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CSP approach and interval computation for the coupling between static and dynamic requirements in the preliminary design of a compression spring

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In this study, a new design approach based on an interval computation method and the constraint satisfaction problem technique (CSP approach) was discussed. It has been applied in the design of a compression spring, implemented in the vehicle suspension system. A design process is proposed and compared with what is done in conventional design. IT allows making static and dynamic sizing in one step. In fact, with the CSP, static and dynamic requirements can be coupled in the same step of sizing. In the CSP all requirements imposed can be integrated from the beginning. So it avoids falling on the loop "design-simulate-back to the initial step in case of failure". In this study, the design parameters values of the compression spring generated by the CSP verify all requirements and the resulting simulation of the system behaviour respects all constraints required. The results obtained in this study affirmed that the suggested method is valid and potentially useful to the size dynamic system and can be applied easily and effectively.

Keywords: design; interval computation; constraint satisfaction problem; compression spring; suspension system; requirements

1. Introduction

Conventional design methodologies based on a loop "design-simulate-back at the initial stage in case of failure" appear to be increasingly obsolete. During the past years, many studies have been conducted to optimise design, more precisely at the pre-sizing step. We are interested here in optimising the design approach (Colton, Mark, & Ouellette, 1994; Hwang et al., 2006; Meyer & Yvars, 2012; Philipp, 2009; Song, Lee, & Choung, 2011; Teorey, Yang, & Fry, 1986; Yvars, Lafon, & Zimmer, 2009) based on tools and methodologies using constraint satisfaction techniques (Edmunds, Feldman, Hicks, & Mullineux, 2011; Eldon, 2002; Granvilliers, Monfroy, & Benhamou, 2001; Montanari, 1974; Moore, 1966). Our approach is applied to optimise the sizing of a compression spring (Deb & Goyal, 1998; Kulkani & Balasubrahmanyam, 1979; Paredes, 2009; Paredes, Sartor, & Daidie, 2005; Yokota, Taguchi, & Gen, 1997). Several software tools are available for sizing springs, in particular for compression springs. In most cases, it is either software validation of a given size, or tools allowing very low variability

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specifications (Deb & Goyal, 1998; Kulkani & Balasubrahmanyam, 1979; Paredes et al., 2005; Yokota et al., 1997).

In general, the design process used in those software tools is alike the process described in Figure 1. In this approach, we commonly find three main steps: the first one consists to estimate the design variables values based on static lows of calculation and expertise. The optimisation of the choice of those values requires a lot of expertise. In the second step, the designer makes a static test and fixes the safety factors according to the requirements imposed. The next step is to achieve the dynamic modelling and then to make the dynamic test. So, the designer is faced with two situations. In the first case, if the resulting behaviour of the system has fulfilled the constraints imposed, the design parameters used in the simulation will be taken as a solution. In the second case, if the system response does not satisfy the constraints imposed, then the designer has to change those parameters by taking into account the previous simulation and the same sizing steps must be repeated until obtaining the optimal solution. This approach has the disadvantage that the connection between the static and dynamic design is missing and depends on a lot of expertise. Also, the passage through two steps of sizing (static and dynamic sizing) leads to the oversizing of the spring, especially when the designer took enormous safety factors after the static modelling. The designer has to go through several simulations to determine an optimal solution without being sure if the retained solution is the global optimal within the solution space. It comes from the fact that the

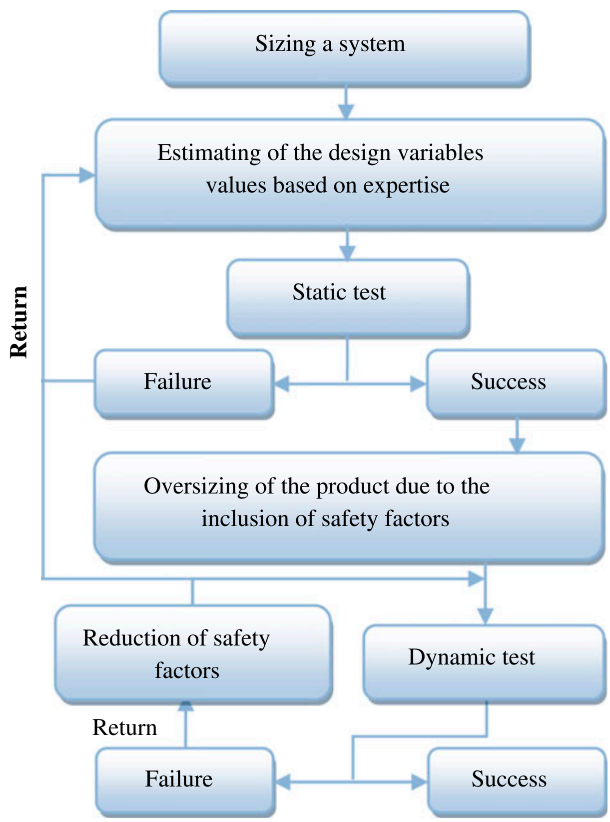

Figure 1. Conventional process of a product design.

number of simulations that can be done is limited by time and cost constraints. To overcome the limits of design process used in the study of (Paredes et al., 2005), we thought to couple the static and dynamic sizing in one step in the aim to optimise the choice of the safety factors, and to use intervals instead of fixed values in order to minimise the number of simulations and to obtain a set of solution instead of a single one. So to realise these goals the interval arithmetic of Moore (1966) are used, and the constraint satisfaction problem (CSP) approach is integrated in a new design process describing in Figure 2.

In Figure 2 a new design approach based on interval computation and the CSP approach is presented. This sizing process is made up of three main stages: the first step is to express design variables by intervals, here the choice of the design variables values does not require expertise but we cannot deny that expertise may reduce the calculation time. In the next step, the designer identifies the requirements that must be satisfied, expresses these requirements as constraints on the design variables, and then implements all types of constraints in the CSP model. Finally, the designer spreads those constraints in the intervals of design variables to frame the areas of parameters defining the product, and here the role of the CSP approach comes in. In fact, the CSP approach

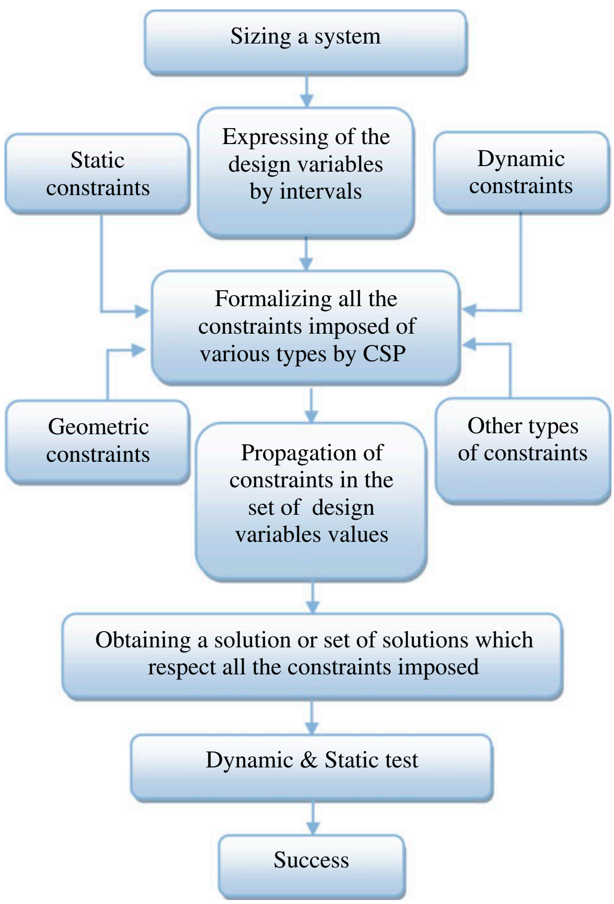

Figure 2. New design process based on CSP and Interval computation methods.

eliminates all values of design variables that do not respect the requirements imposed. The set of values that remain represent the set of solutions generated. This calculation was applied for the design of compression springs made by circular section rods, submitted under the action of static and dynamic forces.

In this paper a review of the various steps of the conventional design process and the proposed new design process is presented, following this review, the technique used to optimise the sizing process was defined. In the third and the forth part all static and dynamic constraints of a compression spring integrated in the vehicle suspension system have been described. Section 5 is devoted to presenting, in detail, the application of the CSP approach for the sizing of the compression spring. Finally, we end with a brief summary and a conclusion is presented.

2. CSP approach

The CSP approach (Edmunds et al., 2011; Eldon, 2002; Granvilliers et al., 2001; Montanari, 1974; Moore, 1966; Teorey et al., 1986; Yvars et al., 2009) is a programming paradigm that emerged in the 1980s for solving combinatorial problems of large sizes such as problems with planning and scheduling. This technique is extensively used to treat problems manipulating intervals and to solve mathematical problems that look for states or objects satisfying a number of constraints. A CSP (Yvars et al., 2009) is defined by a 3-tuple (X, D, C) such that:

- $-X = \{x_1, x_2, x_3, \ldots, x_n\}$ is a finite set of variables which we call constraint variables with n being the integer number of variables in the problem to be solved.
- $-D = \{d_1, d_2, d_3, \ldots, d_n\}$ is a finite set of variable value domains of X such that:

$$\forall i \in \{1, \ldots, n\}, x_i \in d_i \tag{1}$$

- A domain should be a real interval or a set of integer values.
- $-C = \{c_1, c_2, c_3, \ldots, c_p\}$ is a finite set of constraints, p being any integer number representing the number of constraints of the problem.

$$\forall i \in \{1, \ldots, p\}, \exists X_i \subseteq X / c_i(X_i) \tag{2}$$

To solve a CSP requires the instantiation of each variable in X while addressing the whole constraints problem C, and at the same time satisfying all the constraints of the problem C. Here, a constraint is a relationship between one or more variables which limit values that can take each of the variables simultaneously by the constraint. It can be any type of mathematical relationships (linear, quadratic, non-linear, Boolean, etc) covering the values of a set of variables. A solution is an assessment that satisfies all constraints. The constraint propagation is the operation that consists to apply recursively all contractors of a problem in a manner to make an exhaustive reduction of intervals. Here is an example to explain the constraint propagation mechanism.

The following continued CSP is defined by:

$$\begin{array}{c} x \in [-10;10] \text{ and } y \in [-10;10]; \\ \text{Constraints:} \\ (C_1): y = \sin x; \end{array} \tag{3}$$

$$(C_2){:}y = x^3; \tag{4}$$

The constraint propagation gives:

$$(C_1) \Rightarrow y \in [-10;10] \cap [-1;1],\ y \in [-1;1] \tag{5}$$

$$(C_2) \Rightarrow x \in [-10;10] \cap \sqrt{[3][-1;1]},\ x \in [-1;1] \tag{6}$$

$$(C_1) \Rightarrow y \in [-10;10] \cap \sin[-1;1]\ y \in [-0.8414;0.8414] \tag{7}$$

$$(C_2) \Rightarrow x \in [-1;1] \cap \sqrt{[3][-0.8414;0.8414]}\ x \in [-0.944;0.944] \tag{8}$$

The process stops and returns the resulting intervals.

We apply this mechanism to the design of a compression spring. The purpose is to design the compression spring in any environment wherever it is placed. We chose the example of a compression spring integrated into a vehicle suspension. There are too many factors that influence the calculation of the spring and its service life, so preferably to stay in a simplified form within the spring sizing can be quickly done.

3. The problem of the optimal design of a compression spring

Springs are structural elements designed to maintain and store the energy and mechanical work based on principles of the flexible deformation of materials. They are among the components of the most heavily loaded machines and are usually used as: (energy absorbing, dampers in the anti-vibration protection and devices for control, etc). The purpose in this section is to determine the static design requirement of a compression spring (Duchemin, 1985; Paredes, 2009; Paredes et al., 2005; Spaes, 1989) with round wire (Figure 3). The design parameters (Choné, 2007; Paredes, 2000) of the compression spring are classified into three types:

- Variables characterising the material: They are related to the material used. A material set, some variables are known (G, E, ρ). Others vary depending on the values of the geometric variables of the spring (Table 1).

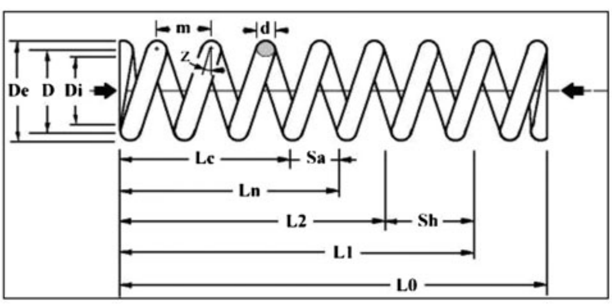

Figure 3. Parameterizing of a compression spring.

Table 1. Mechanical material properties of compression springs.

Material	Steel DH	Stainless steel 302
Limits of the manufacturer (mm)	$0.3 \leqslant d \leqslant 12$	$0.15 \leqslant d \leqslant 15$
G	81,500	70,000
E	206,000	192,000
R_m	2230 −355.94 ln (d)	1919–255.86 Ln (d)
Maximum permissible stress τ_{zul} *(% de* R_m*)*	50	48
ρ	7.85	7.90

- Geometric variables: They (D_e, D_i, R, L_0, L_c, d, n, z and m) are used to define the geometry of a compression spring.
- Operating variables: (F_1, F_2, L_1, L_2 and S_h) The spring is a component whose geometry varies significantly during use. It works between two configurations: one corresponding to the less compressed state, the second corresponding to the most compressed state.

3.1. Technological relations between design variables

The different parameters defining the design of a compression spring are interrelated through a set of equations (Paredes, 2000) which is detailed below.

$$D = D_e - d \tag{9}$$

$$D_i = D - d \tag{10}$$

$$F_1 = R(L_0 - L_1) \tag{11}$$

$$F_2 = R(L_0 - L_2) \tag{12}$$

$$F_{c\,theo} = R(L_0 - L_c) \tag{13}$$

$$F_n = R(L_0 - L_n) \tag{14}$$

$$f_e = \frac{3560d}{nD^2}\sqrt{\frac{G}{\rho}} \tag{15}$$

$$k = \frac{w + 0.5}{w - 0.75} \tag{16}$$

$$L_0 = m_u + (n_i + n_m)d \tag{17}$$

$$L_c = d(n + n_i + n_m) \tag{18}$$

$$L_d = \pi D\left(2 + n_m + \frac{n}{\cos z}\right) \tag{19}$$

$$L_n = d(n + n_i + n_m) + S_a \tag{20}$$

$$L_r = L_0 - \frac{\pi d^3 \tau_{zul}}{8DRk} \tag{21}$$

$$M = \frac{\rho 10^{-3} L_d \pi d^2}{4} \tag{22}$$

$$m = \frac{L_0 - d(n_i + n_m)}{n} \tag{23}$$

$$n = \frac{Gd^4}{8RD^3} \tag{24}$$

$$n_t = n + n_m + 2 \tag{25}$$

$$R = \frac{Gd^4}{8nD^3} \tag{26}$$

$$s_h = L_1 - L_2 \tag{27}$$

$$V_{0l0} = \frac{\pi D_e^2 L_0}{4000} \tag{28}$$

$$V_{0l2} = \frac{\pi D_e^2 L_2}{4000} \tag{29}$$

$$W = 0.5(F_1 + F_2)(L_1 - L_2) \tag{30}$$

$$w = \frac{D}{d} \tag{31}$$

$$\tan z = \frac{m}{\pi D} \tag{32}$$

$$\tau_{k2} = 8DR(L_0 - L_2)k/(\pi d^3) \tag{33}$$

$$\tau_{kc}theo = 8DR(L_0 - L_c)k/(\pi d^3) \tag{34}$$

Also, the additional relationships such as inequalities, compatibility tables and conditional relations were reformulated and taken into account in the design of the spring.

3.2. Choice of extremities

This choice determines the value of the variable n_i. It can take its values from the set {1, 2, 3, 4}. In practice, extremities with simply cut and just grinded ($n_i = 1$) are rarely used because they cause a force dispersion. It is preferable to use closer extremities ($n_i = 3$) or even close and ground ($n_i = 2$).

3.3. Choice of the number of dead spiral turns

It is also possible to add spiral turns named dead spirals to increase the length of the spring without changing its stiffness (n_m).

3.4. Winding ratio

The winding ratio w (also called index of the spring) is the ratio between the average diameter of the spring and the wire diameter. DIN [22] indicates that:

$$4 \leq w \leq 20 \tag{35}$$

3.5. Minimum operational length

The minimum length L_n is the minimum operational length based on geometrical considerations. DIN [22] imposed to respect a minimum distance between the spiral turns named S_a as:

$$s_a = n\left(0.0015\frac{D_2}{2} + 0.1d\right) \tag{36}$$

Moreover, when the number of failure cycles is around $N > 10^4$ then S_a was multiply by a coefficient of 1.5.

3.6. Choice of compressing spring material

The type of material selected imposes values on certain parameters and a restriction for others bounds (Table 1). On the other hand, DIN (DIN, 2088, DIN 2089–1, DIN 2089–2) sets the application scope of these formulas for helical compression springs:

$$d \leq 17 \tag{37}$$

$$D \leq 200 \tag{38}$$

$$L_0 \leq 630 \tag{39}$$

$$n \leq 2 \tag{40}$$

$$4 \leq w \leq 20 \tag{41}$$

The maximum static stress (Del Llano-Vizcaya, Rubio-Gonzalez, Mesmacque, & Banderas-Hernandez, 2007) is defined by the following equation:

$$\tau_{K2} < \tau_{zul} \tag{42}$$

Almost all main constraints that define the static behaviour of the compression spring were included. In the next section the dynamic constraints related to the vehicle suspension system are studied.

4. Vehicle suspension

A vehicle suspension is a set of elements designed to absorb shocks and ensure permanent adhesion of the wheels on the ground. The use of the suspension was imposed by the irregularities of the surface on which the vehicle travels. It lessens the impact on the carrier, avoids excessive wear and breakage, improves ride comfort and maintains contact between the wheels and the ground despite the irregularities. The spring is one of engineering's masterpieces. It is the element that sets the frequency of oscillation of the sprung mass and the amplitude of vertical movements.

A dynamic study was made to obtain the dynamic requirements of the suspension system (Suciu & Buma, 2009) to avoid its destruction in case of resonance. A simple model of the suspension system was treating in the following. Indeed, in Figure 4 a suspension system of a quarter vehicle in vertical mode is exposed; it is composed by the chassis m_2 which is connected to the wheel m_1 by a linear spring of stiffness k_2, and in parallel with a linear viscous damper provided with a damper coefficient η. The wheel-ground contact is modelled by a linear spring of stiffness k_1 (which represents the stiffness of the tyre and the rim).

The purpose here is to determine some conditions under which the system resists and its behaviour responds well in case of disturbance and excitation phenomena. By applying the fundamental principle of the dynamics and by the isolation of each mass, the motion equations are as follows:

$$m_2.\ddot{z_2} + \eta.\dot{z_2} - \dot{z_1}) + k_2.(z_2 - z_1) = 0 \tag{43}$$

$$m_1.\ddot{z_1} + \eta.(\dot{z_1} - \dot{z_2}) + k_2.(z_1 - z_2) + k_1.(z_1 - z_0) = 0 \tag{44}$$

We apply the Laplace transform

$$(m_2.p^2 + \eta.p + k_2)z_2 - (\eta.p + k_2)z_1 = 0 \tag{45}$$

$$(m_1.p^2 + \eta.p + k_1 + k_2)z_1 - (\eta.p + k_2)z_2 - k_1.z_0 = 0 \tag{46}$$

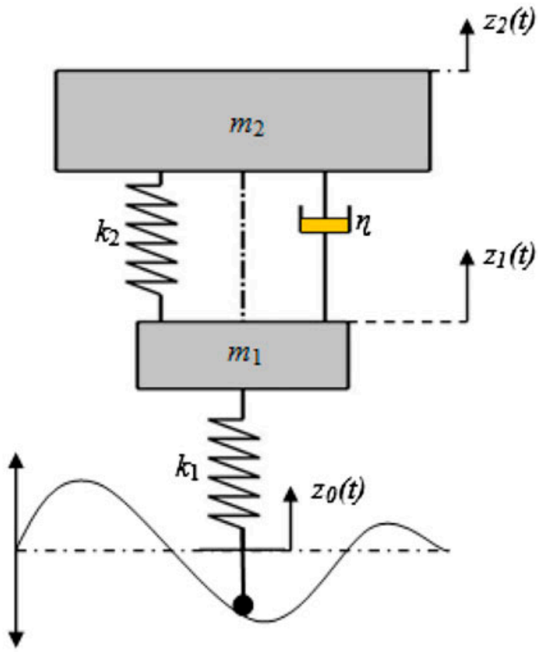

Figure 4. Linear model of a passive suspension system of vehicle.

We obtain the following global transfer function:

$$F_t(p) = \frac{z_2}{z_0} = \frac{(\eta.p + k_2)k_1}{m_2.m_1.p^4 + \eta.(m_2+m_1).p^3 + [m_2(k_2+k_1) + m_1.k_2].p^2 + \eta.k_1.p + k_2.k_1} \tag{47}$$

The harmonic transfer function is obtained simply by substituting p by jω

$$F_t(j\omega) = \frac{k_1.k_2 + j.\omega.k_1.\eta}{(k_2 - m_2.\omega^2)(k_1 - m_1.\omega^2) + j(k_1 - m_1.\omega^2).\omega.\eta} \tag{48}$$

Assume that $x_{\max}$ and $x_{\min}$ represent the maximum and minimum displacements imposed in the requirements of the suspension system and $F(t) = F_0 \sin \Omega t$ is the excitation force type sinusoidal. Then for a best functioning of the suspension system and to avoid that the system oscillates until the destruction, it must respect the following constraints:

$$x_{\min} < x(t) < x_{\max} \text{ with:} |x| = \frac{F_0}{k_1}\sqrt{\frac{(k_1.k_2)^2 + (\omega.k_1.\eta)^2}{[m_2.m_1.\omega^4 - [m_2(k_2+k_1) + m_1.k_2].\omega^2 + k_2.k_1]^2 + [\eta.(m_2+m_1).\omega^3 - \eta.k_1.\omega]^2}} \tag{49}$$

The excitation frequency of the system is different from its natural frequency (the stability study of the suspension system is attempting to determine the roots of the denominator poles of the transfer function depending on the parameters m_1, m_2, k_1, k_2 and η). This constraint can be expressed by the following mathematical relation:

$$m_2.m_1.\omega^4 - [m_2(k_2 + k_1) + m_1.k_2.\omega^2 + k_2.k_1 - j[\eta.(m_2 + m_1).\omega^3 - \eta.k_1.\omega] \neq 0 \quad (50)$$

5. Constraint formalisation and resolution by interval

5.1. Definition of design variables and implementation of technological relationships in CSP

Each design variable presented in the nomenclature of a compression spring (view annex) will be considered as a variable in our CSP that involve an initial range of values. They can be found in the column "initial value" in Table 4. All mathematical relations between design variable types static or dynamic described previously were implemented in CSP code using the ILOG solver library developed by IBM Company. The table of valid combinations of material parameters is modelled as a table constraint. This is called global constraint representing the possible combinations of values for a set of variables constraints. Each row of the table is considered as a constrained tuple of consistent values. For example, with Table 2, if the value of G must be less than 81,500, lines 1 and 2 are automatically removed from the table by propagation. Only the line numbers 3 in the table remain constrained. The advantage of such constraint is to spread an event across the table.

Generally all design variables of the compression spring are expressed by intervals as shown on Table 4. So, it remains to propagate these constraints in the intervals of design variables to determine all possible solution of the spring sizing that satisfied the requirements imposed. All constants used in the calculation by CSP are presented in Table 3.

Table 2. Constraints table.

Material	1	2	3
E	206,000	206,000	192,000
Coeff τ_{zul}	0.5	0.5	0.48
ρ	7.8510–6	7.8510–6	7.910–6
G	81,500	81,500	70,000

Table 3. Constants.

π	3.14159265358979323846264338327950
ε	0.01
m_1	25
m_2	500
k_1	200,000
K_2	20,000
	7000
Ω_e	10

Table 4. Numerical results of the compression spring dimensions obtained by the CSP approach.

Variables	Initials values	Results
The material parameters		
Material	{1, 2, 3}	1
G	{70,000, 81,500}	81,500
E	{192,000, 206,000}	206,000
R_m	[0; 2230]	1455.69189252899
ρ	{7.85e−06, 7.9e−06}	7.85e−06
M	[0; +∞]	[906.159418; 906.159419]
τ_{zul}	[0, 10,000]	727.845961728
μ	[0; 1]	[0.263803680981595; 0.263803680981596]
Principal constructive parameters of the spring		
d	[0.15; 15]	7.00000011026432
D	[0; 200]	50
d_{min}	{0.15, 0.3}	5.599999819776
d_{max}	{12, 15}	7.50000007020544
De	[0; 217]	[58.8056134702269; 58.8056134702269]
Di	[0; 200]	[41.1943865297732; 41.1943865297733]
L_0	[0; 630]	[200; 200.000010986247]
L_c	[0; 630]	105.667361642722
$R = k_2$	[0; 10,000]	49
n	[2; 2e9]	3
n_i	[2; 2e9]	2
n_m	[2; 2e9]	1
m	[0, 315]	12
S_a	[0; +∞]	[0.00076; 0.00077]
z	[-π/2, π /2]	[0.114205511695; 0.1142055148096]
Secondary Constructive parameters of the spring		
L_d	[0; +∞]	[1895.508930; 1895.508932]
Vol_0	[0; 24,000]	[543.1970, 543.1971]
n_t	[4; 2e9]	6
w	[4; 20]	[4.1728433945700; 4.1728433945701]
f_e	[0; +∞]	0.49
M	[0; +∞]	[1677.6568091; 1677.6568096]
Functional parameters of the spring		
F_1	[0; +∞]	[2.763; 2.764]
F_2	[0; +∞]	[4621.7713; 4621.7718]
F_n	[0; +∞]	[4622.2613; 4622.2619]
L_1	[0; 630]	[199.9435; 199.9436]
L_2	[0; 630]	[105.67812; 105.67813]
L_n	[0; 630]	[105.66812; 105.66813]
S_h	[0; 630]	[94.26545; 94.26547]
W	[0; +∞]	217966.956249088
Vol_2	[0; 24,000]	[287.02024, 287.02028]
Performance parameters (static rupture)		
L_r	[0; 630]	[136.46128; 136.46129]
F_c *théo*	[0; +∞]	[4622.2992; 4622.2998]
k	[1; 10]	8.80561375674368
τ_{kc}*théo*	[0; +∞]	[1080.5951; 1080.5953]
τ_{k2}	[0; +∞]	[8.153535; 8.153538]

(*Continued*)

Table 4. (*Continued*).

The material parameters		
Variables	Initials values	Results
Dynamic study		
Ω	[0; +∞]	2
F_0	[0; +∞]	4000
x_0	[0; +∞]	[0.0291543950465661; 0.0291543950465662]

5.2. Tables implementation and Resolution by interval

The static and dynamic requirement imposed on the compression spring was coupled and implemented in CSP code according to the proposed design approach based on intervals and the CSP showed in Figure 2. Then after the step of constraint propagation, results in Table 4 are obtained. According to those results, we notice a drastic reduction in certain intervals and the solution for some parameters is an interval (example: m, L_0, $Fc_{\text{théo}}$ …) and for the other parameters is just one value (example: d, $d_{\min}$, f_e …). We notice also that the material 1 is the only one that can satisfy the requirement imposed. So, we could say that we succeed to size the compression spring taking into account the entire static and dynamic requirement studied previously in the same time without the need to resize the spring which is due to the fact that all the solutions generated and represented with all values (column Results in Table 4) of the parameters that define the possible dimensions of the spring satisfy the constraints imposed.

To sum up, with the new design process based on the CSP approach and interval computation we can couple more than one analysis for designing a system which helps to take an exact decision contrary to a conventional design approach. Also, using this approach can determine a set of solutions instead of one, thanks to a mathematical operator by intervals used in the CSP approach. The calculation was made by intervals that explain the accuracy of some values obtained. Despite the accuracy, which is an advantage in the CSP approach, values of some parameters obtained precisely with this will lead to a problem in the manufacturing step (example: D_e).

6. Conclusion

The study shows that the CSP approach should be applied efficiently to the optimal design of a compression spring. The static and dynamic sizing steps were coupled in the same step of sizing. The design processes allows taking into account all type of constraints from the beginning that avoid resizing the system. The optimisation becomes easier, since with the new design approach proposed in this paper the designer uses intervals instead of fixed values and thus generates a set of solutions instead of a single simulation. The computation time is interesting: in this study, the simulation results are almost obtained immediately on a standard PC. So, it can be claimed that the objectives and the advantages of the proposed design approach compared with the conventional design approach are verified and demonstrated. The method was applied for the design of one component of a linear system. Future work might focus on the validation of the capability of the proposed design based on the CSP approach and intervals computation to optimise the sizing of a full non-linear system (structure + all components …).

Nomenclature

D	mm	mean diameter of spiral turns
D_e	mm	external diameter of spiral turns
D_i	mm	internal diameter of spiral turns
d	mm	wire diameter
E	N/mm^2	modulus of elasticity of the material
F_1	N	spring force for the length L_1
F_2	N	spring force for the length L_2
Fc_{theo}	N	theoretical force of the spring for L_c
F_n	N	spring force for L_n
f_e	Hz	natural frequency of the spring
G	N/mm^2	shear modulus
k	–	coefficient of stress as a function of w
L_0	mm	free length
L_1	mm	spring length in charge, for force F_1
L_2	mm	spring length in charge, for force F_2
L_c	mm	block length
L_d	mm	developed length
L_r	mm	shorter length of eligible work (maximum stress)
L_n	mm	shorter length of eligible work (geometrically)
M	g	mass of the spring
m	mm	step of the spring
n	–	number of active spiral turns
n_i	–	number of spiral turns for the extremities
n_m	–	number of dead spiral turns
n_t	–	total number of spiral turns
R	N/mm	spring stiffness
R_m	N/mm^2	minimum value of the tensile strength
S_a	mm	sum of minimum space between the active spiral turns
S_h	mm	course
Vol_0	cm^3	volume envelope for L_0
Vol_2	cm^3	volume envelope for L_2
W	Nmm	work of the spring
w	–	winding ratio
z	°	winding angle
ρ	kg/dm^3	density
τ_{k2}	N/mm^2	constraint shear adjusted for L_2
τ_{zul}	N/mm^2	maximal eligible constraint
τ_{kc}theo	N/mm^2	theoretical constraint shear adjusted L_c

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