# Active Loss Allocation Systems in Radial Distribution Systems

*Sivkumar Mishra*
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## ABSTRACT

In the restructured power system under deregulation, proper pricing of electricity has emerged as a key issue. The cost of transmission and distribution activities needs to be allocated to the users of these networks. Among others, power losses are one of the costs to be allocated. The main difficulty faced in allocating losses is the nonlinearity between the losses and delivered power which complicates the impact of each user on network losses. The purpose of this article is to review the various loss allocation schemes as applicable to the distribution systems. These schemes have been compared considering a simple radial distribution system. Since cogeneration and distributed generation (DG) plants often generate power for on-site consumption, and a fraction of the output is for export to the grid, developers of cogeneration and DG with substantial on-site consumption can realize significant savings in transmission and distribution losses. Thus, the schemes discussed herein can help a cogeneration or DG developer estimate such distributed-loss cost savings.

**Key Words—***Loss allocation, radial distribution system, pro rata loss allocation scheme, proportional loss allocation scheme, quadratic loss allocation scheme, distribution losses savings, electricity economics*.

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Electricity supply industries worldwide are undergoing major structural changes with the objective of introducing competition and choice in electricity supply. These changes are motivated primarily by the belief that competition will bring better service, at a lower price to electricity consumers. The vertically integrated systems have been restructured and unbundled to one or more generation companies, transmission companies and a number of distribution companies. An essential condition for competition to develop is open access, on a nondiscriminatory basis, to transmission and distribution networks. The central issue in the concept of open access is setting an adequate price for transmission and distribution services. This is because price affects the future sitting of generators and loads, and network operating costs as well as strongly influencing further development of the network. Under such a scenario, there is ever-growing pressure for all components of costs to be clearly identified and assigned equitably to all parties taking care to avoid or minimize any temporal or spatial cross-subsidies. The cost of transmission and distribution activities needs to be allocated to the users of these networks. Allocation can be done through network use tariffs, with a focus on the true impact they have on these costs.

Among others, distribution power losses are one of the costs to be allocated. The main difficulty faced in allocating losses is the non-linearity between the losses and delivered power which complicates the impact of each user on network losses[1]**.** It is impossible to calculate the exact amount of losses in advance, without running a power flow. At the same time, even after computing the power flow solution, there is a strong interdependence among all the users, expressed by the presence of cross terms due to the fact that losses are a nearly quadratic function of the power flows. Hence, allocating the losses to the market participants cannot be carried out in a straightforward way. Different techniques have been published in the literature for allocation of losses, most of them dedicated to transmission networks and can be classified into three broad categories—pro rata procedures, marginal procedures and proportional sharing procedures[2-7]. Costa and Matos[8] have addressed the allocation of losses in distribution networks with embedded generation by considering quadratic loss allocation technique. In general, a first distinction can be made between loss allocation methods dedicated to transmission and distribution systems. The difference between these two classes of methods basically lies in the role given to the slack node. In transmission systems, the generator located in the slack node compensates for all the losses and is explicitly considered in the mechanism of loss allocation. In radial distribution systems, the location of the slack node at the root node of the distribution tree is naturally unique, and the slack node usually represents the connection to the higher voltage network.

In this article, three schemes of active power loss allocation are presented with clearly explaining the methodology. These schemes are Pro rata, proportional sharing and quadratic sπharing. A simple scheme to allocate active power loss in radial distribution systems known as Exact Method[9] is also explained. This method establishes almost a linear relationship among active loss allocated to a consumer and its both active and reactive loads connected, making the loss allocation far simpler. All these four methods are critically compared considering a practical test distribution system.

## METHODOLOGY

For the purpose of explanation, a sample radial distribution feeder as shown in Figure 1 is considered.

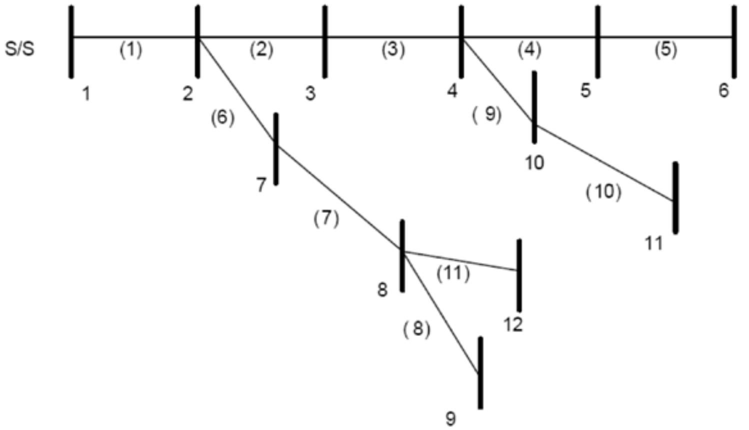

**Figure 1. Sample distribution network with 12 buses**

In Figure 1, branch numbers are shown between parenthesis or (#). The branch number, sending end and receiving end buses are also given in Table-1.

The following notations will be used:

p : Bus number
IL(p) : Load current at the $p^{th}$ bus
ILD(p) : Real part of the load current at the $p^{th}$ bus

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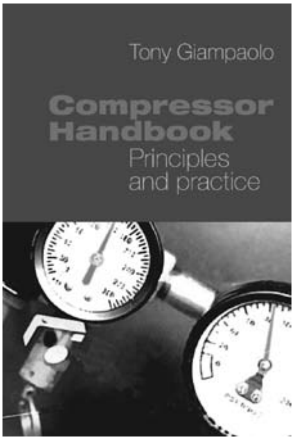

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ILQ(p) : Imaginary part of the load current at the $p^{th}$ bus
PL (p) : Real power load at the $p^{th}$ bus
QL (p) : Reactive power load at the $p^{th}$ bus
Vp : Voltage at the $p^{th}$ bus
NB : Total number of buses
I(jj) : Current of branch- jj
N(jj) : Total number of buses beyond branch-jj
ie(jj,i) : Buses beyond branch-jj, for i=1,2…,N(jj)
IL{ie(jj,i)} : Load current of bus ie(jj,i)
PLOSS (jj) : Real power loss of branch-jj
R (jj) : Resistance of branch-jj
ploss(jj,l) : Real power loss of branch-jj allocated to bus-l
Tploss(jj,l) : Real power loss supported by consumer at bus –l
Tloss : Total active power loss of the system
$kVA_i$ : kVA demand of the consumer ‘i’
TkVA : Total kVA demand

The load current at any bus p, is given as:

$$IL_{(P)} = \frac{PL_p - {}_jQL_p}{V^*_P}, \quad p = 2,3,\ldots,NB \tag{1}$$

Consider branch-5 of Figure 1, number of bus beyond branch-5 is one and this is bus 6. Therefore, current through branch-5 is:

$$I_{(5)} = IL_{(6)} \tag{2}$$

Now consider branch-4. The total number of buses beyond branch-4 is two and these buses are 5 and 6, respectively. Therefore, current through branch-4 is:

$$I_{(4)} = IL_{(5)} + IL_{(6)} \tag{3}$$

Similarly, consider branch-3. Total number of buses beyond branch-3 is five and these buses are 4, 5, 6, 10 and 11. Therefore, current through branch-3 is:

$$I_{(3)} = IL_{(4)} + IL_{(5)} + IL_{(6)} + IL_{(10)} + IL_{(11)} \tag{4}$$

**Table-1. Sample distribution system data**

| Branch no. (jj) | Sending end node IS(jj) | Receiving end node IR(jj) | Total no. of nodes beyond branch-jj N(jj) | Nodes beyond branch –jj ie(jj,i) i=1,2,3……N(jj) |
|---|---|---|---|---|
| 1 | 1 | 2 | 11 | 2,3,4,5,6,7,8,9,10,11,12 |
| 2 | 2 | 3 | 6 | 3,4,5,6,10,11 |
| 3 | 3 | 4 | 5 | 4,5,6,10,11 |
| 4 | 4 | 5 | 2 | 5,6 |
| 5 | 5 | 6 | 1 | 6 |
| 6 | 2 | 7 | 4 | 7,8,9,12 |
| 7 | 7 | 8 | 3 | 8,9,12 |
| 8 | 8 | 9 | 1 | 2 |
| 9 | 4 | 10 | 2 | 10,11 |
| 10 | 10 | 11 | 1 | 11 |
| 11 | 8 | 12 | 1 | 12 |

From (2)-(4), it is clear that, if we identify the buses beyond all the branches and if the load currents of the corresponding buses are known, then it is extremely easy to compute the branch currents. Thus, the general expression of branch current through branch-j is given by:

$$I_{(jj)} = \sum_{k=1}^{N_{(jj)}} IL_{\{ie(jj,i)\}} \tag{5}$$

In Table 1, N(jj) and ie(jj,i); i= 1, 2, …, N(jj)) for branch-jj are given for Figure 1 for the purpose of explanation. Let us define,

$$IL_{\{ie(jj,k)\}} = ILD_{\{ie(jj,i)\}} - {}_{j}ILQ_{\{ie(jj,i)\}} \tag{6}$$

From (5) and (6) we obtain,

$$I_{(jj)} = \sum_{k=1}^{N_{(jj)}} [ILD_{\{ie(jj,i)\}} - {}_{j}ILQ_{\{ie(jj,i)\}}]$$

$$\therefore \quad I_{(jj)} = \sum_{k=1}^{N_{(jj)}} [ILD_{\{ie(jj,k)\}} - j \sum_{k=1}^{N_{(jj)}} [ILQ_{\{ie(jj,i)\}} \tag{7}$$

Magnitude of current in branch-jj is expressed as:

$$\left|I_{(jj)}\right| = \left[\left(\sum_{k=1}^{N(jj)} ILD_{\{ie(jj,i)\}}\right)^2 + \left(\sum_{k=1}^{N(jj)} ILQ_{\{ie(jj,i)\}}\right)^2\right]^{\frac{1}{2}}$$

$$\therefore \quad \left|I_{(jj)}\right|^2 = \left(\sum_{k=1}^{N(jj)} ILD_{\{ie(jj,i)\}}\right)^2 + \left(\sum_{k=1}^{N(jj)} ILQ_{\{ie(jj,i)\}}\right)^2 \tag{8}$$

$$\therefore \quad \left|I_{(jj)}\right|^2 = \sum_{k=1}^{N(jj)} \left[ILD_{\{ie(jj,i)\}}\right]^2 + 2.\sum_{i=1}^{N(jj)-1} \sum_{k=i+1}^{N(jj)} ILD_{\{ie(jj,i)\}} \;.\, ILD_{\{ie(jj,k)\}}$$
$$+ \sum_{k=1}^{N(jj)} \left[ILQ_{\{ie(jj,i)\}}\right]^2 + 2.\sum_{i=1}^{N(jj)-1} \sum_{k=i+1}^{N(jj)} ILQ_{\{ie(jj,i)\}} \;.\, ILQ_{\{ie(jj,k)\}}$$

$$\therefore \quad \left|I_{(jj)}\right|^2 = \sum_{k=1}^{N(jj)} \left[ILD_{\{ie(jj,i)\}}\right]^2 + \left[ILQ_{\{ie(jj,i)\}}\right]^2$$
$$+2.\sum_{i=1}^{N(jj)-1} \sum_{k=i+1}^{N(jj)} ILD_{\{ie(jj,i)\}} \;.\, ILD_{\{ie(jj,k)\}} + \;.\, ILQ_{\{ie(jj,k)\}}.ILQ_{\{ie(jj,i)}$$

Power loss of the branch-jj can be written as:

$$PLOSS_{(jj)} = R_{(jj)}.\ I_{(jj)}{}^2 \tag{9}$$

From (8) and (9), we get,

$$PLOSS_{(jj)} = R_{(jj)}. \left\{ \begin{array}{l} \sum_{k=1}^{N(jj)} \left[ILD_{\{ie(jj,i)\}}\right]^2 + \left[ILQ_{\{ie(jj,i)\}}\right]^2 \\ +2.\sum_{i=1}^{N(jj)-1} \sum_{k=i+1}^{N(jj)} ILD_{\{ie(jj,i)\}} \;.\, ILD_{\{ie(jj,k)\}} \\ \qquad\qquad + \;.\, ILQ_{\{ie(jj,k)\}}.ILQ_{\{ie(jj,i)} \end{array} \right\} \tag{10}$$

The development of expression given by (10) suggests that the consumer ie(jj,i) (consumers beyond branch-jj for i=1,2……, N(jj)) has impact on the terms $[ILD\{ie(jj,i)\}\ ]^2$ and $[IL\{ie(jj,i)\}]^2$,which are exclusively due to consumers at ie(jj,i), for i=1,2…….,N(jj)), whereas the terms

$$\sum_{i=1}^{N(jj)-1}\ \sum_{k=i+1}^{N(jj)} ILD_{\{ie(jj,i)\}}\ .ILD_{\{ie(jj,k)\}} + \ ILQ_{\{ie(jj,k)\}}.ILQ_{\{ie(jj,i)}$$

are the results of the simultaneous influence of consumers ie(jj,i) and remaining consumers ie(jj,k) in the components of the power loss in the branch-jj. The presence of these cross terms in the power loss expression of the branch-jj makes the loss allocation task very difficult among the consumers beyond the branch. However, the allocation of these cross terms can be made in several ways. The crossed terms 2ILD{ie(jj,i)}. ILD{ie(jj,k)} and 2ILQ{ie(jj,i)}.ILQ{ie(jj,k)} are split into two components as follows. Let us define,

$$\begin{gathered}\alpha\{ie(jj,i)\}.[ILD\{ie(jj,i)\}.ILD\{ie(jj,k)\}]+\alpha\{ie(jj,k)\}.[ILD\{ie(jj,i)\}.ILD\{ie(jj,k)\}]\\ = 2[ILD\{ie(jj,i)\}.ILD\{ie(jj,k)\}]\\ \text{for } i = 1,2,\ldots,N(jj) - 1\\ \text{for } k = i + 1,i + 2,\ldots,N(jj)\end{gathered}\tag{11}$$

$$\begin{gathered}\beta\{ie(jj,k)\}.[ILQ\{ie(jj,i\}.ILQ\{ie(jj,k)\}]+\beta\{ie(jj,k)\}.[ILQ\{ie(jj,i)\}.ILQ\{ie(jj,k)\}]\\ = 2[ILQ\{ie(jj,i)\}.ILQ\{ie(jj,k)\}]\\ \text{for } i = 1,2,\ldots,N(jj) - 1\\ \text{for } k = i + 1,i + 2,\ldots,N(jj)\end{gathered}\tag{12}$$

(11) and (12) can be simply written as:

$$\alpha\{ie(jj,i)\} + \alpha\ \{ie(jj,k)\} = 2.0 \tag{13}$$

$$\beta\{ie(jj,i)\} + \beta\ \{ie(jj,k)\} = 2.0 \tag{14}$$

Where $\alpha$ and $\beta$ are the loss allocation factors for the crossed terms of real and reactive component of load current.

## PRO RATA TECHNIQUE OF LOSS ALLOCATION

There is a very simple and straightforward way of allocating the losses, known as Pro rata techniques[2-3]. Losses are assigned to generators and consumers related to its level of power and not considering the kVA demand of each consumer, the total active power loss of the distribution system is allocated to various consumers as

$$\text{ploss}(i) = \text{Tloss}. \frac{\text{kVA}_i}{\text{TkVA}}, \quad \text{for } i = 2,3\ldots\text{NB} \tag{15}$$

Though, Pro rata method is quite simple to understand and implement but they ignore the geographic location of the loads. Due to which, two identical demands located respectively near substation and far away from the substation are equally treated, which is quite unfair for the load located near the substation.

## PROPORTIONAL LOSS ALLOCATION SCHEME

In order to allocate losses in a fair way, the active power loss *PLOSS*(*jj*) in a specific branch-jj is allocated to the subsequent consumers taking into account the crossed terms of (10).The most intuitive and straightforward approach results from the proportionality assumption,

$$\frac{\alpha\{ie(jj,k)\}}{ILD\{ie(jj,i)\}} = \frac{\alpha\{ie(jj,k)\}}{ILD\{ie(jj,k)\}} \tag{16}$$

$$\frac{\beta\{ie(jj,k)\}}{ILD\{ie(jj,i)\}} = \frac{\beta\{ie(jj,k)\}}{ILD\{ie(jj,k)\}} \tag{17}$$

From (13) and (16), it can be written as

$$\alpha\{ie(jj,i)\} = \frac{2ILD\{ie(jj,i)\}}{ILD\{ie(jj,i)\} + ILD\{ie(jj,k)\}} \tag{18}$$

$$\alpha\{\text{ie(jj,i)}\} = \frac{2\text{ILD}\{\text{ie(jj,i)}\}}{\text{ILD}\{\text{ie(jj,i)}\} + \text{ILD}\{\text{ie(jj,k)}\}} \tag{19}$$

Similarly, from (14) and (17), we get

$$\beta\{\text{ie(jj,i)}\} = \frac{2\text{ILQ}\{\text{ie(jj,i)}\}}{\text{ILQ}\{\text{ie(jj,i)}\} + \text{ILQ}\{\text{ie(jj,k)}\}} \tag{20}$$

$$\beta\{\text{ie(jj,i)}\} = \frac{2\text{ILQ}\{\text{ie(jj,k)}\}}{\text{ILQ}\{\text{ie(jj,i)}\} + \text{ILQ}\{\text{ie(jj,k)}\}} \tag{21}$$

So, based on this principle, the power loss of the branch-jj of the network allocated to consumers beyond branch-jj (ie (jj,i) for i =1,2,……N(jj)) are:

$\text{ploss}\{\text{jj,ie(jj,i)}\} = \text{R(jj)}.\ \langle\ [\text{ILD}\{\text{ie(jj,i)}\}]^2 + [\text{ILQ}\{\text{ie(jj,k)}\}]^2$

$$\begin{aligned} ploss\{jj, ie(jj,i)\} = R(jj).\ \Big\langle\ & [ILD\{ie(jj,i)\}]^2 + [ILQ\{ie(jj,i)\}]^2 \\ & + \sum_{\substack{k=1 \\ k\neq i}}^{N(jj)} ILD\{ie(jj,i)\}.ILD\{ie(jj,k)\}.\frac{2ILD\{ie(jj,i)\}}{ILD\{ie(jj,i)\} + ILD\{ie(jj,k)\}} \\ & + \sum_{\substack{k=1 \\ k\neq i}}^{N(jj)} ILQ\{ie(jj,i)\}.ILQ\{ie(jj,k)\}.\frac{2ILQ\{ie(jj,i)\}}{ILQ\{ie(jj,i)\} + ILQ\{ie(jj,k)\}}\ \Big\rangle \end{aligned} \tag{22}$$

## QUADRATIC LOSS ALLOCATION SCHEME

Since power losses grow quadratically with power flows, it is reasonable to impose the following constraint.

$$\frac{\alpha\{\text{ie(jj,i)}\}}{[\text{ILD}\{\text{ie(jj,i)}\}]^2} = \frac{\alpha\{\text{ie(jj,k)}\}}{[\text{ILD}\{\text{ie(jj,k)}\}]^2} \tag{23}$$

$$\frac{\beta\{\text{ie(jj,i)}\}}{[\text{ILD}\{\text{ie(jj,i)}\}]^2} = \frac{\beta\{\text{ie(jj,k)}\}}{[\text{ILD}\{\text{ie(jj,k)}\}]^2} \tag{24}$$

From (13) and (23), it can be written as

$$\alpha\{ie(jj,i)\} = \frac{2[ILD\{ie(jj,i)\}]^2}{[ILD\{ie(jj,i)\}]^2 + ILD\{ie(jj,k)\}]^2} \quad (25)$$

$$\alpha\{ie(jj,k)\} = \frac{2[ILD\{ie(jj,k)\}]^2}{[ILD\{ie(jj,i)\}]^2 + [ILD\{ie(jj,k)\}]^2} \quad (26)$$

Similarly, from (14) and (24), we get

$$\beta\{ie(jj,i)\} = \frac{2[ILQ\{ie(jj,i)\}]^2}{[ILQ\{ie(jj,i)\}]^2 + ILQ\{ie(jj,k)\}]^2} \quad (27)$$

$$\beta\{ie(jj,k)\} = \frac{2[ILQ\{ie(jj,k)\}]^2}{[ILQ\{ie(jj,i)\}]^2 + [ILQ\{ie(jj,k)\}]^2} \quad (28)$$

Thus, based on this principle, the power loss of the branch-jj of the network allocated to consumers beyond branch-jj (ie (jj,i) for i =1,2,……N(jj)) are:

$ploss\{jj,ie(jj,i)\} = R(jj).\ \langle\ [ILD\{ie(jj,i)\}]^2 + [ILQ\{ie(jj,i)\}]^2$

$$\begin{aligned} ploss\{jj,ie(jj,i)\} = R(jj).\ \langle\ & [ILD\{ie(jj,i)\}]^2 + [ILQ\{ie(jj,i)\}]^2 \\ & + \sum_{\substack{k=1\\k\neq i}}^{N(jj)} ILD\{ie(jj,i)\}.ILD\{ie(jj,k)\}.\frac{2\left[ILD\{ie(jj,i)\}\right]^2}{\left[ILD\{ie(jj,i)\}\right]^2+\left[ILD\{ie(jj,k)\}\right]^2} \\ & + \sum_{\substack{k=1\\k\neq i}}^{N(jj)} ILQ\{ie(jj,i)\}.ILQ\{ie(jj,k)\}.\frac{2\left[ILQ\{ie(jj,i)\}\right]^2}{\left[ILQ\{ie(jj,i)\}\right]^2+\left[ILQ\{ie(jj,k)\}\right]^2}\ \rangle \end{aligned} \quad (29)$$

In case of proportional and quadratic allocation methods, the global value of losses to be supported by consumers, results from the sum of the losses allocated to it from the loss in each branch-jj of the network, i.e.

$$\mathrm{Tploss}(\ell) = \sum_{jj=1}^{NB-1} \mathrm{ploss}\ (jj,\ell) \qquad \text{for } \ell = 2,3\ldots,NB \tag{30}$$

## EXACT METHOD OF LOSS ALLOCATION

Savier and Das[9] have proposed a new loss allocation scheme for radial distribution systems, which is termed as "Exact Method." Mishra and Das[10] have used a similar approach for allocating losses in unbalanced radial distribution systems. The scheme is explained as below:

In (5), the load currents can be replaced by the following relation:

$$IL(i) = \frac{PL(i) - j\ QL(i)}{V i^*} \tag{31}$$

Thus, (5) modifies to

$$I(jj) = \sum_{k=1}^{N(jj)} \frac{PL\{ie(jj,k)\} - jQL\{ie(jj,k)\}}{V\{ie(jj,k)\}^*} \tag{32}$$

Real power loss of branch-jj with sending end voltage $V_i$, receiving end voltage $V_j$ and branch current I(jj) is given by:

$$PLOSS(jj) = \mathrm{Re}\ al(V_i - V_j)^*.I(jj) \tag{33}$$

Using (31), the expression in (32) modifies to

$$PLOSS(jj) = \mathrm{Re}\ al\left\{(V_i - V_j)^* \sum_{k=1}^{N(jj)} \left(\frac{PL\{ie(jj,k) - j\ QL\{ie(jj,k)\}}{V\{ie(jj,k)\}^*}\right)\right\} \tag{34}$$

Further arranging,

$$PLOSS(jj) = \mathrm{Re}\,al\left\{\sum_{k=1}^{N(jj)} \left(\frac{V_i - V_j}{V\{ie(jj,k)\}}\right)^* \left(PL\{ie(jj,k) - j\ QL\{ie(jj,k)\}\right)\right\} \tag{35}$$

Let $$\left(\frac{V_i - V_j}{V\{ie(jj,k)\}}\right)^* = A\{ie(jj,k)\} + j\,B\{ie(jj,k)\} \tag{36}$$

$$\therefore PLOSS(jj) = \mathrm{Real}\left\{\sum_{k=1}^{N(jj)} (A\{ie(jj,k)\} + j\,\mathrm{B}\{ie(jj,k)\}).\,(PL\{ie(jj,k)\} - jQL\{ie(jj,k)\})\right\} \tag{37}$$

Hence,

$$\mathrm{PLOSS(jj)} = \sum_{i=1}^{N(jj)} (\mathrm{A\{ie(jj,k)\}.PL\{ie(jj,k)\} + B\{ie(jj,k)\}.QL\{ie(jj,k)\}}) \tag{38}$$

Using (37) active power loss in branch-jj can be allocated to consumers beyond branch-jj. Thus, active power loss of branch-jj allocated to a consumer connected to node {ie(jj,k)} is
given by:

$$\mathrm{ploss\{jj,ie(jj,k)\}) = A\{ie(jj,k)\}.PL\{ie(jj,k)\} + B\{ie(jj,k)\}.QL\{ie(jj,k)\})}$$
$$\text{for jj} = 1,2,\ldots \mathrm{NB} - 1 \text{ and k} = 1,2\ldots \mathrm{N(jj)} \tag{39}$$

The global value of losses to be supported by consumer connected to node $\ell$ results from the sum of the losses allocated to it in each branch-jj of the network, which is given by (30).

## COMPARISON OF THE LOSS ALLOCATION METHODS

To compare the effectiveness of the loss allocation schemes discussed including the proposed scheme, a physically existing 30 node, 11 kV radial distribution as shown in Figure 2 is considered. The line and load data for this system is given in Table-A1. The base voltage and base power are considered to be 11kV and 100kVA respectively. A load flow based on the algorithm[11] is used to carry out the load flow for the distribution system first. The total real power loss of the system is 146.0905 kW. This total active power is then allocated to various buses (consumers) using the above mentioned schemes as shown in Table-2.

In pro rata scheme, losses are allocated to the consumers irrespective of the geographical location of consumers. So, consumers having same load demands are allocated same losses, even though the power loss contribution of the consumer electrically closer to the substation is less as compared to those consumers electrically away from the substation. Thus, pro rata scheme allocates same losses to consumers at bus 10, 15, 16 and 28 i.e. 4.3623 kW. Similarly, same losses are allocated to consumers at bus 14 and 26 i.e.2.7932 kW. However, all other loss allocation schemes allocate lesser power loss to bus 10 than bus 28.

Proportional and quadratic schemes are both based on branch current flow and these two methods ensures that each consumer only has allocated losses at branches for which current it contributes. But, the "Exact method" allocates branch losses to different consumers based on actual contribution of the branch power losses by each consumer beyond that branch.

## CONCLUSIONS

In this article, three schemes of loss allocation for radial distribution systems have been presented. Also, a new loss allocation scheme has been proposed. All these schemes have been compared considering a radial test distribution system. It is evident that pro rata loss allocation scheme is simple and easy to implement. However, in this scheme,

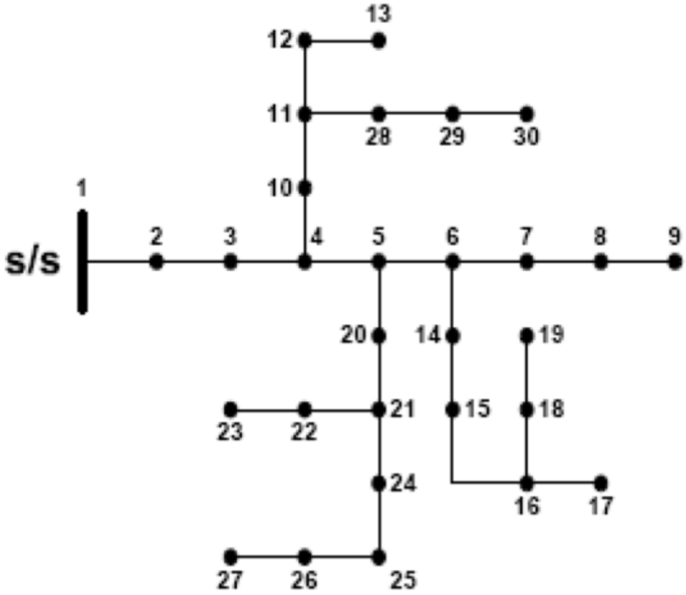
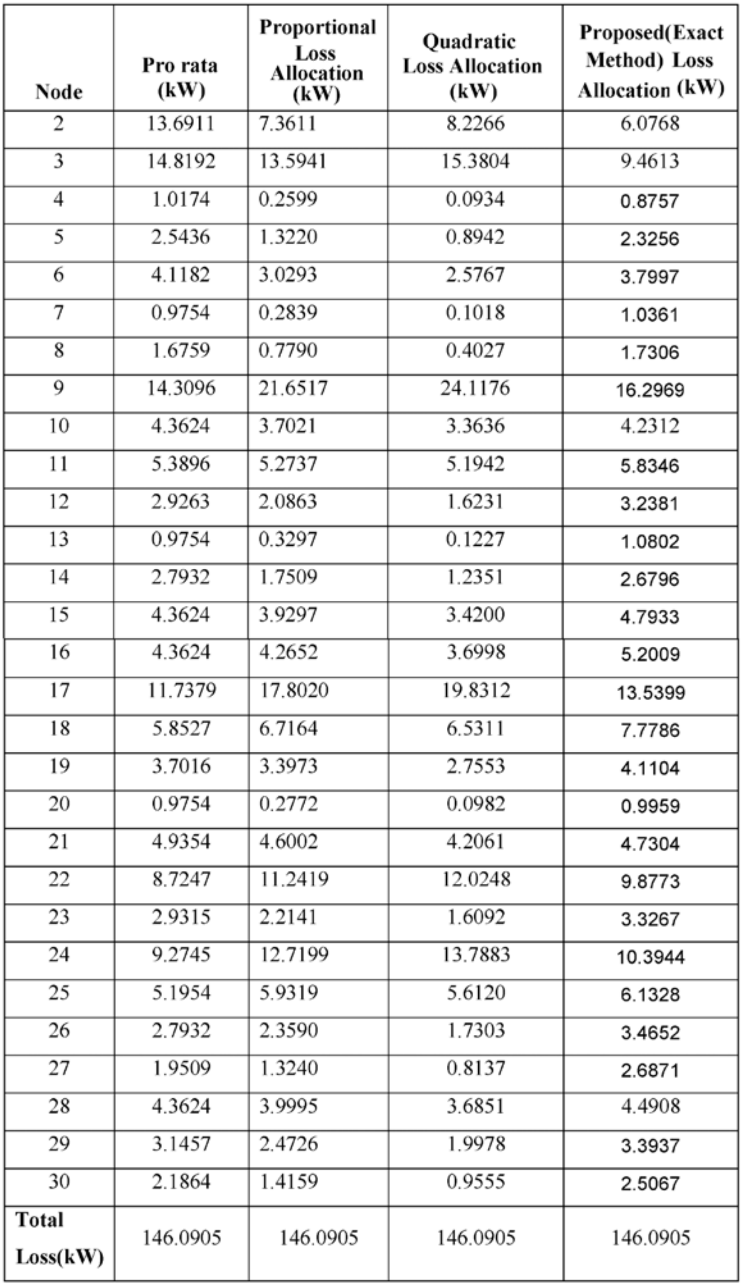


**Figure 2. 30-node radial distribution network**

**Table-2. Loss Allocation to consumers by various methods**



| Node | Pro rata (kW) | Proportional Loss Allocation (kW) | Quadratic Loss Allocation (kW) | Proposed(Exact Method) Loss Allocation (kW) |
|---|---|---|---|---|
| 2 | 13.6911 | 7.3611 | 8.2266 | 6.0768 |
| 3 | 14.8192 | 13.5941 | 15.3804 | 9.4613 |
| 4 | 1.0174 | 0.2599 | 0.0934 | 0.8757 |
| 5 | 2.5436 | 1.3220 | 0.8942 | 2.3256 |
| 6 | 4.1182 | 3.0293 | 2.5767 | 3.7997 |
| 7 | 0.9754 | 0.2839 | 0.1018 | 1.0361 |
| 8 | 1.6759 | 0.7790 | 0.4027 | 1.7306 |
| 9 | 14.3096 | 21.6517 | 24.1176 | 16.2969 |
| 10 | 4.3624 | 3.7021 | 3.3636 | 4.2312 |
| 11 | 5.3896 | 5.2737 | 5.1942 | 5.8346 |
| 12 | 2.9263 | 2.0863 | 1.6231 | 3.2381 |
| 13 | 0.9754 | 0.3297 | 0.1227 | 1.0802 |
| 14 | 2.7932 | 1.7509 | 1.2351 | 2.6796 |
| 15 | 4.3624 | 3.9297 | 3.4200 | 4.7933 |
| 16 | 4.3624 | 4.2652 | 3.6998 | 5.2009 |
| 17 | 11.7379 | 17.8020 | 19.8312 | 13.5399 |
| 18 | 5.8527 | 6.7164 | 6.5311 | 7.7786 |
| 19 | 3.7016 | 3.3973 | 2.7553 | 4.1104 |
| 20 | 0.9754 | 0.2772 | 0.0982 | 0.9959 |
| 21 | 4.9354 | 4.6002 | 4.2061 | 4.7304 |
| 22 | 8.7247 | 11.2419 | 12.0248 | 9.8773 |
| 23 | 2.9315 | 2.2141 | 1.6092 | 3.3267 |
| 24 | 9.2745 | 12.7199 | 13.7883 | 10.3944 |
| 25 | 5.1954 | 5.9319 | 5.6120 | 6.1328 |
| 26 | 2.7932 | 2.3590 | 1.7303 | 3.4652 |
| 27 | 1.9509 | 1.3240 | 0.8137 | 2.6871 |
| 28 | 4.3624 | 3.9995 | 3.6851 | 4.4908 |
| 29 | 3.1457 | 2.4726 | 1.9978 | 3.3937 |
| 30 | 2.1864 | 1.4159 | 0.9555 | 2.5067 |
| **Total Loss(kW)** | 146.0905 | 146.0905 | 146.0905 | 146.0905 |

losses allocated to consumers having the same load demands are the same, which is not desirable as it does not differentiate between two consumers on the basis of its relative location in the distribution system. On the other hand, quadratic and proportional loss allocation schemes are based on branch current flow and these methods allocate branch power loss only to those consumers beyond that branch. Proportional loss allocation scheme makes the assumption that the loss allocation factor is proportional to the real/reactive load current of that consumer whereas quadratic loss allocation scheme assumes that the loss allocation factor is proportional to the square of the real/reactive load current. The presence of cross terms as mentioned before makes the loss allocation very complex. However, the proposed scheme allocates losses to the consumers without making any assumptions and the cross terms are tactically avoided making the scheme simple to implement. Thus, the proposed scheme can be helpful in estimating savings resulting from avoided distribution losses resulting from cogeneration and distributed generation plants with substantial on-site consumption.

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## APPENDIX

**Table-A1. Line and load data 30 bus radial distribution system**

| Sending End Node | Receiving End Node | Branch No. | R (Ω) | X (Ω) | Load at receiving end node | |
|---|---|---|---|---|---|---|
| | | | | | $P_L$(kW) | $Q_L$(kVAR) |
| 1 | 2 | 1 | 1.632 | 1.1019 | 162 | 96 |
| 2 | 3 | 2 | 1.088 | 0.7346 | 150 | 138 |
| 3 | 4 | 3 | 0.544 | 0.3673 | 12 | 7.2 |
| 4 | 5 | 4 | 0.272 | 0.1836 | 30 | 18 |
| 5 | 6 | 5 | 0.544 | 0.3673 | 45.6 | 33.6 |
| 6 | 7 | 6 | 1.3760 | 0.3896 | 12 | 6 |
| 7 | 8 | 7 | 2.752 | 0.7792 | 18 | 14.4 |
| 8 | 9 | 8 | 4.128 | 1.1688 | 156 | 120 |
| 4 | 10 | 9 | 3.6432 | 1.5188 | 48 | 36 |
| 10 | 11 | 10 | 0.9108 | 0.3797 | 64.8 | 36 |
| 11 | 12 | 11 | 0.4554 | 0.1898 | 36 | 18 |
| 12 | 13 | 12 | 0.4550 | 0.1898 | 12 | 6 |
| 6 | 14 | 13 | 0.9108 | 0.3797 | 30 | 24 |
| 14 | 15 | 14 | 1.8216 | 0.7594 | 48 | 36 |
| 15 | 16 | 15 | 1.8216 | 0.7594 | 48 | 36 |
| 16 | 17 | 16 | 0.9108 | 0.3797 | 120 | 108 |
| 16 | 18 | 17 | 1.3760 | 0.3896 | 72 | 36 |
| 18 | 19 | 18 | 1.3760 | 0.3896 | 36 | 36 |
| 5 | 20 | 19 | 0.9108 | 0.3797 | 12 | 6 |
| 20 | 21 | 20 | 1.8216 | 0.7594 | 48 | 48 |
| 21 | 22 | 21 | 2.7324 | 1.1391 | 96 | 72 |
| 22 | 23 | 22 | 0.9108 | 0.3797 | 32.4 | 24 |
| 21 | 24 | 23 | 2.752 | 0.7792 | 96 | 84 |
| 24 | 25 | 24 | 3.0272 | 0.8571 | 54 | 46.8 |
| 25 | 26 | 25 | 2.752 | 0.7792 | 30 | 24 |
| 26 | 27 | 26 | 2.752 | 0.7792 | 24 | 12 |
| 11 | 28 | 27 | 1.376 | 0.3896 | 48 | 36 |
| 28 | 29 | 28 | 1.376 | 0.3896 | 36 | 24 |
| 29 | 30 | 29 | 4.128 | 1.1688 | 26.4 | 14.4 |