Optimum Tilt Angles for Manual Tracking of Photovoltaic Modules

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ABSTRACT

This article reports the investigation for the average optimal tilt angle of solar panels on a monthly basis (single axis tracking) for a wide range of latitudes in the northern hemisphere to collect maximum total solar irradiation. Based on experimentally validated modeling, a new set of empirical relations has been proposed to compute the optimum tilt angle on a monthly basis for the entire year. The accuracy of equations set is evaluated by standard statistical measures. The proposed set of empirical equations is compared with an existing set of empirical equations on various cities within the latitude and has yielded significantly better results. Solar Advisory Model (SAM) has been used to compare—with respect to a fixed Solar Photo Voltaic (SPV) panel—the electricity predicted by (1) a new set of manual solar tracking equations, (2) an established set of solar tracking equations, and (3) data from an automated single axis tracking system by a Programmable Logic Controller or PLC. It is found that, the manual tracking system based on the proposed set of equations generates an annual average increase in electrical energy of (5-8)%, the old set of equation yields annual increase of (2-4)% and the PLC automated single axis tracking system generates a growth of (8-15)% over fixed SPV modules. Based on the proposed empirical set of equation, a manual tracking system has also been designed and commissioned to reaffirm the justification of the proposed equation set.

Keywords: Solar photovoltaic panel, manual single axis tracking, empirical set of equations, optimum tilt angle, electrical energy.

Notation:

I_b: instantaneous beam radiation (Wm^{-2}) on a horizontal surface
I_d: instantaneous diffuse radiation (Wm^{-2}) on a horizontal surface
I_T: instantaneous global radiation (Wm^{-2}) on a tilted surface
I: instantaneous global radiation (Wm^{-2}) on a horizontal surface
I_{iso}, I_{cs}: instantaneous isotropic and circumsolar diffuse radiation (Wm^{-2}) on a horizontal surface
I_{hb}: instantaneous horizontal bighting radiation (Wm^{-2}) on a horizontal surface
I_{ref}, I_o: ground reflected component and extraterrestrial radiation (Wm^{-2})
θ: angle of incidence of sun rays on a tilted surface
θ_z: zenith angle
φ: latitude (°), (φ>0 for northern hemisphere)
ρ: ground albedo
ω: hour angle(°), negative for morning , positive for afternoon
ω_s: hour angle(°) corresponding to sunrise and sunset
β: slope of solar PV module (°)
γ: surface azimuth angle (°)
δ: solar declination angle (°)
R_b: tilt factor for beam radiation
R_d: tilt factor for diffuse radiation
R_r: tilt factor for reflected radiation
ρ: ground albedo
δ: solar declination (°), north positive
n: julian day of the year

INTRODUCTION

Solar PV panels collect the maximum solar radiation when the sun's rays are perpendicular to the surface of the panel. However, the earth's diurnal and seasonal motions affect declination angle of the sun making it dependent on the latitude of the site and the month of the year. Therefore, the optimal tilt angle for a PV panel varies with latitude on a monthly basis and can play a significant role in determining its optimum performance. In the Northern Hemisphere, conventional fixed PV modules are generally installed with a southern orientation with the optimum tilt angle equal to the local latitude [1]. However, SPVs at optimal tilt angles can boost the collected energy considerably over different periods of time and geographical conditions [2,3]. Sensor-based automated sun tracking devices, which change the position of the solar systems keeping the best orientation relative to the sun, are not suitable for small solar panels due to energy losses in the driving system [4] and as well due to high implementation costs. Manual tracking in such cases can be a cost-effective substitute, especially in developing countries where an estimate of 1.3 billion people has no access to electricity primarily in rural areas. Previous research studies over different parts of the world have obtained values of optimal tilt angles of solar trackers based on numerical analysis of observed solar irradiation data and models [5-9] which are site specific on account of localized meteorological conditions, changes in radiation patterns and utilization periods of time [7,8]. Optimal tilts at different sites are usually computed by solar decomposition models [10-15], that transform solar direct and diffuse irradiation received on horizontal surfaces to inclined surfaces to maximize solar efficiency of SPVs. One of the most widely known and commonly used solar radiation isotropic models in this field is the Liu and Jordan model [11, 12], which assumes a uniform distribution of diffuse radiation on the sky dome and that reflection from the ground is also diffused. According to Klutcher [16] and Badescu [17], this model underestimates the value of the diffused (clouded time) irradiation, however by a small amount over clear skies, while working satisfactorily for cloud covered days. Previous researchers such as Tian Pau Chang [18] and Al-Rawahi et al. [19] have successfully adopted this model across different latitudes. In the present study an anisotropic model is applied to compute optimum tilt angles based on measured hourly solar radiation data obtained from different regions within these latitudes. Since the optimum tilt angle values obtained from model simulations seem to be closely related to the model of diffused solar radiation that is used [20], a rigorous evaluation of model performance is conducted in this study before accepting it for optimum tilt estimations. Since observational solar radiation data are limited by low spatial and temporal resolution it is difficult to run solar radiation models for every site. To reduce necessity of modeling which by itself is a complex phenomenon; Nijegorodov et al. [21] presented an analytical method to calculate optimum tilt angles between latitudes 60^{o}N and 60^{o}S based on modeling simulations at different sites. This type of statistical analysis gives a realistic representation of optimal tilt angles since it incorporates modeled data. Although Gunerhan and Hepbasli [22] have found close agreement with the results of optimal tilt computed from these equations in Turkey, Yakup and Malik [23] have obtained some disparities with these estimated values at Brunei, Darussalam. Thus it becomes important to assess optimal tilt angles, at least regionally, to ensure optimal energy efficiency of solar collectors since it seems insufficient to infer on a generalized numerical approach.

The present study found inconsistencies with the optimum tilt angles computed from the previous empirical set of equations [21] within the given domain. Therefore a primary objective of this study was to compute an improved empirical set of equations relating optimum tilt with latitude from validated model simulations across different regions of varied climatic regimes within the prescribed latitudes. The optimal tilt angles from the proposed set of equations can then be effectively used for monthly manual single axis tracking over fixed SPV modules.

This study investigates and confirms the electrical energy generated by using the optimum tilt angles derived from our proposed set of equations. In addition to standard statistical measures, another method is used to evaluate the yield of electrical energy from SPV panels tilted at 84 these angles. And it is well established that sensor based PLC controlled solar panels generate maximum electricity—Abdallah [24], Sungur[25] and Dutta et al. [26].

An inter-comparison of the amount of electricity generated from optimum tilt angles from our proposed equations, and from the equations in [21] with that obtained from PLC controlled solar system was performed to establish the relative efficacy of the manual tracking system based on our enhanced equations. Uncertainties in estimates of yield from SPVs is around 3.9% on account of year-year climate variability [27] which do not have a significant impact on the amount of electricity generated. Therefore this study establishes an improved set of mathematical expressions which can be utilized for cost-effective manual sun tracking system within the specified latitudes in the northern hemisphere.

METHODOLOGY

Model Setup

Study Area

Hourly observational solar radiation on horizontal planes for direct beam and diffused radiation have been used to compute optimum tilt angle for various locations within similar range of latitudes. Details of instrument calibration, data collection and maintenance are discussed in [28-30]. Hourly observations of solar irradiation (direct and diffuse) for cities of United States of America (USA) within these latitudes were obtained from the website of National Aeronautics Space Administration (NASA) [31] and used in the model. These cities are located within the same range of latitudes (13°-39°) covering different climate regimes.

Solar Radiation Model

The instantaneous global radiation on a tilted surface is the sum of beam constituent from direct irradiation on tilted surface, diffused radiation and ground reflected radiation on inclined surface. Solar radiation models simulate the radiation on inclined planes. Although the solar radiation models are of different complexities, they fundamentally differ from each other in the method of computing the diffuse radiation component. The isotropic models assume the homogeneity of intensity of diffuse sky radiation over the sky dome whereas in case of anisotropic model all the diffuse radiation components—isotropic diffuse, circumsolar diffuse and horizontal brightening component have been taken into account. The angular distribution of diffused radiation is to some degree a function of the reflectance (the albedo) of the ground. In a location where there is no snow on the ground, the average value of ground albedo is 0.2 as suggested by Jain [32].

The general equation [1] for computing the instantaneous solar radiation on tilted surface is

$$I_T = I_b + I_{iso} + I_{cs} + I_{hb} + I_{ref.}$$

In general, a reflected component simplified to ground reflectance factor(ϱ). Depending on sky condition the isotropic or anisotropic model has been considered. In case of isotropic model

$$IT = I_b R_b + I_d R_d + I_r R_r$$

$$R_b = cos\ \theta / cos\ \theta_z$$

Various methods for computation of R_d have been provided by different isotropic models. In this study R_d is formulated as Liu Jordan model [12] and is given as

$$Rd = \frac{(1 + \cos \beta)}{2}$$

The value of $R_r = \rho(1-\cos\beta)/2$

$$\cos \theta_z = \cos\phi\cos\delta\cos\omega + \sin\phi\sin\delta$$

Hence in isotropic sky the total radiation can be expressed as [1]

$$I_T = I_b R_b + I_d \frac{(1+\cos\beta)}{2} + I\rho\frac{(1-\cos\beta)}{2}$$

The circumsolar diffuse is due to the forward scattering of solar radiation and it is concentrated in the part of the sky around the sun. Horizontal brightening is concentrated near the horizon which is most pronounced for clear sky condition. This part may be scattered to some extent due to the high reflectance factor of the ground. Under clear sky condition most of the diffuse component is assumed to be forward scattered and hence the value of Anisotropy Index 'A_i' is nearly equal to 1. During cloudy condition the forward scattered component of diffuse radiation reduces and the isotropic component increases. Thus the isotropic diffuse and circumsolar diffuse are expressed as

$I_{iso} = I_d(1-A_i)$ and $I_{cs} = I_d A_i$

$$I_T = I_b R_b + I_d(1 - A_i)\frac{(1 + \cos\beta)}{2} + I_d A_i R_b + I_d(1 - A_i)\frac{(1 + \cos\beta)}{2}$$
$$(1 + f \sin^3 \frac{\beta}{2}) + I_\rho \left(\frac{(1 - \cos\beta}{2}\right)$$

Incidence angle θ is the angle between the beam radiation and the normal to the surface with any orientation. It can be expressed using the following equation,

$$\cos q = \sin\delta \sin\phi \cos\beta - \sin\delta \cos\phi \sin\beta \cos\gamma + \cos\delta \cos\phi\cos\beta\cos\omega +$$
$$\cos\delta \sin\phi \sin\beta \cos\gamma \cos\omega + \cos\delta \sin\beta \sin\gamma \sin\omega$$

In terms of ordinal date, the declination is expressed by the following formula:

$\delta = 23.45\ sin\ ((360(284+n)/365)$

The hour angle (ω_s) corresponding to sunrise or sunset on a horizontal surface if zenith angle is substituted by 90° is

$\omega_s = cos^{-1}(-tan\phi tan\ \delta)$

This equation yields a positive and negative value for ω_s. The positive value corresponds to sunrise and negative value to sunset. Since 15^0 of the hour angle is equivalent to 1 hour, the corresponding day length (in hours) is

$S_{max} = (2/15)\ cos^{-1}(-tan\phi tan\delta)$

A solar simulation radiation model was developed to assess global radiation (Wm-2) on any inclined plane from hourly observations of direct irradiation data and hourly diffuse radiation data on horizontal plane from the above set of equations. The simulations can be conducted for any day and time throughout the whole year. The FORTRAN computer program can compute inclined plane radiation values at 1-minute time intervals at different tilt angles in any location characterized by its latitude and longitude. The program can simulate the diurnal radiation on annual basis over any reference plane.

The model simulations validated throughout the year with hourly solar radiation data collected from the experimental set up as discussed in the following section. The optimum tilt angle was calculated by searching for the values of tilt angle for which the global radiation is a maximum for a particular day of a south facing solar photovoltaic panel. The assessment of the PV output power was the interest of many authors. Osterwald [33] suggested a method to calculate the power provided by a photovoltaic generator. This method assumes that:

$$P_m = P_{m,stc}\left(\frac{G_T}{G_{T,stc}}\right)[1 - \propto_p\ (T_C - T_{C,stc})$$

Where Pm and Pm,stc are respectively the cell maximum power and the cell maximum power in the standard test conditions. Maatallah

[34] in their study used a model to predict the PV array output power which assumes that:

$$P_{PV} = Y_{PV} f_{PV} \left(\frac{G_T}{G_{T,stc}}\right) [1 - \alpha_p \; (T_c - T_{c,stc})]$$

Where Y_{pv}, f_{pv}, are respectively the rated capacity of the PV array, meaning its power output under standard test conditions, the PV derating factor (it is used to account for such factors as shading, snow cover, aging, and so forth), the incident radiation at standard test conditions, the temperature coefficient of power, the PV cell temperature in the current time step and finally the PV cell temperature under standard test conditions. In this study SAM(System Advisor Model) is used to calculate the power the PV array output using the equations of power computation from PV generators.

Experimental Setup for Data Collection for Solar Radiation Model Validation

The first field experiment was carried out at Jadavpur University in Kolkata, India (Latitude 22.57°N and Longitude 88.37°E). The set up is shown in Figs. 1&2. The experimental arrangement comprises of two pyranometers mounted on a platform. The platform consists of one fixed horizontal plane and one tilted plane, whose tilt angle can be set at any angle from 0° to 90° with horizontal plane. The platform has a facility to rotate at any azimuthal angle from 0° to 360°. In this case, to compute the optimum tilt the platform was fixed at perfectly south (azimuth angle 0°). One pyranometer was fixed on the horizontal plane of the platform and the other on the tilted plane. For model validation the experimental set up is fixed to measure south facing (γ=0°) vertical plane (β=90°). From the measured horizontal radiation the south facing vertical radiation is simulated and compared with the measured value.

The data collection was carried out on a continuous basis from June 2010 to May 2011. Pyranometers readings were scanned in 1-minute interval and averaged to 10-minute intervals which are recorded by data logger and stored. To facilitate the modeling the data logging was activated for a 24-hour cycle. The data acquisition system for acquiring the thermocouple temperature consists of programmable data logger (model: HP 34970A) with online transfer of data via RS 232 interface to portable PC (Figure 2).The Data Logger acquisition unit has the provision of using 6.5 digit multi meter accuracy, stability and noise rejection capability. Window based 'HP Bench Link' data logger software has been provided with the acquisition system to upload the data from data logger unit to PC.

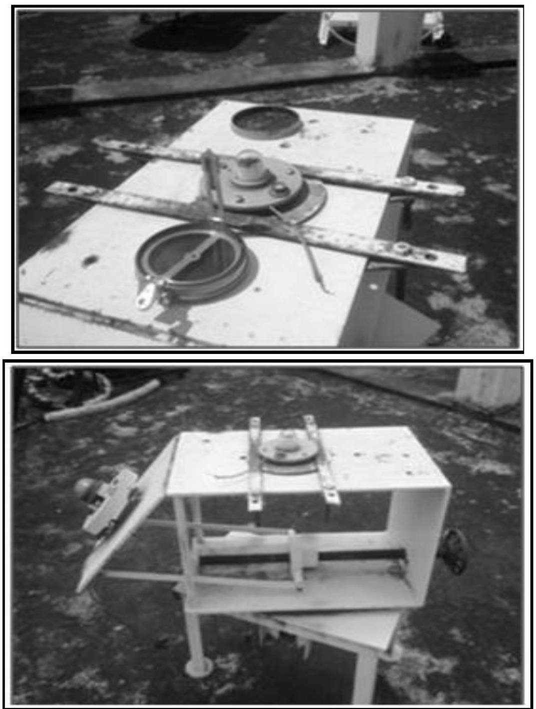


Figure 1. Solar Radiation Measurement set up with pyronometer on revolving platform to measure total radiation on horizontal and tilted

Figure 2. Data Logger unit connected with PC

Solar Advisory Model (SAM)

A comprehensive solar technology systems analysis model, the Solar Advisory Model (SAM), has been developed to support the federal R&D community and the solar industry by staff at the National Renewable Energy Laboratory (NREL) and Sandia National Laboratory. Details of the model can be obtained from technical report [35]. SAM allows users to do complex system modeling with an intuitive graphical user interface (GUI). This model has the capability to compare different solar technologies within the same interface using parameters such as finances, incentives and technological performance, The software also allows renewable generation technologies to be simulated at a device level and allows to estimate annual energy output.

In this article SAM is used to model four different types of rooftop solar photovoltaic system and to estimate the annual energy generation from these SPV units. The model used fixed tilt (at the latitude angle) solar system without any sun tracking mechanism, PLC based single axis sun tracking system, manual solar tracking system using the monthly optimum tilt angles obtained in this study as given in equation set (3) for Kolkata region and manual tracking system applying monthly optimum angles computed from old set of equations given in [21]. A comparative study is also conducted among these three systems. The main purpose of this comparison is to determine that how much more energy can be generated yearly by the PLC tracking system or by the manual tracking system over fixed tilt solar system. The intercomparison is also done with PLC tracking system to gauge relative efficacy of manual tracking system. The Solar advisor model(SAM) gives an idea about which tracking mechanism is more practically viable with respect to overall system cost and annual energy generation.

Statistical Analyses

To estimate monthly optimum tilt angles on the basis of local latitudes, best fit prediction equations have been developed by R software after minimizing the error sum of squares (sum of squared estimation errors). To judge the predictive accuracy of the developed simulation model and the precision of the monthly prediction equations for optimum tilt angles, statistical indicators such as mean absolute error (MAE), mean bias error (MBE) and root mean square error (RMSE), are used, For evaluating the relative accuracy of the model simulations with respect to observations, MBE(%) and RMSE(%) are expressed as a percentage of mean of observed values. These are obtained for each day of the year. Prediction errors of hourly simulated values are averaged over twenty four hourly values. From MBE(%) we can also evaluate the amount of overestimation or underestimation of modeled values with respect to observed values. For assessing the accuracy of estimation of optimum tilt angles from latitudes of the location using our proposed empirical set of equations (3), MAEs and RMSEs are computed considering estimation errors over months and over different cities within specified latitudes(13°-39°) of northern hemisphere. The same has been done with the old equations [21] as well for comparing the errors of the two different sets of prediction equations—the new proposed expressions and the expressions obtained from the model of Nijegorodov et al. [21]. The MBE (%), RMSE (%), MAE and RMSE are expressed as follows:

$$MBE(\%) = \frac{\sum_{i=1}^{24} e_i}{\sum_{i=1}^{24} y_i} \times 100, \qquad RMSE(\%) = \frac{\sqrt{\frac{1}{24}\sum_{i=1}^{24} e_i^2}}{\frac{1}{24}\sum_{i=1}^{24} y_i} \times 100,$$

$$MAE = \frac{1}{n}\sum_{i=1}^{n} | e_i | \quad \text{and} \quad RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n} e_i^2}$$

Where y_i's are the observed values, e_i's are the prediction errors (difference between observed and predicted optimum tilt angles), n is the total number of estimated angles. The MAE indicates the average unsigned discrepancy between estimates and observations. The RMSE predicts average extent of errors, but since the errors are squared before they are averaged, the RMSE gives higher weights to larger errors. This means the RMSE is most useful when large errors are particularly undesirable. Both of these statistical measures can work in combination to measure the extent of errors in predicting optimum tilt for each month.

Manual Solar Tracking System Setup

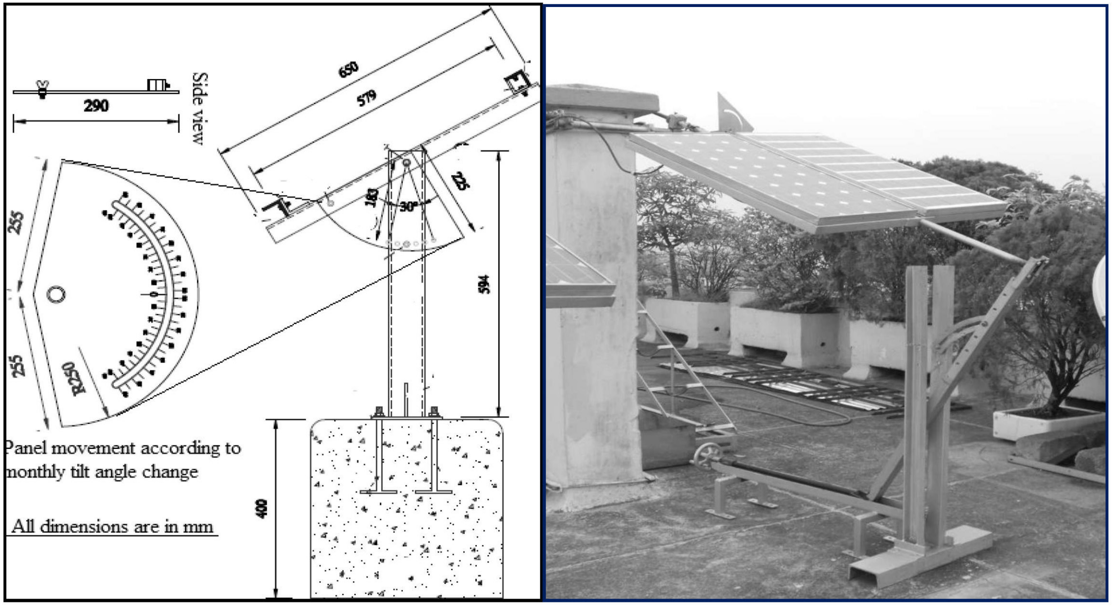
The solar PV system with monthly optimum tilt angle control system is a very simple, much lower cost effective and completely innovative method to increase the unit area illumination on PV panel which results to increase in energy generation through the same panel. The manual tracking enables the solar tracker to be oriented manually at the monthly optimum tilt angles computed in this study from set of equations (3). The SPV yield from this set up has been evaluated with SAM with observed data closely matching modeled output.

Table 1. Solar Panel Specifications used in this study

COMPANY NAME	TATA BP Solar	
PANEL	Serial No	TBP1280
	Type	Monocrystalline
	Peak power(P_{max},W)	80 W
	Max power voltage(V_{mp})	18.44 V
	Max power current(I_{mp})	4.7 A
	Open circuit voltage(V_{oc})	22.51 A
	Short circuit current(I_{sc})	5.13 A
	Module efficiency (%)	13.45
	Module area(m^2)	1
	Working temperature(°C)	25

This type of solar tracking system has been installed on the rooftop of the Indian Institute of Social Welfare and Business Management (IISWBM), Kolkata, India to reaffirm experimentally the justification of the proposed equation set. The schematic layout of the solar tracking system is shown in Figure 3. The position of SPV module can be changed from 0° to 90° manually. The overall positions of the eight solar panels including the solar panels having manual tracking facility and the connection diagram of overall rooftop solar system of IISWBM are shown in Figures 3 and 4.

Here the PV panels of modules (C,D) are fixed at latitude of Kolkata(23.5°) and in case of PV panels of modules(A,B) there is an arrangement of manual tracking. The solar PV panels are positioned at optimum tilt angle as per eqn. (3) and the values are stated at table 3. One pyronometer was fixed on a solar panel of module(C) and other is fixed on another solar panel of module(B) to see the variation in radiation on fixed panel(fixed at latitude) and the panel fixed at optimum tilt angle on monthly basis. The same type of data acquisition system as in Jadavpur University (explained in section 2.1.3) was also installed



(a) (b)

Figure 3. (a) Schematic layout of solar PV module with manual tracking system. (b) Snapshot of Solar module with manual tracking system.

at IISWBM rooftop. To facilitate the modelling the data logging was activated for a 24-hour cycle. Window based 'HP Bench Link' data logger software has been provided with the acquisition system to upload the data from data logger unit to PC. PLC tracking of module has not been incorporated since it is not economically viable for the small 640 watt SPV panel. The data collection was carried out on a continuous basis from April 2011 to May 2012.

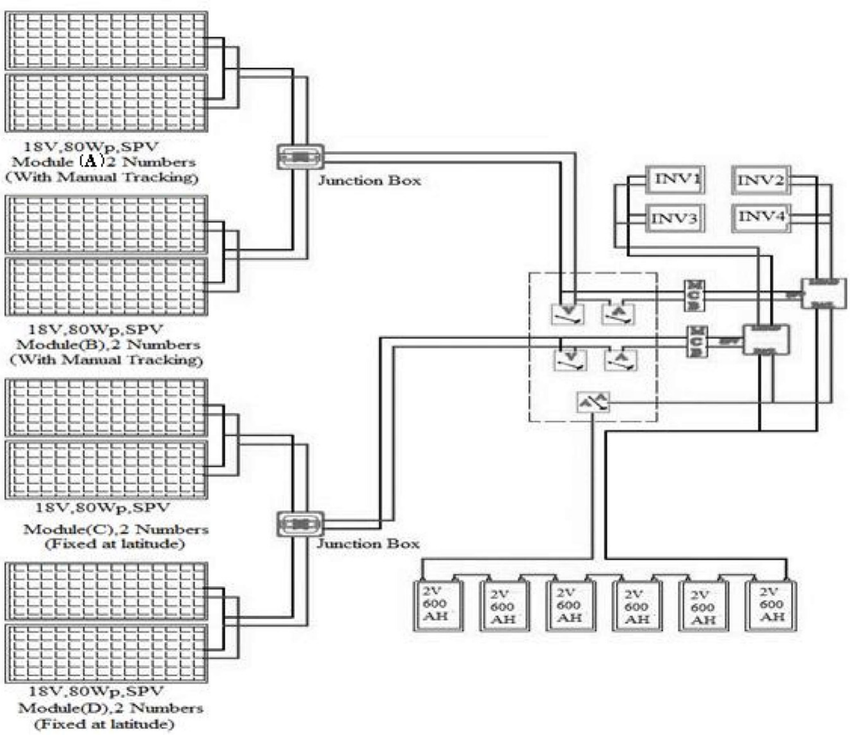


Figure 4. Positions of SPV and the connection diagram of IISWBM rooftop solar system

RESULTS AND DISCUSSION

Solar Model Assessment

To validate the solar simulation model for any tilted surface the experimental set up is fixed to measure south facing (γ=0°) and the tilted plane fixed at 90° with horizontal plane. From the measured horizontal radiation, the total radiation in above said tilted plane is simulated. The measured total radiation at the tilted plane (Figure 2) is then compared with the simulated total radiation of the same tilted plane throughout the year. The MBE(%) and RMSE(%) between the simulated and measured total radiations are estimated and found to be lying between -4.9% to 5.1% and between -2.1%to 6.1% respectively throughout the year. The MBE and RMSE estimated between simulated and measured data along with the other parameters required for simulation for some different days (as sample) has been presented in table2. The table2 and Figs.5 (5a and 5b) (illustrate that the model behaves well seasonally and on a daily basis with respect to observations. The model slightly underestimates solar radiation in summer and overestimates solar radiation in winter for the south facing vertical inclined solar panel. It has been verified that the model also correctly estimates sunrise, sunset and day length hours for both summer and winter. The model also captures the tilted diffused radiation as well. It can be observed from the day length that the PV panel is generating electricity slightly about 10 hour at the winter and to slightly for more than 12 h at the summer. The statistical analysis shows that the model simulates well seasonally and diurnally with respect to observations.

Table 2. MBE and RMSE expressed as percent

DATE	δ (deg)	β(deg)	Sun rise(hr)	Solar Noon(hr)	Sun Set(hr)	Day Length(hr)	MBE (%)	RMSE (%)
15-01-11	-21.4	90.0	6.32	11.76	17.20	10.80	5.1	4.2
10-02-11	-14.95	90.0	6.02	11.20	17.17	11.15	3.8	4.8
15-05-11	18.8	90.0	4.92	11.54	18.16	13.2	-.2.8	.61
25-07-10	19.65	90.0	5.10	11.28	18.25	13.14	-0.38	0.44
28-09-10	-2.91	90.0	5.513	11.44	17.36	11.845	3.8	4.0
25-12-10	-23.397	90.0	6.03	11.35	17.06	10.63	2.6	4.2

After validation of the solar simulation model, the model is used to find out optimum tilt angle for a south facing solar collector for every month of the year.

Computation of Optimal Tilt Angle by Solar Radiation Model

After model validation over the Kolkata region, the solar simulation model was then applied to investigate the monthly favorable

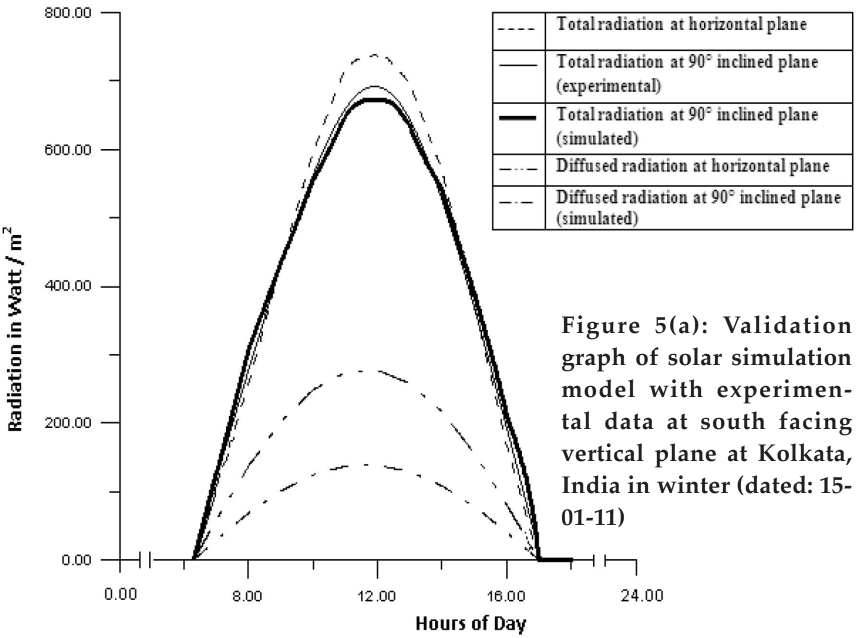


Figure 5(a): Validation graph of solar simulation model with experimental data at south facing vertical plane at Kolkata, India in winter (dated: 15-01-11)

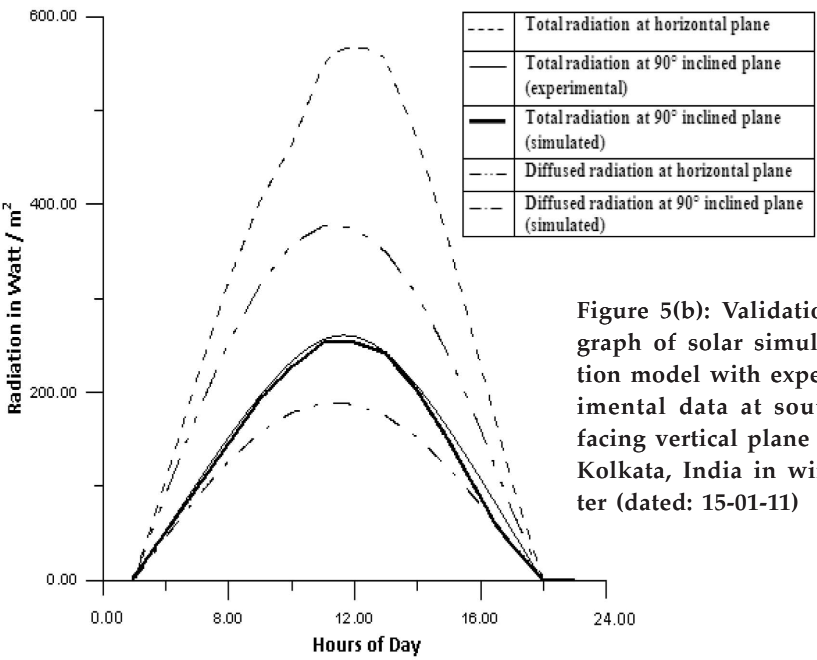


Figure 5(b): Validation graph of solar simulation model with experimental data at south facing vertical plane at Kolkata, India in winter (dated: 15-01-11)

angles of solar panel at southern orientation (γ=0°) to maximize solar collection efficiency over different metropolitan cities of India and USA. For every city, the model was run for 3 days for a particular month (the 1st, the 15th and the 30th, except for the month of February). The average optimum beta angle for maximum solar radiation on an inclined plane was computed by the model for southern orientation (γ=0°). The results for Kolkata are presented in Table 3, which demonstrates that optimal tilt values increases towards the beginning and end of the year, and during the summer months the values are closest to zero. At these select tilt of the solar panel there is variation in monthly increase of inclined radiation values with an average increase of 5% to 12% from horizontal radiation for all regions considered in this study.

Table 3 shows the optimum tilt angles vary from the conventional tilt of fixed panel where the solar panel is tilted at an angle equal to the latitude of the site. The average gain in energy from that position is (2to 6)% when considered for all regions in this study for the entire year. The percentage gain of total radiation from the fixed plane (fixed at latitude) of solar panel is higher in the summer months (approximately 7%) and lowers during winter (approximately 3%). This finding is in good agreement with the finding of other researchers [36,34]. Maatallah [34] explained the cause as the zenith angle of the sun varies from 90° at the sunrise and the sunset to slightly more than 0° at the midday in

Table 3. Simulated Monthly optimum tilt angles for Kolkata, India

Month	Optimum tilt angles (in degree)	Percentage increase of radiation (W/m^2) at Optimum Tilt Angle than fixed panel at solar noon of 15th of each month(measured value)
	φ= 22.57°N Kolkata	
January	39.33	2.5 %
February	31.66	2.07%
March	21.00	0.6%
April	10.66	2.16%
May	3.66	3.92%
June	0.00	7%
July	1.66	6.3%
August	6.00	4.9%
September	14.33	1.8%
October	26.33	0.85%
November	36.33	2.26 %
December	42.00	3.86%

the summer season while it varies only from 90° to slightly less than 60° in the winter season. In India where there is high population density, there are substantial numbers of buildings where solar collectors can be installed. It may be seen from Table 3 that the monthly optimum tilt values vary approximately between latitude ±15 degrees as seen in previous studies over other cities [37] and [38] within the specified latitudes.

The model performance demonstrates that the optimal tilt is credibly simulated for different months of the year. It is seen that the model correctly estimates that the magnitude of solar radiation at optimal inclination. The optimum tilt angle and fixed tilt radiation curves also have been plotted to inter-compare the irradiation of summer and winter (Figure 6(a) and 6(b)) for Kolkata. It is observed that the solar radiation in June at optimum tilt angle is much higher than fixed angle whereas in February the difference is much less. The primary reason for that is the magnitude of difference in the tilt angles between optimum tilt and fixed (latitude) tilt. In June the maximum radiation is available at an optimum tilt of 0°. Hence the difference of angle over fixed latitude is about 23° whereas for February the difference in angles is only 9°. The increase of radiation at optimum tilt angle over fixed (latitude) angle strictly depends on the difference of angles.

Mathematical Expression of Optimal Tilt

After rigorous model evaluation discussed in previous sections, we have developed a set of empirical expressions to describe the relation between optimum tilt angle and latitude (ϕ) of the site using the model simulated outputs for the latitudes from around 13°N to 39°N. Such equations are presented below and estimate monthly averaged optimum tilt values.

$$\left.\begin{array}{llll} \text{January} & 1.28\varphi+11.04 & \text{February} & 1.36\varphi+0.22 \quad \text{March} \; 1.22\,\varphi-7.15 \\ \text{April} & 0.71\,\varphi-5.099 & \text{May} & 0.28\varphi-2.8 \quad \text{June} \; 0.03\varphi+0.2 \\ \text{July} & 0.03\varphi-.098 & \text{August} & 0.63\varphi-6.21 \quad \text{September} \; 1.04\,\varphi-6.57 \\ \text{October} & 1.28\,\varphi-0.66 & \text{November} & 1.28\varphi+8.61 \quad \text{December} \; 1.37\varphi+9.67 \end{array}\right\} \quad ...(3)$$

The expressions were independently validated with model simulated optimum tilt values from Ahmadabad (an Indian city), Fort Lauderdale and Miami (US cities). Figure 7 represents monthly variations of optimum tilt angles over two representative cities (Ahmadabad

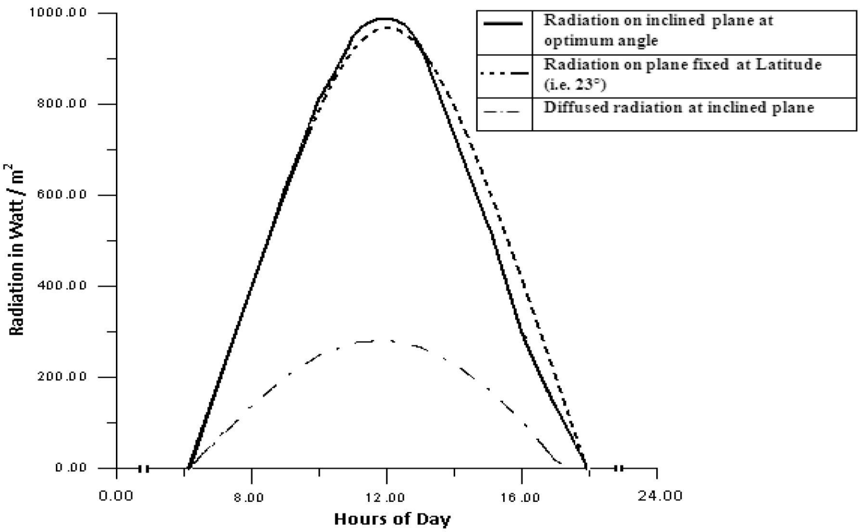


Figure 6(a). Comparison for south facing PV panel for optimum tilt angle and fixed latitude tilt at Kolkata, India in winter (dated:15-02-12)

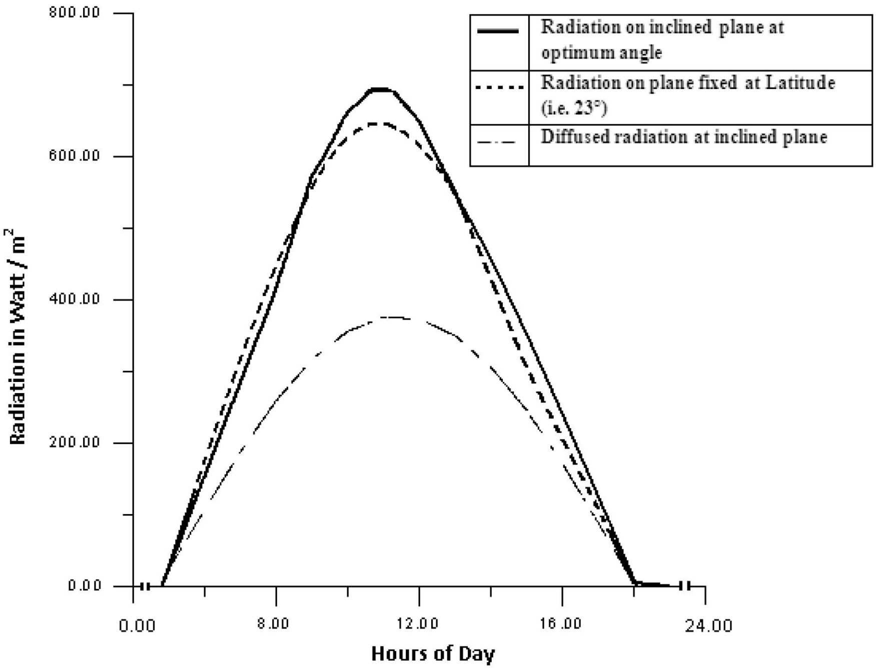


Figure 6(b). Comparison for south facing PV panel for optimum tilt angle and fixed latitude tilt at Kolkata, India in summer (dated 15-06-12)

east of Greenwich and Miami west of Greenwich) where the derived mathematical expressions were validated with model simulated values. It is seen that the optimum tilt angles demonstrates that both the modeled and estimated output capture well the distinctive patterns of the optimum tilt as represented in earlier studies[22,38]. Both the sets of values (modeled and estimated) are in close agreement thus proving the robustness of the set of mathematical expressions. The comparable values in magnitudes of the tilts demonstrate that the values obtained are consistent within these latitudes in the northern hemisphere despite the variability's associated with latitudes, ground albedo, meteorological variables and solar-radiation patterns. The monthly R^2 coefficient of determination linear regression between latitude and optimal tilt are quite significant ranging from 0.83 to 0.98 across all latitudes and seasons. Therefore, optimal tilt angles computed from these set of mathematical expressions can be used to set up manual sun tracking systems where the PV module can be manually set to the estimated optimal tilt angle at the beginning of every month to maximize solar output.

Comparison of Two Sets of Mathematical Expressions

The error estimates of optimum tilt angles from the new empirical set of equations have been inter-compared with the error estimates of

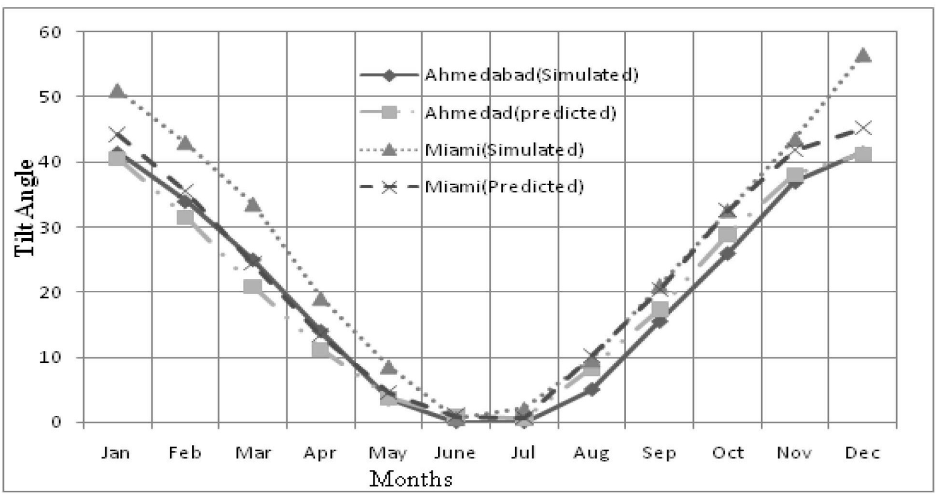


Figure 7: Comparison of simulated and predicted optimum tilt angle for Ahmadabad, India and Miami, USA

the same from the set of equations suggested in [21], and the results are presented in Table 4. The error estimates both spatial and temporal between the two sets of optimum tilt values are quite significant. The overall error combines all MAEs and RMSEs to compare the error scenario of the two sets of equation, Nijegorodov equation (represented as old equation later) and new equation, which shows a significant improvement in estimating the angles by the present regression model over the previous one. This demonstrates that the new mathematical expressions derived in this study seem to be universally applicable within 13°to 39° in the Northern Hemisphere. It is clearly understood that the new empirical set equations are more acceptable for predicting the optimum beta of solar PV on monthly basis between the latitude 13°≤ϕ≤39° in northern hemisphere.

	MAE				RMSE			
	Temporal		Spatial		Temporal		Spatial	
	New	Old	New	Old	New	Old	New	Old
Overall error	2.76	6.54	2.76	6.54	3.46	7.71	3.46	7.71
Min	0.95	2.69	1.12	4.44	0.99	3.38	1.32	2.81
Max	5.06	12.57	4.10	9.93	5.98	13.93	5.54	11.6

Relative Efficacy Assessment of Statistically Estimated Optimum Tilt Angles

The empirical set of equations (3) are further corroborated by calculating the electric energy generated by SPVs at these optimal tilt angles along with inter-comparing with fixed solar panels tilted at latitude of location, the SPV with month wise different angle as per the equations in [21] and PLC single axis tracking by SAM. SAM was revalidated with electrical energy obtained from monthly measured energy data by the manual tracking system (Figures. 3 & 4) based on optimum tilt angle as per the proposed equation set. The MAE and RSME errors of the simulation result and measured energy were within acceptable limit. The MAE and RSME errors of the simulation result and measured energy were also within acceptable limits. The results for inter comparison mentioned earlier have been presented in Figs. 8(a), 8(b) and Table 5.

As can be seen from Figure 7 the solar PV yield from all configurations of solar panel is closest in summer months when optimum tilt is close to zero. The differences are substantial in winter months from fixed position of the SPV. Figure 8(a) depicts that all configurations display consistency with seasonal variability in electric power generation. Figure 8(b) reveals the relative total gain in electric energy from the fixed position. The relative gains in annual electric energy are 8.71% from PLC tracking simulated by SAM, 5.02% from manual tracking base on our proposed set of equations and 2.54% from manual tracking based on equations represented in [21] than the fixed SPV panel. The results reiterate that for manual tracking optimal tilts from our proposed set of equations are superior to fixed solar panel position and optimum tilts obtained from [21]. From table 5 it is clear that the manual tracking can increase the electric energy generated from solar PV with a negligible cost. In summer months the electricity energy can be increased 6% on average whereas in winter months also it shows a good performance of 4% increase on average than the fixed (latitude) solar PV. In case of rooftop solar for a building or where implementation of PLC is not economically viable, the simple construction of manual tracking system (Figs. 3(a)&3(b)) tilted at monthly optimum tilt angles from proposed empirical set of equations (3) can make a radical change in electric energy production for a payback period less than 2 months.

Table 5: Comparison of Output power of PV and gains for different time periods (kWh)

Month	Fixed panel(at Latitude), kWh	Tracked Panel (As per proposed Equation), kWh	Energy Gain, kWh	Energy Gain%
January	113.7	124.5	10.8	9.5
February	113.9	125.5	11.6	10.2
March	126.3	138.3	12	9.5
April	121.4	132.5	11.1	9.1
May	122.6	134.4	11.8	9.6
June	105.1	117.1	12	11.4
July	101.9	114.3	12.4	12.2
August	106.7	118.7	12	11.2
September	93.1	101.4	8.3	8.9
October	99.5	107.8	8.5	8.3
November	90.2	99.6	9.4	10.4
December	107.8	117.8	10	9.2
Total Generation	**1302.2**	**1431.9**	**129.7**	**10**

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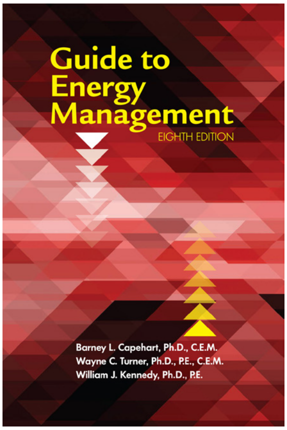

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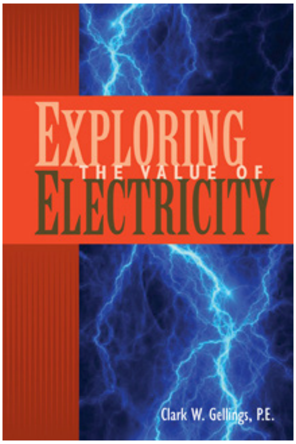

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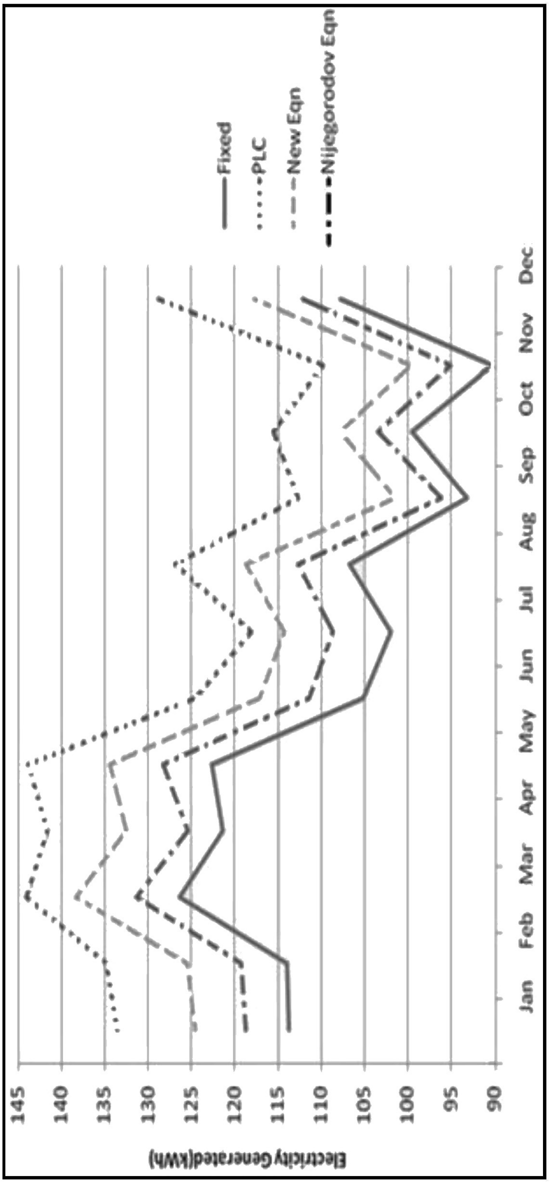

Figure 8a: Month wise generation of electrical energy (kWh)

SAM was further used to gauge the spatial variability in solar PV yields in other regions within the specified latitudes. The average increase in yields in annual electric power over fixed panels is (14-20)% in case of PLC tracking, (8-12)% for manual tracking from optimal tilts from our proposed set of equations and (3-6)% from optimum tilts obtained from [21]. This clearly demonstrates the robustness and validity of the optimum tilts calculated from our proposed set of equations for the study region.

The use of a tracking system is more expensive and more complex than fixed mounts. However they can become cost-effective in many cases because they provide more power output throughout the year [39]. Although the solar PV with PLC tracking system can generate more energy throughout the year, but the main drawbacks are operational complexities and huge installation costs. Dutta et. al. [26] elaborately describe different types of PLC tracking systems and conclude that the different types of PLC control systems are applicable for large scale application such as solar parks where a large number of modules is employed simultaneously.

Thus we can conclude that though the manual tracking with optimum tilt angle control mechanism generates less energy in comparison with PLC tracking system but its installation cost is much lower, a little more than the fixed tilt SPV system and hence the proposed manual tracking system is more economically viable and practically feasible. Manual tracking greatly reduces the costs which would be incurred in case of an automated sun tracking system while at the same time enhancing the benefits from fixed tracking.

CONCLUSION

An anisotropic model was utilized in this study to predict optimal solar collector angles to improve solar electrical energy within latitudes (13°-39°) of northern after rigorous validation with observed data. The range of latitudes in this study encompasses different climate regimes. The most favorable orientation (azimuthal angle) of solar collectors is due south. The model performance captured the diurnal variability of observations and also displayed higher solar intensities at optimal tilts of solar module. It is found that the average solar gain increase of (2-6%) over fixed panel tilted at the latitude. This increase in solar intensity becomes substantial in regions where there is high intensity of solar radiation and in regions where there is high density of buildings. The optimum tilt angles exhibit a strong seasonal trend. A new empirical set of mathematical expressions for optimal tilts across different latitudes has been derived. The optimum tilt values obtained from new equation set has been inter compared statistically with the empirical set of equations proposed by Nijegorodov et al. [21] and is seen to predict more accurate spatial and temporal values for the regions within these latitudes. The coefficients of determination, R^2 for optimal tilts from our proposed equations are also quite large. The robustness of these computed optimal tilts was further corroborated by inter-comparing the electric power yield from SPVs manually oriented at these new proposed tilt angles versus fixed orientation, versus manual orientation of SPV according to estimated angles from [21] and from PLC tracking. Solar Advisory Model (SAM) was used to compare each of these options over the specified range of latitudes in northern hemisphere. The average increase in yields in annual electric power over fixed panels is (14-20)% in case of PLC tracking, (8-12)% for manual tracking from optimal tilts from our proposed set of equations and (3-6)% from optimum tilts obtained from The results obtained from model simulations clearly demonstrate the superiority of the estimated optimum tilts from our proposed set of equations in case of manual tracking for the specified latitudes. Therefore this set of expressions can be used in general to optimize the monthly tilt of a manual sun tracking solar system, which can be cost effective and yield higher electric power than fixed SPV panel tilted at the latitude. Previous studies have also revealed that despite some uncertainties, the model to model variability in prediction of optimal tilt angles is not significant [18, 38] which reaffirms robustness of the derived equations and the equation set can be used as a universal initial estimate to optimize solar energy of solar collectors over regions within the latitudes specified.

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