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# Multi-inspection time optimisation for a cracked component based on a updating reliability approach

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It is well acknowledged that Fatigue Crack Growth (FCG) process is one of the main processes that can produce failure of structures and mechanical components. To ensure the survival of these components a maintenance inspection schedule is needed. The aim of this study is to develop a predicting inspection time model for structural FCG life based on updating reliability analysis. The proposed approach takes into account the additional information generated by the previous inspection results. First-order reliability method and surface response method are used to evaluate the reliability. The uncertainties such as material parameters and geometrical parameters that affect the lifespan of the structure were regarded as random variables. Updating reliability assessment based on Bayesian approach was introduced to determine the updating inspection time for target reliability. Moreover, the proposed approach leads to determine the optimal inspection time strategy based on an economic study under the minimal total inspection and detecting costs using a dynamic method. The optimal inspection time for a single and two inspections was determined. A generalisation of this method is carried out for the case of multiple inspections. In order to illustrate and validate this method two applications are carried out: the first one is applied to the crack growth in mode I, and the second one is applied to the crack growth in mixed mode. The results of these applications are in good agreement with the physical results and show that the proposed method is proved to be feasible and applicable in the general complex fatigue loading and able to give accurate updating framework for scheduling inspections.

**Keywords:** fatigue crack growth; inspection scheduling; updating reliability; First-order reliability method; response surface; optimisation

## 1. Introduction

Mechanical parts are generally subjected to a repetitive loads that are often complex. For a better secure design, practical computation of fatigue behaviour of mechanical components, used in automotive, aerospace, naval structures, nuclear plants is much needed. In structural design, fatigue and damage tolerance analysis have become the most important task for the designers. Two different stages can be considered during the structure fatigue life they are usually decomposed into: (i) a crack initiation period including some micro-crack growth, where the prediction of the high cycle fatigue

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(HCF) behaviour of a mechanical component is carried out, in the majority of cases, by deterministic multiaxial HCF criteria (Papadopoulos IV, Davol, Gorla, Filippini, & Bernasconi, 1997) and (ii) crack growth period, where the crack is growing until complete failure and the prediction of the Fatigue Crack Growth (FCG) behaviour is carried out by linear elastic fracture mechanics (LEFM) approach (Gdoutos, 2005).

The large majorities of FCG models are limited to opening mode and neglect the two other modes, namely, sliding and tearing modes. However, for complex loading, at which two or more loading axes fluctuate with time all crack modes are activated in practice the FCG is mainly a mixed mode propagation.

The stress intensity factor based on LEFM is used for predictions on FCG, where an equivalent stress intensity factor based on energetic method (Destuynder, Djoua, & Lescure, 1983) and maximal shearing stress is used (Erdogan & Sih, 1963).

The traditional analytical method of engineering fracture mechanics usually assumes that crack size, stress level, material property and crack growth rate have all deterministic values. However, due to the randomness of all these properties, the FCG is a process that is naturally considered to be random (Ghonem & Dore, 1987) and requires appropriate inspection plan in order to prevent the risk of failure and improve maintenance strategies.

Considering the development of the reliability fatigue based methods (Cavallini & Lazzeri, 2007; Fourlaris, Ellwood, & Jones, 2007; Shen, 1999; Wirsching, 1980) taking into account the scatterings and the significant dispersions related to the used geometrical and material parameters (Webster & Ezeilo, 2001; Yuan, 2007), it seems to be necessary and with substantial benefit to analyse the HCF and FCG behaviour of the mechanical parts using probabilistic methods.

Several applications dealing with probabilistic FCG (Bea, Doblaré, & Gracia, 1999; Myötyri, Pulkkienen, & Simola, 2006; Newby, 1987) have been carried out where the authors highlighted the advantage of using probabilistic approach to analyse the FCG behaviour of the mechanical parts, but they do not couple reliability and fatigue in their approach.

Because of the increasing complexity of modern engineering systems, reliability computation becomes more and more considered as an engineering design way, several works coupling fatigue reliability assessment leading to useful probabilistic HCF diagrams were carried out (Ben Sghaier, Bouraoui, Fathallah, & Hassine, 2007; Zhao & Ono, 2001).

Different interesting works based on LEFM have been carried out to predict the lifespan of the mechanical part with a probabilistic point of view (Besterfield, Liu, Lawrence, & Belytschko, 1991; Liu,Chen, Belytschko, & Lua,1996; Lua, Liu, & Belytschko, 1992; Yu, Purnendu, & Zheng, 2009). More recently few applications based on coupling FCG reliability assessment have been developed (Leonel, Chateauneuf, Venturini, & Bressolette, 2010). These approaches lead to have a reliable fatigue behaviour of mechanical parts.

For many mechanical structures, such as aircrafts, ships and offshore structures the crack growth period represents the major part of the total fatigue life. In-service inspections are often needed to ensure their safety and availability and prevent sudden failure due to fatigue damage (Nechval, Nechval, Purgailis, & Strelchonok, 2011).

For complex system, it is not easy to identify the failed component upon the failure of the system. In this case, inspection and resource allocation policies are frequently adopted (Kulkarni & Achenbach, 2007; Meng, Li, Sha, & Zhou, 2007; Wu & Ni, 2007).

In the majority of cases, this inspection time strategies are determined according to probabilistic models without taking into account the additional information deduced from previous inspect results.

It is worth noticing that too many inspections cause not only the deterioration of system availability, but also cause an increase in costs. So, it is necessary to provide an optimal inspection strategy for mechanical structures in order to keep these structures safety and reduce their inspection costs.

Many works proposed in the literature have been applied to evaluate the inspection planning time, they use deterministic crack growth approach and propose with an adequate margin of safety an inspection strategy that is easy to implement. However, the deterministic approach does not quantify the risk level of an inspection interval.

In scheduling inspections, in the context of implementing the damage-tolerance few authors proposed some inspection strategies based on reliability approach (Baker & Descamps, 1999; Khaleel & Simonen, 2000; Straub & Faber, 2005). Recently, Tanaka et al. (Tanaka & Toyoda-Makino, 1998) have theoretically investigated the optimal scheduling of a single inspection based on minimising a cost function. Random crack growth is accounted for by the diffusive crack model. This study was later extended by the authors to the case of scheduling multiple inspections. More recently, and in order to increase the confidence level of the inspection planning time, results of previous inspections are used to perform the updating reliability using the collocation method reliability assessment (Riahi, Bressolette, Chateauneuf, Bouraoui, & Fathallah, 2011).

During the lifespan of these mechanical components, inspections are required for the surveillance and monitoring of their state and also to decide on the intervention's nature maintenance's type to be applied to structures (Jiao & Moan, 1990). Several methods are used to define a maintenance optimal inspection programme; there are authors who set a failure probability to not be exceeded and choose an optimal maintenance time. Others use an optimal maintenance programme based on the minimum value of the failure probability in order to ensure the required security (Han, 1999). Economically, the optimal maintenance programme is defined by minimising the total cost function (Laggoune, Chateauneuf, & Aissani, 2009). Most of these studies do not take into account the results of previous inspections of maintenance.

In the present study, a framework for scheduling inspections based on FCG reliability assessment is presented. This approach presents the advantage to carry out time inspection calculation by a more efficient and reliable method that takes into account the dispersion of the different FCG parameters. Based on the previous inspection results, the updating reliability is determined and from which a new updating time inspection is proposed. For illustration, the proposed approach is applied on a centred crack loaded uniaxially perpendicular to the crack direction leading to an opening crack growth mode. This analysis will be helpful and easy to implement for modern engineering systems to optimise, for a given reliability the inspection time under inspection costs reduction. In this study, a new formulation of the cost function in the case of a single inspection is developed. It is based on the decision tree and taking into account the results of previous inspections of maintenance. This formulation is treated in a sequential mode starting from the initial crack. Optimal inspection time is determined by minimising the total cost function. The method is then generalised to the case of two and multiple inspections.

## 2. Background

### *2.1. Fatigue crack growth*

#### *2.1.1. Computation of stress intensity factors*

Usually, concepts of fracture mechanic based on linear elastic behaviour assumption are considered to analyse the behaviour of structures subjected to FCG. The fracture can be described by the use of the stress intensity factors in various modes. This parameter defines the magnitude of the local stresses around the crack tip and depends on loading, crack size, crack shape and geometric boundaries. For a simple geometry specimen, where the fracture follows the opening mode usually named mode I, the stress intensity factors can be expressed under this general form:

$$K_I = \sigma . F(a) . \sqrt{\pi . a} \tag{1}$$

In engineering practice, where the loading is complex and the structures have a complex geometry, the defects are randomly oriented, as a consequence, a mixed mode crack propagation is expected. In this case, the stress intensity factors can not be computed using analytic solution, where several numerical methods have been proposed in the literature, such as the displacement extrapolation technique (Chan, Tuba, & Wilson, 1970; Guinea, Planas, & Eliccs, 2000), the *J*-integral method (Parks, 1974) and the $G$–$\theta$ method (Paris & Erdogan, 1963) in which the author compute the strain energy release rate $G$, representing the decrease of the total potential energy $\pi_c$ during the growth $\Delta a$ of the crack:

$$G = -\frac{\partial \pi_c}{\partial a} = -\frac{\pi_c(a + \Delta a) - \pi_c(a)}{\Delta a} \tag{2}$$

#### *2.1.2. Computation of crack growth direction*

Due to a curved followed path during crack propagation, it is necessary to compute the equivalent stress intensity $K_{\mathrm{Ieq}}$ factor in mode I and crack growth angle $\theta_0$. For this purpose, the maximum circumferential stress criterion (Rackwitz & Fiessler, 1978) is considered, which yields:

$$\frac{\partial \sigma_{\theta\theta}}{\partial \theta} = 0 \text{ and } \frac{\partial^2 \sigma_{\theta\theta}}{\partial \theta^2} < 0 \tag{3}$$

$$\sigma_{\theta\theta} = \frac{2}{\sqrt{2\pi r}} \left[ K_{\mathrm{I}}(1 + \cos(\theta))\cos\left(\frac{\theta}{2}\right) - 3K_{\mathrm{II}}\sin(\theta)\cos\left(\frac{\theta}{2}\right) \right] \tag{4}$$

$$K_{\mathrm{Ieq}} = K_{\mathrm{I}}(1 + \cos(\theta))\cos\left(\frac{\theta}{2}\right) - 3K_{\mathrm{II}}\sin(\theta)\cos\left(\frac{\theta}{2}\right) \tag{5}$$

Based on the maximisation of Equation (3) with respect to the crack orientation angle, the bifurcation angle $\theta_0$ is obtained by:

$$\mathrm{tg}\left(\frac{\theta_0}{2}\right) = \frac{1}{4}\left(\frac{K_{\mathrm{I}}}{K_{\mathrm{II}}}\right) \pm \frac{1}{4}\sqrt{\left(\frac{K_{\mathrm{I}}}{K_{\mathrm{II}}}\right)^2 + 8} \tag{6}$$

#### *2.1.3. Computation of crack growth rate*

In fatigue, the crack growth behaviour is, usually, described by the plot of $\log\left(\frac{da}{dN}\right)$ vs. $\log(\Delta K)$ where $N$ is the number of loading cycles and $\Delta K$ is the stress intensity factor range. This plot is a sigmoïdal curve and is divided in three regions. Majority of the current applications of LEFM concepts to describe crack growth behaviour are associated with the mid-region that is essentially linear.

Paris and Erdogan (1963) proposed an equation which is widely accepted:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C(\Delta K)^m \tag{7}$$

$K_{\max}$ and $K_{\min}$ are the value of the stress intensity factor $K$ at the maximum and the minimum stresses of the loading cycle.

The crack growth life, $N_f$, may be calculated by integrating the Paris equation:

$$N_f = \int_{a_0}^{a_c} \frac{\mathrm{d}a}{C(\Delta K)^m} \tag{8}$$

### **2.2. *Reliability method***

To compute the reliability, one considers a vector of random variables {X} representing uncertain structural quantities. Let $\mathrm{x}_i$ be an element of the random vector {X}, with a probability density function (PDF) $f_X(X_i)$. A performance function $G(\{\mathrm{X}\})$, separating the security and the failure fields is written as follows:

$$G(\{\mathrm{X}\}) = S(\{\mathrm{X}\}) - L(\{\mathrm{X}\}) \tag{9}$$

where $G(\{\mathrm{X}\})=0$ is the limit state function, $S(\{\mathrm{X}\})$ is the strength function and $L(\{\mathrm{X}\})$ is the load function (Lemaire, Chateauneuf, & Mitteau, 2005; Zhao & Ono, 2001). The probability of failure $P_f$ is given by:

$$P_f = \int_{G(X)\le 0} f_X(x_1, x_2, \ldots, x_n)dx_1, dx_2, \ldots dx_n = \Pr(G(\{x\})<0) \tag{10}$$

In that case, if the inequality $G(\{X\})>0$ is satisfied, this indicates a structural safety condition. In the opposite case, if $G(\{X\})<0$, this means a failure of such a structure.

To compute the probability of failure $P_f$ in the Equation (10), one can use :

(1) Analytical resolution: when the joint probability density function (JPDF) of {X} is available ($f_X(x_i)$), however, even when the JPDF is defined, the evaluation of the $P_f$ by multiple integration is very difficult.

(2) Approximate computational methods: such as moments based methods: first order reliability method (FORM) and second order reliability method that are widely used to assess the $P_f$. They use only the information of the first and the second moments of the random variables namely the means and the covariances. In addition the methods need an explicit performance function $G(\{X\})$. These methods are based on the use of the Reliability Index such as Cornell (Lemaire et al., 2005).

$$\beta_{\text{Cornell}} = \frac{E(G(\{x\}))}{\sigma(G(\{x\}))} \tag{11}$$

$$P_f = \varnothing(-\beta_{\text{Cornell}}) \tag{12}$$

This approximation method is used in the case of random Gaussian variables and where the performance function $G(\{X\})$ is linear and its derivates are available under an explicit analytic form in the space of the random variables.

When the limit-state function is nonlinear, there is no unique distance from the performance function to the origin of the variables. The Reliability Index $\beta_{\text{HL}}$ is correctly defined by Hasofer Lind (Hasofer & Lind, 1974). It is defined as the shortest distance between the limit state function and the diagram origin in a standard reduced space of the random uncorrelated variables $\{u\}$.

$$u_i = \frac{X_i - \mu_{X_i}}{\sigma_{X_i}} \tag{13}$$

The index $\beta_{\text{HL}}$ is then evaluated in this space by resolving the following problem of optimization:

$$\beta_{\text{HL}} = \min \sqrt{\sum_i u_i . u_i} \quad \text{Under the constraint } H(\{u\}) \leq 0 \tag{14}$$

where $H(\{u\})$ is the performance function in the standard reduced space (Hasofer & Lind, 1974).

The most probable failure point is:

$$u_i^* = -\alpha_i^* . \beta_{\text{HL}} \tag{15}$$

In which $\alpha_i^*$ are the direction cosines:

$$\alpha_i^* = \frac{\left(\frac{\delta H(\{u\})}{\delta u_i^*}\right)}{\sqrt{\sum_i^n \left(\frac{\delta H(\{u\})}{\delta u_i^*}\right)_*^2}} \tag{16}$$

where the partial derivatives $\left(\frac{\delta H(\{u\})}{\delta u_i}\right)_*$ are evaluated at $\{u\}^* = (u_1^*, u_2^*, \ldots u_n^*)$. And the derivatives are calculated at $\{u\}^* = (u_1^*, u_2^*, \ldots u_n^*)$.

Then, $x_i^* = \sigma_{x_i}.u_i^* + \mu_{X_i} = \mu_{Xi}$

$$= \mu \text{ The reliability } R \text{ is given by the following relationship: } X_i \alpha_i^*.\sigma_{x_i}.\beta_{HL} \quad (17)$$

The solution of the limit state function $G(x_1^*, x_2^*, \ldots x_n^*) = 0$ then yields $\beta_{HL}$.

The results are summarised in the following numerical steps calculations (Rackwitz & Fiessler, 1978):

(1) Assume initial values of $(x_1^*, x_2^*, \ldots x_n^*)$ and obtain $U_i^* = \frac{X_i^* - \mu_{X_i}}{\sigma_{X_i}}$.
(2) Evaluate $\left(\frac{\delta H(\{u\})}{\delta u_i^*}\right)_*$ and $\alpha_i^*$ at $x_i^*$.
(3) Form $x_i^* = \mu_{x_i} - \alpha_i^*.\sigma_{x_i}.\beta_{HL}$
(4) Substitute above $x_i^*$ in $G(x_1^*, x_2^*, \ldots x_n^*) = 0$ and solve for $\beta_{HL}$.
(5) Using the $\beta_{HL}$ obtained in step 4, reevaluate $u_i^* = -\alpha_i^*.\beta_{HL}$.
(6) Repeat step 2 through 5 until convergence is obtained.

The probability of failure $P_f$ is estimated in this case by:

$$P_f = \varnothing(-\beta_{HL}) \quad (18)$$

In the case of random Gaussian variables and with linear performance function $G(\{X\})$ we obtain the following result:

$$P_f = \Pr(G(\{x\})<0) = \varnothing(-\beta_{HL}) = \varnothing(-\beta_{Cornell}) \quad (19)$$

The Monte Carlo Simulation: this method is used when the performance function is defined over a vector of more than two random variables and when the joint PDF of X is practically difficult to find.

This last method remains often the only means of taking into account certain non-linear behaviour. Such procedure is simple, however, it has the drawback to have a large number of runs possibly required to obtain an accurate result. Its convergence speed is low and it is proportional to $\sqrt{N}$ (Ditlevsen & Madsen, 1996).

The reliability $R$ is given by the following relationship:

$$R = 1 - P_f \quad (20)$$

### 2.3. *Surface response method*

The computation of the limit state function or the safety margin $G$ is in general done numerically. But for calculating the Reliability Index we need an explicit relationship of the limit state function. The main idea of the surface response method is to construct a polynomial approximation of the limit state function based on the results obtained by the design of experiments (DoE) method where a full factorial design plan is used.

The second order of this approximation (quadratic surface response) could be considered as a good compromise since it includes a possible calculation of the curves while avoiding a possible fluctuation obtained with a higher order. In the case of the quadratic surface response with three random variables, the response is written as follows:

$$G(\mathrm{X}) = b_0 + \sum_{i=1}^{n} b_i.\mathrm{X}_i + \sum_{i=1}^{n} b_{ii}.\mathrm{X}_i^2 + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} b_{ij}.\mathrm{X}_i.\mathrm{X}_j + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} \times \sum_{k=j+1}^{n} b_{ijk}.\mathrm{X}_i.\mathrm{X}_j.\mathrm{X}_k \quad (21)$$

where $G(\mathrm{X})$ is the response of the model, $\mathrm{X}_i$ are random variables of the model and $b_0$, $b_i$, $b_{ii}$, $b_{ij}$, $b_{ijk}$ are the coefficients of the mathematical model, determined by the regression tool.

### 2.4. Reliability updating based on inspection results

During the cycle life of a component, inspections are scheduled in order to avoid the risk of failure. During inspections, two types of results could be reported: the crack is not detected or detected. These two events can be written as following:

$a < a_d$: Crack not detected and $a \geqslant a_d$: Crack detected

For the case of no crack detection, during an inspection after $N_i$ loading cycles, the failure structure probability $P_f$ should be decreased. The safety margin corresponding to not crack detection could be expressed by the following:

$$M_i = N_d - N_i \quad (22)$$

where $N_d$ is the number of cycles corresponding to the detected crack length and $N_i$ is the number of cycles corresponding to a crack length $a_i$ which is calculated by the equation of the Paris law.

The result of the inspections should be used in order to update the failure probability $P_f$. This probability $P_{f,\mathrm{up}}^d$ could be easily calculated by the Bayesian approach (Jiao & Moan, 1990) as follows:

$$P_{f,\mathrm{up}}^d = P(G(\mathrm{X}) \le 0 | M_i \le 0) = \frac{P(G(\mathrm{X}) \le 0 \cap M_i \le 0)}{P(M_i \le 0)} \quad (23)$$

Based on FORM approximation the probability of failure $P_f$ of the basic safety margin $G$ and the updated probability of failure $P_{f,\mathrm{up}}^d$ considering the inspection event is defined by:

$$P_f \Phi = (-\beta_G) \quad \text{and} \quad P_{f,\mathrm{up}}^d \Phi = (-\beta_{\mathrm{up}}) \quad (24)$$

Jiao and Al. proposed an approximation for calculating the updated Reliability Index $\beta_{\mathrm{up}}$ with:

$$\beta_{\mathrm{up}} = \frac{\beta_G - \rho A}{\sqrt{1 - \rho^2 B}} \quad (25)$$

With $\rho = \alpha_G^T \alpha_{M_i}$ $A = \frac{\varnothing(-\beta_{M_i})}{\Phi(-\beta_{M_i})}$ and $B = A.(A - \beta_{M_i})$

where $\beta_{\text{up}}$ is updated Reliability Index, $\beta_G$ is the Reliability Index corresponding to the basic safety margin $G(\text{X}) \leqslant 0$, $\beta_{Mi}$ is the Reliability Index corresponding to the inspection event $M_i \leqslant 0$, $\rho$ is the correlation coefficient between the events $G(\{\text{X}\}) \leqslant 0$ and $M_i \leqslant 0$, $\alpha_G$, $\alpha_{M_i}$are the direction cosines of the safety margin $G$ and the inspection event $M_i$ respectively. $\Phi$ and $\varnothing$ are the cumulative distribution function and the PDF, respectively, of standard Gaussian random variable.

After expressing $M_i$ using an explicit relationship, the direction cosines $\alpha_{M_i}$ and the reliability index $\beta_{M_i}$ of the inspection event are calculated similarly to the safety margin $G$ as presented previously.

For the case of crack detection, during an inspection, the failure probability should be increased. We notice that the crack detection event $\overline{M_i}$ is the complementary of the no crack detection event $\text{M}_i$. Therefore, the safety margin of crack detection is obtained by reversing the sign of event $M_i$.

$$\overline{M_i} = -M_i = \int_{a0}^{a_d} \frac{\text{da}}{\Delta K^m} - C.N_i \tag{26}$$

Using the total probability theorem (Jiao & Moan, 1990), the updated failure probability $P_{f,\text{up}}^{\text{nd}}$ for this case is:

$$P_{f,\text{up}}^{\text{nd}} = P(G(\text{X}) \leq 0 | \overline{M_i} \leq 0) = \frac{P(G(\text{X}) \leq 0) - P(G(\text{X}) \leq 0 | M_i \leq 0) P(M_i \leq 0)}{1 - P(M_i \leq 0)} \tag{27}$$

## 3. Methodologies of FCG reliability updating

The general procedure of the determination of the FCG updating reliability diagrams is detailed trough the three different steps.

Step1: Determination of crack growth length and the lifespan.

The integration of the Paris law allows to compute the FCG length in terms of the number of loading cycles. Figure 1 shows clearly the flowchart of the crack growth length and the lifespan assessment in the mixed mode.

Step2: Building the explicit limit state functions ($G$(x) and $M_i$(x)) based on surface response method.

The computation of probability of failure in Equation (10) is not easy. For this reason approximate methods based on Reliability Index are developed. An explicit limit state function $G$ (or inspection event $M_i$) depending on different random parameters based on the surface method is then needed. In this study, the DoE based on a full factorial design plan is used. The flowchart, in Figure 2, presents the steps to follow to build an explicit relationship of the basic safety margin (or inspection event $M_i$) by the response surface method using the regression tool.

Step3: Reliability updating assessment.

In the case of a non-linear limited state function the computation of the Reliability Index of Hosfer–Lind $\beta_{\text{HL}}$ is required and an iterative calculation to find the optimal $\beta_G$ index value relative to the $G$ limit state based on Rackwitz-Fiessler (Rackwitz & Fiessler, 1978) algorithm is carry out. The flowchart in Figure 3, describes the necessary steps to determine the Reliability Index $\beta_G$. The same flowchart is used to compute the reliability index $\beta_{M_i}$ of the inspection event relative to the $M_i$ limit state.

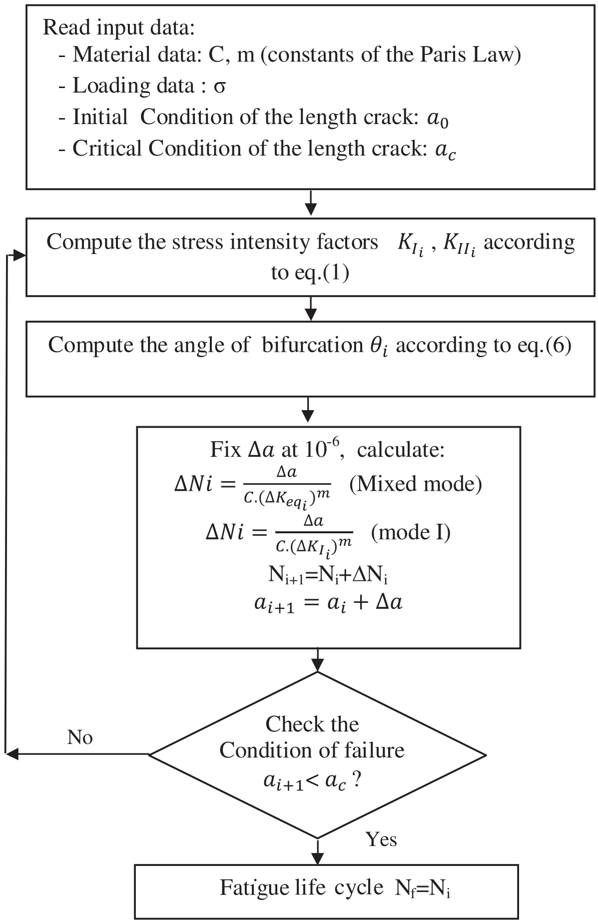


Figure 1. Computation the length of the crack in mixed mode.

Once $\beta_G$ and $\beta_{M_i}$ are determined, the updated probability of failure $P_{f,\mathrm{up}}$ is calculated according to (Equations (24) and (25)) in the case of no crack detection or according to (Equation (27)) in the case of crack detection.

## 4. The optimisation of inspection time based on minimal total cost: dynamic method

We present a methodology to determine the optimal time inspection for a cracked component submitted to a fatigue loading. In that case, the optimisation is realised by minimising the global cost function which represents the failure cost and the inspection cost.

In order to formulate mathematically the cost function, we adopt the following assumptions:

- The lifespan $T_f$ of the component is known.
- The component is failed if $a_i > a_c$

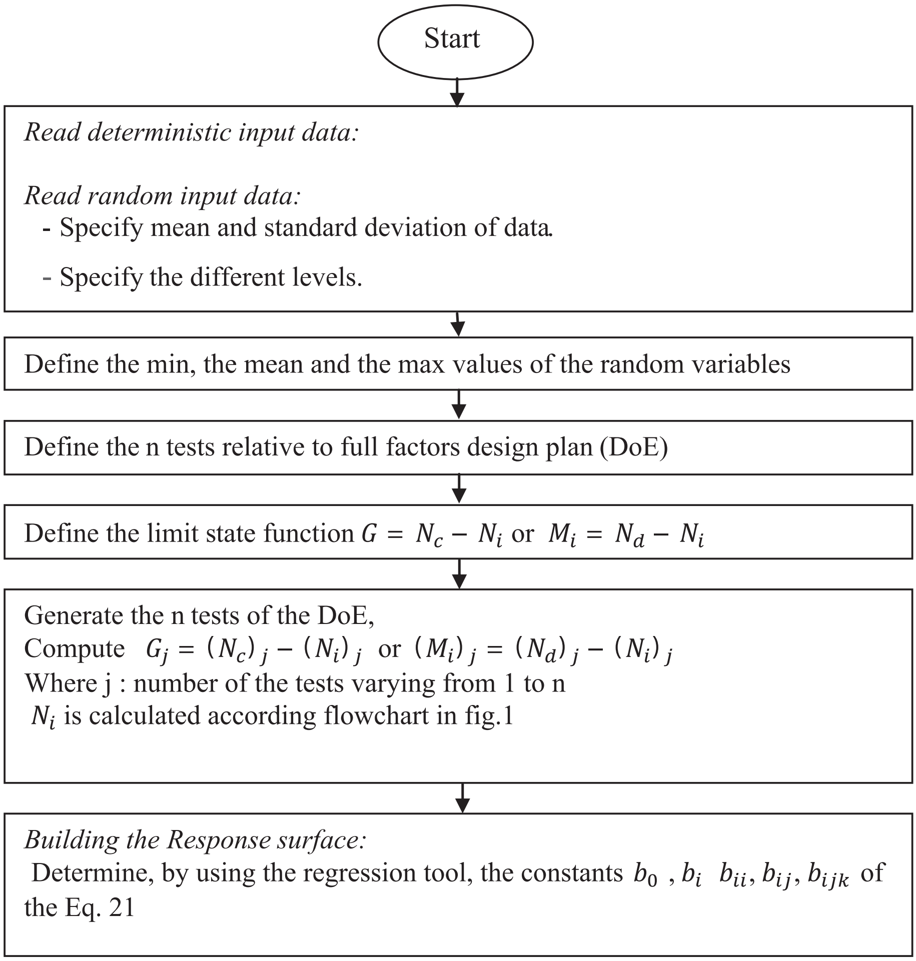


Figure 2. Determining an explicit expression of the safety margin by the method of response surface.

- The inspection is realised without reparation.
- The cost of a failure ($C_f$) and the cost of each inspection ($C_i$) are known.

We describe the formulation of the cost function for the case of one, two and multiple inspections and provide more details for evaluation. The used method in this study is a dynamic method which consists in treating the problem in a sequential way starting with the initial crack.

In this case and based on the decision tree for the case of a single inspection, we define the total cost between $t=0$ and $t_1$ time of the first inspection. Then, we determine the date $t_{1\text{optimal}}$ of the first inspection for which the total cost is minimal.

For the case of two inspections, we define the total cost between $t_{1\text{optimal}}$ and $t_2$ time of the second inspection. Then, by minimising this total cost, we compute $t_{2\text{optimal}}$ corresponding to the optimal time of the second inspection and we do the same way in the case of the multiple inspection.

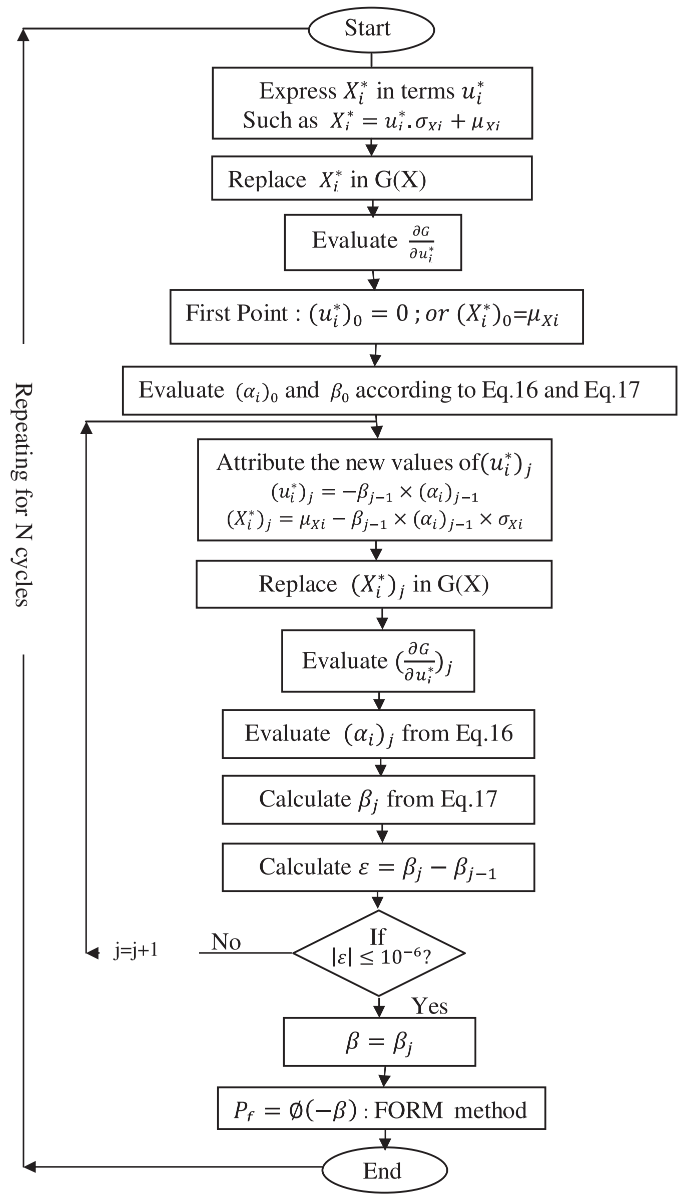


Figure 3. Computation the reliability index by applying the optimization algorithm of Rackwitz-Fiessler.

### *4.1. Case of a single inspection*

The Figure 4 illustrates the decision tree in the case of single inspection realised at the time $t_1$ with $0 < t_1 < T_f$

Based on the decision tree, we can identify two types of cost:

- $C_F(t_1)$: the expected cost of failure in the interval [0, $t_1$] with $0 < t_1 < T_f$
- $C_I(t_1)$: the inspection cost realised at the time $t_1$

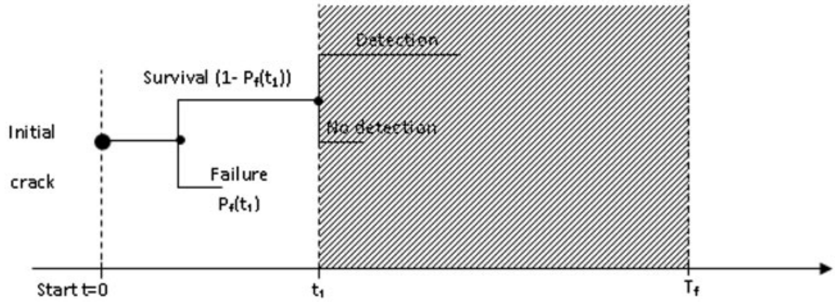


Figure 4. The decision tree in the case of a single inspection.

Being situated in the interval $[0,t_1]$, the determinist total cost is defined as follows:

$$C_T(t_1) = \frac{C_F(t_1) + C_I(t_1)}{t_1} \tag{28}$$

By integrating for each term, the probability of occurrence of each event, the total cost becomes:

$$C_T(t_1) = \frac{C_f.P_f(t_1) + c_i.R(t_1)}{t_1} = \frac{C_f.P_f(t_1) + C_i.(1 - P_f(t_1))}{t_1} \tag{29}$$

where $P_f(t_1)$ is the failure probability at the time $t_1$ and $R(t_1)$ is the survival probability at the time $t_1$. $C_f$ is the cost of a failure and $C_i$ is the cost associated to each inspection. The date $t_{1\text{optimal}}$ of the first inspection is such as $C_T(t_{1\text{optimal}})$ is minimal.

### 4.2. *Case of two inspections*

As in the case of a single inspection, two different costs are considered (see Figure 5):

- $C_F(t_{1\text{optimal}}, t_2)$: the expected cost of failure in the interval $[t_{1\text{optimal}}, t_2]$ with $t_{1\text{optimal}} < t_2 < T_f$

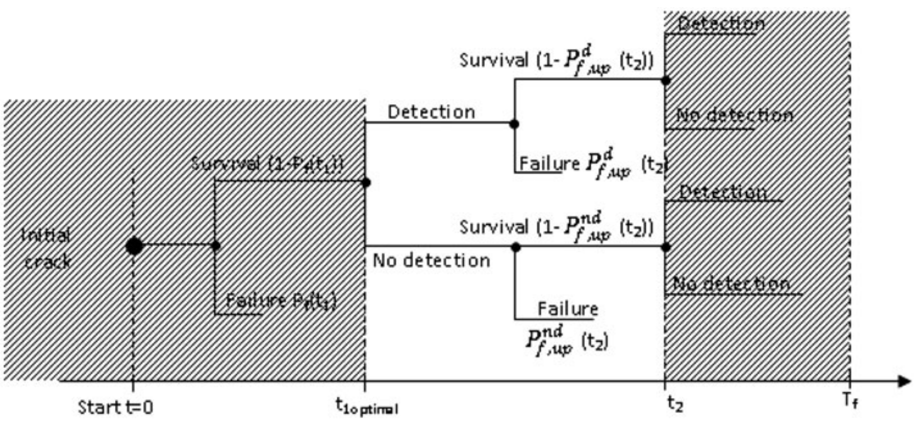


Figure 5. The decision tree in the case of two inspections.

- $C_{\mathrm{I}}(t_{1\text{optimal}}, t_2)$: the inspection cost realised at the time $t_2$.

Being situated in the interval $[t_{1\text{optimal}}, t_2]$, the total cost in the case of the second inspection is defined as follows:

$$C_T(t_{1\text{optimal}}, t_2) = \frac{C_F(t_{1\text{optimal}}, t_2) + C_I(t_{1\text{optimal}}, t_2)}{t_2 - t_{1\text{optimal}}} \tag{30}$$

The result of the first inspection at the time $t_{1\text{optimal}}$ gives two different cases relative to crack detection results:

**Case 1**

In this case, when the crack is detected and by integrating every term the probability of occurrence of each event, the total cost in the interval $[t_{1\text{optimal}}, t_2]$ becomes:

$$\begin{aligned} C_T(t_{1\text{optimal}}, t_2) &= \frac{C_f.P^d_{f,\text{up}}(t_{1\text{optimal}}, t_2) + C_i.R^d_{\text{up}}(t_{1\text{optimal}}, t_2)}{t_2 - t_{1\text{optimal}}} \\ &= \frac{C_f.P^d_{f,\text{up}}(t_{1\text{optimal}}, t_2) + C_i.(1 - P^d_{f,\text{up}}(t_{1\text{optimal}}, t_2))}{t_2 - t_{1\text{optimal}}} \end{aligned} \tag{31}$$

where $P^d_{f,\text{up}}(t_{1\text{optimal}}, t_2)$ is the updated failure probability in the case of detected crack between the time $t_{1\text{optimal}}$ and $t_2$.

**Case 2**

In this case, when the crack is not detected we have:

$$\begin{aligned} C_T(t_{1\text{optimal}}, t_2) &= \frac{C_f.P^{\text{nd}}_{f,\text{up}}(t_{1\text{optimal}}, t_2) + C_i.R^{\text{nd}}_{\text{up}}(t_{1\text{optimal}}, t_2)}{t_2 - t_{1\text{optimal}}} \\ &= \frac{C_f.P^{\text{nd}}_{f,\text{up}}(t_{1\text{optimal}}, t_2) + C_i.(1 - P^{\text{nd}}_{f,\text{up}}(t_{1\text{optimal}}, t_2))}{t_2 - t_{1\text{optimal}}} \end{aligned} \tag{32}$$

where $P^{\text{nd}}_{f,\text{up}}(t_{1\text{optimal}}, t_2)$ is the updated failure probability in the case of not detected crack between the time $t_{1\text{optimal}}$ and $t_2$.

### *4.3. Case of multiple inspections*

Two different costs are defined as follows:

- $C_F(t_{(n-1)\text{optimal}}, t_n)$: the expected cost of failure in the interval $[t_{(n-1)\text{optimal}}, t_n]$ with $t_{(n-1)\text{optimal}} < t_n < T_f$
- $C_{\mathrm{I}}(t_{(n-1)\text{optimal}}, t_n)$: the inspection cost realised at the time $t_n$.

The total cost of the $n$th inspection is defined as follows:

$$C_T(t_{(n-1\text{optimal})}, t_n) = \frac{C_F(t_{(n-1)\text{optimal}}, t_n) + C_I(t_{(n-1)\text{optimal}}, t_n)}{t_n - t_{(n-1)\text{optimal}}} \tag{33}$$

### 4.4. Flowchart of computing the optimal inspection time

A computing the optimal inspection time is determined according to total cost minimisation and presented in a flowchart in Figure 6.

## 5. Numerical application

To validate the proposed approach, two applications are carried out, the first one deals with the crack propagation in mode 1, in which the sensitivity of the different Paris law parameters will be shown, then the computation of the $P_f$ and the updated $P_f$ are determined in the case of detectable and undetectable crack length according to one and multiple inspection.

The second application concerns the crack growth in mixed mode where the $P_f$ and the $P^{d}_{f,up}$ are calculated, then the computed optimal time for the first and the second inspection will be carried out.

### 5.1. Application 1: mode I

A rectangular plate is considered containing an emergent rectilinear crack loaded with a constant amplitude cyclic stress $\sigma$. For this case, the stress intensity factors $K_I$ corresponding to mode I is calculated analytically using the following expression (Riahi et al., 2011):

$$K_I = 1.12.\sigma.\sqrt{\pi.a} \qquad (34)$$

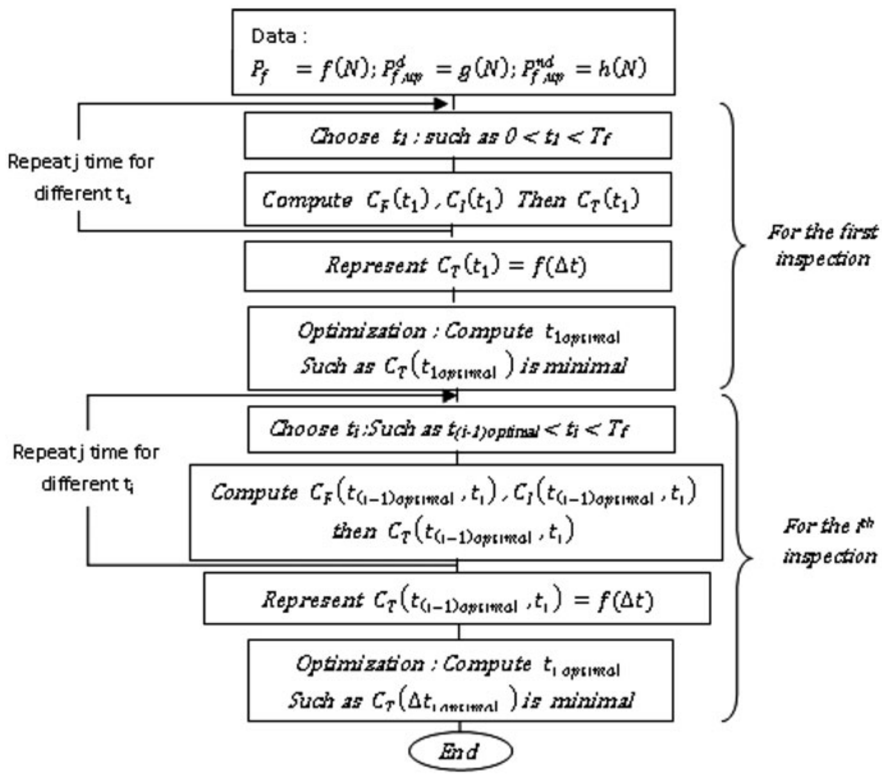


Figure 6. Flow chart of computing the optimal inspection time.

Table 1. Data FCG characteristics.

| Parameters | Law | Average | Coefficient of variation |
|---|---|---|---|
| $a_0$ (m) | Normale | $2.1 \times 10^{-4}$ | 3% |
| $a_c$ (m) | Normale | $5.5 \times 10^{-4}$ | 3% |
| $C$ (m/cycle/MPa m$^{1/2}$)$^m$ | Log-normale | $6.8 \times 10^{-12}$ | 10% |
| $a_d$ (m) | Normale | $4.5 \times 10^{-4}$ | 3% |
| $m$ | Deterministic | 3 | – |
| $\sigma$ (MPa) | Deterministic | 75 | – |

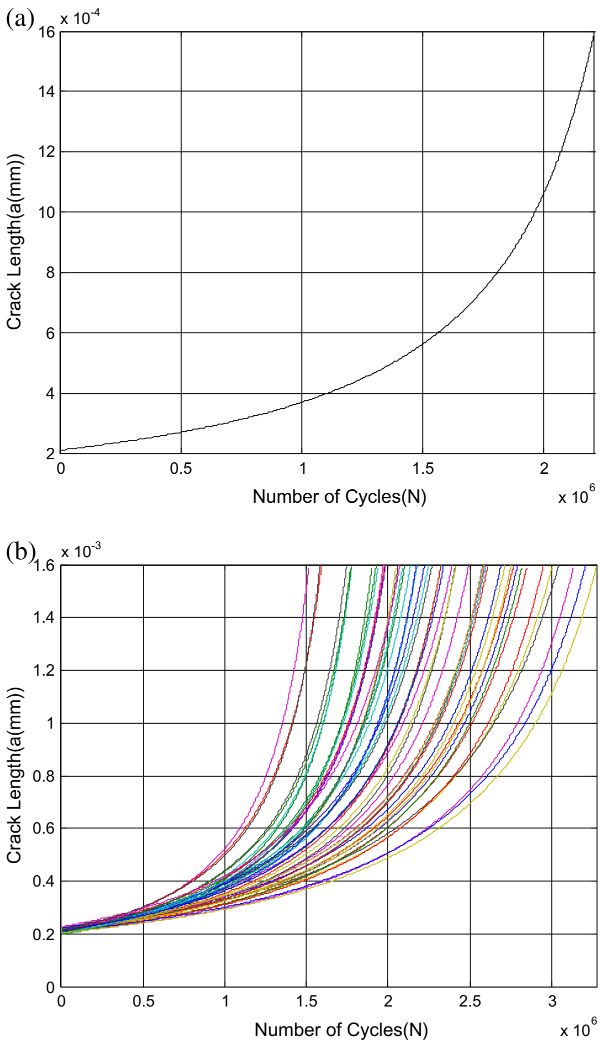


Figure 7. Evolution of crack length vs. the number of cycles $N$: (a) deterministic case and (b) random case.

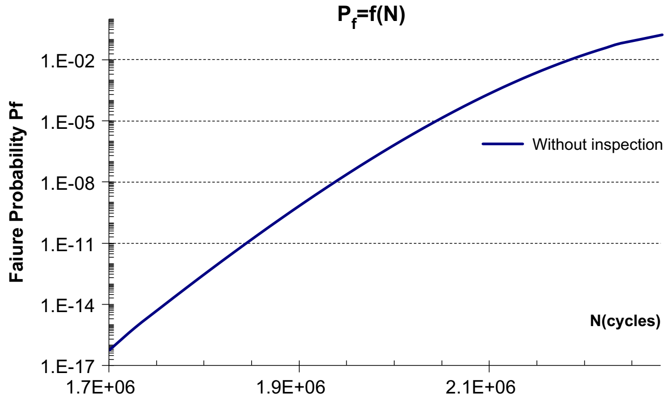


Figure 8. Evolution of $P_f$ vs. the number of cycles $N$.

The crack growth follows the Paris law expressed as:

$$\frac{da}{dN} = C.(\Delta K_{\mathrm{I}})^m \tag{35}$$

The random parameters, their law and their characteristics are given in the Table 1:

The crack growth is performed by using a constant increment length such as $\Delta a = 10^{-6}$m.

Figure 7(a) and (b) shows the evolution of the crack size depending on the cycle's number in the deterministic and random cases, respectively.

In a second step, we calculated the $P_f$ (Figure 8) using the flowchart in Figure 3.

Once the inspection operations is realised at $N = 2 \times 10^6$ cycles, we distinguish two cases:

The first case where the crack is not detected during the inspections therefore the $P_f$ decreases (Figure 9). The second case where the crack is detected then the $P_f$

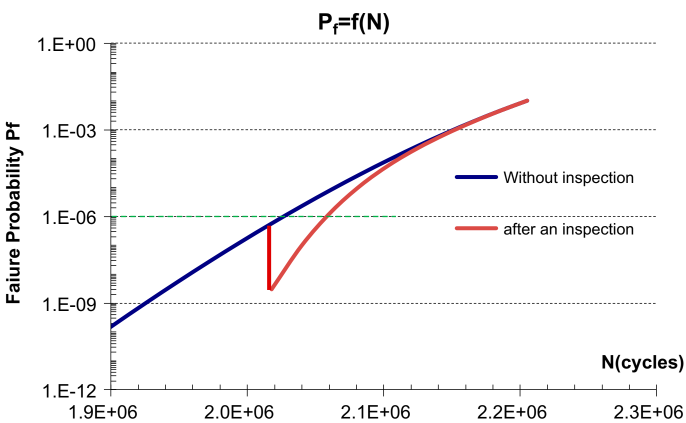


Figure 9. Evolution of $P_f$ and $P^{\mathrm{nd}}_{f,\mathrm{up}}$ vs. the number of cycles $N$.

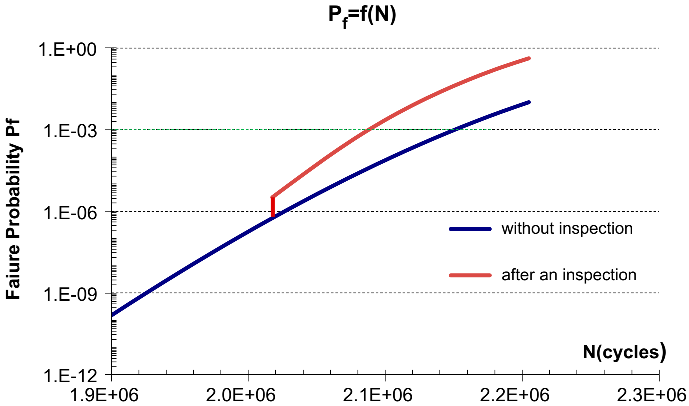


Figure 10. Evolution of $P_f$ and $P_{f,\mathrm{up}}^{\mathrm{nd}}$ vs. the number of cycles $N$.

increases (Figure 10). Using the additional information provided by the inspections results, the $P_f$ is re-evaluated following the Bayesian procedure described previously.

In the case where more than one inspection is carried out, we follow the same steps and we use the same equations (Equations (24), (25), and (27)) to calculate the $P_{f,\mathrm{up}}$ (Figures 11 and 12).

### *5.2. Application 2: mixed mode*

A rectangular plate is considered, of length $2L = 100$ mm and width $2W = 50$ mm of containing a crack rectilinear central of length $2W = 50$ mm and inclined at an angle $\beta = 30°$ to the horizontal loaded with an amplitude cyclic stress $\sigma$. The mechanical properties of the plate is assumed to have an elastic behaviour with a young's modulus $E = 2 \times 10^4$ MPa and a poison's ratio $v = 0.3$ (see Figure 13).

The random parameters, their law and their characteristics are given in the Table 2.

The Figure 14 shows the evolution of the $P_f$ and $P_{f,\mathrm{up}}^{d}$ depending on the cycle's number after an inspection operation in the case of crack detection.

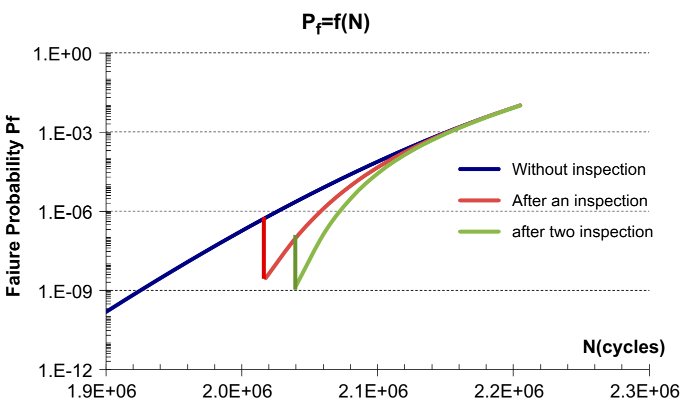


Figure 11. Evolution of $P_f$ and $P_{f,\mathrm{up}}^{\mathrm{nd}}$ vs. the number of cycles $N$ after two inspections.

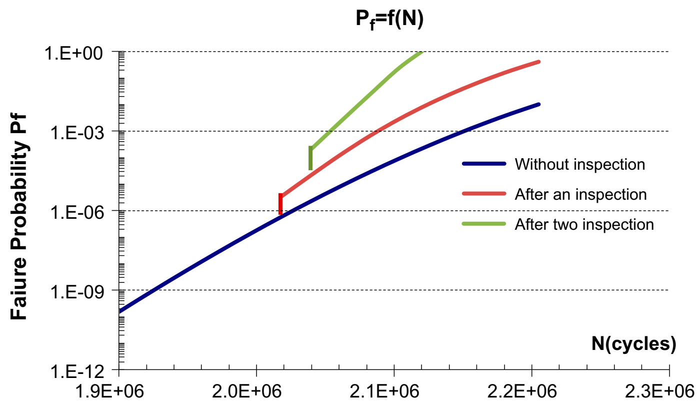


Figure 12. Evolution of $P_f$ and $P_{f,\mathrm{up}}^{\mathrm{nd}}$ vs. the number of cycles $N$ after two inspections.

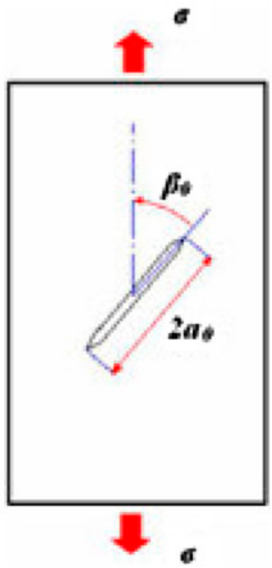


Figure 13. Plate with an inclined central crack.

Table 2. Data FCG characteristics.

| Parameters | Law | Average | Coefficient of variation |
|---|---|---|---|
| $a_0$ (mm) | Normale | 0.2 | – |
| $a_c$ (mm) | Normale | 1.8 | – |
| $C$ (m/cycle/MPa m$^{1/2}$)$^{\mathrm{m}}$ | Log-normale | $6 \times 10^{-12}$ | 10% |
| $a_d$ (mm) | Normale | 0.6 | – |
| $m$ | Deterministic | 3 | 3% |
| $\sigma$ (MPa) | Deterministic | 100 | 3% |

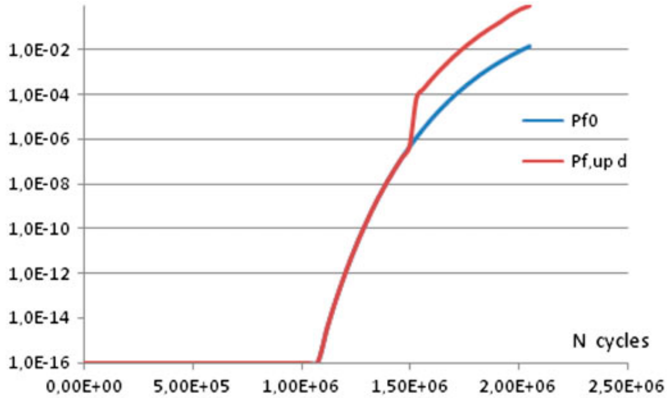


Figure 14. Evolution of $P_f$ and $P^{\mathrm{nd}}_{f,\mathrm{up}}$ vs. the number of cycles $N$.

Optimal time inspection of the first inspection $t_{1\mathrm{optimal}}$, is determined using the equations (Equations (28) and (29)) Where the time $t_1$ is in the interval $[0,T_f]$. This methodology is well explained in the flowchart of Figure 6.

The Figure 15(a) shows the evolution of the total cost of the first inspection depending on the variation of the time inspection $\Delta t$.

The Figure 15(b) shows the evolution of the total cost of the second inspection depending on the $\Delta t$.

The Figure 15(c) presents the evolution of the total cost of the first and the second inspection at the same time vs. on the $\Delta t$.

## 6. Discussion

(1) Referring to Figure 7(a) and (b), for a target critical crack length $a_c$ a very significant scattering of the life cycles is observed, hence the necessity to take into account the uncertain parameters affecting the propagation.

(2) Figure 8 shows that for a cycle's number $N$ less than $10^6$, the $P_f$ is equal to zero, it could be explained by the fact that the crack length a is always much lower than the critical length. For a cycle's number $N$ higher than $10^6$ the $P_f$ increase significatively. Therefore, the inspections operations should be scheduled in order to control (repair, replacement) the evolution of the crack length and thus avoiding the crash of the structure.

(3) Figure 9 shows the evolution of $P_f$ and $P^{\mathrm{nd}}_{f,\mathrm{up}}$ vs. the number of cycles $N$ after an inspection operation in the case of no crack detection. We denote by $t_i$ is the inspection time, $T_v$ is lifespan calculated in the case without inspection and $T_{vi}$ is lifespan calculated in the case with inspection.

For example for $P_f$ of design equal to $10^{-6}$ and at $t_i$ equal to $2 \times 10^6$ we have:

| $P_{f\ \mathrm{design}}$ | $t_i$ (cycles) | $T_v$ (cycles) | $T_{vi}$ (cycles) | $\Delta T = T_{vi} - T_v$ |
|---|---|---|---|---|
| $10^{-6}$ | $2.02 \times 10^6$ | $2.035 \times 10^6$ | $2.065 \times 10^6$ | $3 \times 10^4$ |

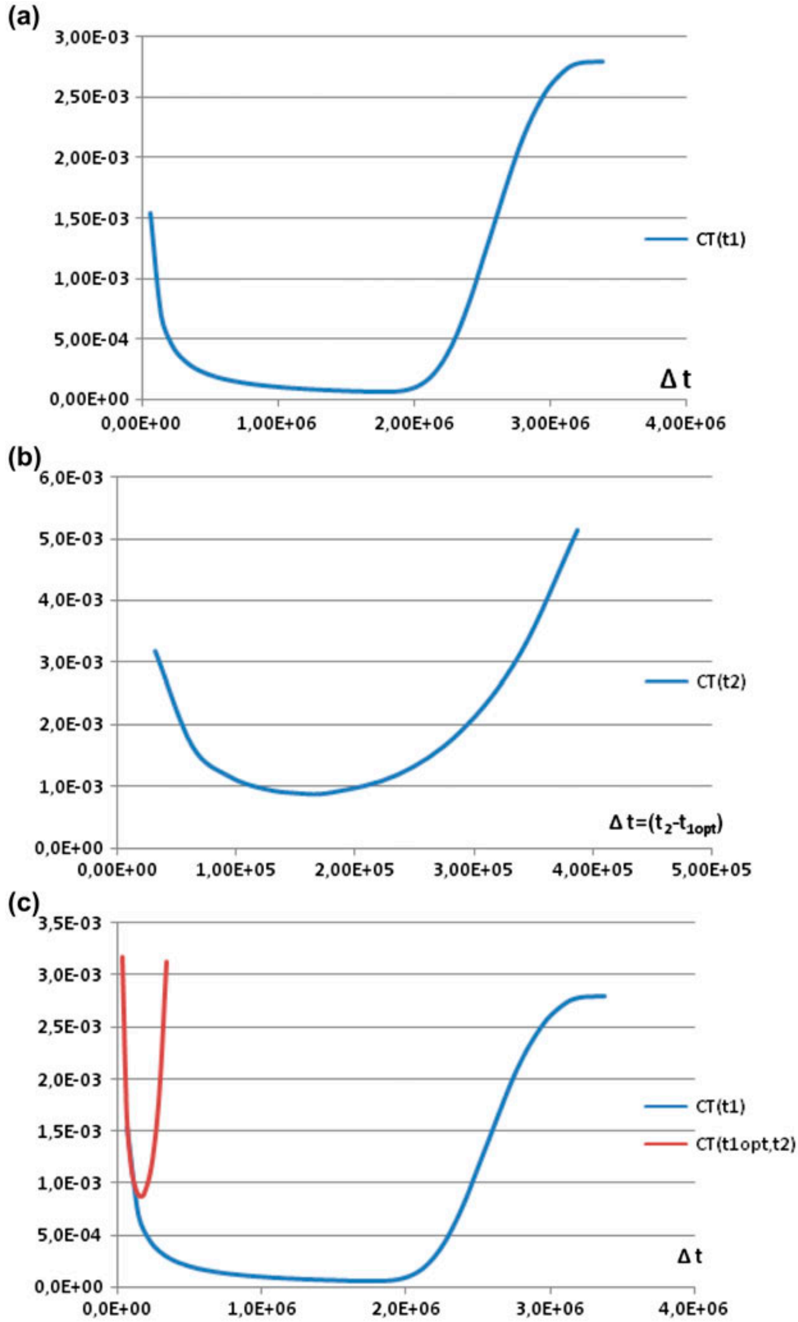


Figure 15. Evolution of the total cost vs. $\Delta t$: (a) first inspection, (b) second inspection, (c) first and the second inspection at the same time.

We note that the no crack detection delay the increase in $P_f$ consequently an additional operational life cycle is gained.

(4) Figure 10 shows the evolution of $P_f$ and $P^{nd}_{f,up}$ vs. the number of cycles N after an inspection operation in the case of crack detection. We note that the crack detection accelerates the increase in $P_f$ thus a gain of the cost of a disaster that would have happened due to that failure, is obtained. For example for $P_f$ of design equal to $10^{-3}$ and at $t_i = 2.10^6$ we have:

| $P_{f\ \mathrm{design}}$ | $t_i$ (cycles) | $T_v$ (cycles) | $T_{vi}$ (cycles) | $\Delta T = T_v - T_{vi}$ |
|---|---|---|---|---|
| $10^{-3}$ | $2.02 \times 10^6$ | $2.155 \times 10^6$ | $2.09 \times 10^6$ | $6.5 \times 10^4$ |

(5) Figures 11 and 12 show, respectively, the evolution of the $P_{f,\mathrm{up}}$ after two inspections for the case of no crack detection and also for the detection crack case. Taking into account the crack detection changes significantly the updating reliability and consequently, the value of the inspection time. This approach leads to improve a maintenance optimal inspection programme

(6) By analysing the curve in the Figure 15(a), the optimal inspection time of the first inspection is $t_{1\mathrm{optimal}} = 1{,}6 \times 10^6$ cycles. In the Figure 15(b), the optimal inspection time of the second inspection $t_{2\mathrm{optimal}}$ is:

$$(t_2 - t_{1\mathrm{optimal}})_{\mathrm{optimal}} = t_{2\mathrm{optimal}} - t_{1\mathrm{optimal}} = 1,6 \times 10^5 \mathrm{cycles}$$

$$t_{2\mathrm{optimal}} = 1,96 \times 10^6 \mathrm{cycles}$$

These results show the benefits of this approach to actualise the optimal time inspection

(7) From the Figure 15(c), we see that the optimal Δt corresponding to the second inspection decreases comparing to the first inspection one. This result is physically coherent since the crack growth increases first slowly then increases exponentially until the rupture. For the case of multiple inspections if Δt optimal is very low, the maintenance programme should be changed from inspection to replacement or repair. In this case, we can deduce the maximum number of inspections to be performed.

## 7. Conclusion

In this study, we have proposed a general approach integrating the coupling of fatigue-reliability and its application to the FCG. Based on the Rackwitz-Fiessler algorithm and the Bayesian approach, the failure probability and the updated failure probability are determined using the FORM method. Taking into account the results of previous inspections and based on the approach of minimising the total cost, an optimal maintenance schedule is implemented. A sequential approach starting by the initial crack is applied to build the total cost function. It is seen that, the inspection of the crack growth length result gives an accurate updating reliability values leading to predict the optimal inspection time. This method can be generalised for complex cases and for multiple inspections scheduling of mechanical component. Two applications have been presented to show the efficiency of this approach. The first one is applied to the crack growth in mode I and the second one is applied to the FCG in a mixed mode.

**Nomenclature**

| | |
|---|---|
| $K_I$, $K_{II}$ | The stress intensity factor in mode I and mode II, respectively |
| $K_{Ieq}$ | The equivalent stress intensity factor |
| $\Delta K$ | The stress intensity factor range |
| $\sigma$ | The applied load |
| $a$ | The crack length |
| $a_0$, $a_d$, $a_c$ | The initial, detectable and the critical crack lengths, respectively |
| $\Delta a$ | The increment of the crack length |
| $F(a)$ | Correction factor |
| $G$ | Strain energy release rate |
| $\pi_c$ | Total potential energy |
| $\theta_0$ | The crack growth angle |
| $\theta$ | The crack orientation angle |
| $\sigma_{\theta\theta}$ | The maximum circumferential stress criterion |
| $N$ | The number of cycles |
| $\Delta N$ | The increment of the number of cycles |
| $N_f$ | The lifespan (the number of cycle at failure) |
| $C$, $m$ | The materials parameters of the Paris low |
| $\{X\}$ | A vector of random variables |
| $x_i$ | Element of the vector $\{X\}$ |
| $f_{X_i}(x_i)$ | Probability density function of the variable $x_i$ |
| $f_{\{X\}}(\{x\})$ | Joint probability density function of the vector $\{X\}$ |
| $G$, $S$, $L$ | Performance, strength and load functions, respectively |
| $P_f$ | Probability of failure |
| $P_{f,up}$ | The updated probability of failure |
| $P^d_{f,up}, P^{nd}_{f,up}$ | The updated probability of failure in the case detectable and non detectable crack, respectively |
| $\beta_G$ | The reliability index |
| $\beta_{up}$ | The update reliability index |
| $M_i$ | Inspection event corresponding to crack detection |
| $\overline{M_i}$ | Inspection event corresponding to no crack detection |
| $\rho$ | The correlation coefficient between the $G$ and $M_i$ |
| $C_F(t_i,t_j)$ | The expected cost of failure in the interval $[t_i, t_j]$ |
| $C_I(t_i)$ | The inspection cost at the time $t_i$ |
| $C_T(t_i,t_j)$ | The total cost in the interval $[t_i, t_j]$ |
| $C_f$ | The cost of a failure |
| $C_i$ | The cost of each inspection |
| $\Delta t$ | The variation of the time inspection |
| FCG | Fatigue crack growth |
| FORM | First Order Reliability Method |
| DoE | The design of experiments |
| HCF | The High Cycle Fatigue |
| LEFM | Linear Elastic Fracture Mechanics |

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