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Estimation of thermal contact resistance in fin–tube heat exchanger using inverse heat transfer methods

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Thermal contact resistance (TCR) is a very important phenomenon in heat transfer problems, such as power generation, air conditioning, refrigeration, aerospace, etc. due to the complex mechanism of heat transfer in the contact surfaces; measurements and calculations of the TCR have many difficulties and problems. Today, one of the effective methods to investigate these problems is the use of inverse heat transfer. The objective of this work is to estimate the contact heat transfer coefficient (reverse of TCR) between the tube and its fin in heat exchanger. Two different methods, consisting of Levenberg–Marquardt for parameter estimation and conjugate gradient with adjoint problem for function estimation conjugate gradient method (CGM), are used. Results show that the CGM is successfully applied for the solution of the inverse problem to determine the unknown time-dependent TCR.

Keywords: thermal contact resistance; fin-tube heat exchanger; inverse heat transfer methods

Introduction

The thermal contact resistance (TCR) is of interest in many fields in industries, such as heat exchangers. The thermal performance of the heat exchanger is dependent on this parameter. In the past years, the study of thermal contact has been of increasing interest to researchers and industrial engineers (Ayers, 2003; Lambert & Fletcher, 1997; Litke, 2002; Madhusudana, 1996; Rosochowska, Chodnikiewicz, & Balendra, 2004; Zhang, Cong, & Fujii, 2006). A number of these researches are related to fixed thermal contact. A study on the effect of TCR in a fin–tube heat exchanger was first attempted by Dart (1959). In his study, TCR was tested with several samples with two passages, which were one for cold and the other for hot water. To minimise the influence of the natural convection, the tube was placed in a vacuumed space. Abuebid (1984) investigated the TCR with plate-finned tube heat exchangers placed in a vacuum. He performed an error analysis, which is similar to the Eckels' method. But the error band was narrower. Shah (1986) investigated the effect of pressure distribution on the collar, analysing the temperature distribution in the fin and collar. Eckels and Rabas (1987) predicted the thermal contact conductance by varying the number of fin, the fin thickness and the diameter of tube in the wet and the dry fin–tube heat exchangers. In addition, they improved the empirical method, based on Draft's method, including error analysis. Sheffield, Sauer, and Wood (1987) considered the

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contact pressure as the significant factor for the TCR. They introduced the influence of surface hardness and studied the correlation between contact pressure and expansion interference (hardness and roughness). Nho and Yovanovich (1989) found that the TCR, as a fraction of the overall resistance in a vacuum environment, ranged from 17.6 to 31.5%. Stubblefield, Pang, and Coundy (1996) studied the heat loss by TCR using insulated pipe and presented a simple method to predict the effect of contact resistance. Salgon (1997) theoretically predicted the thermal contact conductance, which was presented as a function of contact pressure and compared to the experimental data. Kim, Jeong, Young, and Kim (2004) evaluated the thermal contact conductance using experimental–numerical method. They evaluated thermal contact conductance on various fin–tube heat exchangers with 9.25 mm tubes. Therefore, a new correlation between the thermal contact conductance and effective factors, such as expansion ratio, fin type, fin spacing and hydrophilic coating, has been developed. Jeong, Kim, and Youn (2006) investigated the new factors, such as fin types (plate fin, slit fin and wide slit fin) and manufacturing types of the tube (drawn tube and welded tube), affecting the thermal contact conductance and presented a new correlation between the effective factors and the thermal contact conductance in fin–tube heat exchangers with 7 mm tubes. Several studies have discovered the forming of defects at the joint which causes deterioration of the heat transfer. Yang (2007) estimated the TCR and thermally induced optical effects in single-coated optical fibres. He calculated TCR by the conjugate gradient method (CGM) and shows that CGM was successfully applied for the solution of the inverse problem to determine the unknown time-dependent TCR of a carbon-coated optical fibre, while knowing the temperature history at the measurement location in the optical fibre. Subsequently, the temperature distributions, the thermally induced transient micro-bending loss and the refractive index changes of the optical fibre are also calculated. Critoph, Holland, and Turner (1996) examined the tube–fin interface under the microscope and reported that a gap of .01 mm would increase the overall resistance by 10%. ElSherbini and Jacobi (2002) noted that high contact resistance can disappear, when a thin layer of frost fills the gaps between the fins and tubes. Cheng and Madhusudana (2006) evaluated the TCR of tube–fin heat exchanger with direct method by measuring the temperature drop and heat flux across the interface. Ding, Dayong, Yinghong, and Zhaohui (2010) provided a novel method for a preliminary evaluation of the thermal contact conductance with numerical method. They investigated tube/fin-joining status after expansion process and contact pressure, as well as portions of contact area between the fin collar and the tube.

In the research conducted so far, the inverse heat transfer methods to estimate the unknown TCR in fin–tube heat exchangers, as one of the most important design parameters, is not used. Thus, in this paper, the estimation of TCR is investigated using the inverse heat transfer techniques.

The definition of the problem

TCR occurs where two solid specimens (such as pipe and fin in tube–fin heat exchangers) are pressed together. It has long been realised that surfaces are rough on a microscopic scale, which causes the real contact area to be significantly smaller compared to the nominal contact area. This behaviour causes a large temperature drop and TCR in the contacting surface. The existence of TCR has a great influence on the thermal performance of the heat exchangers. Thus, the TCR or

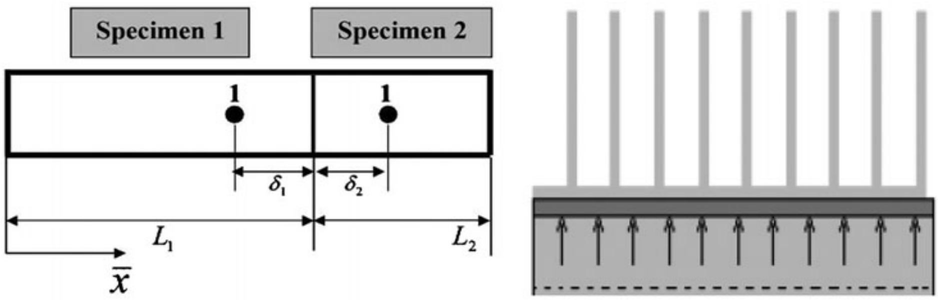

Figure 1. (a) Contacting specimens (b) Circular L-shaped fins.

heat flow at the interface of two specimens must be determined, and then controlled.

Figure 1 shows a simple schematic of the problem. The two cylindrical samples as tube and fin (specimen 1 and specimen 2), are in contact with each other. The thermal contact conductance coefficient between them is $h(t)$, which is the reverse of TCR $R_c(t)$. The end of other samples, one is in constant temperature and the other one is insulated.

Because of the complexity of heat flow mechanism in the connection surface, the heat flux in the contact surface is simulated according to the following equation (Ozisik, 1993):

$$q = h_c(T_{c1} - T_{c2}) \tag{1}$$

Since the aim of this research is mostly the study of TCR, the heat transfer in samples has been considered only for conduction. Therefore, the equations of the discussed problem are:

For specimen 1:

$$K_1 \frac{\partial^2 T_1}{\partial x^2} = \rho_1 c_{p1} \frac{\partial T_1}{\partial t} \ (0 < x < L \text{ and } t > 0) \tag{2-1}$$

$$T_1 = T_A \ (x = 0 \text{ and } t > 0) \tag{2-2}$$

$$-K_1 \frac{\partial T_1}{\partial x} = h(t)[T_{c1} - T_{c2}] \ (x = L \text{ and } t > 0) \tag{2-3}$$

$$T_1(x, 0) = T_{\text{inf}} \tag{2-4}$$

For specimen 2:

$$K_2 \frac{\partial^2 T_2}{\partial x^2} = \rho_2 c_{p2} \frac{\partial T_2}{\partial t} \ (L < x < 2L \text{ and } t > 0) \tag{3-1}$$

$$-K_2 \frac{\partial T_2}{\partial x} = h(t)[T_{c1} - T_{c2}] \ (x = L \text{ and } t>0) \tag{3-2}$$

$$\frac{\partial T_2}{\partial x} = 0 \ (x = 2L \text{ and } t>0) \tag{3-3}$$

$$T_2(x, 0) = T_{\text{inf}} \tag{3-4}$$

Inverse methods

The inverse heat conduction problem of interest is defined using the above equations. The unknown parameter is the thermal contact conductance. The main objective of the IHCP is the calculation of the thermal contact conductance or heat flux in contact surface. In this paper, two powerful techniques, one is Levenberg–Marquardt method for parameter estimation and the other is CGM with adjoint problem, are used for inverse heat transfer problems.

Technique 1: Levenberg–Marquardt

The first technique is an iterative method for solving nonlinear least squares problems of parameter estimation (Ozisik & Orlande, 2000). The solution of inverse heat transfer problem with the Levenberg–Marquardt method can be suitably arranged in the following basic steps:

The direct problem

In the direct problem associated with the physical problem described above, the time-varying thermal contact conductance $h(t)$ is known. The objective of the direct problem is then to determine the transient temperature field $T(x, t)$. For this purpose, the finite volume method is used. The central difference scheme and implicit method is used for temporal and spatial discretisation, respectively. The number of grids on each sample is 20 in this study. In this study, the time step is 1 s.

The inverse problem

For the inverse problem considered here, the thermal contact conductance is regarded as the unknown parameter. The additional information obtained from transient temperature measurements taken at a location $x = x_{\text{meas}}$, at times t_i, $i = 1, 2, \ldots, I$, is then used for the estimation of $h(t)$.

For the solution of the present inverse problem, we consider the unknown function $h(t)$ to be parameterised in the following general linear form:

$$h(t) = \sum_{j=1}^{N} \left(p_j C_j(t)\right) \tag{4}$$

Here, p_j are unknown parameters and $C_j(t)$ are known trial functions (e.g. polynomials, B-plines, etc.). In addition, the total number of parameters, N, is specified. And in this problem, we considered:

$$h(t) = p_1 \times t^n \tag{5}$$

The problem given by Equations (2) and (3) with $h(t)$ unknown, but parameterized as given by Equation (5), is an inverse heat conduction problem in which the coefficient p_j is to be estimated. The solution of this inverse heat conduction problem for the estimation of the N unknown parameters p_j, $j = 1, \ldots N$ is based on the minimisation of the ordinary least squares norm, given by:

$$S(p_j) = \sum_{i=1}^{I} [Y_i - T_i(p_j)]^2 \tag{6}$$

where S: sum of squares error or objective function; $p_j^T \equiv [p_1, p_2, \ldots, p_N]$: vector of unknown parameters; $T_i(p_j) \equiv T(p_j, t_i)$: estimated temperature at time t_i; $Y_i \equiv Y(t_i)$: measured temperature at time t_i; N: total number of unknown parameters; I: total number of measurements, where $I \geq N$. And this problem we considered:

$$S(p_1) = \sum_{i=1}^{I} \sum_{j=1}^{N_1} (T_{1ij} - Y_{1ij})^2 + \sum_{i=1}^{I} \sum_{k=1}^{N_2} (T_{2ik} - Y_{2ik})^2 \tag{7}$$

The estimated temperatures $T_i(p_j)$ are obtained from the solution of the direct problem at the measurement location, x_{meas}, by using the current estimate for the unknown parameters p_j, $j = 1, \ldots N$. Equation (6) can be written in matrix form as:

$$S(p_j) = [Y - T(p_j)]^T [Y - T(p_j)] \tag{8}$$

The iterative procedure for technique

To minimise the least squares norm given by Equation (7), we needed to equate to zero the derivatives of $S(p_j)$ with respect to each of the unknown parameters $[p_1, p_2, \ldots, p_{\mathrm{N}}]$, that is

$$\frac{\partial S(p_j)}{\partial p_1} = \frac{\partial S(p_j)}{\partial p_2} = \ldots = \frac{\partial S(p_j)}{\partial p_N} = 0 \tag{9}$$

Such necessary conditions for the minimisation of $S(p_j)$ can be represented in matrix notation by equating the gradient of $S(p_j)$ with respect to the vector of parameters p_j to zero, that is

$$\nabla S(p_j) = 2\left[-\frac{\partial T^T(p_j)}{\partial p_j}\right][Y - T(p_j)] = 0 \tag{10}$$

The sensitivity or Jacobian matrix, $J(p_j)$, is defined as the transpose of $\left[-\frac{\partial T^T(p_j)}{\partial p_j}\right]$, that is,

$$J(p_j) = \left[\frac{\partial T^T(p_j)}{\partial p_j}\right]^T \tag{11}$$

By using the definition of the sensitivity matrix, Equation (10) becomes

$$-2J^T(p_j)[Y - T(p_j)] = 0 \tag{12}$$

There are several different approaches for the computation of the sensitivity coefficient. We present below the boundary value problem approach for determining the sensitivity coefficients: a boundary value problem can be developed for the determination of the sensitivity coefficients by differentiating the original direct problem with respect to the unknown coefficients. If the direct heat conduction problem is linear, the construction of the corresponding sensitivity problem is a relatively simple straightforward matter (Ozisik & Orlande, 2000). For this problem:

For specimen 1:

$$K_1 \frac{\partial^2 J_1}{\partial x^2} = \rho_1 c_{\mathrm{p1}} \frac{\partial J_1}{\partial t} \quad (0<x<L \text{ and } t>0) \tag{13-1}$$

$$J_1 = 0 \ (x = 0 \text{ and } t>0) \tag{13-2}$$

$$-K_1 \frac{\partial J_1}{\partial x} = t^n[T_{\mathrm{c1}} - T_{\mathrm{c2}}] + p_1 t^n [J_{\mathrm{c1}} - J_{\mathrm{c2}}] \ (x = L \text{ and } t>0) \tag{13-3}$$

$$J_1(x, 0) = 0 \tag{13-4}$$

For specimen 2:

$$K_2 \frac{\partial^2 J_2}{\partial x^2} = \rho_2 c_{\mathrm{p2}} \frac{\partial J_2}{\partial t} \quad (L<x<2L \text{ and } t>0) \tag{14-1}$$

$$-K_2 \frac{\partial J_2}{\partial x} = t^n[T_{\mathrm{c1}} - T_{\mathrm{c2}}] + p_1 t^n [J_{\mathrm{c1}} - J_{\mathrm{c2}}] \ (x = L \text{ and } t>0) \tag{14-2}$$

$$\frac{\partial J_2}{\partial x} = 0 \ (x = 2L \text{ and } t>0) \tag{14-3}$$

$$J_2(x, 0) = 0 \tag{14-4}$$

In the case of a nonlinear problem, the sensitivity matrix has some functional dependence on the vector of unknown parameters p_j. The solution of Equation (12) for nonlinear estimation problems then requires an iterative procedure, which is obtained by linearising the vector of estimated temperatures, $T(p_j)$, with a Taylor series expansion around the current solution p_j^k at iteration k. Such a linearisation is given by:

$$T(p_j) = T(p_j^k) + J^k(p_j - p_j^k) \tag{15}$$

The resulting expression is rearranged to yield the following iterative procedure to obtain (Ozisik, 1993) the vector of unknown parameters p_j:

$$p_j^{k+1} = p_j^k + \left[(J^k)^T J^k\right]^{-1} (J^k)^T \left[Y - T(p_j^k)\right] \tag{16}$$

The iterative procedure given by Equation (16) is called the Gauss method. We require the matrix $J^T J$ to be nonsingular, or $|J^T J| \neq 0$. Where $|.|$ is the determinate. The Levenberg–Marquardt method alleviates such difficulties by utilising an iterative procedure in the form:

$$p_j^{k+1} = p_j^k + \left[(J^k)^T J^k + \mu^k \Omega^k\right]^{-1} \left(J^k\right)^T \left[Y - T(p_j^k)\right] \tag{17}$$

The purpose of the matrix term $\mu^k \Omega^k$, included Equation (17), is to damp oscillations and instabilities due to the ill-conditioned character of the problem by making its components large as compared to those of $J^T J$ if necessary.

The stopping criteria

The following criteria were suggested by Dennis and Schnabel to stop the iterative procedure of the Levenberg–Marquardt method:

$$S\left(p_j^{k+1}\right) < \varepsilon_1 \tag{18-1}$$

$$\|p_j^{k+1} - p_j^k\| < \varepsilon_2 \tag{18-2}$$

where ε_1, ε_2 and ε_3 are user-prescribed tolerance and $\|.\|$ is the vector Euclidean norm, i.e., $x = (x^T x)^{1/2}$, where the superscript denotes transpose.

In this paper we considered $\varepsilon = 10^{-3}$.

The computational algorithm

Different versions of the Levenberg–Marquardt method depend on the choice of the diagonal matrix Ω^k and on the form chosen for the variation of the damping parameter μ^k. We illustrate here a procedure with the matrix Ω^k taken as

$$\Omega^k = \text{diag}\left[(J^k)^T J^k\right] \tag{19}$$

Suppose that temperature measurements $Y = (Y_1, Y_2, \ldots, Y_I)$ are given at times t_i, $i = 1, \ldots, I$. Also, suppose an initial guess P^0 is available for the vector of unknown parameters P. Choose a value for μ^0, ($\mu^0 = 0.001$) and set. Then,

Step 1. Solve the direct heat transfer problem, with the available estimate P^k in order to obtain the temperature vector $T(p_j^k) = (T_1, T_2, \ldots, T_I)$.
Step 2. Compute $S(p_j^k)$.
Step 3. Compute the sensitivity matrix and then the matrix Ω^k by using the current values of P^k.
Step 4. Solve the following linear system of algebraic equation:

$$\left[(J^k)^T J^k + \mu^k \Omega^k\right] \Delta p_j{}^k = (J^k)^T \left[Y - T(p_j^k)\right] \tag{20}$$

In order to compute Δp^k, $\Delta p_j{}^k = p_j^{k+1} - p_j^k$
Step 5. Compute the new estimate p_j^{k+1} as

$$p_j^{k+1} = p_j^k + \Delta p_j{}^k \tag{21}$$

Step 6. Solve the direct problem with the new estimate p^{k+1} in order to find $T(p^{k+1})$. Then, compute $S(p_j^{k+1})$.
Step 7. If $S(p_j^{k+1}) \geq S(p_j^k)$, replace μ^k by 10 μ^k and return to step 4.
Step 8. If $S(p_j^{k+1}) < S(p_j^k)$, accept the new p_j^{k+1} and replace μ^k by 0.1 μ^k.
Step 9. Check the stopping criteria. Stop the iterative procedure if is satisfied; otherwise, replace k by $k+1$ and return to step 3.

Technique 2: CGM with adjoint problem for function estimation

Here, we present a powerful iterative minimisation scheme called the 'conjugate radient method of minimisation with adjoint problem' for solving inverse heat transfer problems of function estimation. In this approach, no a priori information on the functional form of the unknown function is considered available, except for the functional space that it belongs to. To illustrate this technique, the basic steps for the solution of function estimation problem, include:

The direct problem

The direct problem has been described above.

The inverse problem

The inverse problem, on the other hand, is concerned with the estimation of the unknown function $h(t)$ by using the reading taken by a sensor located $x = x_{\text{meas}}$.

The sensitivity problem

The sensitivity function $\Delta T(x, t)$ solution of the sensitivity problem is defined as the directional derivative of the temperature $T(x, t)$ in the direction of the perturbation of the unknown function. The sensitivity function is needed for the computation of the search step size.

The sensitivity problem is obtained from the direct problem in the following manner. It is assumed that when $h(t)$ undergoes a variation $\Delta h(t)$, $T_1(x, t)$ is perturbed by $\Delta T_1(x, t)$ and $T_2(x, t)$ is perturbed by $\Delta T_2(x, t)$. Then, by replacing in the direct problem $h(t)$ by

$[h(t) + \Delta h(t)]$, $T_1(x,t)$ by $[T_1(x,t) + \Delta T_1(x,t)]$ and $T_2(x,t)$ by $[T_2(x,t) + \Delta T_2(x,t)]$, subtracting from the resulting expressions the original direct problem and neglecting second-order terms, the sensitivity problem for the sensitivity functions $\Delta T_1(x,t)$ and $\Delta T_2(x,t)$ is obtained.

For specimen 1:

$$K_1 \frac{\partial^2 \Delta T_1}{\partial x^2} = \rho_1 c_{p1} \frac{\partial T_1}{\partial t} \ (0<x<L \text{ and } t>0) \tag{22-1}$$

$$\Delta T_1 = 0 \ (x = 0 \text{ and } t>0) \tag{22-2}$$

$$-K_1 \frac{\partial \Delta T_1}{\partial x} = h(t)[\Delta T_{c1} - \Delta T_{c2}] + \Delta h_c[T_{c1} - T_{c2}] \ (x = L \text{ and } t> 0) \tag{22-3}$$

$$\Delta T_1(x,0) = 0 \tag{22-4}$$

For specimen 2:

$$K_2 \frac{\partial^2 \Delta T_2}{\partial x^2} = \rho_2 c_{p2} \frac{\partial \Delta T_2}{\partial t} \ (L<x< 2L \text{ and } t>0) \tag{23-1}$$

$$-K_2 \frac{\partial \Delta T_2}{\partial x} = h(t)[\Delta T_{c1} - \Delta T_{c2}] + \Delta h_c[T_{c1} - T_{c2}] \ (X = L \text{ and } t>0) \tag{23-2}$$

$$\frac{\partial \Delta T_2}{\partial x} = 0 \ (x = 2L \text{ and } t>0) \tag{23-3}$$

$$\Delta T_2(x,0) = 0 \tag{23-4}$$

The adjoint problem

In the present inverse problem, the estimated temperatures need to satisfy two constraints, which are the heat conduction problems for specimens 1 and 2. Therefore, two Lagrange multipliers come into image here. To obtain the adjoint problem, Equation (2-1) is multiplied by the Lagrange multiplier $\lambda_1(x,t)$, Equation (3-1) is multiplied by the Lagrange multiplier $\lambda_2(x,t)$ and the resulting expressions are integrated over the time and space domains. Then, the results are added to the right-hand side of Equation (7) to yield the following expression for the function $S[h(t)]$:

$$\begin{aligned} S[h(t)] = {} & \int_0^\tau \left[\sum_{j=1}^{N_1} (T_{1j} - Y_{1j})^2 \right] dt + \int_0^\tau \left[\sum_{k=1}^{N_2} (T_{2k} - Y_{2k})^2 \right] dt + \int_0^\tau \int_o^L \lambda_1(x,t) \\ & \times \left[K_1 \frac{\partial^2 T_1}{\partial x^2} - \rho_1 c_{p1} \frac{\partial T_1}{\partial t} \right] dx dt + \int_0^\tau \int_L^{2L} \lambda_2(x,t) \left[K_2 \frac{\partial^2 T_2}{\partial x^2} - \rho_1 c_{p1} \frac{\partial T_2}{\partial t} \right] dx dt \end{aligned} \tag{24}$$

The variation $\nabla S[h(t)]$ is obtained by perturbing $T_1(x,t)$ by $\Delta T_1(x,t)$ and $T_2(x,t)$ by $\Delta T_2(x,t)$, in Equation (24), subtracting from the resulting expression the original Equation (24) and neglecting second-order terms. Consequently, we find

$$\begin{aligned}\Delta S[h(t)] = & \int_0^\tau \int_0^L \sum_{j=1}^{N_1} 2\Delta T_{1j}(T_{1j} - Y_{1j})\delta(x - x_j)\mathrm{d}x\mathrm{d}t \\ & + \int_0^\tau \int_L^{2L} \sum_{k=1}^{N_2} 2\Delta T_{2k}(T_{2k} - Y_{2k})\delta(x - x_k)\mathrm{d}x\mathrm{d}t \\ & + \int_0^\tau \int_o^L \lambda_1(x,t)\left[K_1 \frac{\partial^2 T_1}{\partial x^2} - \rho_1 c_{\mathrm{p}1} \frac{\partial T_1}{\partial t}\right]\mathrm{d}x\mathrm{d}t \\ & + \int_0^\tau \int_L^{2L} \lambda_2(x,t)\left[K_2 \frac{\partial^2 T_2}{\partial x^2} - \rho_1 c_{\mathrm{p}1} \frac{\partial T_2}{\partial t}\right]\mathrm{d}x\mathrm{d}t \end{aligned} \tag{25}$$

In Equation (25), the last two integral terms are integrated by parts; the initial and boundary conditions of the sensitivity problem are utilised, and then $\Delta S[h(t)]$ is allowed to go to zero. The vanishing of the integrands containing $\Delta T_1(x,t)$ and $\Delta T_2(x,t)$ leads to the adjoint problem for the determination of the Lagrange multipliers $\lambda_1(x,t)$ and $\lambda_2(x,t)$

For specimen 1:

$$K_1 \frac{\partial^2 \lambda_1}{\partial x^2} \rho_1 c_{\mathrm{p}1} \frac{\partial J_1}{\partial t} + \sum_{j=1}^{N_1} 2\Delta T_{1j}(T_{1j} - Y_{1j})\delta(x - x_j) = 0 \ (0<x<L \text{ and } t>0) \tag{26-1}$$

$$\lambda_1 = 0 \, (x = 0 \text{ and } t > 0) \tag{26-2}$$

$$K_1 \frac{\partial \lambda_1}{\partial x} = h(t)[\lambda_2(L,t) - \lambda_1(L,t)] \ (x = L \text{ and } t>0) \tag{26-3}$$

$$\lambda_1(x,\tau) = 0 \tag{26-4}$$

For specimen 2:

$$K_2 \frac{\partial^2 \lambda_2}{\partial x^2} \rho_2 c_{\mathrm{p}2} \frac{\partial J_2}{\partial t} + \sum_{k=1}^{N_2} 2\Delta T_{2k}(T_{2k} - Y_{2k})\delta(x - x_k) = 0 \ (0<x<L \text{ and } t>0) \tag{27-1}$$

$$\lambda_2 = 0 \ (x = 0 \text{ and } t>0) \tag{27-2}$$

$$K_2 \frac{\partial \lambda_2}{\partial x} = h(t)[\lambda_2(L,t) - \lambda_1(L,t)] \ (x = L \text{ and } t>0) \tag{27-3}$$

$$\lambda_2(x,\tau) = 0 \tag{27-4}$$

The gradient equation

In the limiting process used to obtain the adjoint problem above, only one integral term is left; the following expression for the gradient $\nabla S(p)$ is obtained by:

$$\nabla S(p) = \{[\lambda_2(L,t) - \lambda_1(L,t)][T_{c1}(L,t) - T_{c2}(L,t)]\} \tag{28}$$

The iterative procedure

The iterative procedure of the CGM for the minimisation of the norm $S(h(t))$ is given by:

$$h^{k+1}(t) = h^k(t) - \beta^k d^k \tag{29}$$

The direct of descent is a conjugation of the gradient direction, $\nabla S(h^k(t))$, and the direction of descent of the previous iteration, d^{k-1}. It is given as:

$$d^k(t) = \nabla S h^k(t) \; + \gamma^k d^{k-1}(t) \tag{30}$$

The Fletcher–Reeves expression for the conjugate coefficient γ^k is given as (Ozisik & Orlande, 2000):

$$\gamma^k = \frac{\int_0^\tau \{\nabla S\lfloor h^k(t)\rfloor\}^2 \mathrm{d}t}{\int_0^\tau \{\nabla S\lfloor h^{k-1}(t)\rfloor\}^2 \mathrm{d}t} \text{ for } k = 1, 2, \ldots \text{ With } \gamma^k = 0 \text{ for } k = 0 \tag{31}$$

The search step size β^k appearing is obtained by minimising the function $S\left(h^{k+1}(t)\right)$ with respect to β^k. The minimisation with respect to β^k is performed to yield the following expression for the search step size:

$$\beta^k = \frac{\int_{t=0}^\tau \{\sum_{j=1}^{N_1}[(T_{1j} - Y_{1j})]\Delta T_{1j} + \sum_{k=1}^{N_2}[(T_{2k} - Y_{2k})]\Delta T_{2k}\}\mathrm{d}t}{\int_0^\tau \{\sum_{j=1}^{N_1}[\Delta T_{1j}]^2 + \sum_{k=1}^{N_2}[\Delta T_{2k}]^2\}\mathrm{d}t} \tag{32}$$

After computing the gradient direction, the conjugate coefficient and the search step size, the iterative procedure is implemented until a stopping criterion based on the discrepancy principle is satisfied.

The stopping criterion

As for the first technique, the stopping criterion is based on the discrepancy principle, when the standard deviation σ of the measurements is a priori known. It is given by:

$$S(p^k) < \varepsilon \tag{33}$$

In this paper we considered: $\varepsilon = 10^{-3}$

The computational algorithm

The computational algorithm for the CGM with adjoint problem for function estimation can be summarised as follows.

Suppose an initial guess $h^0(t)$ is available for the function $h(t)$. Set $k = 0$ and then:

Step 1: Solve the direct problem and compute $T_1(x, t)$ and $T_2(x, t)$ based on $h^0(t)$.
Step 2: Check the stopping criterion. Continue if not satisfied.
Step 3: Knowing $T_1(x, t)$ and $T_2(x, t)$ and measured temperatures $Y_1(t)$ and $Y_2(t)$, solve the adjoint problem and compute $\lambda_1(l, t)$ and $\lambda_2(l, t)$.
Step 4: Knowing $\lambda_1(l, t)$ and $\lambda_2(l, t)$, compute $\nabla S[h(t)]$.
Step 5: Knowing the gradient $\nabla S[h(t)]$, compute γ^k and then compute d^k.
Step 6: Set $\Delta h^k(t) = d^k(t)$ and solve the sensitivity problem to obtain $\Delta T_1(x, t)$ and $\Delta T_2(x, t)$.
Step 7: Knowing $\Delta T_1(x, t)$ and $\Delta T_2(x, t)$, compute β^k.
Step 8: Knowing $\beta^k$and d^k, compute the new estimate $h^{k+1}(t)$ and return to step 1.

Results and discussion

The purpose of this study is to estimate the thermal contact conductance between two samples (pipe of heat exchanger and the connected fin). The samples are constructed of aluminium. The physical and thermal characteristics of samples are represented in Table 1.

The initial temperatures of two materials are 20 °C. These two substances have connected with each other, one is in a temperature of 60 °C and another one is insulated. Both samples have insulations around them so that they do not lose temperature radially.

It is used from simulated data, because of absence of laboratory data that they are related to temperatures in both samples, in recognised positions. A series of simulated experiments is performed to ensure the ability of the inverse scheme in estimating the space-variable heat transfer coefficient. For this purpose, it is used from simulated data for two conditions:

(1) the thermal contact conduction that is fixed
(2) the thermal contact conduction that is nonlinear.

In order to compare the results for situations involving random measurement error, we assume normally distributed uncorrelated errors with zero mean and constant standard deviation. The simulated inexact measurement data Y can be expressed as:

Table 1. The physical and thermal characteristics of experimented samples.

Materials type	Aluminium
Conductivity coefficient (k)	237 W/m K
Density (ρ)	2702 kg/m3
Heat capacitance (c_p)	903 J/kg K
Length (L)	3 cm

$$Y = Y_{\text{exact}} + \omega\sigma \tag{34}$$

where Y_{exact} is the solution of the direct problem with the exact values of $h(t)$; σ is the standard deviation of the measurements; and ω is a random variable with normal distribution and zero mean. For 99% confidence level we have $-2.576<\omega<2.576$.

We use equidistant time steps of 1 s, both for the time discretisation and for sampling the data. For generation of the data, as well as for solving the inverse problem, a mesh with 40 control volumes is used. Since variations in the temperature are only due to physical conditions of the contacting surface, the mesh is chosen to be finer in the region of interest where the temperature sensors are located. The simulated data (noise-free temperature measurements) are generated by solving direct problem with all considerable state for thermal contact conductance by adding an allowable error. To avoid inverse crimes, the inverse problem is solved using only measurement data which are

Table 2. The results thermal contact conduction fixed by technique 1.

σ (standard deviation)	Number of iterations	CPU time (s)	h_c	Error %
.00	8	.4131	599.8931	.0178
.01	10	.5214	599.7832	.0361
.05	13	.7933	599.5102	.0816
.1	21	1.221	599.2734	.1211

Table 3. The results thermal contact conduction fixed by technique 2.

σ (standard deviation)	Number of iterations	CPU time (s)	h_c	Error %
.00	12	10.489	599.972	.00467
.01	18	22.205	600.069	.0115
.05	26	38.935	600.074	.0123
.1	59	46.261	600.131	.0218

Table 4. The results thermal contact conduction nonlinearly by technique 1.

σ (standard deviation)	Number of iterations	CPU time (s)	RMS error	$V=\text{RMS}^2-D^2$	$D=.0324$
.00	17	4.9931	.0324	0	
.01	24	7.2261	1.1818	1.3956	
.1	31	10.5486	1.9248	3.7038	

Table 5. The results thermal contact conduction nonlinearly by technique 2.

σ (standard deviation)	Number of iterations	CPU time (s)	RMS error	$V=\text{RMS}^2-D^2$	$D=1.0200$
.00	41	20.730	1.0200	0	
.01	81	38.823	2.0855	3.3089	
.1	120	59.341	4.1088	15.4422	

generated with small standard deviation. All numerical tests are realised in the open source finite volume code.

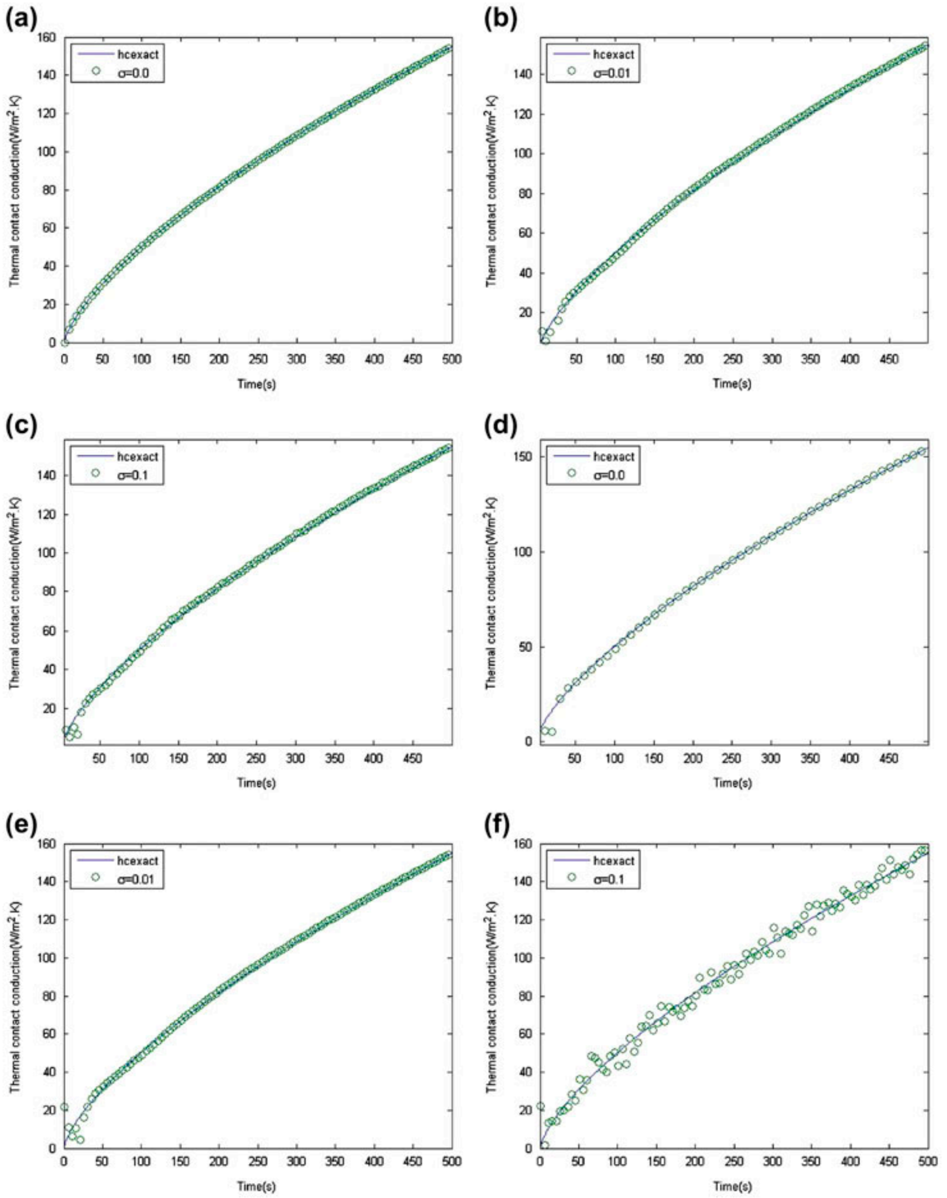

Figure 2. (a) The estimated function of $h(t)$ without adding σ by technique Levenberg–Marquardt; (b) the estimated function of $h(t)$ with $\sigma = 0.01$ by technique Levenberg–Marquardt; (c) the estimated function of $h(t)$ with $\sigma = 0.1$ by technique Levenberg–Marquardt; (d) the estimated function of $h(t)$ without adding σ by technique conjugate gradient with adjoint problem for estimated function; (e) the estimated function of $h(t)$ with $\sigma = 0.01$ by techniquegradient with adjoint problem for estimated function; and (f) the estimated function of $h(t)$ with $\sigma = 0.1$ by technique gradient with adjoint problem for estimated function.

(1) Thermal contact conduction that is fixed ($h = 600$):

The estimated results in this condition are shown in Tables 2 and 3.

According to the estimated data in this table, the CGM method is more exact, but it has more performing time and iterations number. The run time of the first method is less than the CGM method.

(2) Thermal contact conduction that is nonlinearly ($h = 2t^{0.7}$):

The results in this condition are shown in the Tables 4 and 5 and Figure 2. The results represent that the estimated parameter is associated with large errors when the noise data level is more. Bias (index of deviation from the actual value), variance (index of intensity of fluctuations) and sensitivity coefficients are good judgment parameters in determining the best experimental set-up. The final target of the estimations is the heat transfer coefficient on each interval. For each interval, bias (D) is defined for the IHCP as:

$$D = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\hat{h}_{\text{ci,nonoise}} - h_{\text{ci,true}}\right)^2} \tag{35}$$

where $\hat{h}_{\text{ci,nonoise}}$ noise is the estimated heat flux using the measurements containing no noise. The root mean square (RMS) error is also calculated by:

$$\text{RMS} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\hat{h}_{\text{ci,noisydata}} - h_{\text{ci,true}}\right)^2} \tag{36}$$

where $\hat{h}_{\text{ci,noisydata}}$ data is the estimated heat flux using the temperature data polluted with noise. In Figure 2, all the results are displayed simultaneously. Figure 2 shows the time-dependent thermal contact conductance at contacting surfaces of tube and fin for two inverse heat transfer method. From Figure 2, we found that the accuracy of the inverse solution is acceptable, and therefore this assumption can be used in real estimation. In fact, we can validate the estimation of thermal contact conductance with CGM by utilising the simulated measured data in order to use actual measured data containing errors as the input to the analysis.

Conclusions

The results can be expressed as follows:

An inverse heat conduction problem for estimating the thermal contact conductance between one-dimensional, constant property contacting specimens has been investigated with two different methods, which consist of Levenberg–Marquardt for parameter estimation and conjugate gradient with adjoint problem for function estimation. The results are obtained by using inexact (with random errors) simulated measured data. The results obtained with the both inverse methods are in good agreement with the supposed thermal contact conductance, even when inexact measurements with large standard deviation of the measurement errors are utilised in the analysis. The little deviations, which are due to the random errors presented in any simulated temperature measurements, can be negligible. Levenberg–Marquart method is useful for computing of a parameter; but, for computing of a function using of this method, we must give the general form of unknown function to it, to solve of unknown function, but CGM method don't need to give a primitive figure from function form. In all the stages, the CGM with adjoint problem for function estimation has converged later than other methods. Also, the CGM with adjoint problem for function estimation is mostly dependent on the primary guess than other methods.

Nomenclature

D	Direction of descent
$h(t)$	Thermal contact conductance
K	Thermal conductivity
L	Length of specimen
N	Number of sensors
p	Unknown parameters
I	Measurement times
Q	Heat flux
T	Estimated temperature
t	Time
Y	Measured temperature
Greek symbol	
τ	Final time
λ	Lagrange multiplier
β	Search step size
γ	Conjugation coefficient
Ω	Diagonal matrix
μ	Damping parameter
ρ	Density
Subscript	
1,2	Specimen 1,2
$1j$	Specimen 1
$2k$	Specimen 2
C	Contact
Inf	Ambient
Superscript	
k	Number of iterations

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