. 2), respectively. Generally, A is much smaller than P . Therefore, H is critical. Increasing H will shorten the wavelength, thereby tightly binding the electromagnetic waves to the surface of SSPPs units. According to equation (2), only when $k_0 H \geq \pi/4$ can $k_p < k_0$ be obtained. Therefore, in the range of 9 GHz, the theoretical value of H is less than 3.87 mm. In this case, the phase velocity of the electromagnetic wave on the surface of SSPP satisfies $v_p < v_0$, which is a slow wave.

To verify the above analysis, H is initially set to 3 mm, and the tilt angles (α) of the metal strips are changed from 30° to 90° . As Fig. 3 (a) shows, as α decreases, the dispersion curve approaches to the light values at 45° and 30° . Because a small tilt angle will increase the overall lateral size of the antenna, $\alpha = 45^\circ$ is chosen in this paper. Then, A and P are set to 0.3 mm and 1.44 mm, respectively; H is changed from 2.4 mm to 3 mm and 3.6 mm, with a scanning frequency range of 0–25 GHz. From the data in Fig. 3 (b), we can see that, at the same frequency, as H increases, k_p becomes larger, and the corresponding v_p of the leaky wave decreases, causing the dispersion curve to deviate from light. In addition, in the scanning frequency band of >10 GHz, as H increases, the k_p of electromagnetic waves changes smoothly, seemingly confined to the range of <10 GHz. All of the above phenomena indicate that the fundamental mode of the SSPP waveguide is located in the slow wave region. As H increases, the leakage wave field will be more tightly combined with the SSPPs units and cannot be directly radiated into space.

By periodically loading metal circular patches on the SSPPs-TL, spatial harmonics can be introduced to radiate electromagnetic energy. According to the leaky-wave antenna theory, the main beam radiation direction of the SSPP antenna is determined by [10]

$$\theta = \sin^{-1} \left(\frac{k_{-1}}{k_0} \right), \quad (3)$$

where k_{-1} is the propagation constant of the $n = -1$ spatial harmonic, which is defined as

$$k_{-1} = k_p - \frac{2\pi}{p_{\text{unit}}}. \quad (4)$$

$p_{\text{unit}} = D$ is the modulation period of the patches. According to equations (1) and (4), once f changes, k_0 also changes, which in turn adjusts k_{-1} to achieve a phase control, allowing the SSPP antenna to achieve continuous scanning of the beam from back to end over a wide-angle range.

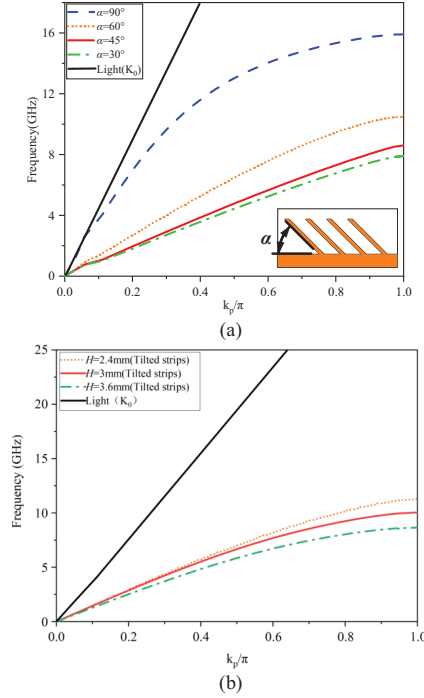


Fig. 3. Dispersion characteristics of the SSPPs-TLs constructed from metal strips with (a) tilt angles and (b) different heights.

Figures 4 (a) and (b) depict the SSPPs-TLs' electric field intensity distributions at different frequencies. The data indicate that the cut-off frequency (f_c) of the SSPPs-TLs is 10 GHz. When the operating frequency $< f_c$ (6–9 GHz), the electric fields near the SSPPs-TLs are tightly bound, which can effectively transfer electromagnetic energy from port 1 to port 2. When the frequency $> f_c$, the electric field gradually weakens. Comparing the electric field intensity distribution characteristics of Figs. 4 (a) and (b), it is evident that the asymmetric structure of SSPPs-TLs provides stronger confinement to electromagnetic waves.

Figure 4 (c) illustrates the electric field intensity distribution after the addition of seven metal circular patches on the upper side of the SSPPs-TL. On the one hand, the patches guide the electric field intensity confined on both sides of SSPP-TL to the edge of the patches and radiate into free space. On the other hand, the introduction of metal patches further improves the impedance matching of the SSPPs-TL and expands its bandwidth, as shown in Fig. 5. As shown in Fig. 5 (b),

when the number of patches increases from 3 to 7, the impedance matching at 9–9.5 GHz becomes better.

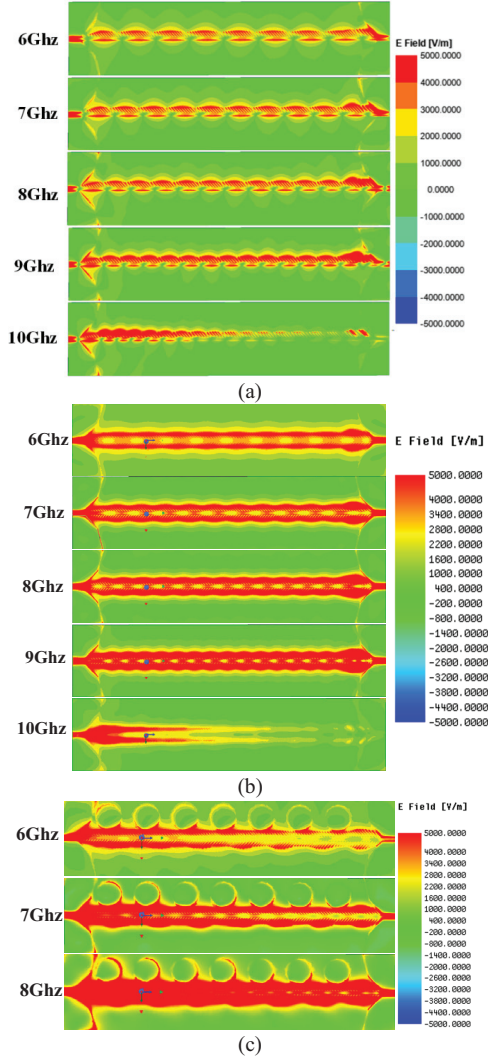


Fig. 4. Electric field intensity distribution diagram of the SSPPs-TLs with (a) symmetric structures, (b) asymmetric structures at $f = 6–10$ GHz and (c) circular metal patches at $f = 6–8$ GHz.

The reflection coefficients shown in Fig. 5 exhibit ripples, and similar phenomena are commonly observed in the reflection coefficient of SSPP antennas [11, 13]. The reason may be that the SSPP structure is composed of many unit slots, resulting in the generation of many resonance points.

In order to radiate the electromagnetic energy of leaky waves, equidistant circular metal patches are introduced to interfere with the transmission characteristics of SSPPs waveguides. According to Fig. 3, when $f = 6.2$ GHz, the guided wavelength λ_g ($\lambda_g = 2\pi k_p$) of SSPPs-TL is 13.8 mm. Therefore, the radius of each

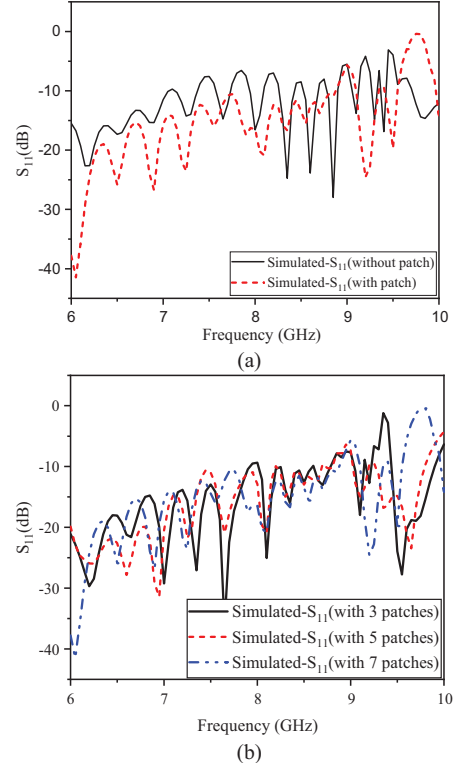


Fig. 5. Comparison of S_{11} values of SSPPs-TLs in the following situations: (a) with and without patches and (b) the number of patches has increased from 3 to 7.

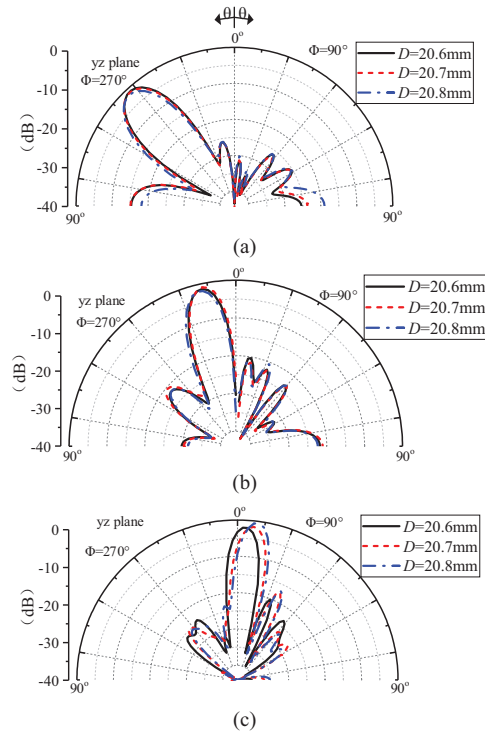


Fig. 6. Radiation patterns at different values of D at (a) $f = 6.2$ GHz, (b) 7.4 GHz and (c) 8.8 GHz.

circular patch is set to $\lambda_g/2$, and the patch spacing is $D = 3\lambda_g/2 = 20.7$ mm [17]. Figure 6 compares the changes in SSPP antenna patterns when D changes from 20.6 to 20.8 mm. Obviously, 20.7 mm is not an optimal value. When $f = 8.8$ GHz with $D = 20.8$ mm, the directional patterns of the SSPP antenna shows a significant rightward rotation.

Figure 7 shows the effect of changing the values of the unit slot A on the antenna. The data shows that when $f = 6.2$ GHz and 8.6 GHz, the beam angle of $A = 0.2$ mm and 0.3 mm is almost the same as that of $A = 0.5$ mm, with no significant change; but when $f = 7.6$ GHz, the beam angle of $A = 0.3$ mm significantly rotates to the right, showing obvious beam scanning characteristics.

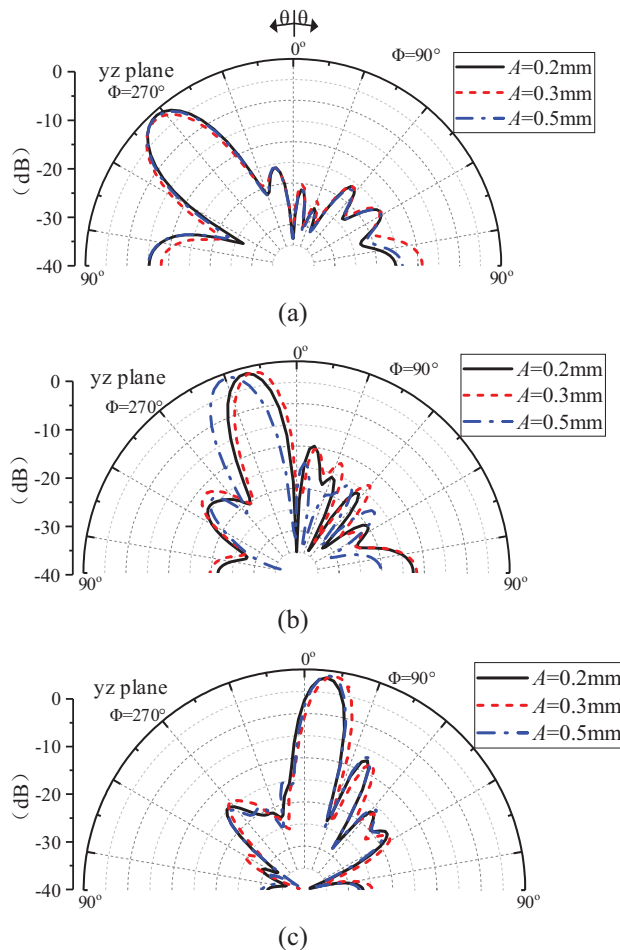


Fig. 7. Radiation patterns at different values of A at (a) $f = 6.2$ GHz, (b) 7.6 GHz and (c) 8.6 GHz.

As shown in Fig. 8, at 6.2 GHz, the influence of parameter P from 1.43 mm to 1.45 mm on the radiation pattern is not significant.

After optimization, the parameters of the SSPP antenna are shown in Table 1.

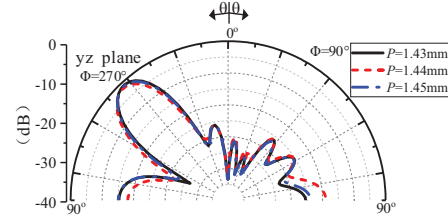


Fig. 8. Radiation pattern at different values of P at 6.2 GHz.

Table 1: Optimized parameters

Parameter	Value (mm)	Parameter	Value (mm)
W_1	1.8	P	1.44
W_q	0.5	H	3
W_f	17.5	H_1	4.2
W_{f1}	14.5	ΔH	0.3
R	7		
D	20.8		
D_1	0.3		
A	0.3		

III. MEASURED RESULTS AND DISCUSSION

The proposed SSPP leaky-wave antenna is fabricated and tested (see Fig. 9). As shown in Fig. 9 (a), two ports are connected to the SMA connector, one end is fed and the other end is connected to a 50-ohm matching load. The radiation characteristics such as radiation pattern and gain of the fabricated antenna prototype are measured by microwave anechoic chamber (see Figs. 9 (b) and (c)).

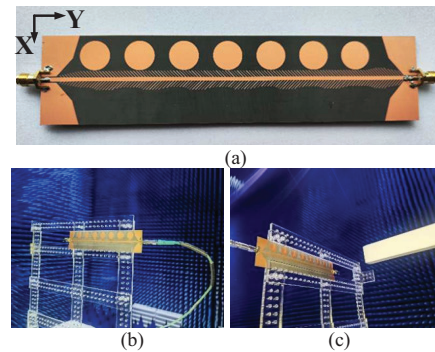


Fig. 9. Physical images and testing environment: (a) antenna physical photo; (b) and (c) testing environment in an anechoic chamber.

An Agilent Network Analyzer (Keysight E5071C) is used to measure the S -parameters of the antenna. The simulated and measured reflection coefficients are compared and shown in Fig. 10. It can be seen that they

match well. The test results show that the amplitude of the reflection coefficient in the range of 5–10 GHz is less than -10 dB, indicating that the antenna achieves good impedance matching and energy transmission in the working frequency range. However, due to the compact and dense structures of the proposed SSPP leaky-wave antenna, the losses caused by fabrication tolerance are very significant. Moreover, radiation loss, dielectric loss, or conductor loss are not considered in simulation, which lead to the measured S_{21} being significantly lower than the simulated ones.

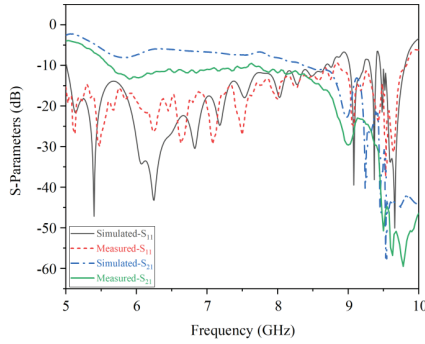


Fig. 10. Comparisons of S -parameters between simulation and measurement of the proposed SSPP antenna.

The simulated and measured radiation patterns of the designed antenna at different frequencies are shown in Fig. 11. Specifically, it can be seen from the diagram that the scanning angle of the symmetrical structure only reaches 56° in the 2.8 GHz frequency band. After the introduction of asymmetric structure, the main beam scans from front to back as the frequency increases in the operating frequency band. The simulated scanning angle range is 43° to 35° ($\Delta\theta = 78^\circ$) and the tested scanning angle range is 41° to 30° ($\Delta\theta = 71^\circ$). Although the angle is slightly offset, the simulation and measurement results are basically consistent.

Figure 12 compares the simulated and tested gains of the SSPP antenna. It can be seen that the simulation gain of the antenna reaches 8.02–11.91 dBi and the test gain is 7.7–10.17 dBi. The low-frequency part matches well. It can also be seen that the gain of the SSPP antenna exhibits oscillatory characteristics, with a gain at 8 GHz going lower, then at 8.5 GHz going higher. Similar features have also appeared in [17, 19, 22]. This is because S_{11} increases from -14.4 dB ($f = 7.5$ GHz) to -10.7 dB at 7.75 GHz and decreases to -16.5 dB at 8.25 GHz. The change of antenna impedance matching affects the radiation efficiency. At the same time, when $f = 7.75$ GHz, the sidelobe of the antenna increases from -5.2 dB ($f = 7.5$ GHz) to -2.4 dB ($f = 8$ GHz). At $f = 8.25$ GHz, the sidelobe level of the antenna decreases to -6.4 dB. The test results and simulation

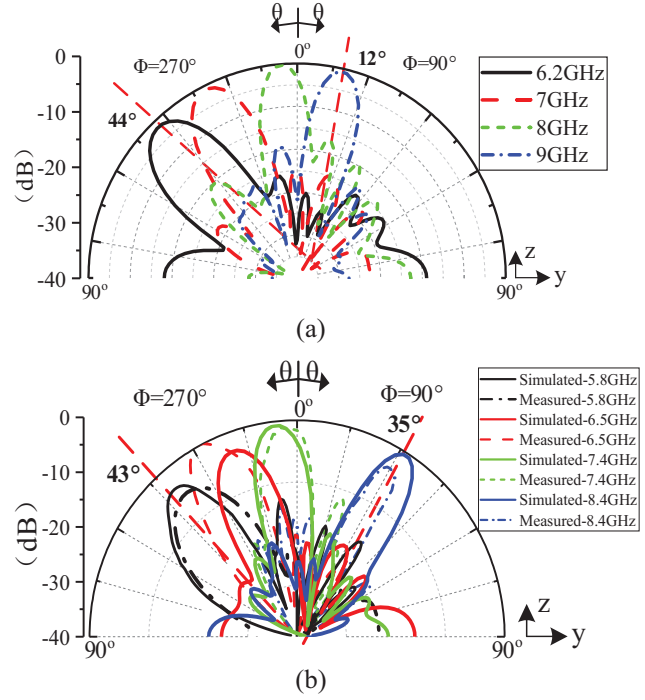


Fig. 11. Comparison of simulation and measurement results of E-plane radiation patterns of the proposed SSPP leaky-wave antenna: (a) simulation results of symmetrical structure at 6.2 GHz to 9 GHz and (b) simulated and measured results at 5.8–8.4 GHz.

results show consistent patterns, but the simulated gain is 1.2 dB higher than the test results. The reason for this phenomenon may be due to the loss and dispersion characteristics of the antenna material changing with frequency.

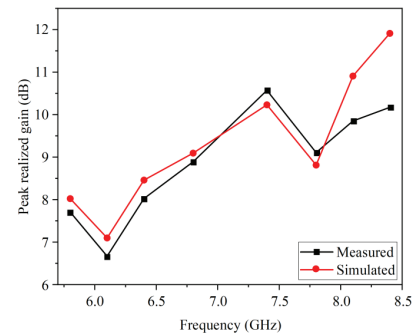


Fig. 12. Comparison of simulated and measured gains of the proposed SSPP antenna.

Table 2 compares the performance of the SSPP antenna designed in this paper with other SSPP antennas listed in [17, 19–22]. It can be seen that while ensuring a wide scanning angle of 71° , the proposed design has a smaller electrical size of only $3 \times 0.7\lambda_{\min}^2$, and its

Table 2: Performance comparison between SSPP antennas

Ref.	Bandwidth (GHz)	Scanning Range ($\Delta\theta$)	Max Gain (dBi)	Antenna Size (mm^2)/($\lambda_{\min}^2$)
17	3 (6–9)	55°	11	320×55.56 (6.4×1.1)
19	16 (8–24)	90°	12.5	404×70 (10.7×1.87)
20	2 (5–7)	81°	19.4	556×66 (9.3×1.1)
21	4.5 (2.87.3)	64.6°	10.23	380×88 (3.5×0.8)
22	5 (7–12)	84°	10.3	246.5×25.54 (5.8×0.6)
This Work	2.6 (5.88.4)	71°	10.17	181×42 (3.0×0.7)

bandwidth is limited to 2.6 GHz, achieving the goal of balancing scanning rate and miniaturization, with better performance.

IV. CONCLUSION

In this paper, a wide angle and high scanning rate leaky-wave antenna based on SSPP structure is designed, fabricated and tested. The side of the proposed structure adopts a uniform metal strip structure, and a periodic radiation patch is loaded above it to couple the electromagnetic energy from the SSPP structure and radiate it into free space. The periodic metal ripple modulation is introduced on the other side, and the overall asymmetric structure is used to improve the scanning angle range, suppress the side lobe and improve the scanning rate.

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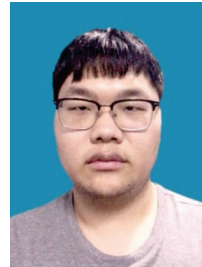
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A Miniaturized Wilkinson Power Divider with Integrated Phase Shifter

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Abstract – This paper reports the design of a compact Wilkinson power divider for use in modern wireless communication systems where circuit size is a critical constraint. The proposed design replaces the conventional quarter-wavelength transmission line with a compact phase-shifting section based on parallel-coupled microstrip lines, providing an impedance transformation factor of $\sqrt{2}$ for proper matching to a 50 Ω system. The power divider is designed to operate at a center frequency of 3.0 GHz on an AD260 substrate, with the phase-shifting function directly integrated into the power-dividing network to reduce the overall circuit footprint. The fabricated prototype occupies an area of 15.367×22.860 mm², corresponding to a size reduction of approximately 22% compared with a conventional Wilkinson power divider. Simulation results obtained using Advanced Design System (ADS) demonstrate strong agreement, with a return loss (S_{11}) better than 15 dB and insertion losses (S_{21} and S_{31}) close to the ideal value of -3 dB at the operating frequency. The fabricated prototype was characterized over the 2–6 GHz frequency range, demonstrating strong agreement with the simulated results. These results indicate that the proposed structure achieves effective power division with good impedance matching while maintaining a compact

size, making it suitable for integration into compact RF and microwave front-end circuits.

Index Terms – Circuit miniaturization, parallel-coupled microstrip lines, phase compensation, Wilkinson power divider.

I. INTRODUCTION

Recent advances in wireless communication technologies have driven the demand for compact, lightweight, and high-performance RF and microwave circuits. Modern applications such as mobile terminals, wearable devices, Internet of Things (IoT) nodes, and vehicle-to-everything (V2X) communication systems require circuit components that not only provide reliable electrical performance but also occupy minimal physical area [1]. As a result, circuit miniaturization has become a key research topic in the development of next-generation wireless front-end systems.

The Wilkinson power divider is a fundamental building block widely used in RF and microwave systems for power distribution, signal combining, and antenna feeding networks. In response to stringent size constraints, numerous miniaturization techniques

Table 1: Summary of reported miniaturization techniques for Wilkinson power dividers

Ref.	Technique	Main Advantage	Limitation
[2]	3-D MMIC integration	Significant size reduction achieved through three-dimensional integration	Complex fabrication process and higher manufacturing cost
[3]	Capacitive loading	Compact circuit realization with reduced physical length	Narrow operating bandwidth
[4]	Active inductors	Effective size reduction using active circuit elements	Additional biasing circuitry and power consumption
[5]	T-shaped transmission lines	Simple planar layout with moderate size reduction	Limited miniaturization capability
[6]	Fractal structure	Harmonic suppression and size reduction	Increased design and optimization complexity
[7]	Resonator-based design	Harmonic control and performance enhancement	Increased layout area compared with simple structures
This work	Coupled-line phase shifter	Compact and planar structure with integrated phase-shifting functionality	Required optimization for impedance and phase balance

for Wilkinson power dividers have been reported in the literature. These include three-dimensional integration technologies [2], lumped-element realizations [8, 9], inductive and capacitive loading methods [3, 4, 10, 11], T-shaped transmission-line configurations [5, 12], fractal-based structures [6], and resonator-assisted designs [1, 7, 13]. While these approaches can effectively reduce circuit size, they often involve trade-offs in bandwidth, fabrication complexity, or design flexibility. A summary of representative miniaturization techniques for Wilkinson power dividers reported in the literature is provided in Table 1. It should be noted that the proposed design employs a lumped capacitor for capacitive compensation, whereas some reported works are based solely on transmission-line structures. The inclusion of a lumped element in this work is intended to provide a simple and practical means for phase adjustment without significantly increasing circuit complexity. Therefore, the comparison focuses on overall design strategy and performance trade-offs rather than strict structural equivalence.

As shown in Table 1, most existing miniaturization approaches achieve size reduction at the expense of increased design complexity, limited bandwidth, or fabrication challenges. This observation motivates the exploration of alternative compact structures that can provide circuit miniaturization while maintaining acceptable electrical performance.

Parallel-coupled microstrip lines have been extensively employed in the realization of compact microwave components, including bandpass filters, impedance transformers, and phase shifters [14–18]. In particular, coupled-line-based phase shifter

configurations, such as Schiffman-type and differential phase shifters, have attracted significant attention due to their planar structure, broadband characteristics, and compatibility with standard fabrication processes. These designs exploit the electromagnetic coupling between adjacent transmission lines to introduce a desired phase delay within a reduced physical length. Despite their advantages, microstrip coupled-line structures inherently suffer from unequal phase velocities between the even and odd propagation modes as a result of dielectric inhomogeneity. This phase imbalance can lead to deviations from the intended phase response, increased insertion loss, and degraded isolation when such structures are integrated into power-dividing or signal-combining networks. To overcome these limitations, a wide range of compensation techniques has been proposed, including capacitive and inductive loading methods, dielectric overlays, and hybrid compensation schemes [19–33]. These approaches provide important design insights for improving the performance of coupled-line-based circuits.

Although a variety of miniaturization techniques have been reported, such as slow-wave structures, defected ground structures (DGS), meandered transmission lines, and lumped-element loading, each approach has certain limitations. For instance, slow-wave and DGS-based techniques often require complex geometrical modifications and may increase fabrication complexity, while lumped-element approaches introduce additional parasitic effects and may limit bandwidth performance. In contrast, the proposed design employs a capacitive-compensated parallel-coupled microstrip structure, which provides a simple planar

implementation without requiring multilayer configurations or intricate layouts. This approach enables effective size reduction while maintaining good impedance matching, power division, and isolation characteristics, making it more suitable for practical RF and microwave circuit integration.

In this work, a compact Wilkinson power divider operating at 3.0 GHz is proposed by replacing the conventional quarter-wavelength transmission line with a 90° phase-shifting section implemented using parallel-coupled microstrip lines. Based on even- and odd-mode analysis, the coupled-line section is designed to provide a characteristic impedance of $1.414Z_0$, enabling the required impedance transformation while occupying a shorter physical length than a traditional $\lambda/4$ line. By integrating the phase-shifting function directly into the power-dividing network, the proposed design achieves circuit miniaturization without significantly compromising electrical performance.

II. DESIGN AND ANALYSIS

A. Design principle of the capacitive compensated coupled-line structure

In RF and microwave circuit design, the physical length of quarter-wavelength ($\lambda/4$) transmission lines often limits circuit miniaturization. To address this issue, a compact phase-shifting structure based on parallel-coupled microstrip lines with capacitive compensation is proposed to emulate the electrical behavior of a conventional quarter-wavelength transmission line.

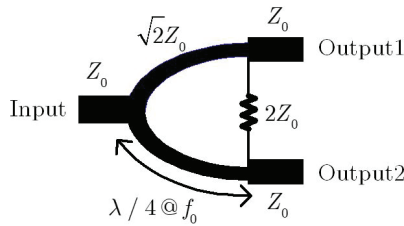


Fig. 1. Conventional Wilkinson power divider with quarter-wavelength transmission lines.

Figure 1 illustrates a conventional Wilkinson power divider, where two $\lambda/4$ transmission lines with characteristic impedance $\sqrt{2}Z_0$ provide impedance matching and equal power division, while an isolation resistor of $2Z_0$ ensures high isolation between output ports. Although excellent electrical performance is achieved, the quarter-wavelength sections result in a relatively large physical footprint.

To achieve size reduction, the proposed design replaces the conventional $\lambda/4$ transmission lines with compact 90° phase-shifting sections, as shown in Fig. 2. Each phase-shifting section is designed to provide an

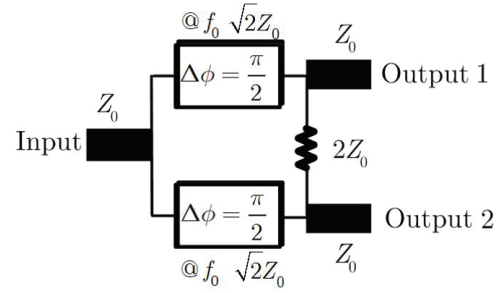


Fig. 2. Proposed Wilkinson power divider using integrated 90° phase-shifting sections.

electrical phase delay of $\pi/2$ and an equivalent characteristic impedance of $\sqrt{2}Z_0$ at the operating frequency f_0 . This approach preserves the required impedance transformation while significantly reducing the circuit size.

The proposed phase-shifting section is realized using a parallel-coupled microstrip line with even- and odd-mode impedances Z_{0e} and Z_{0o} , respectively, and an electrical length $\theta_c < \pi/2$. Due to dielectric inhomogeneity, the even- and odd-mode phase velocities are generally unequal, which degrades circuit performance as in Fig. 3 shows the equivalent two-port network of the capacitive-compensated coupled-line structure. To compensate for this effect, a lumped capacitor with impedance Z_C is connected between the open-circuited ports of the coupled-line section.

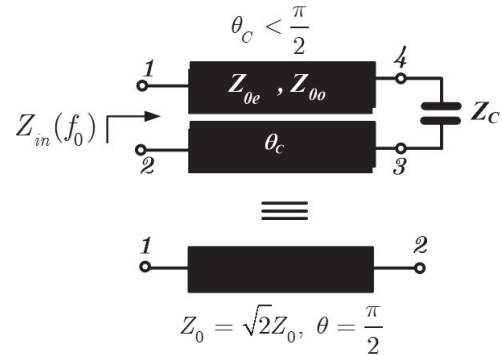


Fig. 3. Equivalent two-port network of the capacitive-compensated coupled-line structure.

For circuit analysis, the coupled-line structure is initially represented as a four-port network. By symmetrically terminating Ports 3 and 4 with shunt capacitive impedances of $Z_C/2$, the network can be reduced to an equivalent symmetric two-port representation, as illustrated in Fig. 4. This shunt capacitive loading primarily introduces an additional phase delay to the odd mode, thereby compensating for the phase velocity mismatch. Physically, the compensation capacitor introduces an additional reactive loading to the coupled-line section,

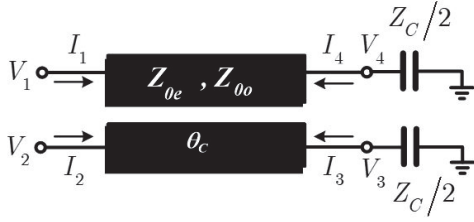


Fig. 4. Voltage-current definition of the capacitive-compensated coupled-line structure for circuit analysis.

which modifies the phase response of the structure. Since the odd mode is more sensitive to the electric-field distribution between the coupled strips, the shunt capacitive loading predominantly affects the odd-mode propagation, introducing an additional phase delay. In contrast, the even mode is comparatively less affected because its field distribution is less concentrated across the coupled region. As a result, the capacitor helps reduce the phase velocity imbalance between the even and odd modes, enabling the shortened coupled-line section to more closely emulate the phase behavior of a conventional quarter-wavelength transmission line.

1. Equivalent two-port impedance representation

Under the assumptions of reciprocity and symmetry, the equivalent two-port impedance parameters of the capacitive-compensated coupled-line structure [29, 33] can be expressed as:

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34} \\ Z_{41} & Z_{42} & Z_{43} & Z_{44} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}. \quad (1)$$

Because the proposed structure is geometrically symmetric with respect to the input and output planes and consists of passive reciprocal elements, the impedance matrix exhibits both symmetry and reciprocity [33]. Accordingly, the diagonal elements satisfy $Z_{11} = Z_{22}$ and $Z_{33} = Z_{44}$, whereas the off-diagonal terms follow $Z_{12} = Z_{21}$, $Z_{13} = Z_{24}$, and $Z_{14} = Z_{23}$. Ports 3 and 4 are terminated by shunt capacitive impedances $Z_C/2$, yielding

$$V_3 = -\frac{Z_C}{2}I_3, \quad V_4 = -\frac{Z_C}{2}I_4. \quad (2)$$

By applying the symmetry conditions and imposing the current relationship at Ports 3 and 4 (i.e., $I_4 = -I_3$), the four-port impedance equations can be reduced to

$$\begin{aligned} V_1 &= Z_{11}I_1 + Z_{12}I_2 + (Z_{13} - Z_{14})I_3, \\ V_2 &= Z_{12}I_1 + Z_{11}I_2 - (Z_{13} - Z_{14})I_3. \end{aligned} \quad (3)$$

After applying the capacitive boundary conditions and reducing the four-port description to an equivalent

two-port network, the resulting voltage-current relations can be expressed as

$$V_1 = \left(Z_{11} - \frac{(Z_{13} - Z_{14})^2}{Z_C + 2(Z_{11} - Z_{12})} \right) I_1 + \left(Z_{12} + \frac{(Z_{13} - Z_{14})^2}{Z_C + 2(Z_{11} - Z_{12})} \right) I_2, \quad (4)$$

$$V_2 = \left(Z_{12} + \frac{(Z_{13} - Z_{14})^2}{Z_C + 2(Z_{11} - Z_{12})} \right) I_1 + \left(Z_{11} - \frac{(Z_{13} - Z_{14})^2}{Z_C + 2(Z_{11} - Z_{12})} \right) I_2. \quad (5)$$

Equations (4)–(5) indicate that the capacitive-compensated coupled-line section can be represented by an equivalent symmetric two-port Z-parameter matrix. For compactness, define

$$Z_H \triangleq \frac{(Z_{13} - Z_{14})^2}{Z_C + 2(Z_{11} - Z_{12})}. \quad (6)$$

The parameter Z_H represents the effective impedance contribution introduced by the combined effect of the coupling difference and the shunt capacitive loading. Thus, the equivalent two-port representation is written as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} - Z_H & Z_{12} + Z_H \\ Z_{12} + Z_H & Z_{11} - Z_H \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}, \quad (7)$$

where Z_H captures the combined effect of the coupling imbalance and the shunt capacitive loading, enabling the shortened coupled-line section to emulate a quarter-wavelength impedance inverter at the design frequency.

$$Z_{11T} = Z_{11} - \frac{(Z_{13} - Z_{14})^2}{Z_C + 2(Z_{11} - Z_{12})}, \quad (8)$$

$$Z_{12T} = Z_{12} + \frac{(Z_{13} - Z_{14})^2}{Z_C + 2(Z_{11} - Z_{12})}.$$

Therefore, the equivalent two-port network can be written in matrix form as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11T} & Z_{12T} \\ Z_{12T} & Z_{11T} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}. \quad (9)$$

Accordingly, the voltage-current relationship of the proposed structure can be written in the standard two-port impedance matrix form as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} - Z_H & Z_{12} + Z_H \\ Z_{12} + Z_H & Z_{11} - Z_H \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}. \quad (10)$$

The parameter Z_H represents the combined effect of coupling imbalance and shunt capacitive loading,

enabling the shortened coupled-line section to emulate a quarter-wavelength impedance inverter.

2. Input matching condition

From the equivalent two-port impedance matrix in (10), the input reflection coefficient can be expressed in terms of the Z-parameters as [29, 33]

$$S_{11} = \frac{(Z_{11T} - Z_0)^2 - Z_{12T}^2}{(Z_{11T} + Z_0)^2 - Z_{12T}^2}. \quad (11)$$

The input reflection coefficient of the equivalent two-port network is given by

$$S_{11} = \frac{(Z_{11T} - Z_0)^2 - Z_{12T}^2}{(Z_{11T} + Z_0)^2 - Z_{12T}^2}. \quad (12)$$

Perfect input matching at the design frequency f_0 is achieved when $S_{11}(f_0) = 0$, which yields a closed-form expression for the required capacitive impedance Z_C .

3. Physical design parameters

The required shunt capacitance can be obtained from

$$C_p = \frac{Z_0}{2\omega_0 |\text{Im}(Z_C)|}, \quad (13)$$

where $\omega_0 = 2\pi f_0$. The electrical length of the shortened coupled-line section is determined by

$$\theta_f = \frac{1}{\pi k} \tan^{-1} \left(\frac{2Z_{0o}}{\sqrt{4Z_{0o}^2 - Z_C^2}} \right), \quad (14)$$

with

$$k = \sqrt{\frac{\epsilon_{\text{effo}}}{\epsilon_{\text{effe}}}}, \quad (15)$$

representing the phase velocity ratio between the odd and even modes.

4. Transmission performance

Finally, the transmission coefficient of the synthesized two-port network is expressed as

$$S_{21} = \frac{2Z_0 Z_{12T}}{(Z_{11T} + Z_0)^2 - Z_{12T}^2}, \quad (16)$$

or equivalently,

$$S_{21} = \frac{2Z_0(Z_{12} + Z_H)}{(Z_{11} - Z_H + Z_0)^2 - (Z_{12} + Z_H)^2}. \quad (17)$$

Equation (17) confirms that the transmission performance of the proposed capacitive-compensated coupled-line structure is directly governed by the synthesized impedance parameter Z_H , enabling equal power division while maintaining a compact circuit footprint.

B. Impedance transformation and electrical length adjustment of the capacitive-compensated parallel-coupled transmission line

To quantitatively describe the compensation behavior, the synthesized values of the shunt capacitance C_p and the shortened electrical length θ_f were extracted for coupling levels ranging from -14 to -8 dB. The coupling range from -8 to -14 dB was selected because it represents a practical design interval for parallel-coupled microstrip lines at 3.0 GHz, where the coupling strength is sufficiently high to support compact phase-shifter implementation while still maintaining realizable physical dimensions and acceptable fabrication tolerance.

In addition to circuit miniaturization, the proposed capacitive-compensated parallel-coupled transmission line must simultaneously satisfy two essential design requirements: impedance transformation and precise electrical length control. In a conventional Wilkinson power divider, each quarter-wavelength ($\lambda/4$) transmission line provides both a 90° phase shift and an impedance transformation from Z_0 to $\sqrt{2}Z_0$. When the physical length of the transmission line is reduced, these two functions must be preserved through appropriate circuit synthesis. In the proposed structure, the impedance transformation is achieved through the even- and odd-mode characteristic impedances of the parallel-coupled microstrip lines. By properly selecting Z_{0e} and Z_{0o} , the coupled-line section can be designed to exhibit an equivalent characteristic impedance of $\sqrt{2}Z_0$ at the operating frequency. This behavior emulates the impedance-inverting property of a conventional quarter-wavelength transmission line while occupying a shorter physical length. However, due to dielectric inhomogeneity in microstrip structures, the even- and odd-mode phase velocities are generally unequal, causing a deviation from the desired 90° electrical phase shift. To compensate for this effect, a shunt capacitor C_p is introduced between the open-circuited ends of the coupled-line section. The capacitive loading effectively introduces an additional phase delay to the odd-mode propagation, while the even mode remains comparatively unaffected. As a result, the overall phase imbalance between the two modes is reduced. By appropriately selecting the value of the shunt capacitance C_p , the shortened coupled-line section can be adjusted to provide an exact 90° electrical phase shift at the design frequency f_0 , despite its reduced physical length. Consequently, the capacitive-compensated coupled-line structure simultaneously achieves impedance transformation and electrical length control, enabling it to serve as an electrically equivalent substitute for a conventional quarter-wavelength transmission line in compact Wilkinson power divider applications. To facilitate practical implementation, the design equations derived

in the previous subsection are employed to determine suitable initial values for the physical and electrical parameters of the proposed structure. Specifically, the even- and odd-mode impedances, the shortened electrical length, and the shunt capacitance C_p are jointly considered to ensure that both impedance matching and phase compensation conditions are satisfied at the operating frequency f_0 . In the synthesis procedure, the coupled-line geometry is first selected to achieve the required coupling coefficient, which directly governs the values of Z_{0e} and Z_{0o} . The physical dimensions of the coupled lines, including the conductor width W , spacing S , and coupling length L , are then adjusted accordingly based on the chosen substrate parameters. In this work, the proposed structures are implemented on FR4 and AD260 substrates in order to investigate the influence of dielectric properties on the impedance transformation and phase response. After defining the coupled-line geometry, the shunt capacitive loading C_p is introduced to compensate for the phase velocity mismatch between the even and odd modes. The value of C_p is initially estimated using the derived closed-form expressions and subsequently refined through parametric optimization. The optimization process focuses on minimizing the input reflection coefficient S_{11} while maintaining a transmission coefficient S_{21} close to -3 dB and achieving a phase shift of approximately 90° at the design frequency. The electrical performance of the synthesized two-port network is evaluated in terms of its scattering parameters, namely S_{11} , S_{21} , and the phase of S_{21} . These parameters provide direct insight into the impedance matching, insertion loss, and phase characteristics of the proposed structure. By systematically adjusting the coupled-line dimensions and the shunt capacitance C_p , an optimal trade-off between size reduction and electrical performance can be achieved. To further elucidate the impedance transformation and phase compensation mechanism, Figs. 5–7 present the parametric synthesis results obtained for the FR4 substrate ($\epsilon_r = 4.55$). Figure 5 illustrates the variation of the required shunt capacitance C_p and the shortened electrical length θ_f as functions of the coupling factor. As the magnitude of the coupling factor decreases (from -8 to -14 dB), the required compensation capacitance is reduced, while the effective electrical length shifts closer to the ideal 90° condition. In general, stronger coupling requires a larger compensation capacitance and a corresponding adjustment of the electrical length to maintain the desired impedance transformation and phase response.

This behavior confirms that stronger electromagnetic coupling requires greater capacitive loading to mitigate the even–odd mode phase imbalance. Figure 6 presents the evolution of the even- and odd-mode impedances before and after optimization. It can be

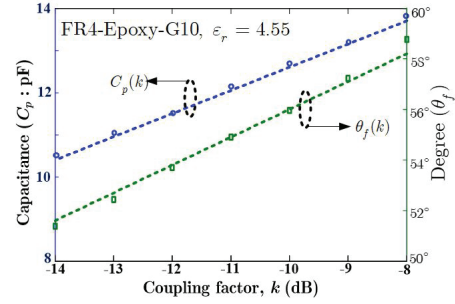


Fig. 5. Relationship between shunt capacitance C_p , shortened electrical length θ_f , and coupling factor for the FR4 substrate ($\epsilon_r = 4.55$).

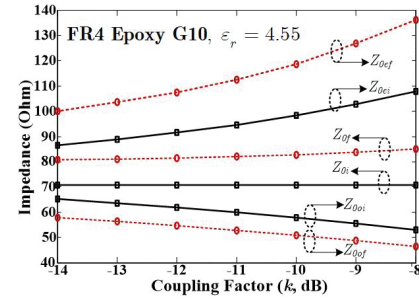


Fig. 6. Initial and optimized even- and odd-mode impedances versus coupling factor for the FR4 substrate.

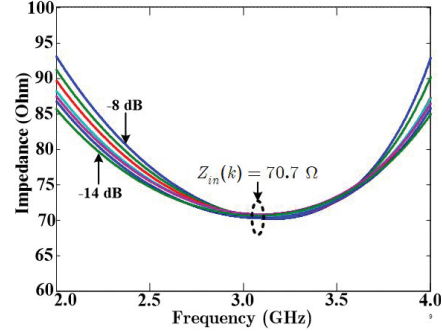


Fig. 7. Input impedance Z_{in} versus frequency for different coupling factors on the FR4 substrate.

observed that the optimized values gradually approach the required impedance transformation condition of $\sqrt{2}Z_0$, ensuring that the synthesized section functions as an equivalent quarter-wavelength impedance inverter. Finally, Fig. 7 shows the input impedance response of the synthesized structure over frequency. All coupling scenarios exhibit an impedance minimum near 70.7Ω at 3.0 GHz, validating that the proposed structure successfully preserves the impedance-inverting property at the design frequency. The corresponding synthesized design parameters for the FR4 substrate are summarized in Table 2. The table presents the initial and optimized

Table 2: Design parameters of the capacitive-compensated parallel-coupled transmission line implemented on FR4 substrate at 3.0 GHz for various coupling levels

Coupling (dB)	Initial Impedance (Z_0, Z_{0e}, Z_{0o})	Initial Dimensions (W, S, L) [mm]	Optimized Impedance (Z_0, Z_{0e}, Z_{0o})	Optimized Dimensions (W, S, L) [mm]	C_p and θ_f
-8.0	$Z_0 = 70.70$ $Z_{0e} = 107.75$ $Z_{0o} = 46.38$	$W = 1.13$ $S = 0.34$ $L = 7.19$	$Z_0 = 59.48$ $Z_{0e} = 86.94$ $Z_{0o} = 40.70$	$W = 1.71$ $S = 0.34$ $L = 8.98$	$C_p = 13.82$ pF $\theta_f = 58.8^\circ$
-9.0	$Z_0 = 70.70$ $Z_{0e} = 102.45$ $Z_{0o} = 48.79$	$W = 1.21$ $S = 0.45$ $L = 7.15$	$Z_0 = 60.46$ $Z_{0e} = 84.75$ $Z_{0o} = 43.13$	$W = 1.74$ $S = 0.45$ $L = 8.84$	$C_p = 13.14$ pF $\theta_f = 57.62^\circ$
-10.0	$Z_0 = 70.70$ $Z_{0e} = 98.09$ $Z_{0o} = 50.95$	$W = 1.27$ $S = 0.57$ $L = 7.11$	$Z_0 = 61.27$ $Z_{0e} = 82.90$ $Z_{0o} = 45.29$	$W = 1.76$ $S = 0.57$ $L = 8.71$	$C_p = 12.69$ pF $\theta_f = 55.87^\circ$
-11.0	$Z_0 = 70.70$ $Z_{0e} = 94.45$ $Z_{0o} = 52.91$	$W = 1.32$ $S = 0.71$ $L = 7.08$	$Z_0 = 61.76$ $Z_{0e} = 80.78$ $Z_{0o} = 47.21$	$W = 1.79$ $S = 0.71$ $L = 8.57$	$C_p = 12.15$ pF $\theta_f = 54.90^\circ$
-12.0	$Z_0 = 70.70$ $Z_{0e} = 91.38$ $Z_{0o} = 54.69$	$W = 1.37$ $S = 0.86$ $L = 7.06$	$Z_0 = 62.20$ $Z_{0e} = 79.02$ $Z_{0o} = 48.96$	$W = 1.81$ $S = 0.86$ $L = 8.45$	$C_p = 11.51$ pF $\theta_f = 53.80^\circ$
-13.0	$Z_0 = 70.70$ $Z_{0e} = 88.78$ $Z_{0o} = 56.30$	$W = 1.40$ $S = 1.02$ $L = 7.04$	$Z_0 = 62.64$ $Z_{0e} = 77.58$ $Z_{0o} = 50.58$	$W = 1.82$ $S = 1.02$ $L = 8.37$	$C_p = 11.05$ pF $\theta_f = 52.83^\circ$
-14.0	$Z_0 = 70.70$ $Z_{0e} = 86.54$ $Z_{0o} = 57.75$	$W = 1.42$ $S = 1.19$ $L = 7.02$	$Z_0 = 62.96$ $Z_{0e} = 76.23$ $Z_{0o} = 52.01$	$W = 1.83$ $S = 1.19$ $L = 8.29$	$C_p = 10.51$ pF $\theta_f = 52.09^\circ$

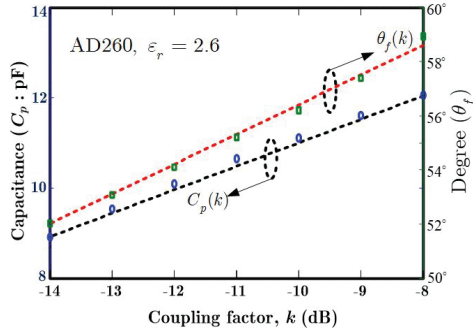


Fig. 8. Variation of compensation capacitance C_f and shortened electrical length θ_f versus coupling factor for the AD260 substrate ($\epsilon_r = 2.6$).

even- and odd-mode impedances, physical dimensions, compensation capacitance, and shortened electrical length for coupling levels ranging from -8 to -14 dB. It can be observed that stronger coupling requires a larger compensation capacitance and a longer electrical length adjustment to satisfy the impedance transformation and phase compensation requirements. Similarly, Figs. 8–10 present the corresponding parametric results for the AD260 substrate ($\epsilon_r = 2.6$). Compared with

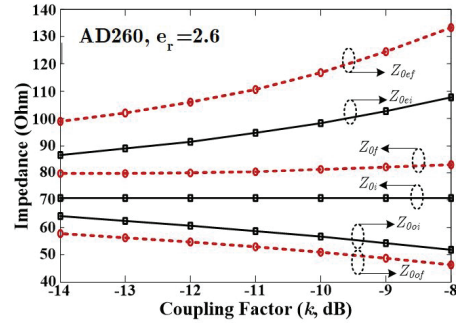
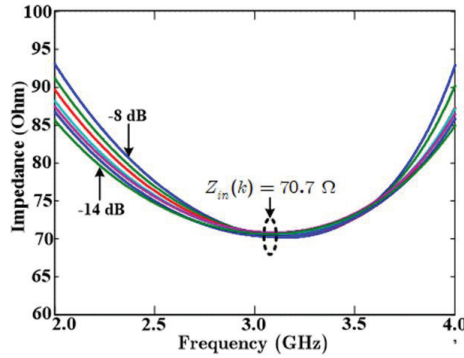


Fig. 9. Initial and optimized even- and odd-mode impedances versus coupling factor for the AD260 substrate.

FR4, the lower dielectric constant of AD260 modifies the even-odd mode field distribution, leading to slightly different compensation capacitance values and electrical lengths. Nevertheless, the same monotonic trend between coupling strength and required capacitive loading is clearly preserved. As shown in Fig. 9, the optimized even- and odd-mode impedances converge toward the target impedance transformation condition of $\sqrt{2}Z_0$, confirming that the proposed synthesis method

Table 3: Design parameters of the capacitive-compensated parallel-coupled transmission line implemented on AD260 substrate at 3.0 GHz for various coupling levels

Coupling (dB)	Initial Impedance (Z_0, Z_{0e}, Z_{0o})	Initial Dimensions (W, S, L) [mm]	Optimized Impedance (Z_0, Z_{0e}, Z_{0o})	Optimized Dimensions (W, S, L) [mm]	Compensation (C_p, θ_f)
-8	$Z_0 = 70.70$ $Z_{0e} = 107.75$ $Z_{0o} = 46.38$	$W = 1.13$ $S = 0.34$ $L = 7.19$	$Z_0 = 59.48$ $Z_{0e} = 86.94$ $Z_{0o} = 40.70$	$W = 1.71$ $S = 0.34$ $L = 8.98$	$C_p = 12.04$ pF $\theta_f = 58.9^\circ$
-9	$Z_0 = 70.70$ $Z_{0e} = 102.45$ $Z_{0o} = 48.79$	$W = 1.21$ $S = 0.45$ $L = 7.15$	$Z_0 = 60.46$ $Z_{0e} = 84.75$ $Z_{0o} = 43.13$	$W = 1.74$ $S = 0.45$ $L = 8.84$	$C_p = 11.62$ pF $\theta_f = 57.34^\circ$
-10	$Z_0 = 70.70$ $Z_{0e} = 98.09$ $Z_{0o} = 50.95$	$W = 1.27$ $S = 0.57$ $L = 7.11$	$Z_0 = 61.27$ $Z_{0e} = 82.90$ $Z_{0o} = 45.29$	$W = 1.76$ $S = 0.57$ $L = 8.71$	$C_p = 11.12$ pF $\theta_f = 56.08^\circ$
-11	$Z_0 = 70.70$ $Z_{0e} = 94.45$ $Z_{0o} = 52.91$	$W = 1.32$ $S = 0.71$ $L = 7.08$	$Z_0 = 61.76$ $Z_{0e} = 80.78$ $Z_{0o} = 47.21$	$W = 1.79$ $S = 0.71$ $L = 8.57$	$C_p = 10.87$ pF $\theta_f = 54.92^\circ$
-12	$Z_0 = 70.70$ $Z_{0e} = 91.38$ $Z_{0o} = 54.69$	$W = 1.37$ $S = 0.86$ $L = 7.06$	$Z_0 = 62.20$ $Z_{0e} = 79.02$ $Z_{0o} = 48.96$	$W = 1.81$ $S = 0.86$ $L = 8.45$	$C_p = 10.12$ pF $\theta_f = 53.92^\circ$
-13	$Z_0 = 70.70$ $Z_{0e} = 88.78$ $Z_{0o} = 56.30$	$W = 1.40$ $S = 1.02$ $L = 7.04$	$Z_0 = 62.64$ $Z_{0e} = 77.58$ $Z_{0o} = 50.58$	$W = 1.82$ $S = 1.02$ $L = 8.37$	$C_p = 9.55$ pF $\theta_f = 53.11^\circ$
-14	$Z_0 = 70.70$ $Z_{0e} = 86.54$ $Z_{0o} = 57.75$	$W = 1.42$ $S = 1.19$ $L = 7.02$	$Z_0 = 62.96$ $Z_{0e} = 76.23$ $Z_{0o} = 52.01$	$W = 1.83$ $S = 1.19$ $L = 8.29$	$C_p = 8.93$ pF $\theta_f = 52.38^\circ$

Fig. 10. Input impedance Z_{in} versus frequency for different coupling levels on the AD260 substrate.

is applicable to substrates with different dielectric constants. Finally, Fig. 10 confirms that the input impedance remains approximately 70.7Ω at 3.0 GHz for all investigated coupling levels. This validates that the impedance-inverting property and phase compensation mechanism of the proposed capacitive-compensated coupled-line structure is robust with respect to substrate variation. The corresponding design parameters obtained for the AD260 substrate are listed in Table 3. Similar trends can

be observed, where the required compensation capacitance and electrical length vary systematically with the coupling level. Compared with the FR4 substrate, the lower dielectric constant of AD260 results in slightly different optimized impedance values and compensation requirements while maintaining the desired impedance transformation characteristics at 3 GHz.

C. Impedance transformation and electrical length adjustment

To quantitatively describe the compensation behavior, the synthesized values of the shunt capacitance C_p and the shortened electrical length θ_f were extracted for coupling levels ranging from -14 to -8 dB. Using the parametric data obtained from Fig. 5, a linear curve-fitting procedure was performed to establish approximate analytical relationships between the coupling factor k (in dB), the required compensation capacitance, and the corresponding shortened electrical length. The fitted expressions are obtained as

$$C_p(k) = 0.55k + 18 \quad (\text{pF}), \quad (18)$$

$$\theta_f(k) = 1.2k + 68 \quad (\text{degree}), \quad (19)$$

where k denotes the coupling factor in decibels (dB). Equations (18) and (19) indicate that both the compensation capacitance and the shortened electrical length exhibit an approximately linear dependence on the coupling level within the investigated design range. Specifically, as the magnitude of the coupling factor decreases (i.e., weaker coupling), the required shunt capacitance is reduced, while the electrical length shifts closer to the ideal 90° condition. This behavior confirms that stronger coupling requires greater capacitive loading to compensate for the even-odd mode phase velocity imbalance. It should be noted that the above expressions are valid within the coupling range of $-14 \leq k \leq -8$ dB and are intended to serve as practical design guidelines for rapid initial parameter estimation prior to full-wave optimization. In addition to the compensation capacitance and electrical length, Fig. 9 also reveals an approximately linear relationship between the coupling factor and the even- and odd-mode characteristic impedances for the AD260 substrate. By applying linear curve fitting within the range $-14 \leq k \leq -8$ dB, the following empirical expressions are obtained:

$$C_p(k) = 0.52k + 16 \quad (\text{pF}), \quad (20)$$

$$\theta_f(k) = 1.1k + 67 \quad (\text{degree}), \quad (21)$$

at the design frequency of 3.0 GHz, as verified in Fig. 10. This result confirms that the proposed capacitive-compensated coupled-line section preserves the impedance-inverting property of a conventional quarter-wavelength transformer while enabling substantial physical length reduction. The impedance transformation and electrical length adjustment of the modified parallel-coupled transmission line can be achieved by appropriately tuning the circuit parameters and introducing a compensating shunt capacitor. The graphical results confirm the relationship between the electrical parameters and the corresponding frequency response obtained from both simulation and experimental measurements.

This study focuses on the impedance transformation and phase adjustment of a microstrip parallel-coupled transmission line with its open ends terminated by a compensating capacitor (C_p). The investigation was conducted at an operating frequency of 3.0 GHz on FR4 and AD260 substrates in order to enhance the transmission-line characteristics, including phase shift and impedance transformation performance. The circuit synthesis and parameter optimization were carried out using parallel-coupled microstrip lines implemented on FR4 and AD260 printed circuit boards. The coupling coefficient was varied within the range of -8 to -14 dB to examine its influence on the required compensation capacitance and electrical phase response.

By systematically adjusting the coupled-line dimensions and the compensation capacitor C_p , the modified structure can be synthesized to exhibit improved phase accuracy and effective impedance transformation at the design frequency. The results demonstrate that proper capacitive compensation enables the shortened coupled-line section to emulate the electrical behavior of a conventional quarter-wavelength transmission line while maintaining a reduced physical size.

III. MATERIALS AND METHODS

A. Proposed circuit configuration

The proposed miniaturized Wilkinson power divider is illustrated in Fig. 11. Unlike the conventional design employing quarter-wavelength transmission lines, the proposed structure integrates a capacitive-compensated parallel-coupled microstrip section to simultaneously achieve impedance transformation and phase shifting. Each branch consists of a shortened coupled-line section characterized by even- and odd-mode impedances (Z_{0e}, Z_{0o}) and electrical length θ_c , with a compensating capacitor C_p connected at the open ends. An isolation resistor of $2Z_0$ is placed between the output ports, preserving the Wilkinson isolation property. The design is targeted for operation at 3.0 GHz with a system impedance of $Z_0 = 50 \Omega$.

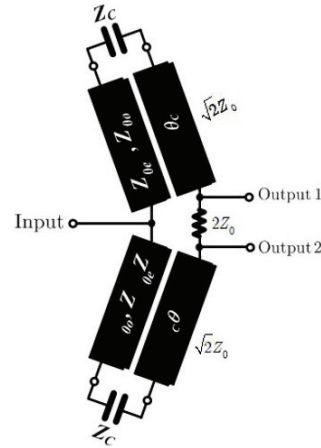


Fig. 11. Proposed miniaturized Wilkinson power divider using capacitive-compensated parallel-coupled transmission lines.

B. Circuit simulation and results

The schematic model of the proposed miniaturized Wilkinson power divider was implemented in Advanced Design System (ADS) 2022 Update 1 (Keysight Technologies), [34], as illustrated in Fig. 12. The substrate parameters were defined using the MSUB (microstrip substrate) block, including the relative dielectric constant, substrate thickness,

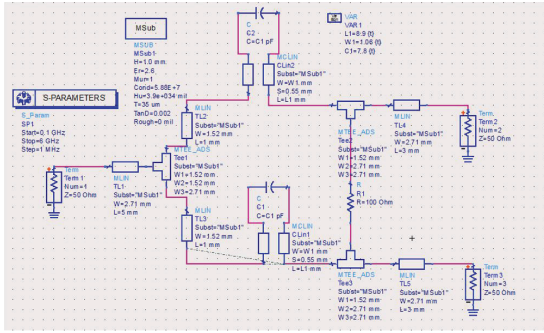


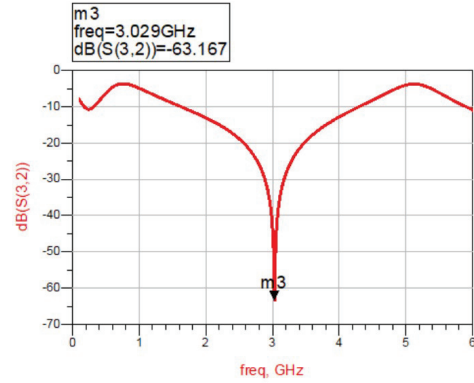
Fig. 12. ADS schematic of the proposed Wilkinson power divider including substrate definition, coupled-line sections, and compensation capacitor.

conductor thickness, and loss tangent corresponding to the AD260 substrate. The coupled-line sections were modeled using microstrip coupled-line elements (MCLIN), while the single transmission lines were implemented using MLIN (microstrip line) components. The shunt compensation capacitor C_p was realized as a lumped capacitor connected between the open-circuited ends of the coupled-line section. A $100\ \Omega$ isolation resistor was placed between the output ports to preserve the inherent isolation property of the conventional Wilkinson topology. S-parameter simulations were performed using the S.Param block in ADS over a frequency range from 0.1 GHz to 6 GHz with a frequency resolution of 1 MHz. All ports were terminated with a reference impedance of $50\ \Omega$ to ensure compatibility with standard microwave measurement conditions.

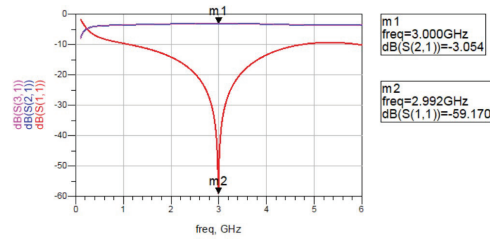
Parametric optimization was conducted to simultaneously adjust the physical dimensions (W, S, L) and the compensation capacitance C_p in order to achieve: (i) proper input impedance matching, (ii) equal power division, and (iii) an electrical phase condition of approximately 90° at the design frequency of 3.0 GHz. The optimization was performed using a combination of parameter sweep, manual tuning, and iterative adjustment to achieve the desired matching, power division, and phase response at 3.0 GHz.

The simulated scattering parameters of the optimized design are presented in Fig. 13. At the center frequency of 3.0 GHz, the proposed power divider exhibits an insertion loss of approximately $-3.05\ \text{dB}$, which is very close to the theoretical value of $-3\ \text{dB}$ for ideal equal power division.

The input return loss is better than 50 dB, indicating excellent impedance matching at the design frequency. Moreover, the isolation between the output ports (S_{23}) is better than 60 dB, confirming that the compensated coupled-line structure successfully maintains the isolation performance of the conventional Wilkinson divider despite the reduced physical length.



(a)



(b)

Fig. 13. Simulated S-parameters of the proposed Wilkinson power divider. (a) Isolation characteristic. (b) Return loss and insertion loss.

C. Fabrication and measurement procedure

After circuit and electromagnetic (EM) optimization, the final layout was exported from ADS and fabricated using a standard printed circuit board (PCB) photolithography process. The microstrip patterns were etched on FR4 and AD260 substrates. For clarity, the AD260 substrate used in this work has a relative dielectric constant of $\epsilon_r = 2.6$, a thickness of 1 mm, and a copper thickness of $35\ \mu\text{m}$.

The compensation capacitor C_p was implemented using a commercial surface-mount chip capacitor soldered between the open ends of the parallel-coupled microstrip lines. The chip capacitor used in the prototype has a tolerance of $\pm 5\%$, which may introduce slight variations in the measured results compared with the simulated values.

A $100\ \Omega$ surface-mount chip resistor was used to realize the isolation resistor between the two output ports, thereby preserving the intrinsic isolation characteristic of the Wilkinson topology.

Standard $50\ \Omega$ SMA (Sub-miniature version A) connectors were mounted at the input and output ports. Particular care was taken during soldering to minimize parasitic inductance and excess conductor loss at the feed transitions, which may otherwise degrade the high-frequency performance.

Table 4: Design parameters of the proposed miniaturized Wilkinson power divider implemented on AD260 substrate at 3.0 GHz

Substrate Parameters (AD260)			
$\epsilon_r = 2.6$	$W_{50} = 2.71 \text{ mm}$	$W_{70.7} = 1.52 \text{ mm}$	$f_0 = 3.0 \text{ GHz}$
Coupled-Line Design Parameters			
Coupling factor C	$Z_0 = 61.96 \text{ }\Omega$	$Z_{0e} = 102.95 \text{ }\Omega$	$Z_{0o} = 61.96 \text{ }\Omega$
-12.09 dB	$K_E = 2.142$	$K_O = 1.819$	
Physical dimensions		$W = 1.06 \text{ mm}, S = 0.55 \text{ mm}, L = 13.3 \text{ mm}$	
Optimized Values			
Compensation capacitor	$C_p = 7.8 \text{ pF}$		
Optimized dimensions	$W = 1.06 \text{ mm}, S = 0.55 \text{ mm}, L = 8.9 \text{ mm}$		
Simulated S-Parameters at 3.0 GHz			
$S_{21} = S_{31}$	-3.05 dB	$S_{11} = -59.17 \text{ dB}$	$S_{23} = -63 \text{ dB}$
Operational bandwidth: 2–6 GHz			

The fabricated prototype was characterized using a calibrated vector network analyzer (VNA) over the frequency range of 2.0–6.0 GHz. Prior to measurement, a full two-port Short-Open-Load-Through (SOLT) calibration was performed. The calibration reference plane was established at the SMA connector interfaces to eliminate systematic errors caused by cables, connectors, and adapters.

The measured S-parameters were subsequently compared with the circuit-level and EM simulation results. Any minor discrepancies are attributed to fabrication tolerances, variations in substrate dielectric properties, and parasitic effects of lumped components.

Table 4 summarizes the key design parameters of the proposed miniaturized Wilkinson power divider implemented on the AD260 substrate at 3.0 GHz. The table presents the initial coupled-line parameters, optimized physical dimensions, and the selected compensation capacitance. It can be observed that the coupling factor of -12.09 dB results in optimized even- and odd-mode impedances that satisfy the impedance transformation requirement of approximately $\sqrt{2}Z_0$. After compensation, the electrical length is shortened from the conventional quarter-wavelength condition, while maintaining excellent impedance matching and isolation performance.

The fabricated prototype of the proposed miniaturized Wilkinson power divider is shown in Fig. 14. The microstrip layout was implemented on the AD260 substrate with a three-port configuration and integrated parallel-coupled transmission-line sections. The overall physical dimensions of the fabricated circuit are approximately $15.4 \times 22.9 \text{ mm}^2$, corresponding to a size reduction of about 22% compared with a conventional quarter-wavelength Wilkinson power divider operating at the same frequency. Surface-mount compensation capacitors were soldered between the open-circuited

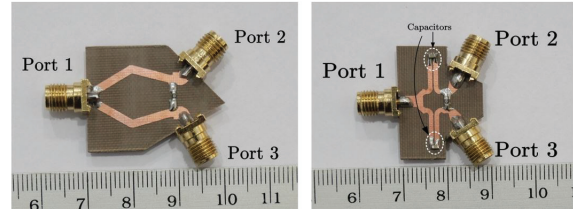


Fig. 14. Fabricated prototype of the proposed miniaturized Wilkinson power divider.

ends of the coupled-line structure, while a 100Ω isolation resistor was mounted between the output ports to preserve the Wilkinson topology. The experimental realization confirms that the proposed capacitive-compensated coupled-line structure can be implemented using standard single-layer PCB fabrication without requiring multilayer processing or three-dimensional integration. The compact planar configuration makes the design suitable for integration into modern RF front-end modules.

IV. RESULTS AND DISCUSSION

The measured S-parameters of the fabricated prototypes, including both the proposed miniaturized design and the conventional quarter-wavelength Wilkinson power divider, were obtained using a calibrated vector network analyzer (Keysight S5065) over the frequency range of 2.0–6.0 GHz.

A. S-parameter performance

Figure 15 compares the measured S-parameters of the proposed and conventional structures. At the design frequency of 3.0 GHz, the proposed miniaturized power divider exhibits S_{21} and S_{31} of approximately -3.2 dB , which are very close to the ideal -3 dB value for equal

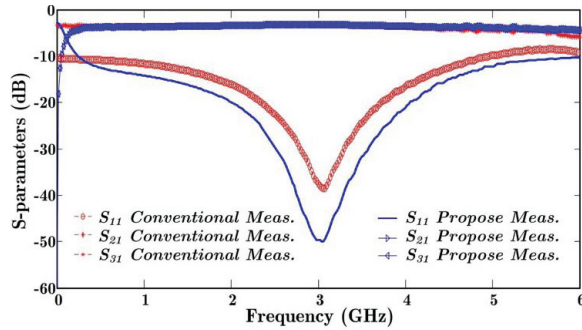


Fig. 15. Measured S-parameters of the proposed and conventional Wilkinson power dividers over the frequency range of 2.0–6.0 GHz.

power division. The slight additional loss compared to simulation is attributed to dielectric loss of the AD260 substrate, conductor loss, soldering parasitics, and SMA connector effects. The measured input return loss S_{11} is better than 20 dB at 3.0 GHz, indicating good impedance matching performance. Although the simulated return loss was below -50 dB, the measured result reflects practical implementation conditions. Compared with the conventional design, the proposed structure maintains comparable power division characteristics, while achieving a significant size reduction of approximately 22%. This confirms that the capacitive-compensated coupled-line section effectively replaces the conventional quarter-wavelength transmission line without degrading fundamental performance.

B. Phase difference analysis

In addition to amplitude characteristics, the phase response between the two output ports was also investigated. Figure 16 shows the measured phase difference

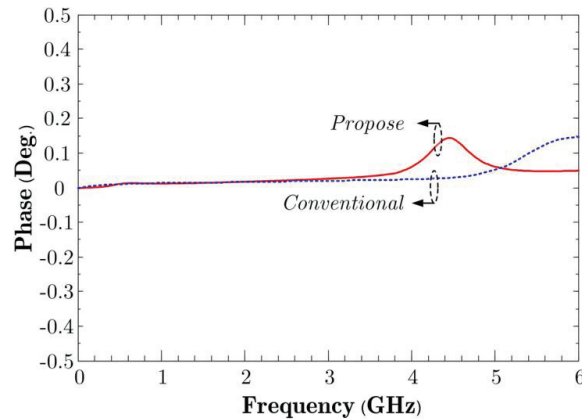


Fig. 16. Measured phase difference between output ports of the proposed and conventional Wilkinson power dividers.

between the output ports for both the proposed and conventional structures. Near the operating frequency of 3.0 GHz, the proposed design maintains a phase condition very close to the desired 90° relationship, demonstrating that the capacitive compensation successfully mitigates the even–odd mode phase velocity imbalance inherent in microstrip coupled-line structures. Across the wider frequency range of 2.0–6.0 GHz, the proposed structure exhibits a stable and controlled phase variation, while preserving performance comparable to the conventional topology. This confirms that the shortened coupled-line section provides accurate electrical-length equivalence to the $\lambda/4$ transmission line. Overall, good agreement between theoretical analysis, simulation, and measurement is observed. The proposed capacitive-compensated parallel-coupled transmission-line structure achieves accurate impedance transformation, controlled phase response, and approximately 22% size reduction, making it suitable for compact RF front-end applications.

C. Discussion

For a clearer quantitative comparison, the measured performance of the conventional quarter-wavelength Wilkinson power divider and the proposed miniaturized design is summarized as follows. At the design frequency of 3.0 GHz, the conventional structure exhibits an input return loss S_{11} of approximately -39 dB, whereas the proposed design achieves an improved return loss of about -50 dB. This indicates that the capacitive-compensated coupled-line section provides more accurate impedance transformation at the operating frequency. Regarding power division performance, the conventional divider shows S_{21} and S_{31} of approximately -3.27 dB, while the proposed structure achieves -3.25 dB for both output ports. The difference between the two designs is negligible (less than 0.02 dB), confirming that the miniaturization technique does not degrade equal power division characteristics. In terms of physical dimensions, the conventional design occupies an area of $16.306 \times 27.584 \text{ mm}^2$, whereas the proposed structure measures $15.367 \times 22.860 \text{ mm}^2$. This corresponds to an overall footprint reduction of approximately 22%. The size reduction is achieved without compromising impedance matching or insertion-loss performance. These results demonstrate that the proposed capacitive-compensated parallel-coupled transmission-line structure successfully replaces the conventional quarter-wavelength section, providing improved matching performance together with significant circuit miniaturization. It should be noted that the potential coupling effects between adjacent microstrip lines were considered in the design. The

Table 5: Performance comparison with representative compact Wilkinson power dividers

Ref.	Frequency (GHz)	Technique	Key Feature	Limitation
[1]	3.0	Resonator-based	Harmonic suppression	Complex structure
[3]	2.0	Capacitive loading	Compact design	Lumped elements required
[4]	2.4	Active inductors	Size reduction	High complexity
[11]	2.45	Stub-based	Compact structure	Limited flexibility
[6]	0.9	DGS / Fractal	Size reduction	Ground modification required
This work	3.0	Coupled-line + capacitor	Compact + integrated phase	Simple planar structure

spacing between transmission lines was carefully optimized to minimize unintended parasitic coupling. Furthermore, full-wave EM simulation inherently accounts for these coupling effects. The good agreement between simulation and measurement confirms that such effects have negligible impact on the overall performance.

The performance of the coupled microstrip line used in the proposed design can be compared with that of a conventional quarter-wavelength transmission line. In the proposed structure, the phase response is achieved through the combined effect of electromagnetic coupling and capacitive compensation, enabling the shortened transmission line to approximate the required 90° phase shift at the design frequency. In terms of insertion loss, the coupled-line section with a lumped capacitor may introduce slightly higher loss compared to a conventional uniform transmission line due to additional coupling effects and component parasitics. However, both simulated and measured results indicate that the insertion loss remains close to the ideal value, demonstrating that the additional loss is minimal. These results confirm that the proposed approach provides an effective trade-off between size reduction and electrical performance, making it suitable for compact RF and microwave circuit applications.

Table 5 compares the proposed design with representative compact Wilkinson power dividers reported in the literature. Various techniques such as resonator-based structures, capacitive loading, DGS, and stub-based configurations have been widely used to achieve size reduction. However, these approaches often introduce additional structural complexity or require modifications to the ground plane or active components. In contrast, the proposed design employs a capacitive-compensated coupled-line structure, which enables size reduction through phase integration while maintaining a simple planar configuration. This makes the proposed design more suitable for practical implementation in RF and microwave circuits.

V. CONCLUSIONS

A compact Wilkinson power divider employing a capacitive-compensated parallel-coupled microstrip transmission-line section has been presented. The conventional quarter-wavelength transmission line was replaced by a shortened coupled-line structure integrated with a shunt compensation capacitor, enabling simultaneous impedance transformation and precise 90° phase control at 3.0 GHz. Based on even-odd mode analysis, closed-form expressions were derived to determine the required compensation capacitance and shortened electrical length. Parametric investigation further revealed an approximately linear relationship between the coupling factor and both the compensation capacitance and phase response, providing practical design guidelines within the range of -14 to -8 dB.

The proposed divider was designed using Advanced Design System (ADS), fabricated on an AD260 substrate, and experimentally characterized using a calibrated vector network analyzer over the frequency range of 2.0–6.0 GHz. At the operating frequency of 3.0 GHz, the measured S_{21} and S_{31} were approximately -3.2 dB, the return loss exceeded 20 dB, and the isolation was better than 30 dB. These results show good agreement with simulation and confirm accurate impedance transformation and phase compensation. Compared with the conventional quarter-wavelength Wilkinson power divider, the proposed structure achieves approximately 22% reduction in physical area while preserving balanced power division and isolation characteristics. The fully planar configuration, together with the simple capacitive compensation technique, makes the design suitable for compact RF and microwave front-end applications.

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Design of Wideband Filtering Power Divider with High Port Isolation Performance for RF Energy Harvesting

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Abstract – This paper presents a novel wideband filtering power divider with designable bandwidth and high port isolation, developed for RF energy harvesting applications. Two classes of structures integrate different shared networks with impedance transformers in their filtering branches to realize Chebyshev transformer functions. This synthesis approach allows for precise control over the bandwidth and return loss (RL). Furthermore, high isolation is achieved by incorporating two resistors between the two output ports. Measured results demonstrate a wide operating band 103.7%, and high isolation level better than 25 dB of the bandwidth.

Index Terms – Filtering power divider, high isolation, RF energy harvesting, wideband power divider

I. INTRODUCTION

Power dividers play a vital role in radio frequency (RF) energy harvesting by enabling efficient signal distribution [1]. In RF energy harvesting setups, power dividers are used to aggregate the collected energy from multiple antennas. Their low voltage standing wave ratio and high isolation between ports are critical for minimizing energy loss and preventing detrimental interactions between the harvesting channels, thereby maximizing the overall power collection efficiency and stability [2, 3]. Figure 1 depicts the block diagram of the system, where RF signals generated by multiple RF sources are captured by antenna. A broadband antenna is used to capture ambient RF energy across ISM/WiFi/5G bands. The power divider is designed to cover the corresponding wide operating bandwidth. The rectifier

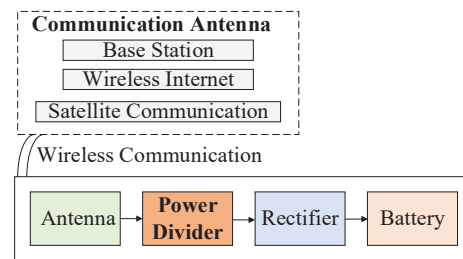


Fig. 1. Wireless system architecture of proposed wideband filtering power divider for RF energy harvesting.

is optimized for large power conversion efficiency and the generated DC power is used to recharge its battery. Among these components, the power divider is essential for realizing the system's overall functionality.

In RF energy harvesting systems, the received ambient RF signals are distributed across multiple frequency bands (e.g., GSM, WiFi, 5G). A multi-pole filtering response is employed to realize a wide passband with good in-band flatness and high out-of-band rejection. This ensures that only useful RF energy within the target frequency range is collected and transmitted to the rectifier, while out-of-band interference and noise are suppressed. This improves the stability and power conversion efficiency of the entire harvesting system. As for port isolation, high isolation between output ports is critical to avoid channel interactions. When port isolation is poor, the harvested RF power from one channel will leak into the other channel, causing unexpected signal reflection, power loss, and phase mismatch. Therefore, it is imperative to improve the bandwidth and isolation performance of power dividers.

The conventional wideband Wilkinson power dividers are designed for the broadband RF energy harvesting system [4–6]. For example, in [4], a multisection Wilkinson wideband power divider is applied to the ultra-wideband energy harvesting circuit, but the isolation is only -10 dB.

Recent research has focused on integrating filtering functions into power dividers to improve overall performance. The resistive isolation technique, which introduces resistors between the output ports to absorb reflections, is demonstrated in designs with improved isolation [7, 8]. Multimode resonator structures utilize multiple transmission poles to improve bandwidth and isolation by adjusting coupling; however, achieving both wide bandwidth and high isolation simultaneously remains challenging. Although power dividers based on substrate-integrated waveguide (SIW) or self-packaged air-filled SIW can achieve good in-band isolation, these designs tend to be complicated and operate over relatively narrow bandwidths [9–11]. Advanced techniques employing composite isolation impedances and surface wave suppression can achieve high isolation of -36 dB, but usually at the expense of operating bandwidth [12]. Recent works in filtering power dividers focus on achieving wideband performance, utilizing approaches such as multilayer slotline structures, direct synthesis techniques, and multilayer stacked stripline designs [13–15]. In conclusion, most approaches suffer from inherent limitations in achieving both wide bandwidth and high isolation. Generally, even though the isolation component is appropriately selected, the crosstalk between output ports will also limit the isolation level of filtering power dividers. For most microstrip wideband filtering power dividers, crosstalk cannot be avoided, and the isolation level is usually below 25 dB [16–20]. Achieving over 25 dB of isolation remains challenging, particularly across a wide and designable bandwidth and, to our knowledge, no existing design has simultaneously met both of these requirements.

In this paper, we propose a novel wideband filtering power divider design featuring designable bandwidth and high port isolation. A novel integrated scheme of shared resonant network and Chebyshev impedance transformer is proposed to realize designable four-pole and five-pole wideband filtering responses. The shared resonant structure broadens matching bandwidth and miniaturizes circuit size, eliminating bulky multi-section cascaded configurations. Different from traditional single-resistor schemes, the design adopts dual isolation resistors cooperating with resonant branches, achieving high wideband isolation without degradation of bandwidth and filtering performance. The synergistic design of the filtering transformer and shared resonant

structure enables precise control over the fractional bandwidth (FBW).

II. THEORETICAL ANALYSIS

To realize wideband responses, two classes of power dividers based on $\lambda/4$ resonators, as shown in Fig. 2, are presented herein. First, two different shared networks before the power dividing junction are designed to enlarge RL bandwidth. Designs I and II realize four-pole and five-pole Chebyshev function, respectively. As Fig. 3 shows, both designs achieve a 20 -dB RL bandwidth of 100% , demonstrating that both topologies manifest good bandpass responses in

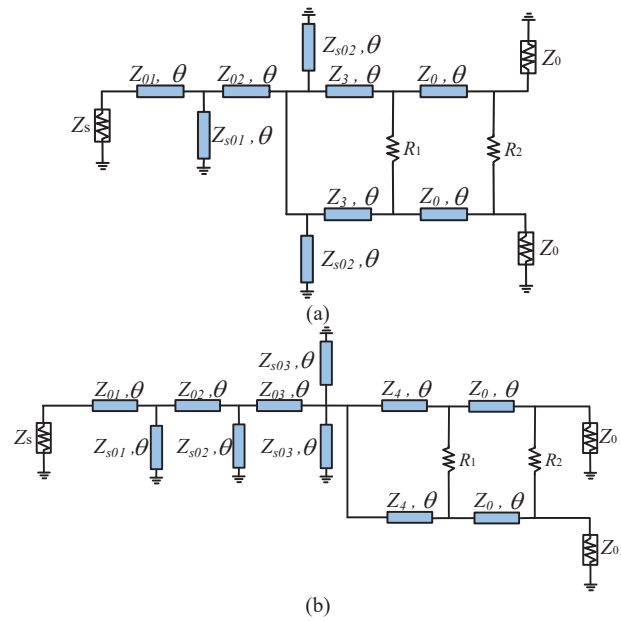


Fig. 2. Proposed wideband filtering power divider with different shared networks. (a) Design I and (b) Design II.

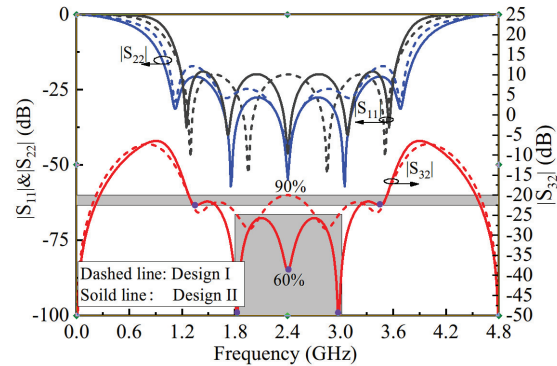


Fig. 3. Theoretical results of proposed Designs I and II with same RL bandwidth of 100% and different isolation bandwidth of 90% (-20 dB) and 60% (-25 dB).

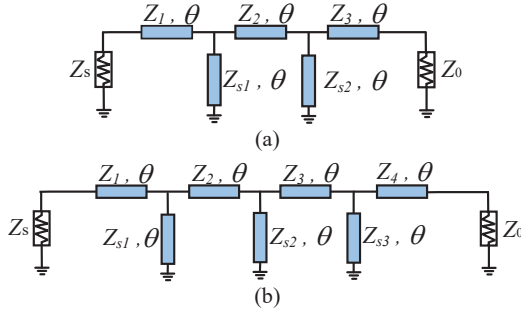


Fig. 4. Proposed wideband filtering transformer I and II. (a) With four-pole Chebyshev response and (b) with five-pole Chebyshev response.

the passband range. Design I features a simple structure and achieves an isolation of -20 dB. In contrast, Design II incorporates increased complexity to achieve an enhanced isolation level of up to -25 dB. Second, all the impedance parameters of the two designs can be determined by the synthesis theory. Third, only two isolation resistors are used to ensure good isolation and port matching.

The working principle for the realization of the wideband power dividers is detailed as follows. To achieve wide matching bandwidth, two classes of filtering transformers shown in Fig. 4 as branches are synthesized. As an example, the filtering transformer I shown in Fig. 4 (a) is discussed in detail. The proposed wideband filtering transformer is designed with impedance transformer ratio $r = Z_s/Z_0$ ($Z_0 = 50 \Omega$). To implement the four-pole equal-ripple response, the Chebyshev polynomial function is adopted in [21–23]. The theoretical S-parameters of the wideband filtering transformer can be expressed by

$$|S_{21}| = \sqrt{\frac{1}{1 + \epsilon^2 \cos^2(3\phi + \xi)}}, \quad (1)$$

where

$$\begin{aligned} \epsilon \cos(3\phi + \xi) = & \left(4\epsilon\alpha^3(\alpha + \sqrt{\alpha^2 - 1}) \right) \frac{\cos^4 \theta}{\sin \theta} \\ & + \left(-\epsilon\alpha(5\alpha + 3\sqrt{\alpha^2 - 1}) \right) \frac{\cos^2 \theta}{\sin \theta} \\ & + \frac{\epsilon}{\sin \theta}. \end{aligned} \quad (2)$$

All the impedance parameters are normalized by the load impedance Z_0 . Then, the theoretical S-parameters can be transformed as

$$|S_{21}| = \sqrt{\frac{1}{1 + |F|^2}} = \sqrt{\frac{1}{1 + \left| \frac{2\sqrt{r}}{A_1 + B_1 + C_1 r + D_1 r} \right|^2}}. \quad (3)$$

The A_1, B_1, C_1 , and D_1 of the entire filtering transformer can be obtained by multiplying the ABCD matrices of each transmission line segment. Through the formula derivation, the function F of the filtering transformer I shown in Fig. 4 (a) can be derived as

$$\begin{aligned} F = & \frac{j}{2\sqrt{r}z_1z_2z_3 \sin \theta} [k_1 \cos^4 \theta + k_2 \cos^2 \theta + k_3] \\ & + k_4 \cos^3 \theta + k_5 \cos \theta, \end{aligned} \quad (4)$$

where

$$\begin{aligned} k_1 = & (r - z_1z_3)(z_1 + z_2 + y_{s1}z_1z_2)(z_2 + z_3 \\ & + y_{s2}z_2z_3), \end{aligned} \quad (5a)$$

$$\begin{aligned} k_2 = & 2z_1^2z_3^2 - 2rz_2^2 + z_1z_2z_3(z_1 + z_2 + z_3) \\ & + z_1z_2z_3(y_{s1}z_1z_3 + y_{s1}z_1z_2 + y_{s2}z_2z_3 + y_{s2}z_1z_3 \\ & + y_{s1}y_{s2}z_1z_2z_3) - ry_{s1}z_1z_2^2 - ry_{s2}z_2^2z_3 \\ & - r(z_1z_2 + z_1z_3 + z_2z_3), \end{aligned} \quad (5b)$$

$$k_3 = -(z_1^2z_3^2 - rz_2^2), \quad (5c)$$

$$\begin{aligned} k_4 = & -((z_2 - rz_0)(z_0 + z_1)(z_1 + z_2 + y_{s1}z_1z_2 \\ & + y_{s2}z_1z_2), \end{aligned} \quad (5d)$$

$$\begin{aligned} k_5 = & z_0z_2^2 + z_1z_2^2 + z_1^2z_2 - r(z_0z_1^2 + z_0^2z_1 + z_0^2z_2) \\ & + z_1^2z_2^2(y_{s1} + y_{s2}). \end{aligned} \quad (5e)$$

By solving equations (1) and (3), the five normalized impedance parameters z_1, z_2, z_3, y_{s1} , and y_{s2} are all determined by the specified FBW, RL , and r . To achieve equal power divider, the filtering transformers with impedance transformer ratio $r = 2$ are designed and simulated. The impedance parameters meet the condition of $Z_{01} = Z_1/2, Z_{02} = Z_2/2$, and $Z_{s01} = Z_{s1}/2$. The shared network reduces the size of the circuit and enlarges the available RL bandwidth of power divider. For enhancing isolation performance, two $\lambda/4$ resonators with impedance of Z_0 and two resistors are added as shown in Fig. 2 (a). Design II as shown in Fig. 2 (b) can be devised using a similar approach. By trading off the output port matching and isolation performance, the values of the isolation resistors can be finally decided. Figure 5 illustrates the simulated results of the proposed wideband filtering power dividers I and II with different numbers of isolation resistors. The wideband filtering power divider I exhibits a narrow isolation bandwidth with one resistor (R_1) and fails to achieve isolation below -25 dB with two resistors (R_1 and R_2). In contrast, the wideband filtering power divider II delivers both good port matching and a wide isolation bandwidth when two resistors are incorporated.

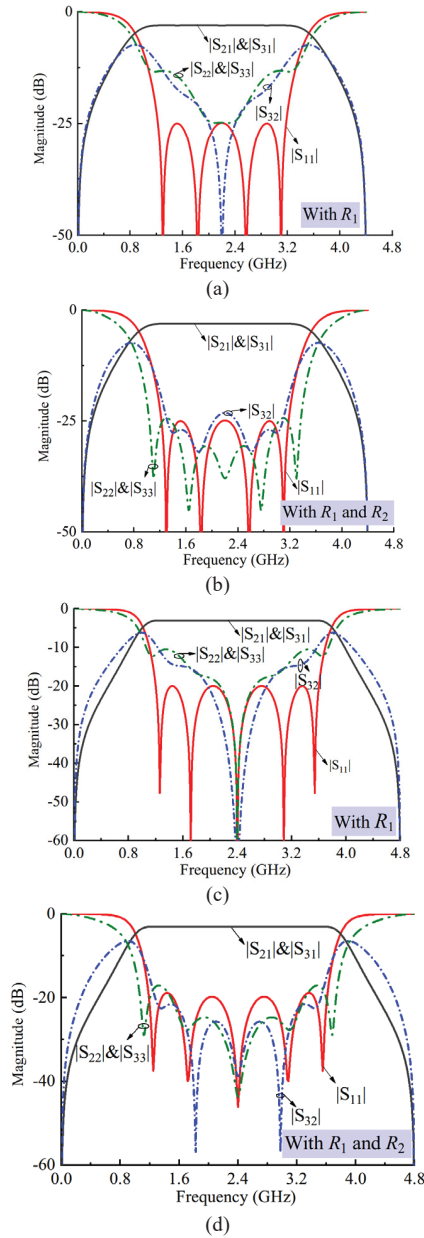


Fig. 5. Simulated results of proposed wideband filtering power dividers. (a) Design I with isolation resistor R_1 , (b) Design I with isolation resistors R_1 and R_2 , (c) Design II with isolation resistor R_1 , and (d) Design II with isolation resistors R_1 and R_2 .

The design method of the proposed wideband filtering power divider is summarized as follows:

- (1) Specify the design goals: center frequency, fractional bandwidth, return loss level, and port isolation requirement;
- (2) Determine the characteristic impedances of the filtering transformer using the proposed Chebyshev synthesis method;

- (3) Synthesize the shared resonant network to extend the matching bandwidth;
- (4) Calculate the initial circuit parameters of all transmission-line segments;
- (5) Introduce two isolation resistors between the output ports and optimize their values for high port isolation;
- (6) Perform fine-tuning in circuit simulation to satisfy specifications;
- (7) Fabricate and measure the prototype for experimental verification.

III. SIMULATION AND EXPERIMENT

For verification, two wideband filtering power dividers are designed, simulated, and fabricated on an RO4003C substrate ($\epsilon_r = 3.55$, thickness = 0.508 mm). The first design, shown in Fig. 2 (a) with center frequency of 2.2 GHz, FBW = 90%, and $RL = 25$ dB is designed, simulated, and fabricated. Prior to simulation and fabrication, detailed theoretical calculations were conducted to determine the key characteristic impedance parameters of the divider's transmission lines, which are critical to its filtering and power division performance. Its theoretical parameters are calculated as: $Z_{01} = 36.5 \Omega$, $Z_{02} = 19.9 \Omega$, $Z_3 = 36.5 \Omega$, $Z_{s01} = 148.8 \Omega$, $Z_{s2} = 31.4 \Omega$. The second wideband filtering power divider shown in Fig. 2 (b) with center frequency of 2.4 GHz, FBW = 100%, and $RL = 15$ dB is designed. Theoretical parameters are calculated as: $Z_{01} = 33.5 \Omega$, $Z_{02} = 20.2 \Omega$, $Z_{03} = 14.3 \Omega$, $Z_4 = 33.5 \Omega$, $Z_{s01} = 57.7 \Omega$, $Z_{s02} = 34.8 \Omega$, $Z_{s03} = 31.4 \Omega$, $Z_{s04} = 31.4 \Omega$. After global tuning, the dimensions displayed in Figs. 6 (a) and 7 (a) are listed. The chip resistors in Fig. 6 (b) are selected as $R_1 = 68 \Omega$ and $R_2 = 270 \Omega$. The chip resistors in Fig. 7 (b) are selected as $R_1 = 56 \Omega$ and $R_2 = 270 \Omega$. The fabricated prototypes are measured by a Keysight N9918A vector network analyzer before measurement, two-port calibration are performed via electronic calibration module to eliminate the effects of test cables and connectors. For the 2.2 GHz design in Fig. 6 (b), the measured FBW is 108.6% with RL better than 10.6 dB and in-band isolation over 18.6 dB (90% isolation bandwidth). The 2.4 GHz design in Fig. 7 (b) achieves 103.7% FBW, RL better than 12.8 dB, and in-band isolation over 25 dB (60% isolation bandwidth). As observed in Figs. 6 (b) and 7 (b), slight discrepancy between simulated and measured S_{11}/S_{22} can be reasonably attributed to several inevitable practical factors. First, fabrication tolerance of microstrip line width and etching deviation introduces impedance variation. Second, SMA connector soldering parasitic effect and assembly misalignment bring extra stray inductance and capacitance. These non-ideal practical factors are common in microstrip circuit implementation and lead to

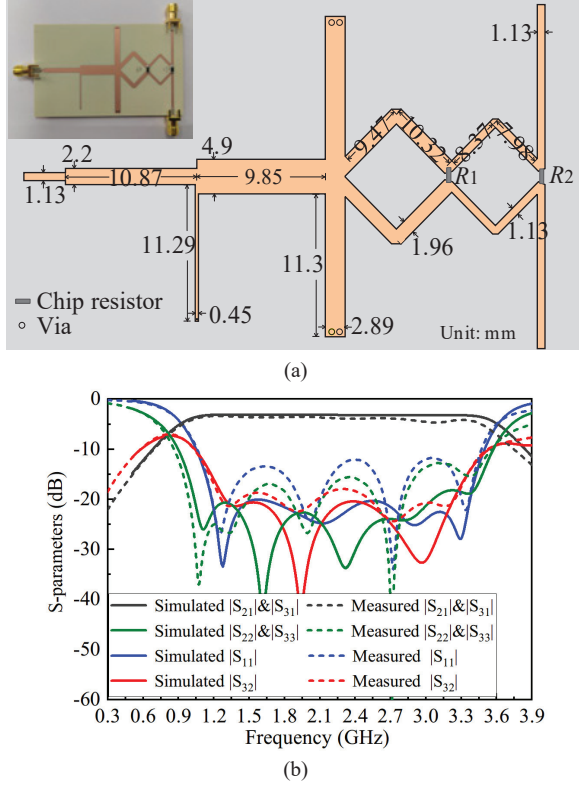


Fig. 6. (a) Physical layout and photograph of Design I and (b) simulated and measured results of Design I.

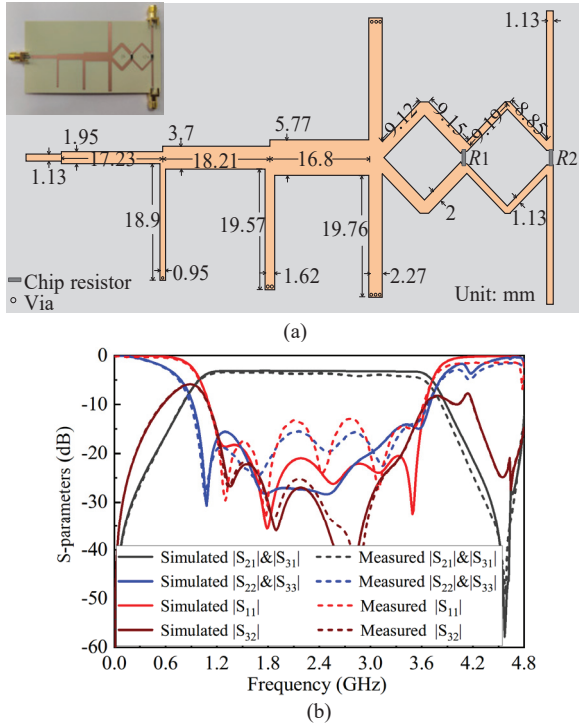


Fig. 7. (a) Physical layout and photograph of Design II and (b) simulated and measured results of Design II.

Table 1: Comparison with recently published power dividers

Ref.	Freq. (GHz)	FBW (%)	Isolation (dB)	RL (dB)
[12]	1.28	12.5	-36	-20
[8]	2	33	-10.1	-15.1
[20]	30	46.67	-12	14.1
[18]	2.35	55.32	-17	-14
[16]	1.8	73	-18.5	-10
[15]	3	84	-19.5	-15
[6]	6.5	138.5	-10	-10
This	2.2	108.6	-18.6	-10.6
Work	2.4	103.7	-25	-12.8

the observable minor mismatch between simulation and measurement.

Table 1 summarizes the key performance parameters of the two proposed wideband filtering power dividers, along with a comparison with other state-of-the-art designs reported in the literature. The comparison clearly demonstrates that the proposed designs exhibit superior performance, including ultra-wide bandwidth, embedded filtering functionality (eliminating the need for additional filtering components), and high-level in-band isolation.

IV. CONCLUSION

In this paper, two novel wideband filtering power dividers with improved isolation level have been proposed. The designs achieve Chebyshev transformer functions, enabling wideband impedance matching and equal power division. The two resistors further enhance end port isolation without compromising insertion loss. The proposed wideband filtering power dividers demonstrate designable bandwidth and high isolation, thereby offering an efficient solution for broadband RF energy harvesting systems.

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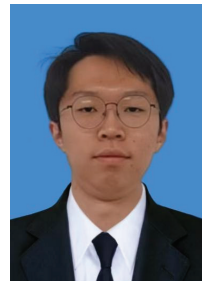
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A Fast LO Leakage Calibration Method for MIMO Transmitter

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Abstract – Local oscillator (LO) leakage caused by in-phase/quadrature (I/Q) offsets severely degrades the performance of direct-conversion multiple-input multiple-output (MIMO) transmitters in 5G and emerging 6G systems. Existing LO leakage calibration methods often require many measurements or suffer from slow convergence, making them inefficient for multi-channel and multi-frequency applications. To address this challenge, this paper proposes a fast LO leakage calibration method based on Newton's method. For each channel at each frequency point, the I/Q offset compensation values are determined from only nine groups of I/Q settings and the corresponding LO leakage measurements. Simulations and experiments on an 8-channel MIMO transmitter show that the proposed method achieves an average LO leakage suppression of -55.6 dBc while reducing the number of calibration samples by more than 95% compared with traditional methods, demonstrating its efficiency and practicality for large-scale MIMO transmitters in 5G and 6G communications.

Index Terms – Calibration, in-phase/quadrature offset, local oscillator leakage, multiple-input multiple-output transmitter, Newton's method.

I. INTRODUCTION

Massive multiple-input multiple-output (MIMO) technology is widely regarded as one of the key enabling technologies for 5G and future 6G systems. A MIMO system configured with hundreds or even thousands of antennas can significantly enhance

wireless communication performance [1–3]. In MIMO systems, direct-conversion transmitters (DCTs) [4] are commonly employed due to their high integration, low cost, and low power consumption. However, the in-phase/quadrature (I/Q) DC offsets in DCTs cause local oscillator (LO) leakage, which results in transmitter performance degrade [5, 6]. The LO leakage signal cannot be directly eliminated by a filter, since it falls within or near the signal bandwidth in a DCT. Therefore, it is essential to calibrate the I/Q offsets to effectively suppress LO leakage signal.

For large-scale MIMO systems, a key challenge is the trade-off between calibration effectiveness and calibration time. In practical systems, calibration must be performed over multiple transmitter channels and multiple operating frequency points. As a result, the total calibration time scales with channels, frequency points, and measurements required at each frequency point. Although accurate LO leakage calibration is essential, many existing methods require excessive calibration time, which limits their practical applicability in large-scale MIMO systems. Therefore, it is highly desirable to develop a fast calibration method that achieves a favorable trade-off between calibration effectiveness and calibration time.

Existing LO leakage calibration methods can be broadly classified into traversal-based and estimation-based techniques. In traversal-based methods, [7] uses a detector feedback loop to search for proper I/Q offsets, while [5] employs a complex receiver feedback loop with frequency offset for LO leakage calibration. Although effective, such methods usually require

exhaustive searches over the calibration space, resulting in long calibration time, especially in multi-channel and multi-frequency MIMO systems.

Estimation-based techniques have also been investigated. A blind technique based on higher-order statistics introduced in [4] estimates the calibration parameters using gradient descent, but may suffer from slow convergence and sensitivity to initialization and noise. The method in [8] performs compensation using specially designed training sequences, which limits implementation flexibility. Joint transmitter and receiver calibration is performed in [9, 10] with high computational complexity and unavoidable error propagation. Reference [11] combines least squares and gradient descent methods, but multiple candidate solutions and the need for phase information further increase calibration complexity.

Although these studies provide useful solutions for LO leakage calibration, several limitations remain for large-scale MIMO transmitters. Many existing methods mainly focus on single-channel calibration and do not explicitly consider the stringent calibration-time requirement in multi-channel systems. Moreover, the trade-off among low sample complexity, fast calibration speed, and sufficiently high LO leakage suppression has not been adequately addressed for practical deployment.

To overcome these limitations, this paper proposes a fast LO leakage calibration method based on Newton's method for MIMO transmitter systems. The main contributions of this work are summarized as follows: a fast LO leakage calibration method for MIMO transmitters is developed, which is particularly suitable for systems with numerous channels and frequency points; a Newton-based estimation scheme is developed to determine the I/Q offset compensation values; a nine-point measurement strategy is designed to balance calibration effectiveness and calibration time; experimental results demonstrate the effectiveness and efficiency of the proposed method.

The rest of the paper is organized as follows. Section II introduces the system model and the mechanism of LO leakage. Section III presents the proposed Newton-based LO leakage calibration method. Section IV presents the simulation and experimental results of the LO leakage in an 8-channel MIMO transmitter system. Section V concludes the paper.

II. SYSTEM MODEL

The system model of the LO leakage calibration of DCTs in a MIMO transmitter system is illustrated in Fig. 1. The I/Q signals $I^k(t)$ and $Q^k(t)$ are mixed with the LO signals f_{LO}^k to obtain the RF output signals y_{RF}^k , where k represents the k th transmitter channel, d_i^k and d_q^k represent the DC offsets corresponding to I and Q branch

introduced due to the I/Q mixer or I/Q modulator in the k th transmitter channel and, accordingly, $d_{i_set}^k$ and $d_{q_set}^k$, generated from digital-to-analog converters (DACs), are the target compensating voltages to the k th transmitter channel for the LO leakage calibration.

The RF output signal of the k th transmitter channel, ignoring the influence of I/Q imbalance, can be expressed as

$$y_{RF}^k(t) = A^k \{ [I^k(t) + d_i^k] \cos(2\pi f_{LO}^k t) + [Q^k(t) + d_q^k] \sin(2\pi f_{LO}^k t) \}, \quad (1)$$

where A^k represents the channel gain of the k th transmitter channel.

Ideally, the power of LO leakage signal is

$$P_{LO_0}^k = 10 \lg \{ (A^k)^2 [(d_i^k)^2 + (d_q^k)^2] \}. \quad (2)$$

The simulated variation of the LO leakage level with I/Q offsets is shown in Fig. 2, where I and Q offsets correspond to d_i^k and d_q^k in (2). It is clearly shown that the LO leakage level is minimized when d_i^k and d_q^k are zero. However, d_i^k and d_q^k cannot be equal to zero in practice, and therefore should be compensated by $d_{i_set}^k$ and $d_{q_set}^k$ to minimize the LO leakage level. After calibration, equation (2) can be adjusted to

$$P_{LO}^k = 10 \lg \{ (A^k)^2 [(d_i^k - d_{i_set}^k)^2 + (d_q^k - d_{q_set}^k)^2] \}. \quad (3)$$

Obviously, the LO leakage calibration can be completed by finding out the values satisfying $d_{i_set}^k = d_i^k$ and $d_{q_set}^k = d_q^k$.

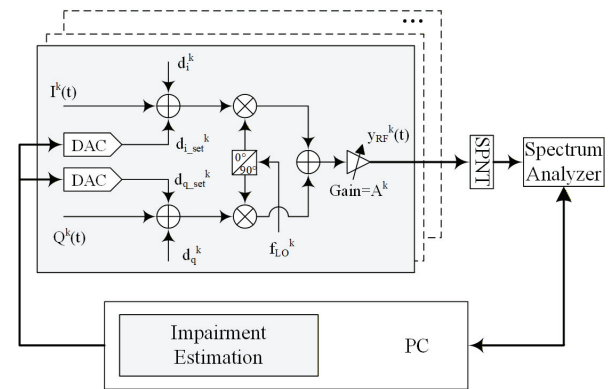


Fig. 1. System model of MIMO DCT system with calibration circuit.

III. CALIBRATION ALGORITHM

As shown in Fig. 1, the PC controls the DACs to generate a set of compensating voltages for various

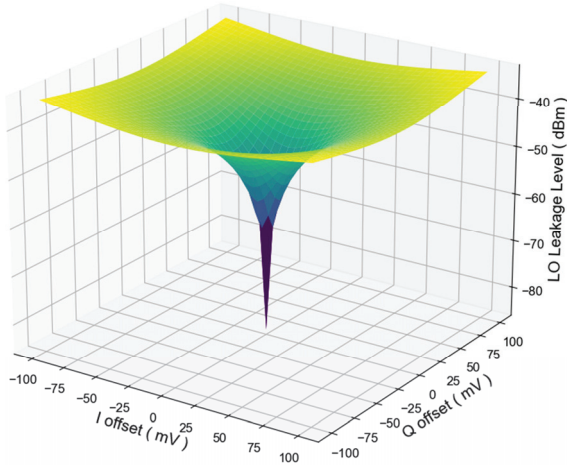


Fig. 2. Simulated LO leakage level versus I/Q offsets.

transmitter channels. The spectrum analyzer measures the corresponding LO leakage levels and reports back to the PC. The proposed calibration algorithm based on Newton's method computes the optimized compensating voltages required to minimize the LO leakage levels.

Taking one transmitter channel as an example, for the n th measurement, $n = 0, 1, \dots, N$, applying different I/Q offsets $(d_{i_set_n}, d_{q_set_n})$ and measured LO leakage power P_n , a set of equations can be obtained

$$\begin{cases} P_0 = A(d_i^2 + d_q^2) \\ P_1 = A((d_i - d_{i_set_1})^2 + (d_q - d_{q_set_1})^2) \\ \vdots \\ P_n = A((d_i - d_{i_set_n})^2 + (d_q - d_{q_set_n})^2) \end{cases}, \quad (4)$$

where, for simplicity, the channel superscript k is omitted, and the gain A is a fixed constant for this channel. Set the 0th measurement as the reference, then

$$\begin{cases} \frac{P_n}{P_0} = \frac{(d_i - d_{i_set_n})^2 + (d_q - d_{q_set_n})^2}{d_i^2 + d_q^2} \\ \frac{P_{n-1}}{P_0} = \frac{(d_i - d_{i_set_n-1})^2 + (d_q - d_{q_set_n-1})^2}{d_i^2 + d_q^2} \end{cases}. \quad (5)$$

Subtracting the two equations in (5), i.e., $\frac{P_n}{P_0} - \frac{P_{n-1}}{P_0}$, yields

$$\begin{aligned} & 2 \left(\frac{d_{i_set_n}}{z_n} - \frac{d_{i_set_n-1}}{z_{n-1}} \right) d_i \\ & + 2 \left(\frac{d_{q_set_n}}{z_n} - \frac{d_{q_set_n-1}}{z_{n-1}} \right) d_q \\ & = \frac{d_{i_set_n}^2}{z_n} - \frac{d_{i_set_n-1}^2}{z_{n-1}} + \frac{d_{q_set_n}^2}{z_n} - \frac{d_{q_set_n-1}^2}{z_{n-1}}, \end{aligned} \quad (6)$$

where $z_n = 1 - \frac{P_n}{P_0}$ and $z_{n-1} = 1 - \frac{P_{n-1}}{P_0}$.

Under the ideal noiseless model, all $n - 1$ lines in (6) will intersect at a single point (d_i, d_q) . The target value for the LO leakage calibration can be obtained by determining the intersection point. However, the lines cannot intersect at a single point due to non-ideal physical conditions, such as hardware noise and measurement errors. Instead, an optimization algorithm can be employed to identify a point (d_x, d_y) that minimizes the sum of the squares of the distances from this point to the set of lines. This approach yields the point with the smallest error relative to the theoretical target point (d_i, d_q) .

Let $(d_i, d_q) = (x, y)$, equation (6) can be simplified as

$$a_n x + b_n y = c_n. \quad (7)$$

For a candidate point (x, y) as the target point, the square of its distance from the lines in (7) can be expressed as

$$dist_n^2(x, y) = \frac{(a_n x + b_n y - c_n)^2}{a_n^2 + b_n^2}. \quad (8)$$

For convenience, define $w_n = \frac{1}{a_n^2 + b_n^2}$, $r_n(x, y) = a_n x + b_n y - c_n$, the objective function is the sum of (8)

$$f(x, y) = \sum_{n=1}^{N-1} \frac{(a_n x + b_n y - c_n)^2}{a_n^2 + b_n^2} = \sum_{n=1}^{N-1} w_n r_n^2(x, y), \quad (9)$$

where N represents n set of I/Q offsets, $N - 1$ represents $n - 1$ lines in (6).

The target point for the LO leakage calibration will be found as soon as the point (x, y) is found which can minimize $f(x, y)$ in (9).

First, the gradient of the objective function is calculated

$$\nabla f(x, y) = \begin{bmatrix} \frac{\partial f(x, y)}{\partial x} \\ \frac{\partial f(x, y)}{\partial y} \end{bmatrix} = 2 \sum_{n=1}^{N-1} w_n r_n(x, y) \begin{bmatrix} a_n \\ b_n \end{bmatrix}. \quad (10)$$

Second, calculate the Hessian matrix

$$H = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{bmatrix} = 2 \sum_{n=1}^{N-1} w_n \begin{bmatrix} a_n^2 & a_n b_n \\ a_n b_n & b_n^2 \end{bmatrix}, \quad (11)$$

Since $f(x, y)$ is quadratic, the Hessian matrix is constant and positive semidefinite. When the measurement points are chosen such that the resulting lines are not all parallel, the Hessian becomes positive definite, and the minimizer is unique. Therefore, the solution of (9)

can be approximated gradually by iteration based on Newton's method. The iterative formula for Newton's method is

$$\begin{bmatrix} x^{(t+1)} \\ y^{(t+1)} \end{bmatrix} = \begin{bmatrix} x^{(t)} \\ y^{(t)} \end{bmatrix} - H^{-1} \nabla f(x^{(t)}, y^{(t)}). \quad (12)$$

The iteration starts from $(x^{(0)}, y^{(0)}) = (0, 0)$ as the values of I/Q offsets for LO leakage calibration are usually very small, and stops when the norm of the update falls below a prescribed threshold.

The impact of the number of I/Q offset configurations on LO leakage calibration results under different LO leakage power errors, based on the proposed method, is simulated first. As shown in Fig. 3, the smaller the LO leakage power measurement error (e.g., when $\sigma = 0.1$), the better the LO leakage calibration results. The trend indicates that increasing number of I/Q offset points can improve the LO leakage calibration result. When 8–10 sets of I/Q offset configurations are selected, the calibration performance is about 10 dB worse compared to selecting 20–40 sets. Nine sets of I/Q offset configurations are adopted in this work as a compromise between the LO leakage calibration performance and the total calibration time.

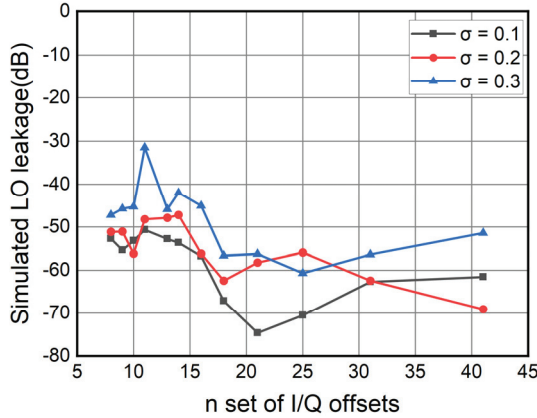


Fig. 3. Simulation of the impact of the number of I/Q offset configurations on LO leakage calibration results under different LO leakage power errors.

The detailed procedure of the proposed LO leakage calibration method for MIMO transmitters is illustrated in Fig. 4. The calibration process is summarized as follows.

S1: Select channel k and the target frequency point. Apply nine sets of I/Q compensating voltages $(d_{i_set_n}^k, d_{q_set_n}^k)$, and measure corresponding LO leakage power P_n^k .

S2: Based on the measured data, derive a set of linear equations according to (4)–(7).

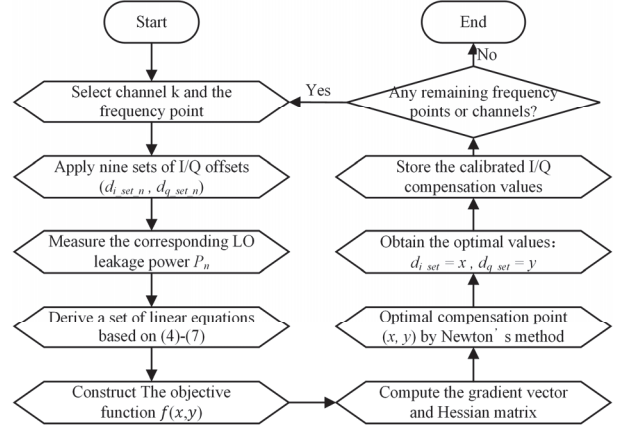


Fig. 4. Flowchart of the proposed Newton-based LO leakage calibration method for the MIMO transmitters.

S3: Construct the objective function $f(x, y)$, which is defined as the sum of the squared distances from a candidate point (x, y) to the derived lines.

S4: Compute the gradient vector and Hessian matrix of $f(x, y)$, and use Newton's method to determine the optimal compensation point (x, y) . The obtained result is taken as the optimal compensation values.

S5: The same procedure is repeated for all remaining channels and frequency points until the calibration process is completed.

IV. EXPERIMENTAL RESULTS

Figure 5 shows the experimental scenario for LO leakage calibration, which consists of an 8-channel MIMO transmitter system, a PC running the calibration algorithm, a signal analyzer for measuring LO leakage levels under different I/Q offsets in each channel, and a switching matrix for selecting and switching between different channels.

Table 1 lists the LO leakage levels corresponding to different I/Q compensating voltages $(d_{i_set_n}, d_{q_set_n})$ in a single transmitter channel. The nine I/Q offset combinations $(0, -0.1, +0.1)$ are carefully chosen to evenly cover the range of possible I/Q offsets, balancing calibration accuracy with calibration time. Following the steps in section III, seven lines are derived, and then the point that can minimize the sum of the squared distances from the seven lines is identified using Newton's method. The seven lines that roughly intersect at one point are depicted in Fig. 6. The point calculated using Newton's method $(-68.70, 19.82)$ is very close to the manually measured point $(-66.10, 17.90)$.

Table 2 compares the I/Q compensating voltages for LO leakage calibration between manually measured and the proposed method for different transmit channels

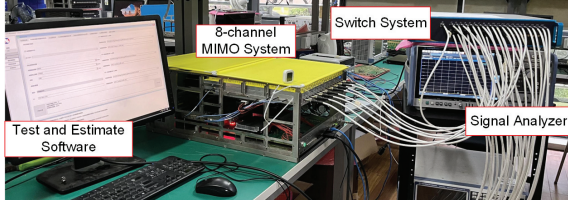


Fig. 5. Experimental scenario.

Table 1: Measured LO leakage level at 9 I/Q offsets

No.	I Offset (mV)	Q Offset (mV)	LO Leakage Level (dBm)
1	0	0	-19.18
2	0	+0.1	-15.54
3	0	-0.1	-13.30
4	+0.1	0	-11.33
5	+0.1	+0.1	-10.88
6	+0.1	-0.1	-9.34
7	-0.1	0	-25.32
8	-0.1	+0.1	-15.71
9	-0.1	-0.1	-15.30

in the 8-channel MIMO system. The proposed method exhibits high consistency with the manually measured values, with an average error of only 2 mV. The results demonstrate the effectiveness of the proposed calibration technique for MIMO transmitter LO leakage suppression in MIMO transmitter systems.

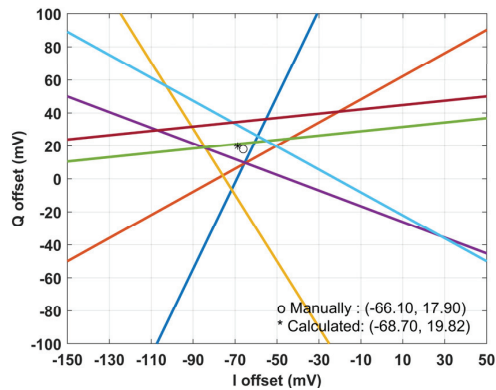


Fig. 6. Calculated and manually measured I/Q compensation values for LO leakage calibration.

The eight I/Q offset calibration values for the 8-channel transmitter, obtained using the proposed method, are applied to the MIMO transmitter system for LO leakage testing. For comparison, the LO leakage without calibration and that with manually calibration are also measured under the same experimental conditions, as shown in Fig. 7.

Table 2: Comparison of calculated and manually measured I/Q offsets

Channel	Newton's Method		Manually Test	
	I(mV)	Q(mV)	I(mV)	Q(mV)
1	-68.70	19.82	-66.1	17.9
2	-49.05	-15.78	-51.2	-17.3
3	-57.31	-10.76	-59.3	-12.5
4	-62.16	7.01	-63.7	7.4
5	-54.30	-3.45	-55.7	-3.3
6	-51.10	-10.80	-52.5	-12.3
7	-65.51	32.27	-68.8	31.3
8	-55.29	-3.73	-57.3	-3.3

It can be observed that the proposed method significantly suppresses the LO leakage for all eight transmitter channels compared with the uncalibrated case. Although manual calibration achieves a lower average LO leakage of about -70 dBc, the proposed Newton-based method reduces the average LO leakage of the eight channels to approximately -55.6 dBc, while requiring only nine samples. These results demonstrate that the proposed method provides a favorable trade-off between calibration performance and measurement efficiency in multi-channel MIMO transmitter systems.

Generally, measurement instruments require a signal-to-noise (SNR) in the range of 40 to 52 dBc [12, 13], while the most stringent SNR requirement specified in 3GPP for 5G mobile communication is 35 dBc [14]. Therefore, the calibrated LO leakage level of -55.6 dBc satisfies both instrument-grade SNR requirements and practical communication system requirements.

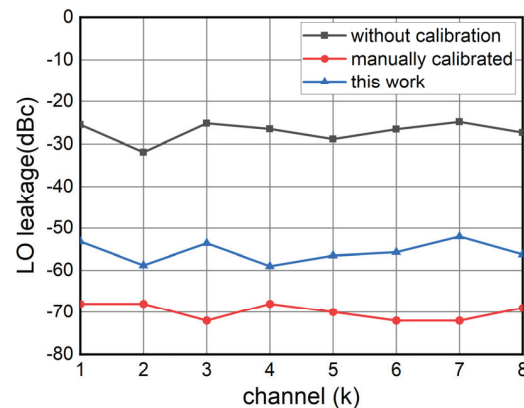


Fig. 7. Measured LO leakage of the 8-channel MIMO transmitter without calibration, with traversal-based and with proposed Newton-based calibration.

Table 3 further compares the proposed method with representative existing methods in terms of LO leakage

Table 3: Comparison of calibration results

Item	Calibration Method	LO Leakage (dBc)	Type	Samples per Frequency Point	Channels	Total Calibration Time* (hour)
[4]	Gradient descent	−52	Chip-level tested	10000	1	1111
[5]	Traversal-based	−100	Simulated	512	1	5.73
[7]	Traversal-based	−45	Chip-level tested	180	1	2
[9]	Least squares	−52	Chip-level tested	1000	1	11.07
[11]	Least squares + gradient descent	−53	System-level tested	40000	1	444
[10]	Joint digital online compensation	−58	Chip-level tested	64	1	/(online)
This Work	Newton's method	−55.6 (average)	System-level tested, 8-channel, RF out	9	8	0.1

*Total calibration time is calculated for an 8-channel system with 100 frequency points per channel, assuming that each set of I/Q compensation values requires 50 ms.

suppression, sample number, applicability, and total calibration time. The proposed method achieves an average LO leakage of −55.6 dBc with only nine samples per frequency point per channel, which is comparable to or better than the reference methods while significantly reducing the sample number by more than 95%. In particular, under the unified assumption of eight channels, 100 frequency points per channel, and 50 ms for each set of I/Q compensation values, the total calibration time of the proposed method is only about 0.1 hour (6 minutes). By contrast, the reported methods cost more than 2 hours due to their much higher sample complexity. This result demonstrates the advantage of the proposed method in achieving a good trade-off between calibration performance and calibration time, especially for multichannel MIMO transmitter systems.

V. CONCLUSION

A novel fast LO leakage calibration technique for the MIMO transmitter system based on Newton's method is proposed in this paper. By measuring nine sets of I/Q offsets and the corresponding LO leakage levels, the I/Q offset values for its LO leakage calibration can be accurately and efficiently computed. Experiments on an 8-channel MIMO transmitter system demonstrate that the proposed method achieves an average LO leakage of −55.6 dBc after calibration, requiring only nine samples per frequency point per channel. These results underscore the method's fast and effective performance, making it well-suited for largescale MIMO systems in 5G and 6G communications.

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