Cost-Effective Method of Optimization of Stacking Sequences in the Cylindrical Composite Shells Using Genetic Algorithm

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Abstract

Buckling is one of the common destructive phenomena, which occurs in composite cylinders subjected to external pressure. In this paper, different methods to optimize the stacking sequence of these cylinders are investigated. A finite element model is proposed in order to predict critical buckling pressure, and the results are validated with previous experimental data. Theoretical analysis based on NASA SP-8007 solution and the simplified equation for cylinder buckling of American Society of Mechanical Engineers ASME RD-1172 are presented and discussed. The results of theoretical and finite element analysis and experimental tests are compared for both glass and carbon epoxy cylinders. Using NASA and ASME formulations, optimal laminations of cylinders in order to maximize buckling pressure, are obtained by the genetic algorithm method. Suggested laminations and the values of corresponding critical buckling pressure calculated by finite element analysis, are presented and compared in various states. Obtained results show that

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while predicted buckling loads of finite element analysis are reliable, NASA formulation can be used in a very cost-effective method to optimize the buckling problems.

Keywords: Cylindrical composite shells, external pressure, finite element analysis, buckling formula, genetic algorithm.

1 Introduction

Cylindrical composite shells are extensively used in different industries due to their good mechanical properties such as corrosion resistance and high strength-to-weight ratio. Composite structures are subjected to compressive loads in many cases; for instance, underwater composite cylindrical vessels subjected to high external pressure. Buckling is one of the common destructive phenomena which might occur in these structures and causes catastrophic failure in them. Therefore, accurately designing of these structures against buckling load is vital. In the design of composite structures, parameters such as the thickness, fiber orientation for each layer, and the number of layers should be precisely determined to meet the design requirements while achieving the minimum level of material consumption.

The focus of attention in many research articles has been the investigation of buckling behavior in composite cylindrical shells subjected to external pressure [1–6]. Various studies have been carried out for the optimization of composite layer orientations to achieve maximum buckling load. The genetic algorithm (GA) as an optimization method has been used in some researches to optimize composite cylinders against buckling loads. The use of GA for optimization of the stacking sequence in a composite laminate in order to maximize buckling load was studied [7]. The same authors improved and revised GA for minimum thickness composite laminate design [8]. A generalized elicit selection procedure for the design of composite laminate was proposed [9]. The stability limits were maximized by using GA with an analytical model in order to obtain fiber orientations of composite cylinders [10]. The application of a GA to a material- and sizing-optimization problem of a plate was studied [11]. Kim et al. [12] used GA for the composite wing subjected to a random gust to maximize the strength and failure index of the wing.

By utilizing GA and generalized pattern search algorithm, the optimal stacking sequence of a composite panel was investigated [13]. Gillet et al. [14] used GA to derive simple design rules that would reliably obtain a high-performance configuration of composite Structures. Lee et al. [15] used a microgenetic algorithm to maximize an allowable design load in relation to buckling and static failure loads. Da Silva et al. [16] presented an optimum design of laminated composite risers using a GA. Gyan et al. [17] used GA in order to minimize the weight of a laminated composite structure using dispersed ply angles for a given loading. Chen et al. [18] proposed a method for laminate stacking sequence optimization based on a two-level approximation and GA in such a way that discrete variables were optimized through the GA. Geng et al. [19] worked on the optimization of winding angle for filament-wound cylindrical vessels under internal pressure. Almeida Jr et al. [20] used GA accounting progressive damage in order to optimize stacking sequence in composite tubes under internal pressure. Pathan et al. [21] presented a constrained GA for the optimization of the damping response of composite laminates.

Solving the buckling problem of a cylindrical shell is a complicated procedure that requires a remarkable computational cost. On the other hand, the fitness function is called several times during the optimization process. Therefore, the optimization of these kinds of problems can be very complicated and time-consuming. In this paper, other cost-effective alternatives are investigated in order to optimize the stacking sequence of a composite cylindrical shell subjected to external pressure. For this purpose, a FE model is proposed to predict the critical buckling pressure of composite cylinders. This model is validated with previous experimental tests; hence, it can be used to investigate the accuracy of the results. Theoretical analysis based on NASA SP-8007 solution and the simplified equation for cylinder buckling due to external pressure in ASME RD-1172 are presented and discussed. Then GA has been used in combination with the mentioned theoretical analysis in order to optimize the stacking sequence. Optimal laminations are presented for both glass and carbon epoxy cylinders. Corresponding critical buckling pressure of optimal laminations is evaluated using the presented FE model. By using a validated FE as an independent method of checking the optimized solutions, the advantages and disadvantages of using GA in combination with these analytical approaches are discussed. The comprehensive study presented in this work could give engineers and companies a new insight for the use of GA based on the mentioned formulations, and the same procedure could be used instead of other time-consuming and complex optimization methods that might not be easily available.

2 Theoretical Analysis

In this section, formulations of classical lamination theory are presented in order to introduce the ABD matrix. Then the elements of the ABD matrix are used to calculate the critical buckling pressure of cylinders using NASA and ASME formulations.

2.1 Classical Lamination Theory

The stress–strain relationship of a lamina in three-dimensional space, with three orthogonal planes of material symmetry, is given by Equation (1) where C_{ij} are stiffness matrix components [22].

$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \tau_{23}\\ \tau_{13}\\ \tau_{12}\end{bmatrix} = \begin{bmatrix}C_{11} & C_{12} & C_{13} & 0 & 0 & 0\\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0\\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & C_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & C_{55} & 0\\ 0 & 0 & 0 & 0 & 0 & C_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\\ \gamma_{23}\\ \gamma_{13}\\ \gamma_{12}\end{bmatrix} \tag{1}$$

When a problem with plane stress conditions is investigated; for example, a membrane or a shell, Equation (1) can be reduced to a two-dimensional relation. If the lamina is thin and does not carry any out-of-plane loads, one can assume plane stress conditions for the lamina and Equation (1) for an orthotropic plane stress problem can then be written as [22]:

$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{bmatrix} = \begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{bmatrix} = \begin{bmatrix}\frac{E_1}{1-v_{12}v_{21}} & \frac{E_2 v_{12}}{1-v_{12}v_{21}} & 0\\ \frac{E_2 v_{12}}{1-v_{12}v_{21}} & \frac{E_2}{1-v_{12}v_{21}} & 0\\ 0 & 0 & G_{12}\end{bmatrix}\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{bmatrix} \tag{2}$$

Stress–strain relationship for a two-dimensional angle lamina can be expressed as [22]:

$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{bmatrix} = [T]^{-1}[Q][R][T][R]^{-1}\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{bmatrix} = \begin{bmatrix}\overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16}\\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26}\\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{bmatrix} \tag{3}$$

where $[T]$ and $[R]$ are transformation matrix and Reuter matrix, respectively and $\overline{Q}_{ij}$ are the elements of the transformed reduced stiffness matrix of angle lamina. A laminate consists of more than one lamina bonded together through their thickness. For a laminate made of *n* plies, A, B, and D matrices are defined as follows [22]:

$$A_{ij} = \sum_{k=1}^{n}[\overline{Q}_{ij}]_k(h_k - h_{k-1})$$

$$B_{ij} = \frac{1}{2}\sum_{k=1}^{n}[\overline{Q}_{ij}]_k(h_k^2 - h_{k-1}^2)$$

$$D_{ij} = \frac{1}{3}\sum_{k=1}^{n}[\overline{Q}_{ij}]_k(h_k^3 - h_{k-1}^3) \tag{4}$$

where h_k and h_{k-1} are z coordinates of top and bottom surfaces of each lamina, respectively, which z coordinate directed along the laminate thickness. The extensional stiffness matrix *[A]* relates the resultant in-plane forces to the in-plane strains, and the bending stiffness matrix *[D]* relates the resultant bending moments to the plate curvatures. The coupling stiffness matrix *[B]* couples the moment and force terms to the mid-plane strains and mid-plane curvatures, respectively.

In the following section, rigorous NASA SP-8007 solution and the simplified equation for cylinder buckling due to external pressure in ASME RD-1172 are demonstrated [4].

2.2 NASA Formulation

Jones [23] substituted the variations in forces and moments in Donnell-type stability differential equations, which were solved to yield a closed-form stability criterion in terms of the geometric and material properties of the stiffened multilayered circular cylindrical shell. Based on the classical lamination theory and derived formulations by Jones ([23], The National Aeronautics and Space Administration (NASA) suggests the following procedure to calculate the maximum allowable external pressure for a stiffened orthotropic cylinder under hydrostatic pressure. Firstly, c_{ij} are calculated from Equations (5)–(13), then the critical buckling pressure is calculated from Equation (14) [24].

$$c_{11} = A_{11}\left(\frac{m\pi}{L}\right)^2 + A_{66}\left(\frac{n}{R}\right)^2 \tag{5}$$

$$c_{22} = A_{22}\left(\frac{n}{R}\right)^2 + A_{66}\left(\frac{m\pi}{L}\right)^2 \tag{6}$$

$$c_{33} = D_{11}\left(\frac{m\pi}{L}\right)^4 + 2(D_{12} + 2D_{66})\left(\frac{m\pi}{L}\right)^2\left(\frac{n}{R}\right)^2 + D_{22}\left(\frac{n}{R}\right)^4 + \frac{A_{22}}{R^2} + \frac{2B_{22}}{R}\left(\frac{n}{R}\right)^2 + \frac{2B_{12}}{R}\left(\frac{m\pi}{L}\right)^2 \tag{7}$$

$$c_{13} = \frac{A_{12}}{R}\left(\frac{m\pi}{L}\right) + B_{11}\left(\frac{m\pi}{L}\right)^3 + (B_{12} + 2B_{66})\left(\frac{m\pi}{L}\right)\left(\frac{n}{R}\right)^2 \quad (8)$$

$$c_{31} = c_{13} \quad (9)$$

$$c_{12} = (A_{12} + A_{66})\left(\frac{m\pi}{L}\right)\left(\frac{n}{R}\right) \quad (10)$$

$$c_{21} = c_{12} \quad (11)$$

$$c_{32} = c_{23} \quad (12)$$

$$c_{23} = (B_{12} + 2B_{66})\left(\frac{m\pi}{L}\right)^2\left(\frac{n}{R}\right) + \frac{A_{22}}{R}\left(\frac{n}{R}\right) + B_{22}\left(\frac{n}{R}\right)^3 \quad (13)$$

$$P_{cr} = \frac{R}{F\left[n^2 + \frac{1}{2}\left(\frac{m\pi R}{L}\right)^2\right]} \frac{\det\begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}}{\det\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}} \quad (14)$$

Geometrical parameters including *R* and *L* are the outside radius of shell and design length of a vessel section, respectively. Parameter *F* is the design factor and is equal to one because of the fact that the exact values of buckling loads without any extra safety factors are considered. Parameters *n* and *m* are integers that their combination, which gives the lowest buckling, pressure, should be chosen.

2.3 ASME Formulation

In comparison with the rigorous NASA SP-8007 solution, a simplified equation for cylinder buckling due to external pressure is presented in ASME RD-1172. The American Society of Mechanical Engineers (ASME) suggests the following procedure to calculate the maximum allowable external pressure for a fiber-reinforced plastic pressure vessel under external pressure [25].

$$v_1 = \frac{-(ABD)^{-1}{}_{5,4}}{(ABD)^{-1}{}_{4,4}} \quad (15)$$

$$v_2 = \frac{-(ABD)^{-1}{}_{5,4}}{(ABD)^{-1}{}_{5,5}} \quad (16)$$

where v_1 and v_2 are flexural Poisson's ratios in the axial and hoop directions, respectively.

$$E_{at} = \frac{A_{11}A_{22} - A_{12}^2}{t(A_{2,2})} \tag{17}$$

$$E_{af} = \frac{12}{t^3(ABD)^{-1}{}_{4,4}} \tag{18}$$

$$E_{hf} = \frac{12}{t^3(ABD)^{-1}{}_{5,5}} \tag{19}$$

where E_{af} and E_{hf} are axial and hoop flexural moduli, respectively, and E_{at} is the axial tensile modulus.

Finally, the critical buckling pressure is calculated as follows [25]:

$$P_{cr} = \frac{KD \cdot 0.8531 \cdot \gamma \cdot E_{hf}^{\frac{3}{4}} \cdot E_{at}^{\frac{1}{4}} \cdot t^{\frac{5}{2}}}{(1 - v_x v_y)^{\frac{3}{4}} \cdot L \cdot (\frac{D_o}{2})^{\frac{3}{2}} \cdot F} \tag{20}$$

Geometrical parameters including L, D_o, and t are design length of a vessel section, outside diameter and wall thickness of the cylinder, respectively. The parameter F is the design factor and is equal to one. *KD* in Equation (20) is the knockdown factor to cover all data points and is equal to 0.84 [25]. The parameter γ is a reduction factor developed to better correlate theoretical predictions and test results, and it is obtained from Equations (21) and (22). The value of γ depends on the parameter Z_p which is calculated from Equation (23).

$$\gamma = 1 - 0.001 \cdot Z_p(Z_p \leq 100) \tag{21}$$

$$\gamma = 0.9(Z_p > 100) \tag{22}$$

$$Z_p = \frac{E_{hf}^{\frac{3}{2}} \cdot E_{at}^{\frac{1}{2}}}{E_{af}^2}(1 - v_1 v_2)^{\frac{1}{2}} \frac{L^2}{(\frac{D_o}{2}t)} \tag{23}$$

3 FE Analysis

In this study, FE simulations are performed using eigenvalue buckling analysis of ABAQUS commercial software in order to obtain critical buckling pressures of composite cylinders. Eigenvalue buckling analysis is a static stress analysis used to estimate the critical (bifurcation) load of stiff structures by linear perturbation procedure. In this method, based on the initial geometry

Table 1 Mechanical properties of cylinders

	E1 (Gpa)	E2 (Gpa)	E3 (Gpa)	G12 (Gpa)	G13 (Gpa)	G23 (Gpa)	v_{12}	v_{13}	v_{23}
Glass epoxy	45.6	16.2	16.2	5.83	5.83	5.78	0.278	0.278	0.4
Carbon epoxy	156	9.65	6.57	5.47	2.8	3.92	0.27	0.34	0.492

of the structure, eigenvalues are extracted. Actually, after the formation of the structural stiffness matrix and loads, the eigenvalues and eigenvectors of this matrix are calculated in such a way that eigenvalues and eigenvectors represent buckling loads and corresponding buckling modes, respectively.

3.1 FE Simulations

In current FE simulations, the internal diameter, the length and thickness of cylinders are assumed 175 mm, 400 mm, and 10 mm, respectively. Four node shell elements with reduced integration (S4R) are assigned to the model. Except for longitudinal translation for one side of the cylinder, all other translational degrees of freedom for both sides are fixed in order to prevent rigid body motion. Cylindrical local coordinate system is also defined for the model. Based on the presented data by [10], the mechanical properties of cylinders are listed in Table 1.

3.2 Validation of FE Models

In this section, glass and carbon epoxy cylinders are modelled based on geometries of France manufacturers, STRAGLEN and CNIM, respectively [10].

In order to validate the results of the presented FE model and compare them with the experimental tests carried out by Messager et al. [10], stacking sequences are selected based on their optimum results. Thus, for glass epoxy cylinder with 7 layers, stacking sequence of [90_3/15_2/90_2] and for carbon epoxy cylinder with 10 layers, stacking sequence of [90_2/60/30_5/60/90] are selected. In addition, critical buckling pressures for these cases are calculated based on NASA and ASME formulations. The results of these methods are presented and compared in Table 2.

It can be comprehended from the presented values of critical buckling pressure in Table 2 that the results of NASA, experimental tests and presented FE model correlate reasonably well. As a result, the FE analysis model predicts the buckling pressure of cylinders accurately.

Table 2 Comparison of the buckling pressure values obtained from different methods

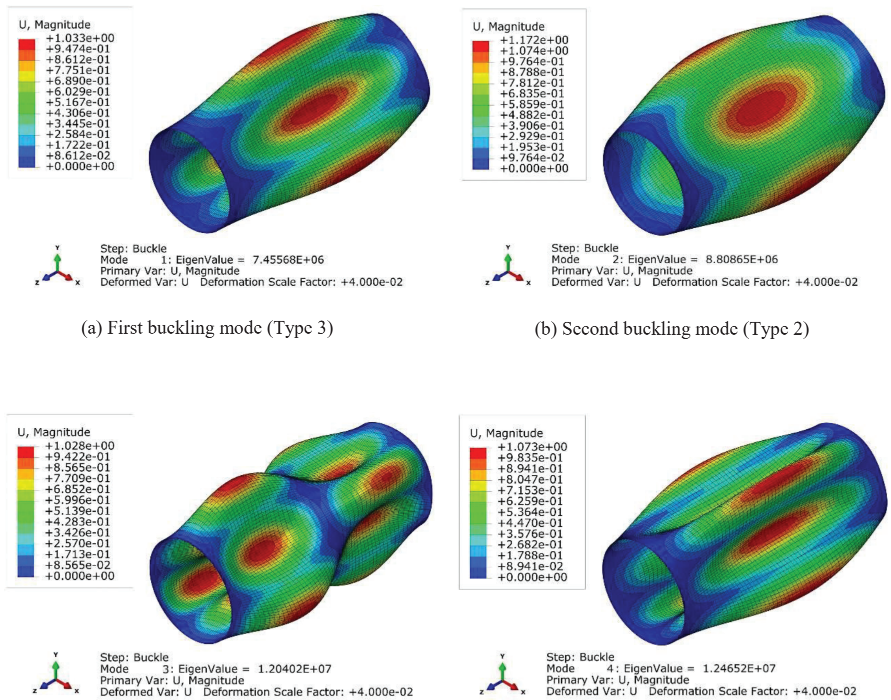
		Buckling Pressure (MPa)			
Composite Cylinder	Stacking Sequence	Calculated by Present FE Simulations	Calculated by NASA Formulation	Calculated by ASME Formulation	Experiments [10]
glass epoxy	$[90_3/15_2/90_2]$	7.45	6.05	4.52	8.27
Carbon epoxy	$[90_2/60/30_5/60/90]$	23.88	24.44	15.01	21.7

The values of critical buckling pressure calculated based on ASME formulations are less than other methods, and the discrepancies between the results are considerable. Actually, the ASME RD-1172.1 is a simplified equation of NASA SP-8007 solution for lateral and longitudinal pressure.

The calculated and experimental buckling modes of the lengthy vessels are characterized by one longitudinal half-wave and two (mode called "type 2") or three ("type 3") circumferential half-waves [10]. Figure 1 shows the first four buckling modes of glass epoxy cylinders. As can be seen in the figure, eigenvalue, as an indicative of critical buckling pressure of the cylinder, is equal to 7.455 MPa for the first mode, which is lower than the eigenvalues of other subsequent modes. Similar to the experimental results of [10], the first buckling mode for this case is type 3.

4 Genetic Algorithm

Nowadays, many optimization methods are used to optimize engineering designs. GA as a metaheuristic inspired evolutionary algorithm is employed to solve many optimization problems. Unlike most of optimization methods that need to have the exact equation of objective function and its relation to design variables, finding the value of objective function is enough to achieve the proper solution of optimization problems using GA. This algorithm can also be used for discrete optimization problems where the design variables can have only integer values. Since the GA has several starting point, it can search several different directions in the space of the problem solution. Hence, GA is appropriate for problems that have a large search space and most of them are nonlinear and complicated optimization problems.



(a) First buckling mode (Type 3)

(b) Second buckling mode (Type 2)

(c) Third buckling mode

(d) Fourth buckling mode

Figure 1 Buckling modes obtained from presented FE model for $[90_3/15_2/90_2]$ glass epoxy cylinder.

Firstly, a fitness function is defined which its minimization is the final aim of the optimization procedure. Potential solutions of the problem in the form of simple chromosomes are encoded. GA creates random changes in the selected solutions and uses fitness function for the evaluation of these changes. GA is a population-based model. Firstly, a population is generated stochastically; afterwards, its fitness is evaluated and scored by the value of fitness function. The new generation is created by conducting crossover/mutation operations on the elite individuals selected from the previous generation. Parents are selected from the certain individuals of current population who have better fitness values to create children in the next generation. GA quits when the termination condition for the main loop of program occurs or the improvement of fitness function during successive generations, is ceased. The settings of optimization are presented in Table 3.

Table 3 Genetic algorithm settings

Description	Maximum Number of Generations	Population Size	Crossover Percentage	Mutation Rate
Value	25	50	0.8	0.02

4.1 Stacking Sequence Optimization Method

Finding the minimum buckling pressure is challenging for designers and understanding this value is vital for them in order to determine safety factors of the structure. Regarding the fact that the value of this pressure for the first buckling mode is less than other modes, first buckling mode is focused in this paper.

Orientation of layers, influences buckling load and even buckling mode, considerably. Thus, an optimization process is employed to find fiber orientation of layers to maximize critical buckling pressure of each cylinder.

In this study, critical buckling pressure of composite cylinders is defined as fitness function for the optimization problem. Fiber orientations of each layer are considered as design variables. These orientations based on manufacturing limitations can have only integer values between 20 and 90 degrees with 5 degrees of intervals. The optimization method based on NASA and ASME formulations discussed in Section 2, is provided.

As shown in Figure 2, a GA is used to obtain the best stacking sequence of composite cylinders. Then the critical buckling pressure related to these obtained stacking sequences are evaluated by FE analysis have been validated in Section 3.2.

4.2 Optimal Laminations of Composite Cylinders

In this section, using the procedure shown in Figure 2, the best stacking sequences obtained from NASA and ASME formulations are investigated. Obtained results for glass and carbon epoxy composite cylinders with the number of layers increasing from 1 to 5 layers are presented in Table 4.

NASA optimal laminations have the characteristic pattern of $[90_{m1}, \theta_{m2}$, $20_{m3}, \theta_{m4}$, $90_{m5}]$, which θ values represent a mild slope between layers of 90 and 20 degrees. ASME optimal laminations have the characteristic pattern of $[90_{n1}, 20_{n2}, 90_{n3}]$, while in most cases, the values of $n1$ and $n3$ are equal. According to the ASME formulations, hoop flexural modulus is increased considerably when orientations of inner and outer layers of cylinders are 90 degrees. As a result, ASME formulations predict 90 degrees' orientations for inner and outer layers of cylinders with optimal laminations. As can be seen

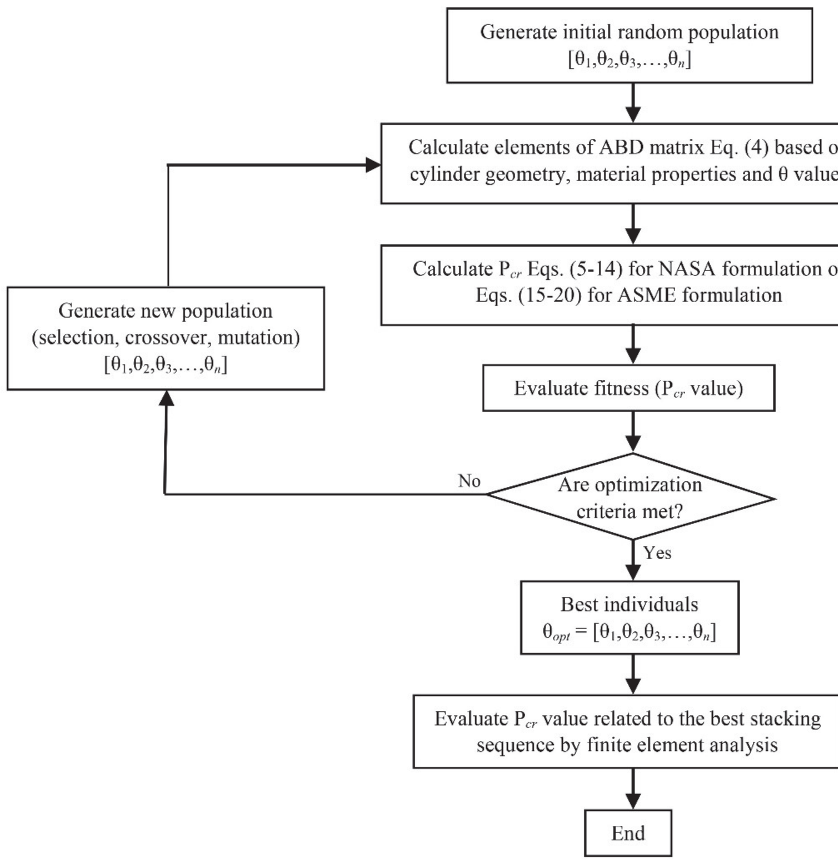


Figure 2 The GA used for optimization of stacking sequence in composite cylinders.

from optimal laminations of Table 4, for glass and carbon epoxy cylinders, characteristic patterns of optimal laminations are generally similar, although some values of θ for NASA formulations and the values of $n1$ for ASME formulations are different. Actually, for the NASA formulations, a maximum of 10 degrees difference of θ values between the glass and carbon epoxy cylinders are seen.

While using ASME formulations for both glass and carbon epoxy cylinders, the values of $n1$ remain constant $(n1 = n3 = 1)$, except for the case of glass epoxy cylinder with 5 layers $(n1 = n3 = 2)$.

In order to evaluate the optimality of stacking sequences, the values of corresponding critical buckling pressure obtained from FE analysis for glass and carbon epoxy cylinders are shown in Figures 3 and 4, respectively.

Table 4 NASA and ASME optimized stacking sequence of composite cylinders

	Glass Epoxy Cylinders		Carbon Epoxy Cylinders	
Number of Layers for the Cylinder	NASA Optimized Stacking Sequence	ASME Optimized Stacking Sequence	NASA Optimized Stacking Sequence	ASME Optimized Stacking Sequence
1	[90]	[90]	[90]	[90]
2	[90,65]	[90_2]	[90,70]	[90_2]
3	[90,20,90]	[90,20,90]	[85,20,90]	[90,20,90]
4	[90,25,20,90]	[90,20_2,90]	[90,30,20,90]	[90,20_2,90]
5	[90,50,20_2,90]	[90_2,20,90_2]	[90,40,20_2,90]	[90,20_3,90]

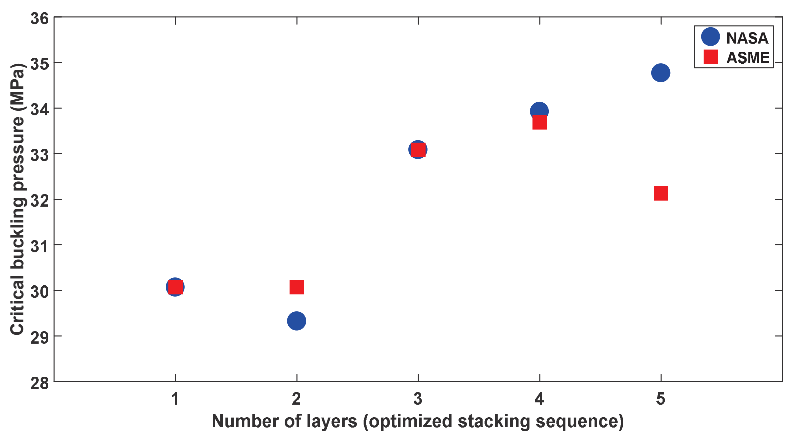


Figure 3 The values of critical buckling pressure obtained from FE analysis corresponding to optimal laminations of NASA and ASME formulations for glass epoxy cylinders.

By comparing the values of critical buckling pressure shown in Figures 3 and 4, it is understood that due to the greater values of elasticity modulus in the first direction for carbon epoxy cylinders in comparison with glass epoxy cylinders, the values of critical buckling pressure of carbon epoxy cylinders are considerably greater than glass epoxy cylinders. For instance, buckling pressures of optimized 5-layer glass and carbon epoxy cylinders are 34.766 MPa and 71.151 MPa respectively, and the difference is about 50%.

As can be seen in these figures, NASA optimal laminations for glass and carbon epoxy cylinders with 5 layers showed 15% and 32% increase of buckling pressure in comparison with the lamination of [90], respectively. Therefore, optimization of the stacking sequence influences the corresponding

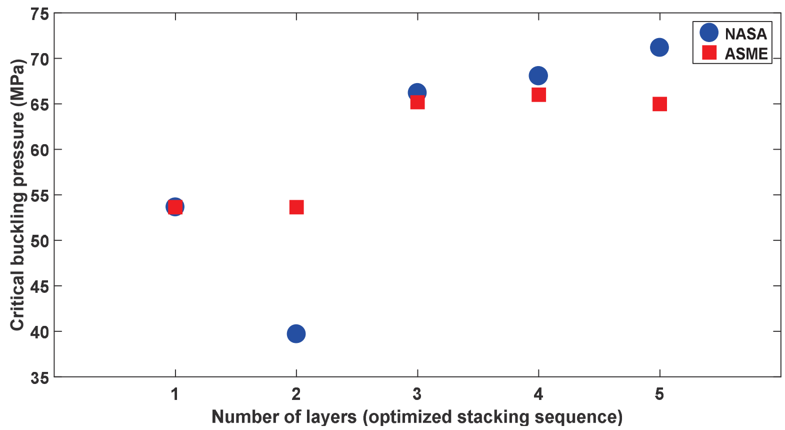


Figure 4 The values of critical buckling pressure obtained from FE analysis corresponding to optimal laminations of NASA and ASME formulations for carbon epoxy cylinders.

critical buckling values considerably. For the carbon epoxy cylinders, the influence of lamination optimality on the buckling pressure is more considerable than glass epoxy cylinders owing to the fact that fiber mechanical properties, especially the ratio of longitudinal to the lateral elastic moduli, in carbon epoxy cylinders are greater than glass epoxy cylinders.

Based on Figures 3 and 4, for the case of glass epoxy cylinder with 5 layers, critical buckling pressure of NASA optimal lamination (34.766 MPa) is 8% greater than its value corresponding to ASME optimal lamination (32.126 MPa), and for the case of carbon epoxy cylinder with 5 layers, critical buckling pressure of NASA optimal lamination (71.151 MPa) is 10% greater than its value corresponding to ASME optimal lamination (64.984 MPa). Critical buckling values corresponding to optimal laminations of NASA are greater than ASME optimal laminations. As a result, optimal laminations predicted by NASA are more accurate and reliable.

4.3 Optimization Histories

In this section, GA histories are produced in order to observe the optimization process in successive generations. Cost function values calculated based on NASA and ASME formulations versus generation numbers are shown in Figures 5 and 6 for glass and carbon epoxy cylinders, respectively.

The final aim of the optimization process is maximization of critical buckling load. As a result, the values of the cost function presented in Figures 5 and 6, are negative. Based on the presented data in Table 3, the

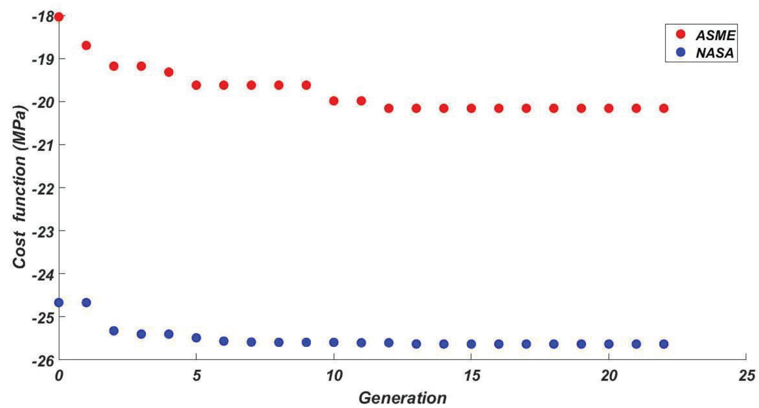


Figure 5 Optimization histories of glass epoxy cylinders with 5 layers.

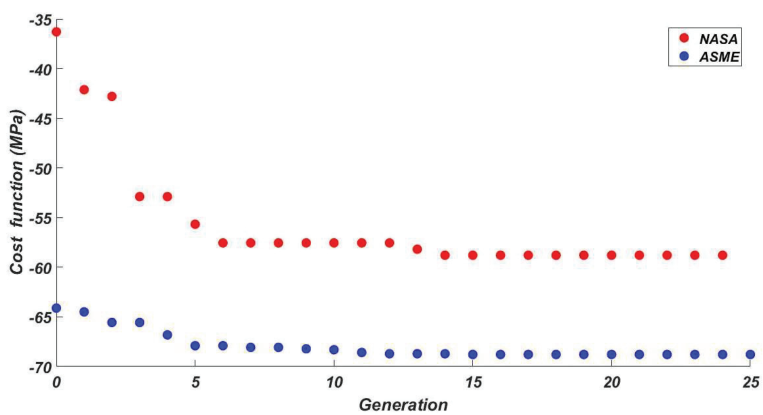


Figure 6 Optimization histories of carbon epoxy cylinders with 5 layers.

population size in GA is set at 50 being indicative of 50 function evaluations per generation and the total number of $25 * 50 = 1250$ function evaluations after 25 generations according to Figures 5 and 6. Please note that due to manufacturing limitations, for the mentioned discrete optimization problem, the fiber orientations of each layer are only integer values between 20 and 90 degrees with 5 degrees of intervals. The total number of cost function evaluations while considering all orientations of layers for the mentioned composite cylinder with 5 layers is equal to $15^5 = 759375$. The significant reduction of cost function evaluations from 759375 to 1250 shows the fact that the GA is a very cost-effective and time-saving method that can optimize stacking sequence efficiently.

4.4 Sensitivity Analysis

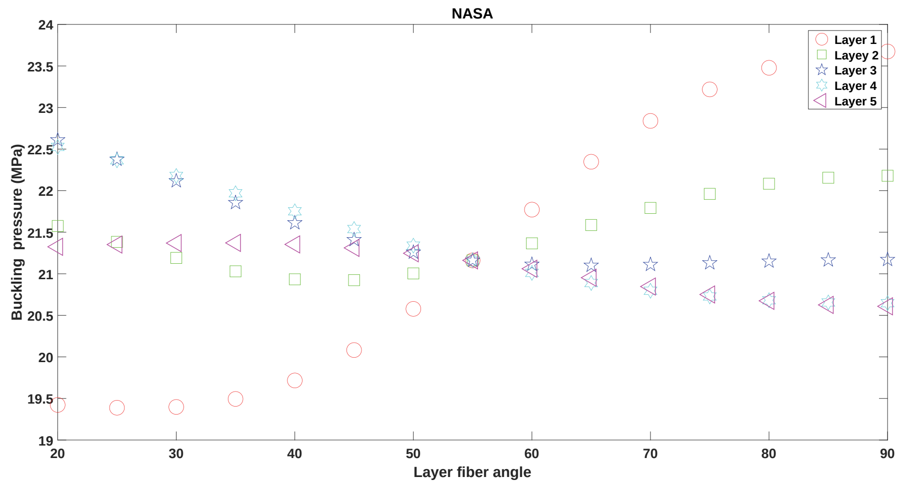
A sensitivity analysis is used in order to have a better understanding of results and determine the relationships between input and output variables. In the current method for sensitivity analysis, by moving one input variable in the specified range (20 to 90 degrees) and keeping others constant at their mean values, the effect of that input on the outputs is observed. Repeating this step for all inputs in the same way, the sensitivity analysis is completed. The input parameters for all layers have the same scale. In this paper, the input variables are the orientations of layers, and the output is critical buckling pressure. The sensitivity analysis of critical buckling pressure corresponding to optimal laminations of NASA and ASME formulations for glass and carbon epoxy cylinders with 5 layers are presented in Figures 7 and 8, respectively.

It can be seen in Figures 7 and 8 that the trends of sensitivity figures are compatible to the results. For instance, similar to the optimization results, the sensitivity analysis in glass epoxy cylinders with 5 layers show that by increasing the angle in the first layer, the magnitude of the buckling load increases. As mentioned in Section 4.2, the values of n_1 and n_3 in ASME characteristic pattern are equal. Therefore, in the sensitivity analysis while using ASME formulations in GA, the fourth and fifth layers have the same trend similar to the first and second layers, and for the sake of simplicity, they are not plotted in Figures 7(b) and 8(b).

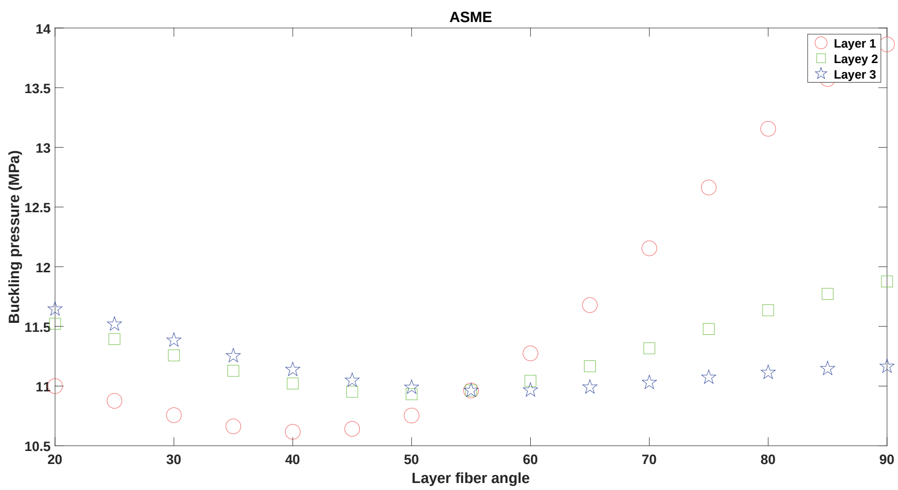
4.5 Evaluation of Optimization Method

In this section, the optimization problem has become simpler than previous sections in such a way that orientations can have only integer values of [30, 45, 60, 75, 90] degrees. Hence, each layer can only have 5 states of orientations.

In order to evaluate optimization methods based on NASA and ASME formulations, FE analyses are performed for all possible orientations of layers in this optimization problem. By changing the fiber orientations automatically, the buckling pressure related to each stacking sequence is obtained. For composite cylinders with 3 layers, 4 layers and 5 layers, respectively $5^3 = 125$, $5^4 = 625$ and $5^5 = 3125$ states for the orientation of layers are checked out, and the results are compared. Among all FE outputs, for each model, the best stacking sequence (true global optimum) and its related buckling pressure is extracted and presented in Table 5. For each model, the optimal laminations obtained by NASA and ASME formulations are also calculated and presented in Table 5.



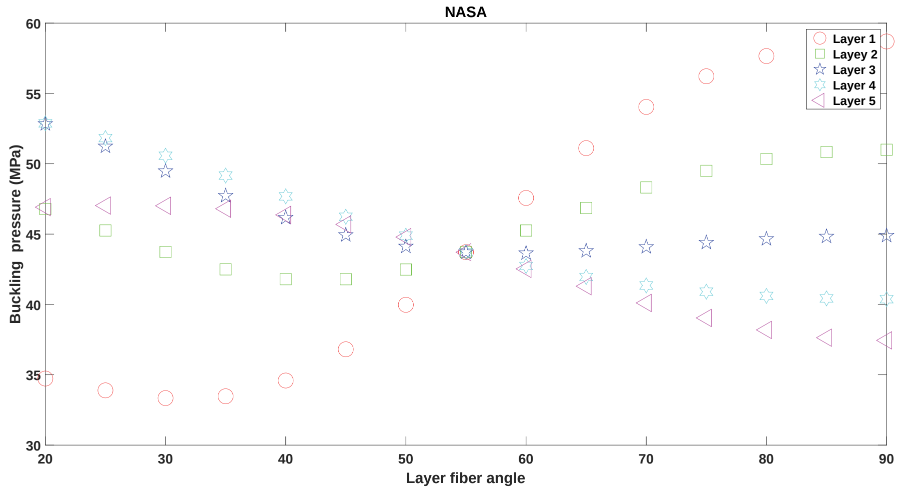
(a) Variations of buckling load calculated based on NASA formulations among different orientation of layers



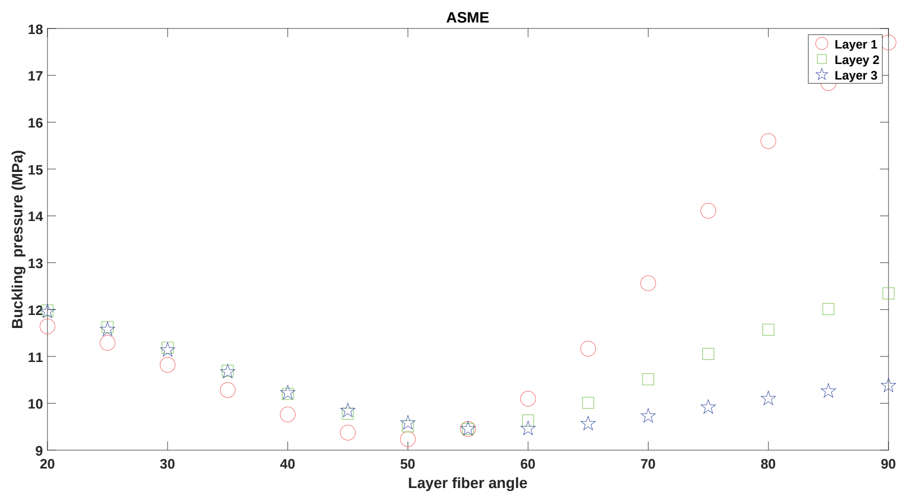
(b) Variations of buckling load calculated based on ASME formulations among different orientation of layers

Figure 7 Sensitivity analysis for glass epoxy cylinders with 5 layers.

The presented values of critical buckling pressure in Table 5 shows that by checking out all possible orientations of layers, maximum optimality is observed for the case of a cylinder with 5 layers, where optimality of critical buckling pressure is only 2% more than critical buckling pressure obtained from NASA prediction of optimal lamination. Hence, for this case, without checking all time-consuming 3125 solutions of the problem, NASA



(a) Variations of buckling load calculated based on NASA formulations among different orientation of layers



(b) Variations of buckling load calculated based on ASME formulations among different orientation of layers

Figure 8 Sensitivity analysis for carbon epoxy cylinders with 5 layers.

formulation predicts the optimal laminations efficiently, and is applicable for the solution of more complicated optimization problems. This method leads to the optimal laminations that correlate very well with the results of experiments and FE simulations.

For more complicated optimization problems, deriving exact solutions or checking all possible orientations of layers for the model would be

Table 5 Optimal laminations and corresponding critical buckling pressure of glass epoxy cylinders for a simple optimization problem

Number of Layers for the Cylinder	Global Optimum by FE	Critical Buckling Pressure by FE Analysis	NASA Optimized Stacking Sequence	Critical Buckling Pressure by FE Analysis	ASME Optimized Stacking Sequence	Critical Buckling Pressure by FE Analysis
1	[90]	30.069	[90]	30.069	[90]	30.069
2	[90_2]	30.069	[90,60]	29.222	[90_2]	30.069
3	[75,30,90]	33.373	[90,30,90]	33.177	[90,30,90]	33.177
4	[90,45,30,90]	33.893	[90,30_2,90]	33.865	[90,30_2,90]	33.865
5	[75,45,30_2,90]	34.358	[90,75,30_2,90]	33.560	[90_2,30,90_2]	32.180

unpractical and impossible. In these cases, NASA formulation can be used in combination with a GA. This optimization method can predict optimal lamination of cylindrical composite shells efficiently and is applicable for the design of various composite structures such as underwater composite cylindrical vessels.

5 Conclusions

A FE model was presented in order to calculate the critical buckling pressure of composite cylinders subjected to external pressure. The values of critical buckling pressure were also obtained using NASA SP-8007 and ASME RD-1172 formulations. Obtained results were compared with previous experimental tests for both glass and carbon epoxy cylinders, and it was observed that:

- The results of the FE method and NASA formulations were in a good agreement with experimental data.
- Obtained values from ASME formulations were lower than other methods, and this simple formulation should not be used in the prediction of critical buckling pressure without corrective safety factor.

GA was used to optimize the stacking sequences based on the NASA and ASME formulations in order to maximize the critical buckling pressure. Corresponding buckling values of optimized laminations were obtained by the presented FE model, and the optimality of stacking sequences was evaluated and verified. The results showed that:

- Optimization of stacking sequence influenced the corresponding critical buckling value considerably
- For discrete optimization problem with θ values starting from 20 degrees ending to 90 degrees, NASA and ASME characteristic patterns of optimized laminations were $[90_{m1}, \theta_{m2}, 20_{m3}, \theta_{m4}, 90_{m5}]$ and $[90_{n1}, 20_{n2}, 90_{n3}]$, respectively, that the values of $n1$ and $n3$ were equal in most cases
- The values of critical buckling pressure for cylinders with optimal laminations predicted by NASA formulation were greater than ASME ones that showed the preference of NASA formulation in optimization
- For the carbon epoxy cylinders, the influence of lamination optimality on the buckling pressure was more than glass epoxy cylinders due to more directed properties

It can finally be concluded that the stacking sequence optimization of cylindrical composite shells by NASA formulation in conjunction with an optimization method such as GA, results in reliable and applicable predictions via a cost-effective and simple manner.

Conflict of Interest Statement

On behalf of all authors, the corresponding author states that there is no conflict of interest.

References

[1] Moon C-J, Kim I-H, Choi B-H, Kweon J-H, Choi J-H. Buckling of filament-wound composite cylinders subjected to hydrostatic pressure for underwater vehicle applications. Composite Structures 2010;92:2241–51.

[2] Lopatin A, Morozov E. Buckling of a composite cantilever circular cylindrical shell subjected to uniform external lateral pressure. Composite Structures 2012;94:553–62.

[3] Cagdas IU, Adali S. Buckling of cross-ply cylinders under hydrostatic pressure considering pressure stiffness. Ocean engineering 2011;38:559–69.

[4] Jung H-Y, Cho J-R, Han J-Y, Lee W-H, Bae W-B, Cho Y-S. A study on buckling of filament-wound cylindrical shells under hydrostatic external pressure using finite element analysis and buckling formula. International Journal of Precision Engineering and Manufacturing 2012;13:731–7.

[5] Govindaraj M, Murthy HN, Patil S, Sudarsan K, Nandagopan O, Kumar A, et al. Buckling behaviour of underwater vessels by experimental, numerical and analytical approaches. Journal of Naval Architecture and Marine Engineering 2014;11:15–28.

[6] Dey P, KM Pandey A. A Computational Study of Buckling Analysis of Filament Wound Composite Pressure Vessel Subjected to Hydrostatic Pressure. Global Journal of Researches In Engineering 2014;14.

[7] Le Riche R, Haftka RT. Optimization of laminate stacking sequence for buckling load maximization by genetic algorithm. AIAA journal 1993;31:951–6.

[8] Le Riche R, Haftka R. Improved genetic algorithm for minimum thickness composite laminate design. Composites Engineering 1995;5:143–61.

[9] Soremekun G, Gürdal Z, Haftka R, Watson L. Composite laminate design optimization by genetic algorithm with generalized elitist selection. Computers & structures 2001;79:131–43.

[10] Messager T, Pyrz M, Gineste B, Chauchot P. Optimal laminations of thin underwater composite cylindrical vessels. Composite Structures 2002;58:529–37.

[11] Costa L, Fernandes L, Figueiredo I, Júdice J, Leal R, Oliveira P. Multiple-and single-objective approaches to laminate optimization with genetic algorithms. Structural and multidisciplinary optimization 2004;27:55–65.

[12] Kim T-U, Shin JW, Hwang IH. Stacking sequence design of a composite wing under a random gust using a genetic algorithm. Computers & structures 2007;85:579–85.

[13] Karakaya Ş, Soykasap Ö. Buckling optimization of laminated composite plates using genetic algorithm and generalized pattern search algorithm. Structural and Multidisciplinary Optimization 2009;39:477–86.

[14] Gillet A, Francescato P, Saffre P. Single-and multi-objective optimization of composite structures: the influence of design variables. Journal of composite materials 2010;44:457–80.

[15] Lee G-C, Kim J-S, Kweon J-H, Choi J-H. Optimal Design of Composite Stiffened Cylinder Subject to External Hydrostatic Pressure. Advanced Science Letters 2012;15:297–300.

[16] da Silva R, da Rocha I, Parente Jr E, de Melo A. Optimum design of composite risers using a genetic algorithm. Blucher Mechanical Engineering Proceedings 2012;1:2578–97.

[17] Gyan S, Ganguli R, Naik GN. Damage-tolerant design optimization of laminated composite structures using dispersion of ply angles by genetic

algorithm. Journal of Reinforced Plastics and Composites 2012;31: 799–814.
[18] Chen S, Lin Z, An H, Huang H, Kong C. Stacking sequence optimization with genetic algorithm using a two-level approximation. Structural and Multidisciplinary Optimization 2013;48:795–805.
[19] Geng P, Xing J, Chen X. Winding angle optimization of filament-wound cylindrical vessel under internal pressure. Archive of Applied Mechanics 2017;87:365–84.
[20] Almeida Jr JHS, Ribeiro ML, Tita V, Amico SC. Stacking sequence optimization in composite tubes under internal pressure based on genetic algorithm accounting for progressive damage. Composite Structures 2017;178:20–6.
[21] Pathan M, Patsias S, Tagarielli V. A real-coded genetic algorithm for optimizing the damping response of composite laminates. Computers & Structures 2018;198:51–60.
[22] Kaw AK. Mechanics of composite materials: CRC press; 2005.
[23] Jones RM. Buckling of Circular Cylindrical Shells with Multiple Orthotropic Layers and Eccentric Stiffeners. Aerospace Corporation San Bernardino Operations; 1967.
[24] NASA. Buckling of thin-walled circular cylinders. SP-8007. Virginia1968.
[25] ASME. Boiler and pressure vessel code. X (Fiber-reinforced plastic pressure vessels). New York2013.

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