Part 2 of 2

The CHP Space: A Basic Model

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> "Essentially, all models are wrong, but some are useful."
> —*George E.P. Box*

ABSTRACT

The second part of this article shows the development of an economic design model for grid-interconnected cogeneration systems. The design model advocates a graph- and math-based methodology. Such a methodology is the foundation upon which future, more complex engineering-economic models of cogeneration systems will be built. The underlying work focuses on deriving a set of linear equations to represent the technical and economic performance of a basic combined heat and power (CHP) system. The vector space spanned by such equations is called the base CHP space. The CHP space is instrumental to building comprehensive design-optimization models with which cogeneration systems take into consideration (a) equipment performance characteristics, (b) regulatory requirements, (c) CHP demand or loads, (d) financial constraints, (e) operational and reliability goals, and (f) environmental requirements.

Keywords: CHP Space, Cogeneration, Combined-Heat and Power, Design, Economic Design, Graphical Model

ECONOMIC DESIGN MODEL

In this first model, the economic aim is to select a constant output CHP with plant of nominal size Pc^* (kWe), which will realize the greatest life-cycle value. So, next we formulate a life cycle annual worth AW objective function (Equation 4) for the CHP system with constant output and subject to constant loads.

$$AW = AC + FC + EC + MC + RC + ES \qquad (4)$$

Where:

AW = annual worth or before tax net profits (\$/yr)

AC = annual amortization cost of ownership or lease cost (\$/yr).

FC = annual fuel cost for the plant (\$/yr) calculated as

= $c_f\,[(Pc/\eta_e.)\ t_1 + (Hd\text{-}Hc)/\eta_t\ t_2];\ Hd > Hc$ (5)

c_f =unit cost of fuel (\$/MMBtu); $c_f < 0$

Pc= nominal size of CHP system or power output capacity (kWe)

Pd = Electrical power demand (kWe)

η_e= fuel-to-electricity efficiency

Hd = Heat demand (kWt or MMBtu/hr)

Hc = Heat output capacity (MMBtu/hr)

η_t= fuel-to-auxiliary firing thermal efficiency

r_c= Hc/Hd = system heat-to-power ratio or system curve slope

t_1= CHP system operating hours per year

t_2= auxiliary heat operating hours per year

t_3= electricity deficit time, hours per year

EC = the cost of electricity consumed from the grid for Pd>Pc (\$/yr)

= $(Pd\text{-}Pc)\ c_e\ t3$ (6)

c_e,= unit electricity cost (<0) to the grid (\$/kWh)

s_e= sale price (>0) to the grid (\$/kWh)

MC = annual operation and maintenance cost (w/o fuel), (\$/yr) computed as

= $c_m\ (Pc.\ t_1)$ (7)

c_m= unit cost of operation and maintenance, $c_m < 0$ (\$/kWh)

RC = heat rejection cost or cooling tower cost (\$/kWht or \$/MMBtu)

= $Rc\ c_r\ (Hc\text{-}Hd)\ (t_1 - t_2);\ Hc > Hd$ (8)

c_r= unit cost of heat rejection, c_r <o (\$/kWh)

ES= Electricity sales to the grid (\$/yr)

= $(Pc - Pd)\ (t_1\text{-}t_3)\ (se);\ Pc > Pd$ (9)

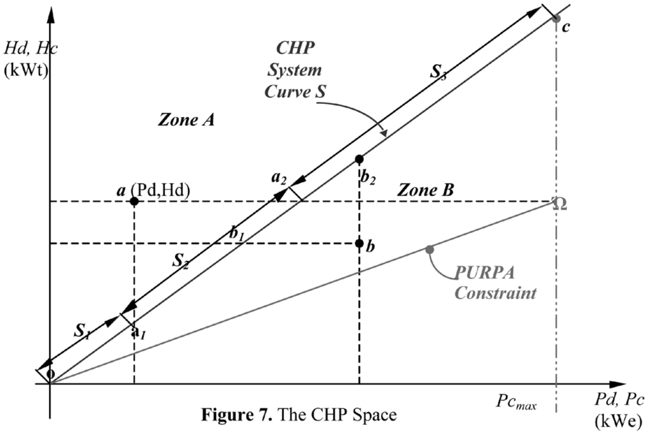

Figure 7. The CHP Space

The States of Nature of an Operating CHP System

Any fixed point on Figure 7 (e.g. points **a** or **b**) represents constant heat and power loads. A fixed demand point is based on the assumption that the host-facility CHP loads are constant over time (i.e. deterministic or non-stochastic). However, most processes and plants have variable CHP loads over time, as shown in Figure 5. So, a real operating CHP system has different operating scenarios: Sometimes the system needs to import heat or power and sometimes it generates excess heat or power, see Table 1.

Table 1. States of Nature of an Operating CHP System

STATE	Operating Decisions	
	Heat ΔHc is	**Electrical power ΔPc is**
W1	Rejected, Hc>Hd	sold to utility, Pc > Pd
W2	Rejected, Hc > Hd	purchased from utility, Pc < Pd
W3	auxiliary Fired, Hc < Hd	sold to utility, Pc > Pd
W4	auxiliary Fired, Hc< Hd	purchased from utility, Pc> Pd

Table 1 lists the four possible states of nature for a variable load/output system. The actual operating point roams randomly the CHP space of Figure 7. The economic and performance model for such operating system is in turn represented by equations 4 through 9. In that case, the CHP loads and output capacities are said to be stochastic or probabilistic. Such a situation requires stochastic modeling which will be addressed in future articles in this journal. Also, planned and unplanned grid and CHP system outages will be incorporated in a future analysis.

Discussion of the CHP Space Shown in Figure 7

We are concerned with determining the optimal plant size Pc* (kWe) for a feasible CHP technology ***S***. Thus, two plant sizing strategies, defined in Figure 7 by *Zone A* and *Zone B (to the left and right of **S**),* are possible in the linear CHP space.

In Zone A, point a represents the CHP demands Pd and Hd. Next, two points a_1 and a_2 represent two "extreme" systems with fixed output capacities defined by:

$\mathbf{a_1}$: $Pc = Pd$ and $Hc = Pd/r_c$ with $Hc < Hd$ (10)

$\mathbf{a_2}$: $Pc = r_c Hd$ and $Hc = Hd$ with $Pc > Pd$ (11)

Similarly in Zone B, the demand point **b** can be projected to $\mathbf{b_1}$ and $\mathbf{b_2}$ resulting in

$\mathbf{b_1}$: $Pc = r_c Hd$ and $Hc = Hd$ with $Pc < Pd$ (12)

$\mathbf{b_2}$: $Pc = Pd$ and $Hc = Pd_{/}r_c$ with $Pc > Pd$ (13)

In addition, we shall define at the extreme right of the system curve S, the uppermost point *c*, which is defined by the PURPA constraint (Figure 7), which defines the largest possible CHP qualifying plant. It states that for a topping cycle CHP plant to be a qualifying facility, it must have an overall (electrical and thermal) efficiency larger than, or equal to, 42.5 %, based on lower heating value. To be conservative in meeting PURPA, we use the following equation based on a higher heating value. Then, it can be shown that the CHP plant resulting of projecting point c (or 'Ω) on the Pc axis meets the constraint

c: $Hd \geq Pc(c)\ (0.425/\eta_e - 1)$ (14)

In addition to the origin point o, note that points **o**, $\mathbf{a_1}$, $\mathbf{a_2}$, $\mathbf{b_1}$, $\mathbf{b_2}$ and **c** lay on the system curve ***S***. Then, with the aid of the CHP states of nature listed on Table 1 and by inspecting equations 4 through 14 above, we notice these six (6) points constitute potential "break points" for the objective function (Equation 4). In other words, the value of any of the right-hand-side terms of Equation 4 may change at any of these break points. Such points, where a change of basis of Equation 4 is possible, are called "extreme points" in linear programming and math optimization. Note these points constitute optimal sizing CHP alternatives.

Fixed CHP Demand Optimality Condition.

The projection of the of the set S^* ={**o**, $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{b}_1$, $\mathbf{b}_2$, **c**} in the Pc axis Pc* = {Pc(**o**) Pc($\mathbf{a}_1$), Pc ($\mathbf{a}_2$), Pc($\mathbf{b}_1$), Pc($\mathbf{b}_2$), Pc(**c**)} means there are potential optimum system sizes. Pc * is the set which contains all the possible optimal sizes. So, the objective function (Equation 4) should be evaluated only at these six points. Consequently, we conclude that a linear optimization model exists for a constant output CHP system subject to constant loads, as represented by one fixed CHP demand point.

CONCLUDING REMARKS

This article has defined and depicted the CHP demand and supply model in the Cartesian space. Next, some relevant design and evaluation issues were discussed to set up the stage for model formulation. Then, we formulated an economic optimization criterion (Equations 1 through 7). Finally through Figures 7, Table 1 and equations 4 through 14 we have shown that the CHP space model is a basis for linear optimization of the CHP system size Pc (kWe). Future articles will show applications and extensions of the design optimization model proposed here.

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